Urban Structure and Rank-Size Rule of Cities

—An examination of cases in Japan from 1975 to 1995—

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SUMMARY: The aim of this paper is to examine the rank-size rule of cities or the constant elasticity of city's rank to city's size in the case of Japan from 1975 to 1995. The main results obtained are as follows.
① The Pareto distribution indicates the good fitness for the relation between the city's rank and city's size.
② However, the value of elasticity of city's rank to city's size, i.e. Pareto exponent, considerably depends on the definition of city and the sample size (the number of cities adopted as samples in order of city's rank).
③ When we increase the sample size, the value of elasticity increases at first and soon reaches the peak and decreases gradually, without regard to the definition of city adopted in this paper.
④ When we adopt two criteria, i.e. the weak criterion (with the range of 0.5 in the elasticity of city's rank to city's size) and the strong criterion (with the range of 0.2 in the elasticity), neither the rank-size rule nor the constant elasticity is valid even on the basis of the weak criterion for the first category of city, i.e. "cities, towns and villages". For the second category, i.e. "cities", the rank-size rule is not valid but the constant elasticity is valid on the basis of the weak criterion, whereas neither the rank-size rule nor the constant elasticity is valid on the basis of the strong criterion. For the third category of city, i.e. "areas", both of the rank-size rule and the constant elasticity are almost valid on the basis of the weak criterion, although neither the rank-size rule nor the constant elasticity is valid on the basis of the strong criterion. For the forth category, i.e. "prefectures", neither the rank-size rule nor the constant elasticity is valid even on the basis of the weak criterion, so this is similar to the first category.
⑤ Generally speaking, we can say that the rank-size rule and the constant elasticity are apt to be valid for the middle category of city which is formed from point of economic activities, whereas neither the rank-size rule nor the constant elasticity is valid for the small or large category of city which is the district formed from point of administration.
⑥ The facts from ① to ⑤ mentioned above are all valid for the period of 1975-95 in Japan.
⑦ For the first, second and forth categories of city which are administrative districts, the values of elasticity of city's rank to city's size increase among the "upper ranking" members ("large" cities) of each category, whereas the values decrease among "all" members, as time passes. For the third category (areas), the values decrease both among the "upper ranking" members and among "all" members, as time passes. In other words, the small cities have turned out to be smaller relatively, and the cities of upper-middle size have grown larger, therefore differences of city's size as a whole increased from 1975 to 95.

1. INTRODUCTION

One of the main themes of urban economics is to prove the real state of urbanization or agglomeration of cities and its mechanism. As concerns the mechanism, we have the theory of transport cost, the theory of spatial competition, the central place theory, the spatial production and consumer theory, and the theory of agglomeration economies (externalities in an urban context), and so on. (See, for example, Hotelling[2] and Mills[3].) On the other hand, there is the empirical studies of urban size and structures of urban areas from point of real state of urbanization. The typical one of them is the rank-size rule or the constant elasticity of city's rank to city's size. (See, for example, Beckman[1] and Rosen[4].) Here, the rank-size rule means that the product of the city's rank and its size is a constant.

(Preto distribution) We can derive the rank-size rule from the Pareto distribution like this,

$$R(S)=AS^{-a} \cdots\cdots\cdots\cdots \text{(1)},$$

where $R(S)$ is the number of urban areas (cities) with at least S-city' size (S or more population), and two coefficients (A and a) are constants which will be estimated from the data. So $R(S)$ means the rank of the city with S people. Here, the ranking is formed in order of large population. In this paper, the city's size is measured by the population, and I showed its validity in my previous paper[12]. We use the data of population in *Basic Data of Residents*[14].

R: city's rank
S: city's size
A and a: constants

If we take logs of both sides of the above expression (1), we have the following log-linear type equation.

$$\log R=-a\log S+\log A \cdots\cdots\cdots\cdots \text{(2)}$$

So, we have this expression,

$$Y=-aX+b \cdots\cdots\cdots\cdots \text{(3)},$$

where $Y=\log R$, $X=\log S$, and $b=\log A$.

In this case, the coefficient (a) of X is Pareto exponent or the elasticity of city's rank (R) to city's size (S), i.e.,

$$a=-(dR/R)/(dS/S).$$

(constant elasticity and rank-size rule) Therefore, the constant elasticity of city's rank to city's size means that the coefficient (a) is constant, and the rank-size rule means that $a=1$, so the rule is the special case of the constant elasticity.

We will examine whether the rank-size rule and the constant elasticity are valid or not, through the estimation of coefficient (a) of X by computing the least squares regression of above equation (1), (2), or (3), on the basis of data from 1975 to 95 in Japan.

2. THE DEFINITION OF CITY AND THE SAMPLE SIZE

In this estimation, we are faced with two problems, one is the definition of city and the other is the sample size, because we will expect that the values of coefficients to be estimated vary with the definition of city or the sample size.

(definition of city) As concerns the definition of "city" which means an urban area, we will adopt following four categories.

four categories of "city":
- category 1: cities, towns and villages
- category 2: cities
- category 3: areas
- category 4: prefectures

Category 1, 2 and 4 are administrative districts in Japan. Category 1 is the basic administrative unit and consists of 3235 members in 1995. The number of members of category 1 is apt to decline, because the number of towns and villages is decreasing. Category 2 is the part of category 1 which consists of only cities. This category has 664 members in 1995, and the number of members is increasing gradually. Category 4 is the more comprehensive administrative unit than category 1 and

2, which includes some cities, towns and villages. Category 4 has 47 members, and the number of members is constant for the period considered here.

We have to note two points. The first concerns the special ward. In addition to cities, towns and villages, we have 23 special wards in Tokyo metropolitan area as basic administrative unit in Japan. We will put these 23 special wards together into one, i.e. Tokyo-special-ward, and category 1 and 2 include this Tokyo-special-ward as one of members. The second concerns category 4. Category 4 includes not only 43 prefectures but also four members, i.e. Tokyo metropolitan area, Osaka metropolitan prefecture, Kyoto metropolitan prefecture and Hokkaido special prefecture.

Category 3, "area", is not the administrative district, but an economic distribution area. "Area" usually has one central city, and the central city has powerful urban functions in the area from points of administration, education, culture, health and welfare as well as economic, distributive and industrial activities.

We have no clear and official definition of "area", because it is not the administrative unit. We will adopt the concept in Minryoku[15] as "area" here, as same as my previous papers[10),11),12)] and[13)]. The number of areas and the members of an area in Minryoku sometimes vary slightly as time passes. But we will adopt 110 areas in Minryoku[15] through this paper.

As we can know through the explanation above, the region of city becomes wider, according as we move from category 1 to 4.

(sample size) Now, as concerns the other problem, i.e. the sample size, we could not decide a priori the size (the number of cities) which we should adopt. Therefore we will examine many cases depending on necessity where the number of cities to be adopted spreads from 5, 10, 20, 30, ⋯, 100, ⋯, 500, ⋯, 1000, ⋯, to all samples. Through this examination of many cases, we can understand the whole feature of relation between city's rank and city's size.

3. CATEGORY 1 (CITIES, TOWNS AND VILLAGES)

(relation between city's rank and city's size) When we adopt category 1 (cities, towns and villages) as the definition of city, we can find the whole feature of the relation between city's rank and city's size at Fig. 1-1. From this figure, we can find out the first important observation that the relation between city's rank and city's size shows almost the straight line (log-linear line), if we exclude the exceptional part close to 8 on the horizontal axis. We can verify the good fitness to the log-linear relation by the high value of coefficients of determination adjusted for the degree of freedom in Appended Table 1. And we can know that the domain of exceptional part is the part with about 8.04 or more value of log of city's rank, by observing Fig. 1-6. (Figures from Fig. 1-2 to Fig. 1-6 are the expanded ones of a part in Fig. 1-1.) Here, $8.04 \fallingdotseq \log 3100$, so we can understand that the exception is about 140 samples which are small towns and villages, by the simple algorism: the number of all samples (about 3240) minus 3100. The exception is about 4.3% of all samples.

We are able to know details of Fig 1-1 through from Fig. 1-2 to Fig. 1-6. As the value of log of city's rank on the horizontal axis grows from zero, the value of log of city's size (population) on the vertical axis decreases considerably at first, then decreases gently, and decreases steeply at last.

