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The Perspective of Construction and Innovation of Didactics of Mathematics as a Scientific Discipline: The Case of Japan and The Didactical Issue

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Abstract

In a programmatic discussion to establish mathematical education as a scientific discipline in the early 1970s in Europe, we could point out the academical construction rationale of Object-Method-Problem in mathematical education and the dialectical innovation perspective of Practice vs. Theory in it. It also follows that both are complementary to each other in research of mathematical education.

Based on the dialectical innovation perspective, it is reported that the research of mathematical education in Japan had begun with the reform movement of mathematical education and it has been institutionalized gradually for a scientific discipline after the modernization movement of it.

From the dialectical innovation perspective, Wittmann, E. introduces "Design Science" into Didactics of Mathematics. He succeeds in clarifying both structure and core of it. If, however, each component of the core is peculiar to Didactics of Mathematics, their peculiarity must be explained by "Design Science". But it has not had such capability because originally it is not a science for education. There must be educational idea or an awareness of issues which integrates each component of the core. We seek it in the transition between elementary education and secondary education. Based on this idea, each of them could be an issue in Mathematics Didactics in the proper meaning of these words.

1. Introduction

College Education is prone to a type of academicism through its fundamental concepts.

Mathematical education may, as part of College Education, also be vulnerable to a certain criticism of this kind. At the same time, given that Teacher Education generally has lain within the fiefdom of the universities and colleges, it would be argued that some kind of academicism in mathematical education is almost inevitable. If we view mathematical education as a scientific discipline in its own right, then any scholarly study must start with an analysis of how the discipline is organized and functions in its own terms. If, however, we take a wider view, we would also need to consider the historical development of teacher education in order to gain a clear perspective on mathematical education.

It is now in Europe and the United States of America (U.S.A.) that mathematics educators and researchers could develop various kinds of discussion about mathematical education as a scientific discipline under a type of academicism. As a matter of fact, "Didactics of Mathematics as a Scientific Discipline" (Biehler, 1994) has been published in Europe 1994 and "Handbook of Research on Mathematics Teaching and Learning" (Grouws, 1992) in the U.S.A. 1992. Moreover, "International Handbook of Mathematical Education" (Bishop, 1996) will be issued in Europe 1996. It, however, seems to us that mathematical education in Japan is a few decades behind them on scientific framework and philosophical base when we look at the table of contents of those books. We, therefore, will put ourselves in the early 1970s of Japan and begin with a reconfirmation of the academic base for mathematical education in those days. This is what Wittmann remarks briefly on the matter like this:

> "The early seventies have witnessed a vivid programmatic discussion on the role and nature of mathematical education in the German speaking part of Europe. Since then the status of mathematical education has not been considered on a large scale despite the contributions by Bursheid, Bigalke and Winter. So the time is overdue for redefining the basic orientation for research." (Wittmann, 1995, p. 355)

Next the authors would like to review the history in which mathematical education has been necessarily getting academic in Japan from an active perspective of scientific discipline.

## 2. Basic Problem on Institutionalization of Mathematical Education

The first degree in mathematical education was conferred on Jackson, L.L. and Stamper, A.W. by Smith, D.E. at Teachers College in Columbia University of the U.S.A. in 1906 (Hirabayashi, 1994). The conferment was done only five years after the distinguished lecture by Perry, J. of launching the Reform Movement of Mathematical Eeducation. It is considered that the academic climate of the U.S.A. at the time was relatively free compared to that of Europe. It was probably in the First ICME (International Congress on Mathematical Education) held in Lyon in 1969 that an agenda over conferring the degree was discussed for the first time in the Continent. The last heading, which was composed of five items, facilitated the institutionalization of mathematical education from the viewpoint of the degree as follows:

> "The theory of mathematical education is situated in the scientific realm involving bilateral problems between mathematics and pedagogy. Mathematical education should be recognized as an academic subject in universities or institutes. At the same time researchers in mathematical education should be also able to receive the degree." (Shibagaki, 1970)

In 1971 a symposium was held in Bayreuth and the theme was "The Relationship between Theory and the Degree in Mathematical Education." (Steiner et al, 1971) Steiner presided at the symposium

and Freudenthal, Griesel, Pickert, Krygowska, Papy, Verga, Weis and Kirsh were also present at the symposium as panelists. Steiner said in advance that they should reach a consensus in their discussion. Each of them, however, insisted on their own view. During the discussion, Weis often put the emphasis on a general organizational schema for getting a degree in mathematical education. However, it is now considered that the crucial problem is to gain a common recognition of its organization.

