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RESEARCH INTO RELATIONSHIP BETWEEN THE COMPUTATIONAL ESTIMATION ABILITY AND STRATEGY AND THE MENTAL COMPUTATION ABILITY: ANALYSIS OF A SURVEY OF THE FOURTH, FIFTH, AND SIXTH GRADERS IN JAPAN*

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Abstract

The purpose of this research is to obtain performance data on the computational estimation and mental computation skills of Japanese students in grades 4, 5, and 6, to clear effects of the teaching and learning of rounded numbers and rounding strategies on computational estimation, and to inquire the relationship between computational estimation ability (CEA), strategy (CES) and mental computation ability (MCA). This paper shows that: (1) the teaching and learning has strong effects on computational estimation containing only numerical data positively and on computational estimation in problematic situations negatively; and (2) there are significant relationships both between CEA and MCA, and CEA and CES, but there is no significant relationship between CES and MCA. These results suggest that the reasonable and efficient computational estimation requires flexible rounding of numbers based on sound number sense as well as mental computation ability.

BACKGROUND FOR RESEARCH

Recently estimation has been emphasized in school mathematics curricula in Japan (Japanese Ministry of Education, 1989) as well as in the United States (National Council of Teachers of Mathematics, 1989). The major emphasis on estimation in elementary and intermediate mathematics curricula is based on such recognition that in the age of calculator/computer students should learn estimation to make sense of number or quantity and decide if answers are reasonable.

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In general estimation is divided into three categories; i.e. computational estimation, measurment estimation, and numerosity estimation (Sowder, 1992). Among them computational estimation has been a main topic addressed by mathematics researchers (Reys, Bestgen, Rybolt, and Wyatt, 1982; Rubenstein, 1985; Reys, 1986; Reys, Reys, Nohda, Ishida, Yoshikawa, and Shimizu, 1991; Dowker, 1992). As a result of this research, the three general cognitive processes and a number of specific strategies in computational estimation were identified (Reys et al., 1982). Moreover it was suggested that computational estimation would be closely related with mental computation and number sense (Rubenstein, 1985; Sowder, 1992; Dowker, 1992).

In this research computational estimation is defined as a process consisting of rounding numbers and mental computation with these rounded numbers without paper and pencil or other external aids. Therefore, in a problematic situation which requires computational estimation, it is very important to round numbers in consistent with the purpose of the situation (reasonable rounding) and in accordance with estimator's ability of mental computation (efficient rounding).

PURPOSE

In Japan, according to the previous curriculum of elementary school mathematics (Japanese Ministry of Education, 1977), it was necessary for students to practice mental computation in the third grade and to know the rounded numbers and how to round numbers by using strategies of "kiriage" (rounding up), "kirisute" (rounding down), and "shishagonyu" (if the following digit is more than 5 round up, and if less than 5 round down) in the fourth grade. Estimation was not highly emphasized. Moreover the relationship between mental computation and estimation was not explicitly stated. On the other hand, in the current curriculum (Japanese Ministry of Education, 1989) which has been implemented in 1992 school year, estimation is emphasized to a greater degree while the emphasis on mental computation is diminished.

The purpose of this research was to obtain performance data on the computational estimation and mental computation skills of Japanese students in grades 4, 5, and 6, to clear the effects of the teaching and learning of rounded numbers and rounding strategies at the fourth grade in the previous curriculum on computational estimation, and to explore the relationship between computational estimation ability, strategy, and mental computation. A total of 351 upper graders in elementary school were administered a computational estimation and mental computation test in June 1990.

METHOD

Subjects A total of 124 fourth graders, 143 fifth graders, and 84 sixth graders in four public elementary schools participated in this research. The schools were selected to represent a range of social and economic backgrounds. One school in a small town (Hyogo), one school in a city (Nara), and two schools in a large city (Hiroshima) were involved. In all classes students were heterogeneously grouped, as is the custom in elementary schools in Japan.

Material The computational estimation and mental computation test (MCE test) consisted of three parts and was based on a pilot test and previous research in Japan (Itoh et al., 1987, 1988). The first part of the MCE test contained 3 computational estimation problems (one was open−ended and the other two were multiple−choice) in problematic situations. The second part included 6 computational estimation items containing only numerical data and required students to provide explanations of how they estimated. The third part included 12 mental computation items. All items were related to four operations with whole numbers relevant to Japanese students.

The first and second parts were included on two sheets of paper. These papers were distributed to students with uniform instructions in each school by a classroom teacher. Students were given 30 minutes to complete these items.