(elasticity of city's rank to city's size) As concerns such relation, the coefficients (a) of X are useful, as we show them in Fig 2-1, Fig 2-2 and Appended Table 1. Fig 2-2 is the expanded figure of a part of Fig 2-1. If we neglect the small decline of the elasticity at the first stage, the elasticity of city's rank to city's size (a: coefficient of X) on the vertical axis grows steeply at first, and it reaches the peak at the neighbourhood of sample size 100, then declines gradually, as the sample

Fig. 1-1 City's rank and its population for 1975-95 in Japan
cities, towns and villages---1

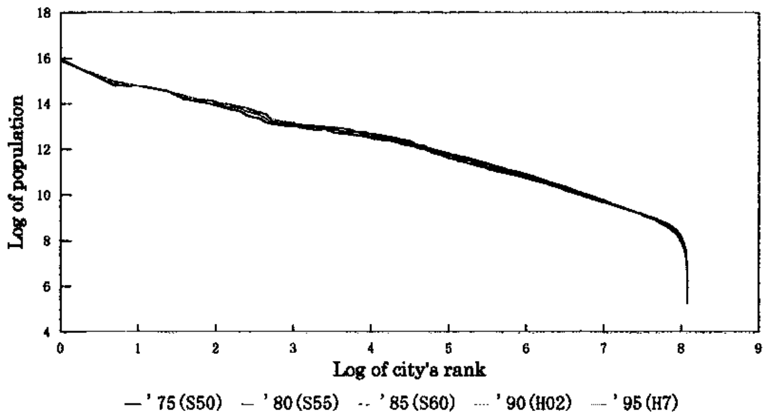


Fig. 1-2 City's rank and its population for 1975-95 in Japan
cities, towns and villages---2

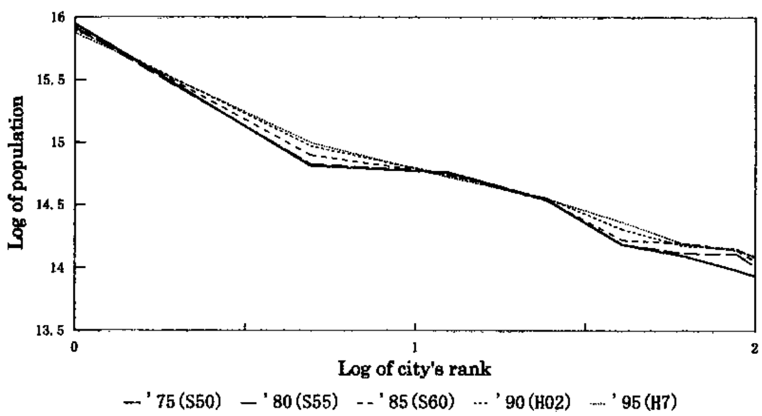


Fig. 1-3 City's rank and its population for 1975-95 in Japan
cities, towns and villages---3

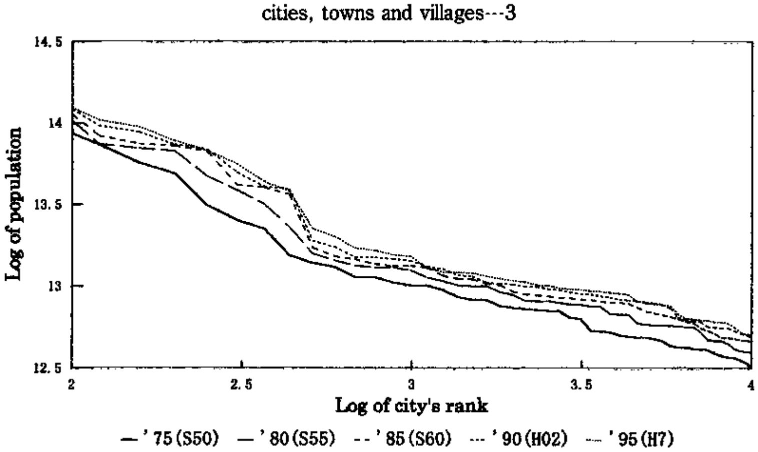


Fig. 1-4 City's rank and its population for 1975-95 in Japan
cities, towns and villages---4

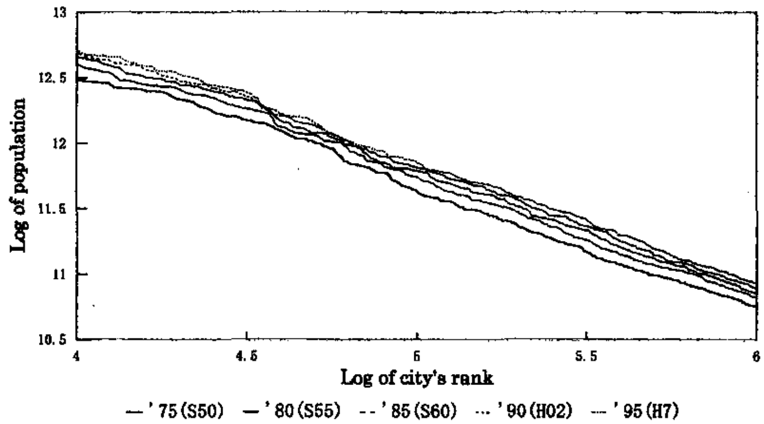


Fig. 1-5 City's rank and its population for 1975-95 in Japan
cities, towns and villages---5

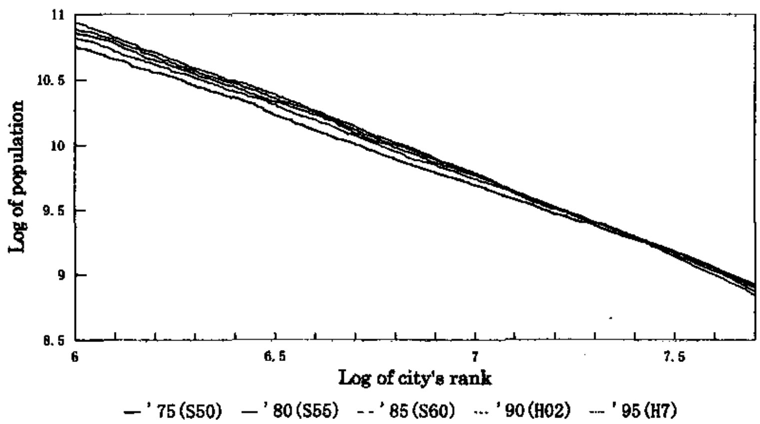


Fig. 1-6 City's rank and its population for 1975-95 in Japan
cities, towns and villages---6

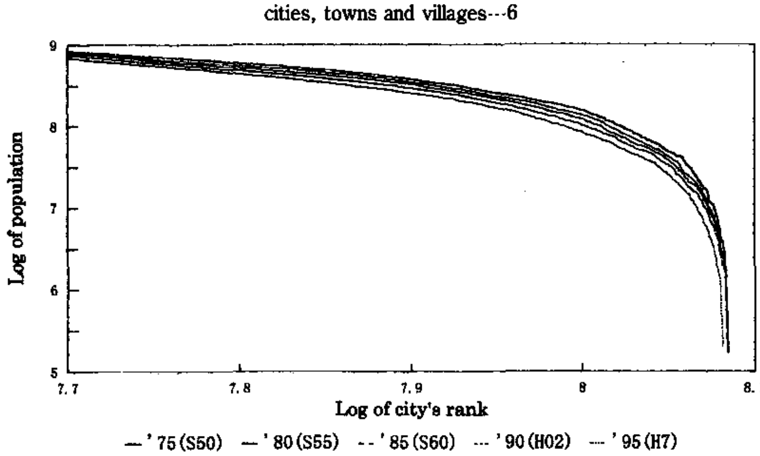


Fig. 2-1 The number of samples and the size elasticity of city's rank cities, towns and villages---1

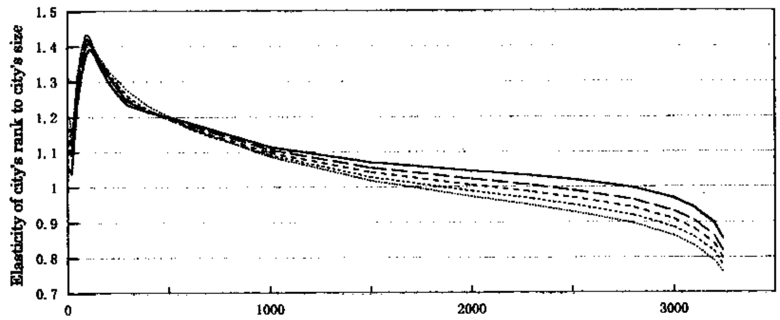
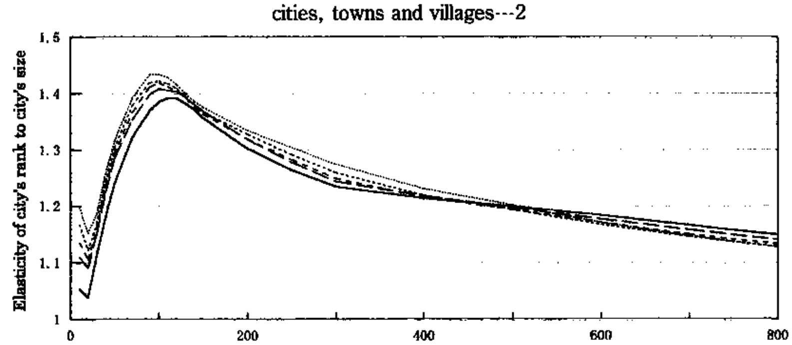


Fig. 2-2 The number of samples and the size elasticity of city's rank cities, towns and villages---2



size (the number of samples) on the horizontal axis increases. The sample size shows the city's rank, because the samples are taken in order of city's rank. In other words, for example, the sample size 100 means that we take 100 cities from city's rank 1 to city's rank 100 as the samples to estimate the relation between the city's rank and city's size.