A few years before this symposium, Steiner had proposed an organizational schema (See the below diagram), in which he located DM (didactics of mathematics) neighboring to PM (pure mathematics) and detached from PL (psychology of mathematics thinking and learning) (Kaufman and Steiner, 1969, p. 317). But he could not get agreement among researchers. Concerning the importance of mathematics in the degree, Pickert said that it might be enough for the Ph.D. candidate in mathematical education to teach mathematics in a gymnasium. Krygowska, on the other hand, insisted that he had to be able to give a lecture in college or university. Weis put the emphasis on psychology or pedagogy rather than mathematics, whereas Steiner objected that mathematics cannot be explained by psychological research.

$$FM - EM - PM - AM - HM$$
$$DM$$
$$GD - DMT - MH$$
$$GP - PL - EL$$
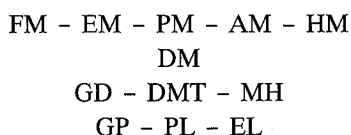
Source: Kaufman and Steiner: "The CSMP Approach to Content-Oriented, Highly Individualized Mathematics Education", Educational Studies in Mathematics vol.1, 1969

(The abbreviations denote: PM: pure mathematics, EM: epistemology of mathematics, FM: foundations of mathematics, AM: applied mathematics, HM: history of mathematics, DM: didactics of mathematics, DMT: didactics and methodology of mathematics teaching, GD: general didactics, MH: mathematical heuristics, PL: psychology of mathematical thinking and learning, GP: general pedagogy, EL: evaluation of mathematics learning.)

It has already been twenty-five years since this symposium was held, though of course it is impossible for us to know to what extent mathematical education has improved on an international scale. Even in Japan the number of researchers has increased remarkably during these twenty-five years. This implies that mathematical education expands and differentiates into special fields of research which satisfy the profound interests of researchers. However, compared to Europe and the U.S.A. it does not seem that we give an enough reflection or reconsideration to scientific aspect of mathematical education. Thirty years ago, Shiomi made a study on it with the title "A Conception of Mathematical Education as a Science" (1967, pp. 1–9). He featured mathematical education in that paper as follows:
"Mathematical education is often considered as a mixture of mathematics and pedagogy because of its name, however it should be viewed in terms of the relationship between mathematical education and scientific principle." (Shiomi, 1967, p. 2)
According to him, mathematical education should base itself on an educational science centering on human beings and it is not the result combining other scientific or pedagogic subjects which have different principles, but rather should be considered as one academic subject. Although this point of his argument of thirty years ago is quite similar to the critical mind of Wittmann (1995) of today, we have not had such kind of research in Japan since then unfortunately.

3. The Concepts of Bigalke and Griesel in Mathematical Education

(1) Concepts of Bigalke in Mathematical Education

The paper of Bigalke "Sense and Reference of Mathematical Education" was organized as follows:
1. Objective
2. Method
3. Problem
4. Content of Mathematical Education

It seems quite normal that he discussed objectives, methods and problems in his paper. Bigalke, however, proposed his own points of view on objectives, methods and problems. He referred to particular methods as follows:

"Mathematical Education does not stand on an exact science. The objectives are connected not only with facts but also with human beings or the environment in a broad sense. Mathematical education, therefore, does stand on an empirical science as it takes not only a hypothetical-deductive approach but also a hypothetical-constructive approach." (Bigalke, 1974/3, pp. 111–112)

Besides he also discussed that only the results got by this sort of approach can produce a normative model bridging a gap between theory and practice in mathematical education.