The third part was presented either in a written test or in an oral test. In the written test, all items were included on a single sheet of paper and then this paper was distributed to students with the instruction that they mentally compute exact answers for all problems and that if they cannot compute mentally, they mark X in a given frame. They were given 10 minutes to complete the written test. For the oral test, all items were prerecorded on a recorder and presented to students at a specified period of time (5−10 seconds per item) with the instruction that they compute mentally exact answers (without copying the problem) and write their answer on a sheet of paper provided.

RESULTS AND ANALYSIS

The First Part of MCE Test

Table 1 summarizes the percent answers to the problem 1; "There are 48 boys and 54 girls in a park. You will give two candies to everyone. About how many candies are needed?" This table shows that 69.4% of the fourth graders, 69.9% of the fifth graders, and 76.2% of the

Table 1 *Percent Answers to The Problem 1*

Grade	Percent Answers				
	200	**204**	**210**	**220**	**other**
4 (n=124)	14.5	19.4	26.6	8.9	30.6
5 (n=143)	51.7	9.1	7.0	2.1	30.1
6 (n=84)	53.6	10.7	8.3	3.6	23.8

Table 2 *Percent Answers to The Problem 2*

Grade	Percent Answers								
	80×40	**80×50**	**90×40**	**90×50**	**100×40**	**85×45**	**other**	**enough**	**not**
4 (n=124)	12.1	21.0	1.6	33.1	2.4	10.5	17.7	88.7	0.0
5 (n=143)	7.0	75.5	0.0	7.0	1.4	4.9	3.5	83.2	4.9
6 (n=84)	0.0	73.8	2.4	14.3	1.2	4.8	3.6	90.5	2.4

sixth graders gave answers ranging from 200 to 220 candies and no significant difference was found among them. Focusing on reasonable answers (204, 210 or 220) for this problem, Table 1 shows 54.9% of the fourth graders gave reasonable answers, but only 18.2% of the fifth graders and 22.6% of the sixth graders gave them. About half of students at fifth and sixth grades made their estimation using "shishagonyu" strategy and answered 200 candies. A chisquare test showed a significant difference between fourth graders and fifth graders ($\chi^2=53.47$, $df=4$, $p<.0001$). Considering the fact that the fourth graders had not learned any rounded numbers and rounding strategies in schools, this analysis suggests that the teaching and learning of rounding strategy "shishagonyu" has an effect on a stereotyped estimation.

Table 2 summarizes the answers to the problem 2; "When you buy 46 apples at 83 yen, is 4500 yen enough for you to buy them? Choose one way among the followings to check by mental computation whether 4500 yen is enough or not, and then mark one of two alternatives of enough or not enough". In this problem, it is a reasonable and efficient estimation strategy to round up both numbers. Whereas 33.1% of the fourth graders chose this strategy (90×50), only 7.0% of the fifth graders and 14.3% of the sixth graders chose it. About 75% of students at fifth and sixth grades chose the "shishagonyu" strategy (80×50). Although the majority of students at each grade level chose a correct alternative "enough", some students seemed to do it based on an unreasonable estimate. A chisquare test showed a significant difference between the fourth graders and the fifth graders in regard of estimation strategies ($\chi^2=62.47$, $df=2$, $p<.0001$). This analysis also suggests a negative effect of the teaching and learning of rounding strategy "shishagonyu" on computational estimation in a problematic situation.

Table 3 *Percent Answers to The Problem 3*

Grade	Percent Answers								
	456÷90	450÷87	450÷80	450÷90	500÷90	500÷100	other	about 5	other
4 (n=124)	38.0	6.5	8.1	8.1	17.0	4.8	17.7	34.7	65.4
5 (n=143)	11.9	5.6	4.9	11.9	52.4	5.6	7.7	54.5	45.5
6 (n=84)	14.3	4.8	7.1	21.4	25.0	6.0	21.4	78.6	21.4

Talbe 3 summarizes the answers to the problem 3; "When 456 candies are shared with 87 persons, about how many candies does one person get? Choose one way among the followings to solve this problem by mental computation, and then write your answer". This table shows that the fourth graders tended to choose a way (456÷90) similar to an exact way (456÷87) in this problem. It is very interesting that only 8.1% of the fourth graders, 11.9% of the fifth graders, and 21.4% of the sixth graders chose a most reasonables and efficient strategy to estimate the number of candies. This fact tells us that it is difficult for the majority of students at each grade level to round up one number and round down the other at the same time in a flexible manner.