(relation between the rank-size and the elasticity) Fig .3-1 and Fig. 3-2 show such a relation. Fig. 3-1 corresponds to Fig. 2-1, and Fig. 3-2 corresponds to Fig. 1-1. We should note that the vertical axis in Fig. 1-1 and Fig. 3-2 is X in equation (3), on the other hand the horizontal axis is Y in Equation (3). In addition to this, we also note that the horizontal axis in Fig. 1-1 has the log-values, but this is not so in the horizontal axis in Fig. 3-2.

Fig. 3-1 and Fig. 3-2 summarize observations appropriately which are stated above about the relation between city's rank and city's size and the elasticity of city's rank to city's size on the basis of Fig. 1-1, Fig. 2-1 and Appended Table 1. There are two observations. First, the city's size (population) decreases considerably at first, then decreases gently, and decreases steeply at last, as the city's rank drops. Second, the elasticity of city's rank to city's size grows steeply at first, and it reaches the peak at the neighbourhood of sample size 100, then declines gradually, as the sample size increases.

(change of the relation between city's rank and city's size) We can observe the change of the relation between city's rank and city's size, as time

Fig. 3-1 The elasticity of city's rank to city's size and the sample size

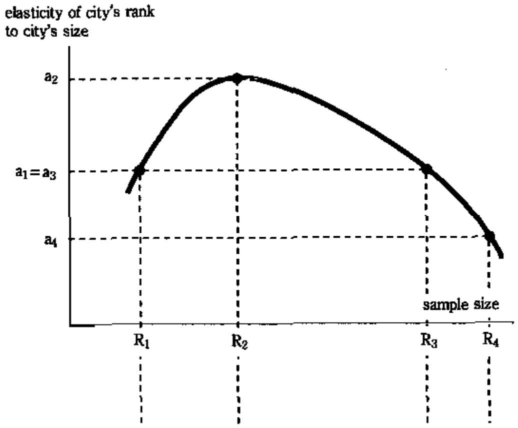


Fig. 3-2 The city's rank and the city's size

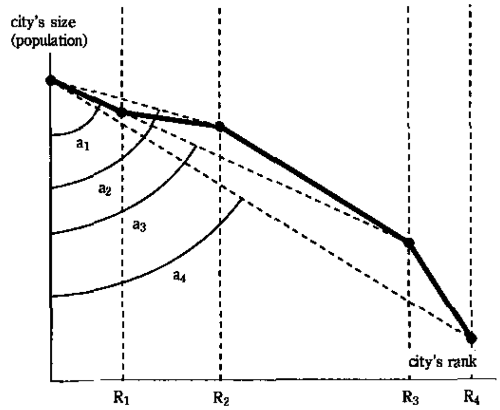


Fig. 4-1 Change of the relation between the elasticity and the sample size

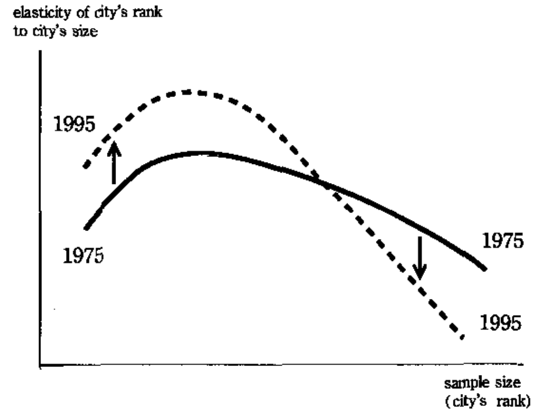
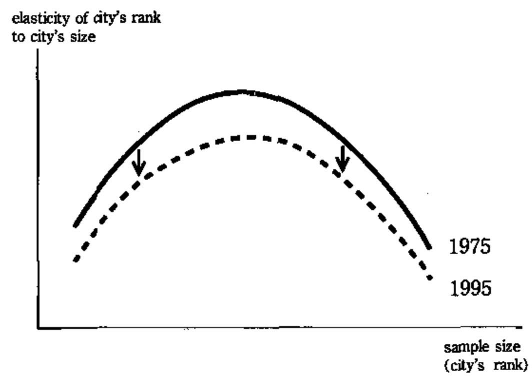


Fig. 4-2 Change of the relation between the elasticity and the sample size



passes from 1975 to 95. According to figures form Fig. 1-2 to Fig. 1-6, populations of cities with upper ranks have grown, on the other hand, populations in small towns and villages with lower ranks have shrunk, for the period from 1975 to 95. The log-value of critical rank is about 7.4 as we can find it in Fig. 1-5. Therefore cities with upper ranks than 1650 are expanding, and the other small cities with lower ranks than 1650 are shrinking, because $7.4 \fallingdotseq \log 1650$. This means that half members of cities, towns and villages are expanding and the other half are shrinking, for there are about 3240 cities, towns and villages in Japan.

(change of the elasticity) As concerns the elasticity, the critical sample size is about 450 as we see Fig. 2-2. The elasticity for sample sizes with less than 450 samples is apt to increase, on the other hand, the elasticity for more sample sizes than 450 is apt to decrease.

We will show this change of the elasticity in Fig. 4-1. The elasticity of city's rank to city's size means how many percent the city's rank changes when the city's size changes by one percent. Therefore, the high elasticity means the large change of rank for a fixed change of size. In other words, the high elasticity means that the differences of sizes of members are small, that is to say, the equalization of size. We can know the tendency of equalization of size of cities with upper ranks and the tendency of expansion of differences of all samples.

(rank-size rule and constant elasticity) As concerns the rank-size rule and the constant elasticity, we will show criteria of the validity of the rule and the constancy at first.

criteria for the rank-size rule:

strong criterion: If all the elasticity is in the domain from 0.9 to 1.1, that is to say, in the domain of width 0.2 with the central value 1.0, the rule is valid.

weak criterion: If all the elasticity is in the domain from 0.75 to 1.25, that is to say, in the domain of width 0.5 with the central value 1.0, the rule is valid.

criteria for the constant elasticity:

strong criterion: If all the elasticity is in the domain of width 0.2, the constancy is valid.

weak criterion: If all the elasticity is in the domain of width 0.5, the constancy is valid.

On the basis of these criteria, we can say as follows.

(category 1) When we think of all samples from rank 5 to the bottom in 1975 at Appended Table 1, the maximum of the elasticity is 1.392, which corresponds to sample size 110 and 120, and the minimum is 0.853, which corresponds to all samples. The difference is 0.539, so the constancy is not valid even for the weak criterion.

If we restrict these samples to the case where samples from rank 3000 to the bottom are excluded, the minimum is 0.927, which corresponds to the sample size 5, while the maximum is the same value, i.e. 1.392, so the difference is 0.465. Therefore, the constancy is valid for the weak criterion, whereas not for the strong criterion.

Now we think of all samples from rank 10 to the bottom in the same year, i.e., 1975 in Appended Table 1, the maximum and the minimum of the elasticity are the same as the first case, where we have all samples from rank 5 to the bottom. So, the constancy is not valid even for the weak criterion, either.

On the other hand, if we restrict these samples to the case where samples from rank 3000 to the bottom are excluded, the maximum and the minimum of the elasticity are the same as the second case, where we have all samples from rank 5 to 3000. Therefore, the constancy is valid for the weak criterion, whereas not for the strong criterion.

If we exclude the domain with sample size from about 50 to 300, both of the constant elasticity and the rule are valid for the weak criterion. Furthermore, if we restrict the sample size to the domain from about 1000 to 2500, both are valid for the strong criterion.

As concern 1980, the results are the same as the cases in 1975, as we showed just above. We will show these situations in Table 1.

However, in 1985,90 and 95, the constancy is invalid for all cases even for the weak criterion, as showed in Table 1.