(2) Concepts of Griesel in Mathematical Education

Griesel's paper "Consideration on Mathematical Education as a Science" is composed of two parts, theory and practice in mathematical education. In the former part he discussed objectives, mathematical recognition and didactic-oriented analysis. In the latter part, he discussed the structure and development of curriculum. He also insisted on a complementary relationship between practice and theory. The research integrating them is therefore developed dialectically, as can be seen in the following quotation:

"Researchers in mathematical education always have to consider the temporality of results. They should not persist in their own view. They have to be prepared to throw away and replace their own findings with something else." (Griesel, 1974/3, p. 118)

The dialectical relationship between theory and practice is therefore the base producing issues in mathematical education. As a result we can see that such issues can improve objectives and methods.

(3) The Relationship between Bigalke's and Griesel's Concepts

In Bayreut Symposium in 1971 previously mentioned above (Bigalke, 1971 & Griesel, 1971), both of them gave a lecture on mathematical education, which preceded their own articles now mentioned above. Let us make an abstract of their lecture. Interestingly, the lecture of Bigalke (1971) was developed on the base of Griesel's point of view (1974) outlined above, and vice versa. We can notice that some elements of their respective theories have changed places mutually after three years. In short there might be a complementary relationship between "objectives-methods-problems" and "theory-practice" in mathematical education. It therefore seems that the former relation could be characterized as a stative part, and the latter as an active part in a type of academical dynamics.

4. The Origin and Development of Educational Studies in Mathematics in Japan

(1) The Beginning of Educational Studies in Mathematics: The Influence of the Reform Movement of Mathematical Education

It dose not seem to have been so many years since the term "Didactics of Mathematics" began to be used in Japan. It can, however, be found in a book "Method of Teaching Mathematics" (Fujisawa, 1900) published in Meiji Era about 100 years ago. In this book FUJISAWA Rikitarou (1861~1933) used the term "mathematical pedagogy" (p. 30). He studied abroad in Berlin University 1880s and edited the first national textbook of Arithmetic, which had been used for thirty years (1905~1935) during which it was revised three times. He also made "Summary Report on the Teaching of Mathematics in Japan" for the Fifth International Congress of Mathematicians held in Cambridge 1912. We cannot know to what extent he recognized the mathematical pedagogy as an academic subject. It seems, however, that he had entrusted it to his successors as one of the problems for the future.

The first step to addressing this problem of mathematical education as a scientific discipline was taken by the Japan Conference of Mathematics Teachers in 1918. A motion for the establishment of institutes on mathematical education in secondary level was made at this conference and approved unanimously. According to this decision, the Mathematical Association of Japan for Secondary Education was inaugurated in 1919, the antecedent of Japan Society of Mathematical Education. The aim of this Association was mentioned in Article 1 as follows:

"The purpose of this association is to investigate mathematics and its teaching methods in secondary education and also try to improve them." (JSME, 1968, p. 123)

At the same time a bulletin of the association was started with an English title "Journal of the Mathematical Association of Japan for Secondary Education" which implies a scientific journal.

The reform movement of mathematical education was able to contribute to awakening of studies in teaching of mathematics. As a result, many books were translated and published as a reference. "Modernized Mathematics" translated from "Lehrbuch der Mathematik nach modernen Grundstzen" by Behrendsen, D. & Goetting, E. was, for example, published by the Ministry of Education in 1915. "Mathematical Education in Germany" written by Klein, F. was translated by HAYASHI Tsuruichi (1873 ~ 1935) in 1922 who was the first president of the Mathematical Association of Japan for Secondary Education mentioned above. And "Fundamental Problems in Mathematical Education" was written by OGURA Kinnosuke (1885~1962) in 1924.

In this way the reform movement of mathematical education began to filter into the public arena. However it is doubtful to what extent its educational theories were matched with the educational practice, because attention was often limited to materials and methods. For example in the general meeting of the Mathematical Association of Japan for Secondary Education, fruitless discussion took place concerning the teaching of how to use graphs of functions. The discussion was as follows: someone asked "For what do we need to teach how to use graphs?" but the head of the meeting answered "Your question is nonsense." (Kakizaki, 1924, p. 5)

This is a very symbolic scene about the reform movement in our country. The use of graph was encouraged without considering the significance and objectives of teaching function in Japan of those days. However, what we really have to understand is an educational concept loaded into that content matters. In this point, the work of Lietzmann who supported Klein's educational approach seems

particularly important. However, obviously from the episode above, we did not understand the reform movement deeply at that time.