Moreover only 34.7% of the fourth graders and 54.5% of the fifth graders made a reasonable number "about 5" candies, while 78.6% of the sixth graders made it. The 29.0% of the fourth, 21.0% of the fifth, and 15.5% of the sixth graders made an unreasonable number near "about 50" candies. This fact suggests that it is difficult for them to mentally compute a division of 3−digit number by 2−digit number which could be reduced to a division of 2−digiti number by 1−digit number and that suggests a lack of number sense.

The Second Part of MCE Test

The second part of MCE test included 6 items containing only numerical data. In order to obtain both performance and strategy data, students were asked to roughly compute each of items without using paper and pencil and then to write a method of rough computation. The strategies used by students could be classified into three types. The R type was that student firstly rounded number(s) and then computed them mentally (e.g. 347−248→350−250=100). The C type was that student computed an exact answer mentally (e.g. 347−248=99). The M type was that student firstly rounded number(s), then computed them mentally, and finally rounded an answer again (e.g. 357+245→350+250=600→610).

Performance levels on 6 items and the range for acceptable estimates for each of them are shown in Table 4. The percentages of the three types of strategies used by students who could estimate reasonably are also shown in this table. The number in a total column means the percentage of students who gave an estimate inside of acceptable interval.

Table 4 *Percent Correct on 6 Items and Types of Strategies Used for Computational Estimation*

Item	Acceptable Interval	Grade 4 (n=124)				Grade 5 (n=143)				Grade 6 (n=84)			
		Total	R	C	M	Total	R	C	M	Total	R	C	M
375+245	[590, 610]	80.6	34.7	40.3	5.6	83.9	66.4	14.0	3.5	92.9	75.0	15.5	2.4
307+699	[1000, 1010]	75.2	38.0	31.5	5.6	81.8	63.6	15.4	2.8	94.0	76.2	15.5	2.4
347−248	[99, 100]	61.3	40.3	18.5	2.4	83.2	70.0	12.6	0.7	90.5	75.0	15.5	0.0
21×48	[900, 1100]	21.8	8.9	11.3	1.6	55.2	46.9	7.7	0.7	73.8	61.9	9.5	2.4
98÷9	[10, 11]	58.9	32.3	20.2	6.5	70.6	57.3	6.3	7.0	89.3	64.3	11.9	13.1
16+58+83+41	[180, 210]	57.3	30.0	24.2	3.2	84.6	66.4	14.0	4.2	96.4	78.6	13.1	4.8

The fact that the percent correct on each of items increased gradually with grade level showed that performance of computational estimation improved with grade. Table 4 also showed that the percentage of using the R type strategy dramatically increased from the fourth grade to the fifth grade. A chisquare test showed significant differences at all items (for the third item $p<.001$, for the others $p<.0001$) between students using the R strategy and those using the C strategy. This analysis suggests a positive effect of the teaching and learning of rounded numbers and rounding strategy "shishagonyu" on computational estimation containing only numerical data.

The Third Part of MCE Test

Performance levels on 12 items of mental computation in the written test and the oral test are shown in Table 5. As expected, this table showed that performance on both the written and oral tests generally improved with grade. In grades 4 and 5, performance on the written test varied by operation, with addition being the easiest and division the most difficult. There

Table 5 *Percent Correct on 12 Items of Mental Computation*

Item	Written Test			Oral Test	
	Grade 4 (n=92)	Grade 5 (n=106)	Grade 6 (n=84)	Grade 4 (n=32)	Grade 5 (n=37)
43+58	96	87	96	47	81
375+32	92	89	95	72	65
209+542	90	85	94	25	27
53−8	95	87	93	78	87
42−13	83	84	93	31	73
145−46	82	80	87	56	57
15×4	79	81	90	53	89
324×2	90	87	93	44	76
54÷9	84	84	98	72	87
96÷8	62	73	92	38	76
603÷3	60	72	93	50	81
284÷4	65	70	91	28	70

was a significant difference between the performance on the written and oral tests. This difference likely resulted from a variety of factors including the format of presentation and timing of each test. Other factors which may have contributed to better performance on the written test include a decreased need to "memorize" numbers. This conjecture seemed to be supported by the performance on the item 209+542. If students had used a mental version of the paper/pencil algorithm, it would be difficult to memorize the numbers and carry out this strategy in only 5−10 seconds. On the other hand, if they had computed the problem mentally by using the "toukaho" strategy (e.g. 209+542=209+500+42=709+42=751) the difference in performance might not have been so great.