4. CATEGORY 2 (CITIES)

(relation between city's rank and city's size) As concerns category 2 (cities) of city, we can find the whole feature of the relation between city's rank and city's size at Fig. 5-1. From this figure, we can find out almost the same observation as the case of category 1, where the relation between city's rank and city's size shows almost the straight line (log-linear line), if we exclude the exceptional part close to 6.5 on the horizontal axis. We can verify this by looking at the high value of coefficients of determination in Appended Table 2. And we can know that the domain of exception is located in the neighbourhood of 6.48 in the horizontal axis by observing Fig. 5-5. (Figures from Fig. 5-2 to Fig. 5-5 are the expanded ones of Fig. 5-1.) Here, $6.48 \fallingdotseq \log 650$, so we can understand that the exception is about 5 or 14 samples, be-

Table 1 Pareto exponent: Results of examination of the rank-size rule and the constant elasticity

		category 1				category 2				category 3				category 4			
		cities, towns and villages				cities				areas				prefectures			
samples		5-	5-3000	10-	10-3000	5-	5-600	10-	10-600	5-	5-90	10-	10-90	5-	5-40	10-	10-40
1975	min	0.853	0.927	0.853	0.927	0.927	0.927	1.039	1.039	0.632	0.632	0.781	0.781	1.265	1.359	1.265	1.359
	max.	1.392	1.392	1.392	1.392	1.392	1.392	1.392	1.392	1.191	1.191	1.191	1.191	2.152	2.152	2.035	2.035
	dif.	0.539	0.465	0.539	0.465	0.465	0.465	0.353	0.353	0.559	0.559	0.410	0.410	0.887	0.793	0.770	0.676
	result	×	△	×	△	△	△	△	△	×	×	△△	△△	×	×	×	×
1980	min.	0.818	0.932	0.818	0.932	0.948	0.948	0.950	0.950	0.637	0.637	0.783	0.783	1.243	1.337	1.243	1.337
	max.	1.408	1.408	1.408	1.408	1.408	1.408	1.408	1.408	1.182	1.182	1.182	1.182	2.426	2.426	2.159	2.159
	dif.	0.590	0.476	0.590	0.476	0.460	0.460	0.458	0.458	0.545	0.545	0.399	0.399	1.183	1.089	0.916	0.822
	result	×	△	×	△	△	△	△	△	×	×	△△	△△	×	×	×	×
1985	min.	0.797	0.908	0.797	0.908	0.969	0.969	1.104	1.136	0.635	0.635	0.781	0.781	1.222	1.317	1.222	1.317
	max.	1.419	1.419	1.419	1.419	1.419	1.419	1.419	1.419	1.172	1.172	1.172	1.172	2.499	2.499	2.171	2.171
	dif.	0.622	0.511	0.622	0.511	0.450	0.450	0.315	0.283	0.537	0.537	0.391	0.391	1.277	1.182	0.949	0.854
	result	×	×	×	×	△	△	△	△	×	×	△△	△△	×	×	×	×
1990	min.	0.776	0.858	0.776	0.858	1.022	1.022	1.084	1.135	0.629	0.629	0.777	0.777	1.203	1.295	1.203	1.295
	max.	1.422	1.422	1.422	1.422	1.422	1.422	1.422	1.422	1.158	1.158	1.158	1.158	2.606	2.606	2.188	2.188
	dif.	0.646	0.564	0.646	0.564	0.400	0.400	0.338	0.287	0.529	0.529	0.381	0.381	1.403	1.311	0.985	0.893
	result	×	×	×	×	△	△	△	△	×	×	△△	△△	×	×	×	×
1995	min.	0.758	0.861	0.758	0.861	1.060	1.060	1.063	1.137	0.620	0.620	0.774	0.774	1.190	1.295	1.190	1.295
	max.	1.434	1.434	1.434	1.434	1.434	1.434	1.434	1.434	1.156	1.156	1.156	1.156	2.796	2.796	2.211	2.211
	dif.	0.676	0.573	0.676	0.573	0.374	0.374	0.371	0.297	0.536	0.536	0.382	0.382	1.606	1.501	1.021	0.916
	result	×	×	×	×	△	△	△	△	×	×	△△	△△	×	×	×	×
1975-1995	min.	0.758	0.858	0.758	0.858	0.927	0.927	0.950	1.039	0.620	0.620	0.774	0.774	1.190	1.295	1.190	1.295
	max.	1.434	1.434	1.434	1.434	1.434	1.434	1.434	1.434	1.191	1.191	1.191	1.191	2.796	2.796	2.211	2.211
	dif.	0.676	0.576	0.676	0.576	0.507	0.507	0.484	0.395	0.571	0.571	0.417	0.417	1.606	1.501	1.021	0.916
	result	×	×	×	×	×	×	△	△	×	×	△△	△△	×	×	×	×

"5-" in "samples" means all samples from rank 5 to the bottom. "5-3000" in "samples" means all samples from rank 5 to rank 3000.
"3000, 600, 90 and 40" mean the numbers of samples except about 10% from the bottom, in category 1, 2, 3 and 4 respectively.
"×" in "result" means that neither the rule nor the constancy is valid for the weak criterion.
"△" in "result" mean that the constancy is valid, whereas the rule is not valid, for the weak criterion.
"△△" in "result" means that the rule (of course, the constancy, too) is valid for the weak criterion.
In all cases, neither the constancy nor the rule is valid for the strong criterion.