The reform movement has brought about various innovations in mathematical education with detailed proposals. Its real fruit is, however, to give a didactical thought to the practical field like mathematics teaching. To put it in another way, it encourages us to realize the theoretical perspective to reconstruct the practice of mathematical education. In those points it can be said that the reform movement was very significant in the historical terms of mathematical education in Japan (Hirabayashi, 1987, p. 47).

(2) Development of Educational Studies in Mathematics: The Influence of New Math Movement of Mathematical Education

After the second world war the educational system changed drastically in Japan. The prewar association of mathematical education had changed its name to "Japan Society of Mathematical Education (hereafter cited as JSME)" from "The Mathematical Association of Japan for Secondary Education" in 1948 and JSME built the new aim in Article 3 as can be seen in the following quotation:

"The purpose of this association is not only to investigate mathematics and mathematical education but also to improve the view of members, and then promoting the development of mathematical education." (JSME, 1968, p. 126)

The journal of JSME was also renewed as "Mathematical Education" and its first volume was published in 1947. Activity curriculum based on living and learning was strongly required in elementary education as New Education after the war. It was, therefore, necessary to place importance on arithmetic education. As a result the journal of JSME "Arithmetic Education" started in 1952.

Since 1955 mathematical education has also begun to improve academically on a full scale. JSME moved to the publication of research bulletin in mathematical education to answer the basic question "what is the scientific research in mathematical education?". The bulletin called "Reports of Mathematical Education" made its first appearance in 1961. The eighth president of JSME, SATO Ryoichiro, made the preface concerning the basic question there as follows:

"Its purpose is to promote the study of mathematics and mathematical education and to assist its members in acquiring their scientific and general vision in mathematics and mathematical education and further to improve mathematical education in all levels." (Sato, 1961, p. 1)

In order to attract as many participants as possible, there must be a certain amount of room for researchers to discuss and criticize about mathematical education. In 1966 the first conference of theoretical research in mathematical education was held by JSME in Tokyo, at which speakers were required to make high-quality scientific presentation.

Because of the scientific aspects of mathematical education, in 1970 the Japanese name of JSME was slightly changed to "Nihon Sugaku Kyoiku Gakukai" from "Nihon Sugaku Kyoiku Kai" although the English name was unchanged. The new Japanese name has a kind of academical meaning in point of "Gakukai" while "Kai" of old one meant only "meeting" in Japanese, and its objectives were proposed anew in Article 3 as follows:

"In our society it is required to make a presentation and offer or exchange information about theory and practice on mathematical education. We aim to promote the improvement of mathematical

education in our school level, and also contribute to the development of the academical aspects of mathematical education." (JSME, 1988, p. 52)

Mathematical education takes root deeply in our society and history because it is not just mathematics but mathematical education, while it is a subject without boundaries because of mathematics. We therefore can find social, historical and international aspects in mathematical education on the background of such an academical inclination as mentioned above. "The Fifty Year of JSME", which is the supplementary special issue for the fiftieth anniversary of JSME, puts various topics concerning academic movement in the headline of "Modernization of Mathematical Education" (JSME, 1968, pp. 103–106). This just coincides with the name of worldwide movement (New Math Movement) in mathematical education. It would be better to say that this headline implies its influence to mathematical education in Japan of those days.

Modernization of mathematical education may have resulted in failure as Kline, M. criticized severely in "Why Johnny Can't Add?: The Failure of the New Math" (1973). It might, however, also be true that its incompleteness caused academic reflection among mathematics educators. As far as we accept what mathematicians say and do, or adopt the viewpoint of psychologists, it might be possible to undermine the relative autonomy of mathematical education. If we give further consideration to the relation between a human being and mathematics, we could recover "The Failure of the New Math" and find the value of mathematical education. Problem-solving and constructivism, for example, are researches and arguments for it, and moreover we could also say that the latter equips the didactical thought and scientific base to the former.