This analysis suggests that although performance on mental computation of four operations with whole numbers improved, many students seem to employ a mental version of the paper/pencil algorithm instead of making use of the advantage of Japanese numerical system (Shigematsu, Iwasaki, & Koyama, In press).

RELATIONSHIP BETWEEN THE COMPUTATIONAL ESTIMATION ABILITY AND STRATEGY AND THE MENTAL COMPUTATION ABILITY

Using data on the second and third parts of MCE test, the relationship between computational estimation ability (CEA), computational estimation strategy (CES), and mental computation ability (MCA) was analyzed. In this research CEA, CES, and MCA were defined as the followings.

CEA: It was measured by the frequency of reasonable estimation on 6 items in the second part of MCE test. Students who made more than 5 reasonable estimates were grouped in the high−ability class (EH) and the others were in the low−ability class (EL).

CES: It was determined by the type of strategy used to estimate 6 items in the second part

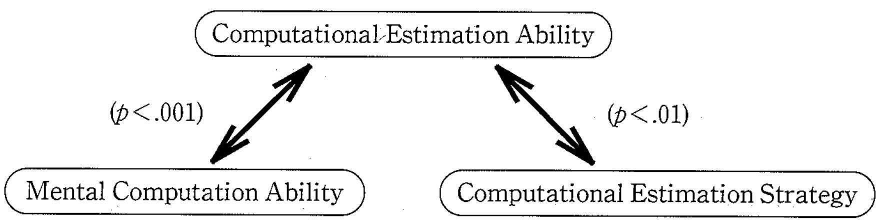

Fig.1 *Relationship between CEA, MCA, and CES*

of MCE test. The R type strategy and the C type strategy were differentiated.

MCA: It was measured by the frequency of correct mental computation on 12 items in the third part of MCE test. Students who computed more than 10 correct answers were grouped in the high-ability class (MH) and the others were in the low-ability class (ML).

As a result of the chisquare tests, there was a significant relationship between CEA and MCA in the written and oral tests ($\chi^2=28.24$, $df=1$, $p<.001$; and $\chi^2=15.01$, $df=1$, $p<.001$, respectively). There were also significant relationships at each of items except for the fifth item "98÷9" ($7.05 \leqq \chi^2 \leqq 23.58$, $df=1$, $p<.01$) between CEA and CES. There was no significant relationship between CES and MCA. These results are illustrated in Figure 1. This suggests that the mental computation ability contributes to the computational estimation ability. For flexible computational estimation strategies, however, the reasonable and efficient rounding of numbers is more important than the mental computation ability. In other words, it might be said that the reasonable and efficient computational estimation requires flexible rounding of numbers based on a sound number sense as well as mental computation ability.

CONCLUSIONS AND IMPLICATIONS

This research suggests the following findings:

(1) The teaching and learning of rounded numbers and rounding strategies in the fourth grade had strong effects on performance on computational estimation at the fifth and sixth grades in Japan. Whereas in the computational estimation containing only numerical data the teaching and learning of rounded numbers and rounding strategy "shishagonyu" had a positive effect, it had a negative effect on the computational estimation in some problematic situations and led to stereotyped estimation.

(2) There were significant relationships both between computational estimation ability and mental computation ability, and computational estimation ability and computational estimation strategy. There was, however, no significant relationship between computational estimation strategy and mental computation ability. These results suggest that the reasonable and efficient computational estimation requires flexible rounding of numbers based on a sound number sense as well as mental computation ability.

These findings are consistent with the observations reported by Reys et al. (1991) which focused on the computational estimation performance and strategies used by fifth－and eighth－grade Japanese students. "Indeed it is possible that the overlearning of written algorithms may inhibit a number of important factors that contribute to success at estimation, namely, flexible use of numbers, tolerance for error, use of multiple strategies, and adjustment techniques. Students at both grade levels tended to apply learned algorithmic computational procedures mentally" (p. 55). The second finding in this research, however, is rather new and could support that computational estimation would be closely related with mental computation and number sense.

Although all results of this research may be of limited generality, they suggest that the formal teaching of rounded numbers and rounding strategies could lead students to a rigid and stereotyped estimation. Therefore the teaching computational estimation should help students round numbers flexibly, estimate value reasonably, and make an adequate decision in a problematic situation. To do it we must put more emphasis on mental computation and number sense than the complicated paper/pencil computation in elementary school mathematics curricula.

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