Fig. 5-1 City's rank and its population for 1975–95 in Japan cities---1

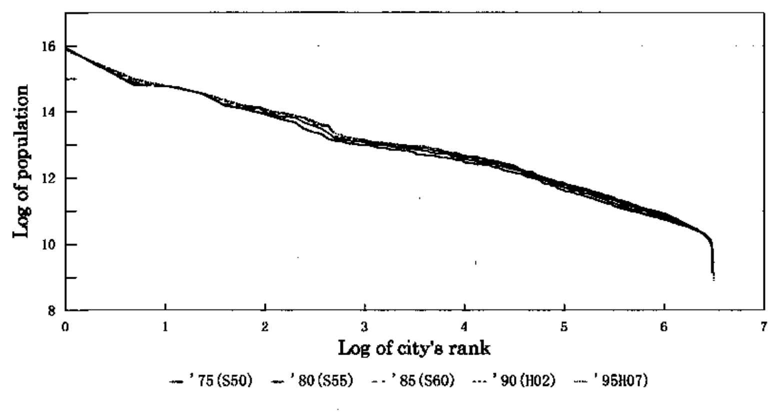


Fig. 5-2 City's rank and its population for 1975–95 in Japan cities---2

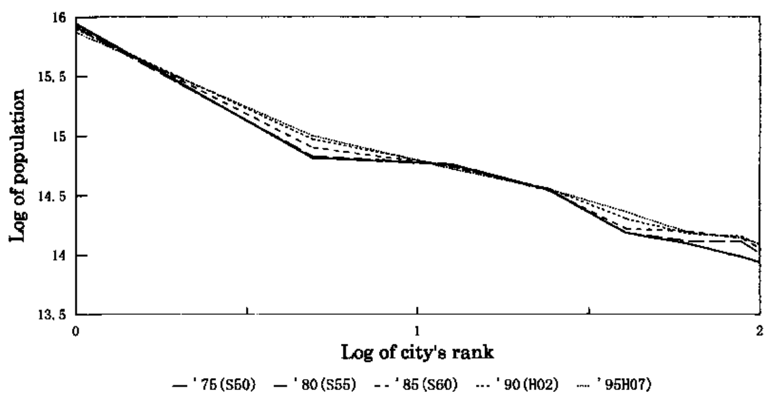


Fig. 5-3 City's rank and its population for 1975–95 in Japan cities---3

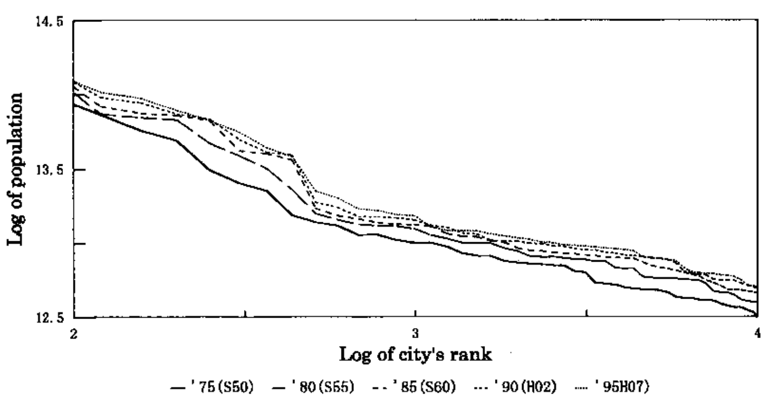


Fig. 5-4 City's rank and its population for 1975-95 in Japan cities---4

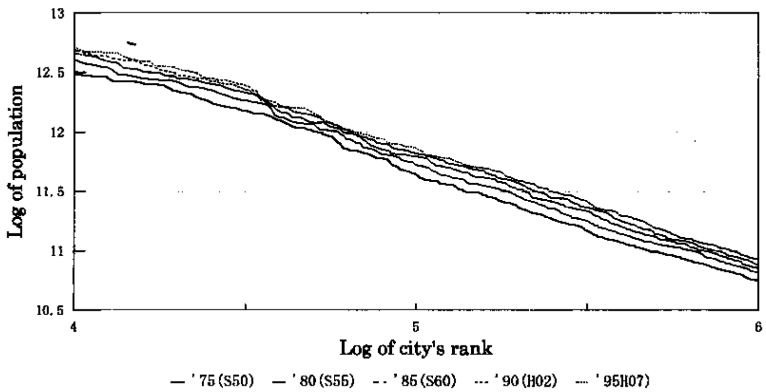
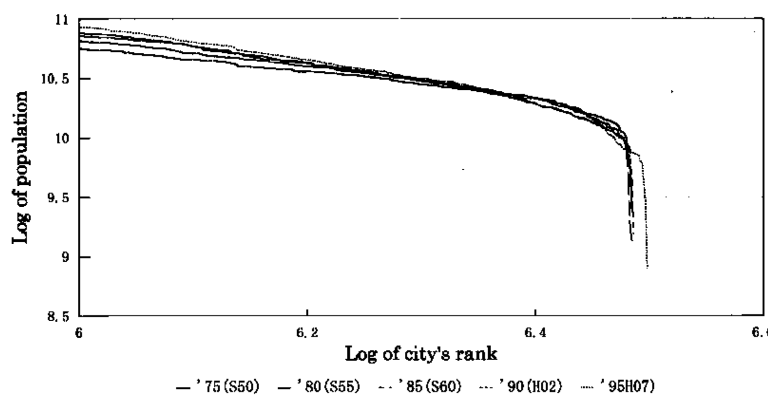


Fig. 5-5 City's rank and its population for 1975-95 in Japan cities---5



cause the number of all samples is about 655 or 664. The exception is about 1.5% of all samples.

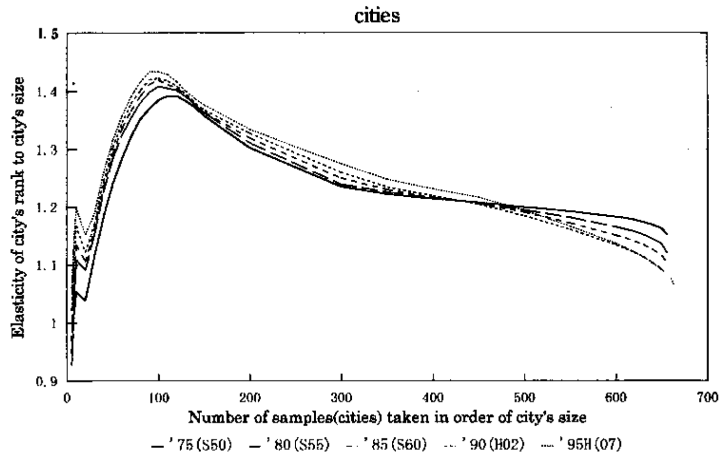
According to figures from Fig 5-2 to Fig. 5-5, as the value of log of city's rank on the horizontal axis grows from zero, the value of log of city's size (population) on the vertical axis decreases considerably at first, then decreases gently, and decreases steeply at last. This is the same as category 1.

(elasticity of city's rank to city's size) We will show the elasticity in Fig 6 and Appended Table 2. If we neglect the small decline of the elasticity at the first stage, the elasticity grows steeply at first, and it reaches the peak at the neighbourhood of sample size 100, then declines gradually, as the sample size increases. This is the same as category 1, too. However, all the values of elasticity are larger than 1.0. This point is different from category 1.

(relation between the rank-size and the elasticity) As concerns the relation between the rank-size and the elasticity, the relation showed in Fig. 3-1 and Fig. 3-2 is valid in category 2, too. Therefore, two observations stated in category 1 are also valid here.

(change of the relation between city's rank and city's size) According to figures form Fig. 5-2 to Fig. 5-5, populations of cities with upper ranks

Fig. 6 The number of samples and the size elasticity of city's rank



have grown, on the other hand, populations in small cities with lower ranks have shrunk, for the period from 1975 to 95. The log-value of critical rank is about 6.35 as we can find it in Fig. 5-5. Therefore cities with upper ranks than 570 are expanding, and the other small cities with lower ranks than 570 are shrinking, because $6.35 \fallingdotseq \log 570$. This means that 85% of cities is expanding and the other 15% is shrinking.

(change of the elasticity) As concerns the elasticity, the same things as category 1 are valid, as we show in Fig. 4-1. Therefore, we can find out that the difference of size of cities with upper ranks is apt to become smaller and the difference of size of all cities is apt to become larger.

(rank-size rule and constant elasticity) As concerns the constant elasticity, the situation is different from category 1, as we are able to see in Fig. 6, Table 1 and Appended Table 2.

In category 2, let's consider the case where there are samples from rank 5 to the bottom in 1975. The maximum is 1.392, which corresponds to sample size 110 and 120, on the other hand, the minimum is 0.927, which corresponds to sample size 5. So the difference is 0.465, and the constancy is valid for the weak criterion, whereas not for the strong criterion.

At all cases of each year in category 2, the constancy is valid for the weak criterion, whereas not for the strong criterion, as we can see at Table 1 and Appended Table 2.

As concerns the whole period from 1975 to 95, at the cases with samples from rank 10 to the bottom or to rank 600, the constancy is valid for the weak criterion, whereas not for the strong criterion. However, at the cases with samples from rank 5 to the bottom or to rank 600, the constancy is invalid even for the weak criterion, because the maximum is 1.434, and the minimum is 0.927, so the difference is 0.507, in these cases.

As concerns the rank-size rule, it is invalid even on the basis of the weak criterion at all cases in category 2.

5. CATEGORY 3 (AREAS)

(relation between city's rank and city's size) Fig. 7-1 shows the whole feature of the relation between city's rank and city's size in category 3. From this figure, we can find out almost the same observation as the case of category 1. The relation between city's rank and city's size in Fig. 7-1 shows almost the straight line (log-linear line), if we exclude the exceptional part close to 4.5 on the horizontal axis. We can verify this by looking at