5. Consideration on the issues in the Didactics of Mathematics: On Core of Didacitcs of Mathematics in Wittmann

As mentioned in the footnote, this thesis bases itself upon the presentation during ICME-8, WG25. The Chief Organizer, Malala, N. at the opening of this WG, referred to "Mathematical Education As a 'Design Science'" by Wittmann and stressed the importance of theme of WG25. Furthermore, it was impressive that the majority of participants read Wittmann's thesis in advance, and esteemed his critical mind for problem and his idea towards its solution. In this section, we would like to analyze critically the core of didactics of mathematics which Wittmann conceived and figured out in his paper. We evaluate that this core should, at the same time, be a central issue of mathematics didactics as a scientific discipline, because any scientific discipline loses its foundation if it has no sense of issue.

Originally speaking, the name "Didactics of Mathematics" may have been created by some means in order to facilitate the institutionalization of teacher training within the realm of university. The university had nothing to do with the job training by nature, and it is a historical fact that neither the training of teachers nor the training of technicians, civil servants, doctors, was a duty of university. Naturally it is nothing but a social demand that forced the university to accept them as a part of duty, and this meant at the same time that each job training was forcibly casted into the mold of academicism.

There is, however, no rationale for self-assertion as 'Didactics of Mathematics', if the old academic norms remains the same as before. Wittmann made a vigorous effort so as to avert this dilemma by establishing a new set of academic norms as a 'design science' in the mathematical education, referring to the Nobel laureate, Herb Simon and etc. He also tried to give the 'raison d'etre' within the

university to the mathematical education as a job training. That is,

"In terms of the prevailing norms (in university), academic respectability calls for subject matter that is intellectually tough, analytic, formalizable and teachable." (Wittmann, 1995, p. 362, The word inside parenthesis is by the authors.)

Faculty of Teacher Education has been dealing with this demand from the point of Faculty of Science or Faculty of Literature. However,

"The professional schools will reassume their professional responsibilities just to the degree that they can discover a science of design, a body of intellectually tough, analytic, partly formalizable, partly empirical, teachable doctrine about the design process." (Wittmann, 1995, p. 362)

It is not just to accommodate the mathematical education with a place within the university why Wittmann characterizes it as a design science. Or rather, it is because the academic development of mathematical education at the core of subject should correspond closely with its practice in classroom. The design science so far has not been regarded to suit the classroom situation or more widely the education because of its mechanistic paradigm. However,

"we are presently witness to the rise of a new paradigm for the design sciences that is based on the systemic-evolutionary development of living systems and take the complexity and self-organization of these systems into account" (Wittmann, 1995, p. 363)

Hence according to him, the core of didactics of mathematics can be clearly defined under the perspective of design science. He describes that the core consists of the eight components listed below, and it can be regarded likewise as a central issue of mathematics didactics.

① analysis of mathematical activity and of mathematical ways of thinking,

② development of local theories (for example, on mathematizing, problem solving, proof and practising skills),

③ exploration of possible contents that focus on making them accessible to learners,

④ critical examination and justification of contents in view of the general goals of mathematics teaching,

⑤ research into the pre-requisites of learning and into the teaching/learning processes,

⑥ development and evaluation of substantial teaching units, classes of teaching units and curricula,

⑦ development of methods for planning, teaching, observing and analysing lessons, and

⑧ inclusion of the history of mathematical education. (Wittmann, 1995, pp. 356–357, the numbering was given by the authors for later use.)

From the point of design science, the issues as components of the didactics of mathematics can be clarified, but the principles governing these components are still in the mist. It is not a matter of design science but a matter of education, and Wittmann did not prepare a philosophical answer to it. This means, if the above eight components are regarded as the core of didactics of mathematics, the core must have such characteristic as to justify mathematical education to be a scientific discipline and then there should be a philosophy as well to explain this characteristic. It is because each component can be located properly within mathematics didactics as a characteristic component or as an issue once it goes through the scrutiny of the philosophy or the consciousness of fundamental problems. Naturally a series of random remarks can never be a component or an issue of the core. In short the objective of this section is to give a solution to the remaining problem, which is untouched by Wittmann.