Fig. 7-1 City's rank and its population for 1975–95 in Japan
areas···1

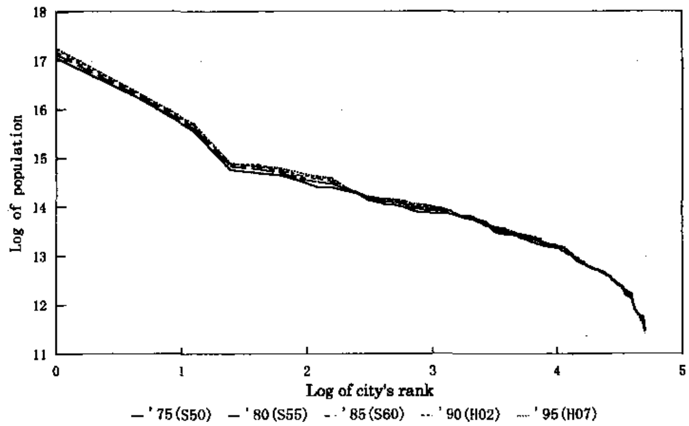


Fig. 7-2 City's rank and its population for 1975–95 in Japan
areas···2

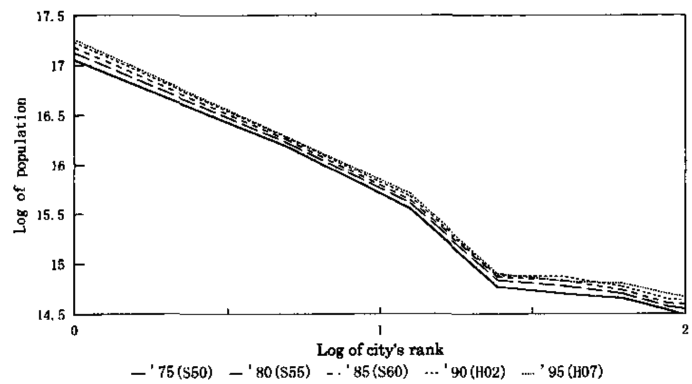


Fig. 7-3 City's rank and its population for 1975–95 in Japan
areas···3

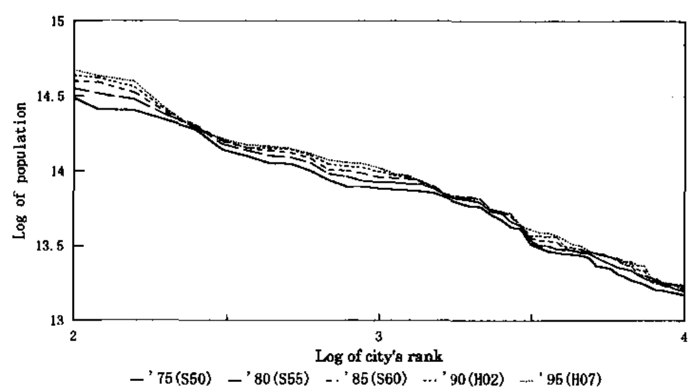
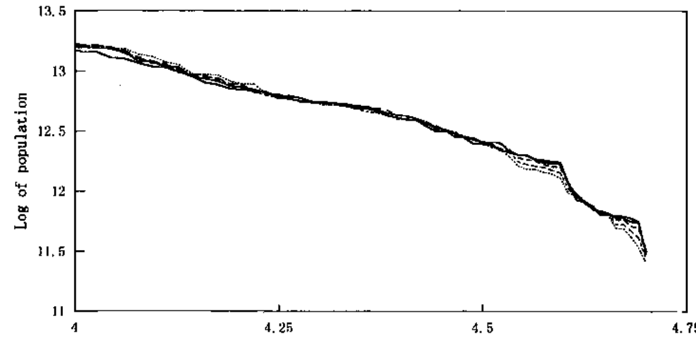


Fig. 7-4 City's rank and its population for 1975-95 in Japan

areas---4



the high value of coefficients of determination in Appended Table 3. And we can know that the domain of exception is located in the neighbourhood of 4.6 in the horizontal axis by observing Fig. 7-4. Here, 4.6≒log 100, so we can understand that the exception is about 10 samples, because the number of all samples is 110. The exception is about 10% of all samples.

According to figures from Fig 7-2 to Fig. 7-4, as the value of log of city's rank on the horizontal axis grows from zero, the value of log of city's size (population) on the vertical axis decreases considerably at first, then decreases gently, and decreases faster at last. This is almost the same as category 1 and 2.

(elasticity of city's rank to city's size) Let's turn our eyes to the elasticity in Fig 8 and Appended Table 3. Here we find a little difference from Fig. 2-1 in category 1 and Fig. 4-1 in category 2. As the sample size increases, the elasticity grows uniformly at first, and it reaches the peak at the neighbourhood of sample size 60, then declines gradually. This shape is the upper-wards convex without twists. Furthermore, almost the values of elasticity are rather near to 1.0.

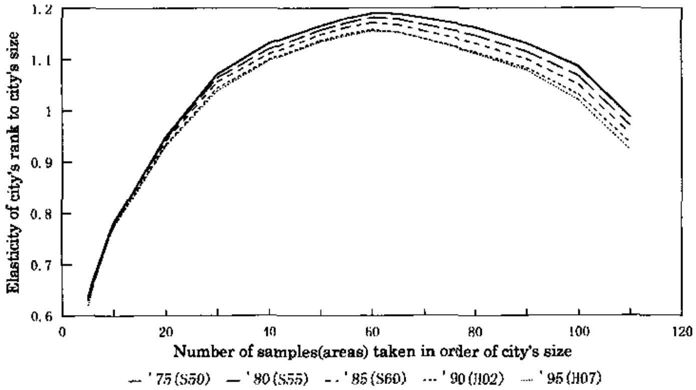
(relation between the rank-size and the elasticity) As concerns the relation between the rank-size and the elasticity, the relation showed in Fig. 3-1 and Fig. 3-2 is valid in category 3, too.

(change of the relation between city's rank and city's size) According to figures form Fig. 7-2 to Fig. 7-4, populations of areas with upper ranks have grown, on the other hand, populations in small areas with lower ranks have shrunk, for the period from 1975 to 95. This is the same as category 1 and 2. The log-value of critical rank is about 4.25 as we can find it in Fig. 7-4. Therefore areas with upper ranks than 70 are expanding, and the other small areas with lower ranks than 70 are shrinking, because 4.25≒log 70. This means that about three fourth of areas are expanding and the other one fourth are shrinking.

(change of the elasticity) As concerns the elasticity, the situation is a little different from category 1 and 2. We show Fig. 4-2 which corresponds to category 3 and is different from Fig. 4-1 corresponding to category 1 and 2. For all sample sizes, the elasticity decreases from 1975 to 95. That is to say, the difference of sizes (population) among areas have uniformly increased. We can think that the difference of city's size is apt to increase when we consider the city form point of economic activities.

(rank-size rule and constant elasticity) When we think of all samples from rank 5 to the bottom in

Fig. 8 The number of samples and the size elasticity of city's rank areas



1975 at category 3, the maximum of the elasticity is 1.191, which corresponds to sample size 60, and the minimum is 0.632, which corresponds to sample size 5. The difference is 0.559, so the constancy is not valid even for the weak criterion.

However, if we restrict these samples to the case with samples from rank 10 to the bottom, the minimum is 0.781, which corresponds to the sample size 10, while the maximum is the same value, i.e. 1.191, so the difference is 0.410. Therefore, the constancy is valid for the weak criterion, whereas not for the strong criterion.

In addition to this, the rank-size rule is also valid for the weak criterion, whereas not for the strong criterion, because all the elasticity is in the domain from 0.75 to 1.25, that is to say, in the domain of width 0.5 with the central value 1.0.

As showed in Table 1, for all samples which include upper ranks than 10 (smaller numerical values than 10), the constancy is invalid even for the weak criterion. On the other hand, for samples which exclude upper ranks than 10, the rank-size rule (of course, the constancy, too) is valid for the weak criterion, whereas not for the strong criterion.

Here, we should note two points. First, as concerns the constant elasticity, to be exact, "rank 10" stated just above is able to be altered to "rank 8". Because the constancy is valid for samples with sample size 8 and 9, too, from point of the weak criterion, as we can see in Appended Table 3. Second, as concerns the rank-size rule, to be exact, "rank 10" stated just above is able to be altered to "rank 9". Because the rule is valid for samples with sample size 9, too, from point of the weak criterion, as we can see in Appended Table 3.

6. CATEGORY 4 (PREFECTURES)

We know sufficiently that there is some question about dealing with prefectures as "cities". Generally speaking, prefectures are too large to regard as cities. However, we will deal with prefectures here, because they are very important local governments functioning really.

(relation between city's rank and city's size) Fig. 9-1 shows the whole feature of the relation between city's rank and city's size in category 4. From this figure, we can find out almost the same observation as the case of category 1, 2, and 3. The relation between city's rank and city's size in Fig. 9-1 shows almost the straight line (log-linear line). We can verify this by looking at the high value of coefficients of determination in Appended Table 4. However, the lines in Fig. 9-1 are curved a little compared with Fig. 2-1, Fig. 5-1 and Fig. 7-1. So the values of coefficient of determination in Appended Table 4 are slightly smaller than those in Appended Table 1, 2 and 3.