Wittmann classifies the natural science and the design science in terms of the object of research. The former deals with natural things and the latter with artificial things. And the closeness of relation-

ship between design science and mathematical education is explained by the following reason:

"the core of mathematical education concentrates on constructing artificial objects, namely teaching units, sets of coherent teaching units and curricula as well as the investigation of their possible effects in different educational ecologies. Indeed the quality of these constructions depends on the theory-based constructive fantasy, the ingenium, of the designers, and on systematic evaluation, both typical for design sciences." (Wittmann, 1995)

Henceforth, the discussion is directed at the design and development of teaching units, and finally converges towards the substantial teaching units. Meanwhile the curriculum always stays latently in the background to support their significance. The design and development of teaching units without a well planned curriculum, can be compared to the purposeless voyage without compass, and hence the essential issue of curriculum is understood to correspond precisely with the critical mind in the core of didactics of mathematics.

An essential problem of school curriculum is the discrepancy between elementary and secondary education in teaching objectives, contents, and method. (Iwasaki, 1988, pp. 77-79). Historically speaking everywhere in the world, the elementary education has stood by itself as a compulsory education and its objectives have been to enable all children to acquire the basic knowledge and skill which are required as a member of society in future. Originally it does not aim at the production of scholars, but corresponds to the necessities of daily life. The arithmetic with this tradition did not assume in the beginning the transition to the higher mathematics. On the other hand, the secondary education traditionally has aimed at giving the humanistic education, and it has originally had no relation with knowledge and skill of daily life. The present secondary education has not been analyzed or identified from this historical point of view, and hence forces the traditional mathematics as it is, to the children without improvement of its original nature, under the assumption that the natural extension of arithmetic to it can be possible.

At the times when there is a clear philosophical division between elementary education and secondary education, there may exist the arithmetic and the mathematical education, but not the mathematics didactics. That is, the arithmetic is the content which can be considered in the teachers college but not the object of research in the university, and vice versa for the mathematics. Their difference had been rigidly and clearly retained. In other words, the main concern of university was a research on content of mathematics and the one of teachers college was a research on teaching method of mathematics. As mentioned in Section 4 (1), the beginning of educational studies of mathematics in Japan is characterized by the Mathematical Association of Japan for Secondary Education, and its journal was highly science-oriented in nature. From this point of view, the academic nature of mathematical education at the time can be easily understood. Naturally, the didactics of mathematics, which is conceived and figured out by Wittmann, can not exist in this situation.

So long as taking this difference for granted, there is no necessity to institutionalize mathematical education and seek its foundation within university. However, today whether in the developed or developing countries, the transition from elementary to secondary education is being devised worldwide from the point of educational system. The junior high school in the U.S.A. and the post war Japan, the comprehensive school in the United Kingdom, and the Gesamtschule in the Germany are some of the typical examples and efforts in this direction. Unfortunately it is sometimes evaluated that the last two cases as a new enforcement against the tradition are not successful. That indicates the system will not function properly unless the software of the system is adjusted and improved accord-

ing to the change of the hardware of the system. In that sense, the universalization of perspective and framework has been long awaited to mediate the decisive difference between arithmetic and mathematics. The traditional academicism under the assumption of this difference cannot provide a tool for the solution of this new generation of problems and thereby the necessity naturally comes out for the academic restructuring of mathematical education.

When the core of the didactics of mathematics by Wittmann is re-examined from this point, it is understood with ease that it is an issue of paramount importance for mathematics didactics. For example, ① and ② concern about the transition from arithmetic to mathematics and conversely ③, ④, ⑤, ⑥, ⑦ concern about the transition from mathematics to arithmetic. This is because arithmetic has not in nature included ① and ② in its objectives and they have been the objectives of mathematical education of the humanistic goal. On the other hand, mathematical education has not had an obligation to take ③ to ⑦ into its scope and they have been regarded as a part of arithmetic education. The last component, ⑧ is for the transition between the two, and it is a critical issue of historical necessity.