Fig. 9–1 City rank and its population for 1975–95 in Japan prefectures---1

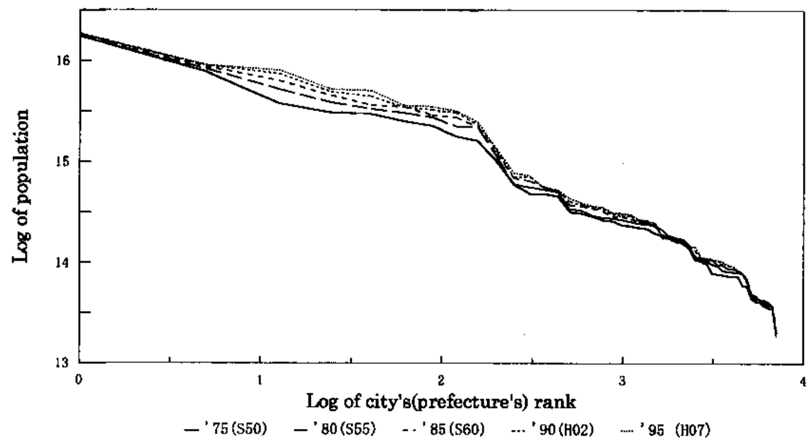


Fig. 9–2 City rank and its population for 1975–95 in Japan prefectures---2

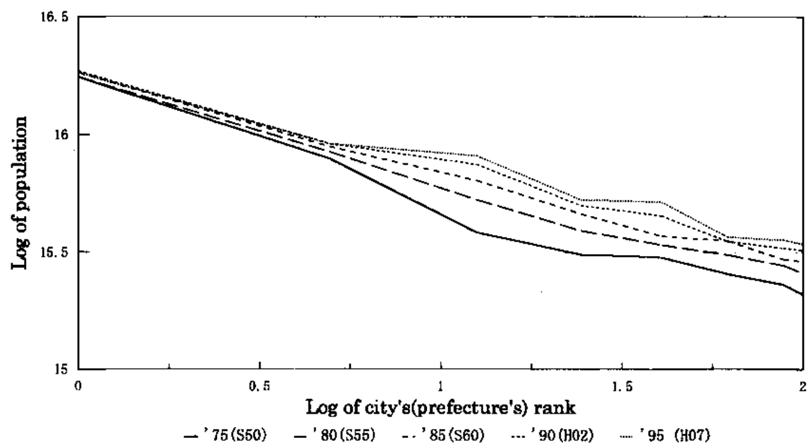


Fig. 9–3 City rank and its population for 1975–95 in Japan prefectures---3

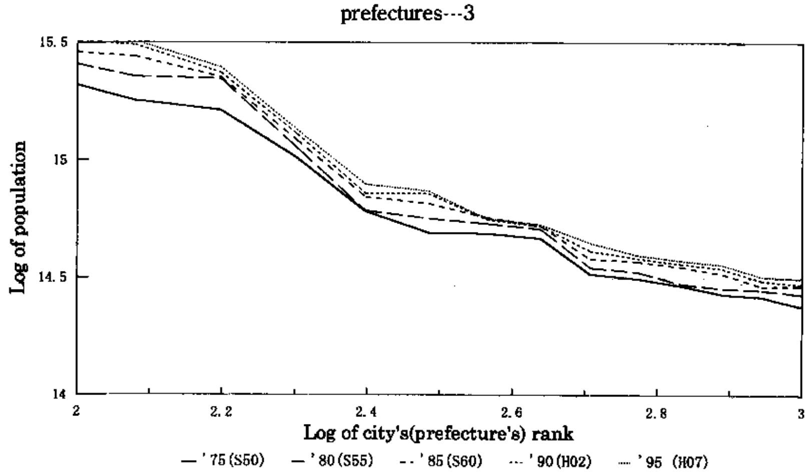


Fig. 9-4 City rank and its population for 1975-95 in Japan
prefectures---4

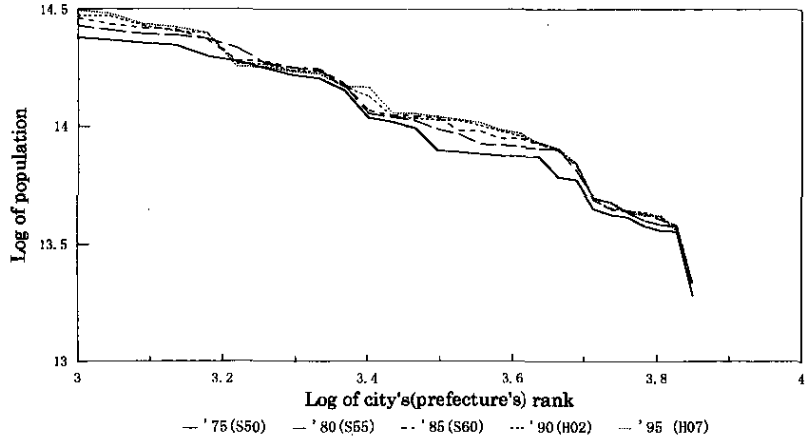


Fig. 10-1 *The number of samples and the size elasticity of city's rank*
prefectures---1

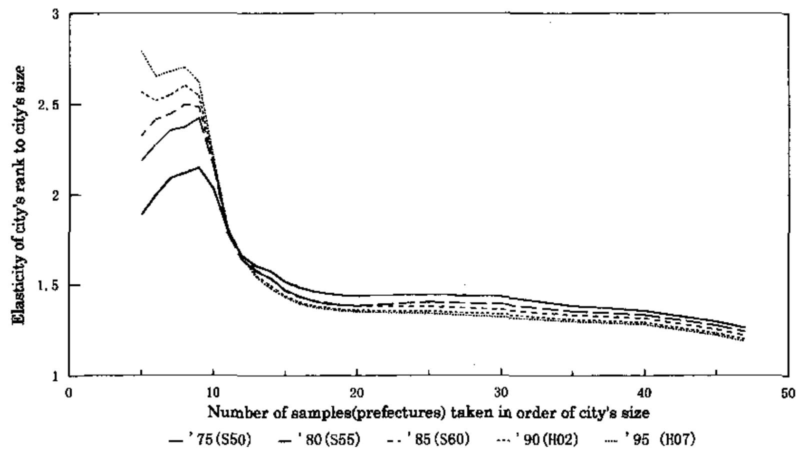


Fig. 10-2 The number of samples and the size elasticity of city's rank
prefectures---2

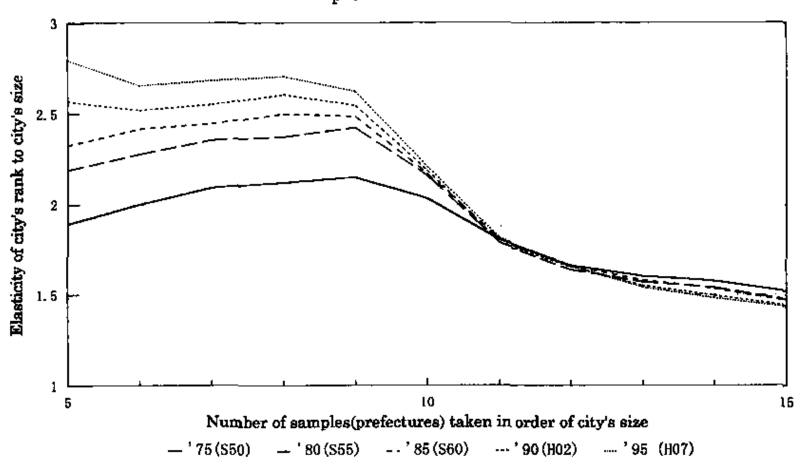
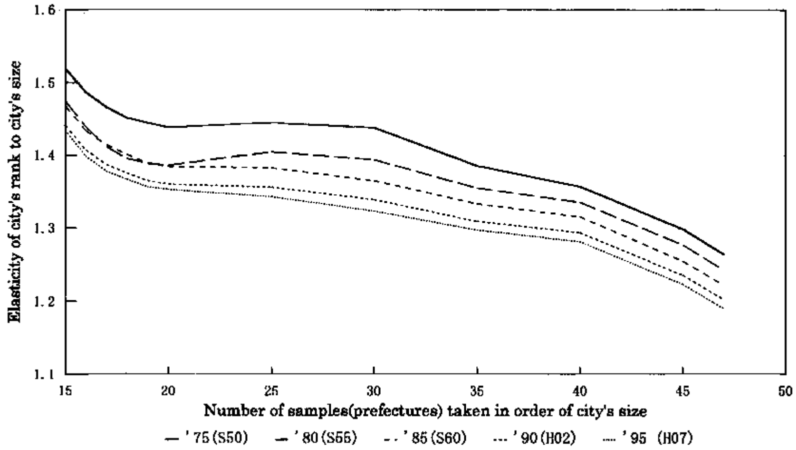


Fig. 10-3 The number of samples and the size elasticity of city's rank prefectures···3



(elasticity of city's rank to city's size) We can know the situation of the elasticity through Fig. 10-1 and Appended Table 4. Here we find some difference from category 1, 2 and 3. As the sample size increases from 5, the elasticity grows at first, and it reaches the peak at the neighbourhood of sample size 8, then declines very sharply, and for samples with more than 15, the values of elasticity hardly vary. In addition to this, all the values of elasticity are larger than 1.0, and they are considerably larger than those in category 1, 2 and 3.

(relation between the rank-size and the elasticity) As concerns the relation between the rank-size and the elasticity, the relation showed in Fig. 3-1 and Fig. 3-2 is basically valid in category 4, but the shape is fairly skew, as we can see in Fig. 10-1.