Moreover, reviewing Wittmann's view of mathematics, it is understood to give a suitable philosophy to the issue of transition.

"Work in the core must start from 'mathematical activity as an original and natural element of human cognition'. Further, it must conceive of "mathematics" as a broad societal phenomenon whose diversity of uses and modes of expression is only in part reflected by specialized mathematics as typically found in university departments of mathematics. I suggest a use of capital letters to describe MATHEMATICS as mathematical work in the broadest sense; this includes mathematics developed and used in science, engineering, economics, computer science, statistics, industry, commerce, craft, art, daily life, and so forth according to the customs and requirements specific to these contexts." (Wittmann, 1996, pp. 358–359)

However, there is regretfully no further discussion beyond this about 'mathematical activity as an original and natural element of human cognition'. The unavoidable contact between human thought and MATHEMATICS can be presumed to be mathematical activity. The feature of the activity lies in the point of dealing with things semi-concretely or idealistically at first and reconstructing afterwards the *virtual reality* under the rational thought. To put them in one word by all means, it can be called a *rational virtuality*, and whether success or failure there are myriad of examples especially in the thought experiment (Kaneko, 1986).

It should be again brought to our attention how the mathematical activity is being understood in didactics of mathematics. The last problem at ICME8, WG25 was a metaphysical one that what is a 'fact' in the didactics of mathematics. Steiner, H.G., who had led the discussion of WG25 throughout, stressed on the theoretical framework and asserted that the 'fact' can be defined clearly only under that framework. On the other hand, Michele Pellerry, who attended this WG from Italy, emphasized his stance that the framework by itself cannot come first and it can be established only after 'facts' in the classroom are accumulated.

In this section, we pointed out that the core of didactics of mathematics can be characterized clearly under the critical application of the concept of transition. This means that the viewpoint of mathematics and the 'fact' in the mathematics didactics should be included at the same time. We would like to make a further research on this in future and conclude this thesis by making a brief discussion about the research situation on the didactics of mathematics.

6. By way of Conclusion

As we have discussed above, a research into mathematical education in Japan has not been motivated spontaneously but was always induced by the following wind out of Europe and the U.S.A. such as the movement of improvement or modernization. As a consequence, it does not have a constructive tradition for a scientific discipline based on theoretical and philosophical framework, but has an innovational tradition based on practical framework as a whole.

About hundred years have already passed since FUJISAWA Rikitarou used the word "mathematical pedagogy" for the first time in 1900, which are probably enough time to organise academically mathematical education. However, the number of researchers who have taken a degree in mathematical education in Japan is still less than ten so far since the first Ph.D. in mathematical education was conferred in 1979.

As far as the transition between elementary education and secondary education does not function properly on the hardware side of system, the software side also continues to have a problem. Or rather the problem on the hardware may be solved by the solution on the software. Under this critical understanding of problem, the core of didactics of mathematics, proposed by Wittmann, may become an issue of mathematics didactics in its true sense of the word. Therefore the design science, also by Wittmann, has an orientation towards the academical expansion of mathematical education. For the fulfillment of this purpose, the responsibility of training of researchers is vested upon the shoulder of university. Moreover this training's including the social demand, the following words by Krygowska have a significance and an applicability even today.

Krygowska (1971, pp. 121–122) has proposed three working principles for research in mathematical education. Now that it is time to give a clear condition and guidance of mathematical education as a scientific discipline, we have to take them into consideration. The authors would like to conclude this article by citing them.

The First Working Principle: To regard mathematical education as an interdisciplinary subject

"Mathematical education as an interdisciplinary subject could not prevent a scientific inconfidence and suspicion from traditional well-established disciplines." (Krygowska, 1971, pp. 121–122)

The Second Working Principle: To make the perspective of an effective method in mathematical education clear

"An effective method can only be evaluated by a result of research. We should consider the results as a verified hypothesis." (Krygowska, 1971, p. 122)

The Third Working Principle: To identify the autonomy of mathematical education

"Mathematical education should be independent relatively from the dogmatic application of mathematics, pedagogy and psychology. Mathematical education should be based on mathematical activities by children." (Krygowska, 1971, p. 122)

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