(change of the relation between city's rank and city's size) According to figures form Fig. 9-2 to Fig. 9-4, populations of prefectures have increased for almost all city's ranks during the period from 1975 to 95. The growth is faster for the domain from about 1.0 to 2.0 of log-value of city's rank. As $\log 3 = 1.1$ and $\log 8 = 2.1$, prefectures with high growth are ones with city's ranks from rank 3 to 8. They have big cities, i.e. ordinance-designated cities, which are poles of "poly-pole" in Japan.

(change of the elasticity) As concerns the elasticity, the situation of category 4 is the same as category 1 and 2, as we can see in Fig. 10-1. The type of change is one of Fig. 4-1. The critical value is 11 or 12 of the horizontal axis. The differences among prefectures with ranks of upper than 12 is shrinking, on the other hand, it is expanding among domain including more than 12 prefectures.

(rank-size rule and constant elasticity) At all cases in category 4, the constancy is invalid even for the weak criterion, because the values of maximum are too high, as we can see through Fig 10-1, Table 1 and Appended Table 4.

7. CONCLUSIONS

In Table 1, we will show conclusions of examination of the rank-size rule and the constant elasticity of city's rank to city's size in Japan from 1975 to 95. And we will add Table 2 which is the summary of Table 1. The main results are showed in SUMMARY from ① to ⑦ at the beginning of this paper.

Table 2 Summary of results

		the constant elasticity (a) of city's rank to city's size		the rank-size rule of cities	
criterion		strong	weak	strong	weak
		max. − min. of a 〈 0.2	max. − min. of a 〈 0.5	0.9 〈 all a 〈 1.1	0.75 〈 all a 〈 1.25
category 1	cities, towns, and villages	×	× (note 1)	×	×
category 2	cities	×	○ (note 2)	×	×
category 3	areas	×	△ (note 3)	×	△ (note 4)
category 4	prefectures	×	×	×	×

"×" means "to be invalid for all or almost cases examined".

"△" means "to be valid for almost cases examined".

"○" means "to be valid for all cases examined".

(note 1) The constant elasticity is valid for cases in 1975 and 1980 of samples 5-3000 of category 1.

(note 2) As concerns each year, the constant elasticity is valid for all cases.
But, as concerns whole period 1975-95, it is valid except for a case with rank 5 in 1975.
In fact, it is valid for samples from rank 6 to the bottom, as we can see through Appended Table 2.

(note 3) The constant elasticity is valid for all cases with samples from 10 to the bottom.
To be exact, "rank 10" is able to be altered to "rank 8", as we can see through Appended Table 3.

(note 4) The rule is valid for all cases with samples from rank 10 to the bottom.
To be exact, "rank 10" is able to be altered to "rank 9", as we can see through Appended Table 3.

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(1996.1.6.)

Appended Table 1 Relation between city's rank and city's size

(cities, towns and villages)

sample size	1975(S50)			1980(S55)			1985(S60)			1990(H02)			1995(H07)		
	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value
5	0.927	0.903	6.2	0.948	0.911	6.5	0.969	0.941	8.1	1.022	0.959	9.7	1.060	0.964	10.5
6	0.965	0.926	8.0	0.989	0.931	8.3	1.021	0.946	9.4	1.054	0.968	12.3	1.086	0.973	13.5
7	0.998	0.939	9.7	1.041	0.930	9.0	1.072	0.943	10.0	1.100	0.964	12.7	1.125	0.973	14.6
8	1.021	0.949	11.4	1.055	0.944	10.9	1.086	0.954	12.1	1.120	0.970	15.0	1.149	0.976	16.8
9	1.037	0.956	13.3	1.080	0.950	12.3	1.108	0.960	13.8	1.146	0.971	16.3	1.177	0.976	17.9
10	1.054	0.961	15.0	1.110	0.950	13.2	1.136	0.960	14.7	1.168	0.972	17.8	1.198	0.977	19.4
20	1.039	0.984	34.2	1.091	0.978	28.8	1.106	0.974	26.5	1.123	0.979	29.4	1.153	0.983	33.3
30	1.116	0.978	36.2	1.154	0.979	37.2	1.160	0.979	37.1	1.171	0.982	40.3	1.188	0.986	45.3
40	1.183	0.972	36.8	1.225	0.971	36.4	1.234	0.970	35.4	1.245	0.972	37.0	1.260	0.976	40.2
50	1.241	0.966	37.4	1.285	0.966	37.3	1.293	0.966	37.1	1.304	0.968	38.3	1.316	0.972	41.3
70	1.322	0.964	43.2	1.354	0.969	46.7	1.367	0.968	45.6	1.379	0.969	46.3	1.391	0.972	49.1
90	1.371	0.968	51.8	1.399	0.973	56.4	1.412	0.972	55.4	1.420	0.973	57.1	1.434	0.976	59.6
100	1.385	0.970	57.0	1.408	0.975	62.3	1.419	0.974	61.4	1.422	0.976	63.6	1.433	0.978	66.4
110	1.392	0.973	62.9	1.405	0.978	69.0	1.412	0.977	67.7	1.418	0.978	70.4	1.429	0.980	73.4
120	1.392	0.976	68.9	1.403	0.980	75.7	1.405	0.979	73.9	1.409	0.980	76.5	1.416	0.981	78.9
130	1.383	0.977	74.4	1.393	0.981	81.3	1.394	0.980	79.1	1.395	0.981	81.3	1.400	0.982	83.3
140	1.373	0.978	79.4	1.381	0.982	86.0	1.376	0.980	82.3	1.380	0.981	85.3	1.387	0.982	87.7
150	1.357	0.979	82.8	1.369	0.982	90.5	1.364	0.981	86.7	1.367	0.982	89.5	1.376	0.983	92.7
200	1.302	0.981	100.8	1.318	0.984	109.9	1.319	0.983	108.2	1.327	0.985	113.0	1.335	0.985	115.7
250	1.264	0.982	117.4	1.278	0.984	125.3	1.282	0.984	125.5	1.292	0.986	130.7	1.304	0.987	136.5
300	1.235	0.983	132.9	1.244	0.985	138.0	1.250	0.985	138.7	1.260	0.986	144.1	1.275	0.987	151.3
400	1.214	0.987	173.8	1.219	0.987	177.4	1.216	0.987	173.6	1.220	0.987	174.6	1.231	0.988	179.4
500	1.200	0.989	213.3	1.198	0.989	212.2	1.195	0.989	207.5	1.195	0.988	205.6	1.202	0.988	206.2
600	1.185	0.990	248.0	1.178	0.990	240.7	1.172	0.989	232.0	1.168	0.988	226.2	1.172	0.988	221.3
700	1.167	0.991	271.6	1.159	0.990	263.7	1.151	0.989	251.9	1.147	0.988	245.1	1.148	0.988	238.0
1000	1.115	0.990	309.2	1.107	0.989	301.7	1.098	0.988	290.8	1.091	0.988	281.8	1.087	0.986	268.4
1500	1.070	0.991	395.4	1.055	0.989	369.1	1.042	0.988	351.3	1.029	0.986	330.2	1.018	0.985	309.1
2000	1.046	0.992	484.7	1.023	0.990	436.4	1.006	0.988	406.8	0.988	0.986	377.6	0.973	0.984	347.6
2300	1.032	0.992	522.2	1.005	0.989	460.8	0.986	0.988	427.3	0.967	0.986	396.0	0.948	0.983	361.7
2500	1.021	0.991	534.9	0.990	0.988	461.8	0.970	0.986	426.0	0.950	0.984	394.8	0.930	0.981	361.0
2800	0.998	0.989	499.6	0.964	0.985	433.8	0.941	0.983	400.8	0.919	0.980	372.2	0.897	0.976	340.2
3000	0.968	0.982	408.9	0.932	0.978	364.8	0.908	0.975	341.8	0.885	0.972	320.1	0.861	0.967	295.5
3100	0.941	0.974	338.0	0.905	0.968	308.5	0.881	0.965	293.0	0.858	0.962	279.0	0.833	0.956	260.6
3200	0.898	0.955	259.2	0.861	0.949	242.8	0.839	0.945	235.3	0.817	0.942	227.6	0.790	0.935	214.3
all	0.853	0.926	202.2	0.818	0.920	193.6	0.797	0.918	190.7	0.776	0.915	186.7	0.758	0.913	183.8

city's size (S): the number of population in cities, towns and villages

city's rank (R): the ranking of the city's size

regression equation: $Y=-aX+b$, where $X=\log S$, $Y=\log R$, $b=\log A$, $(R=AS^{-a})$

elas. (a): the elasticity of city's rank to city's size: Pareto exponent

: the absolute value of the coefficient of X in the regression equation

deter.: the coefficient of determinnation adjusted for the degree of freedom

sample size: the number of samples (cities, towns and villages) taken in order of city's rank

all: all samples: all of cities, towns and villages, including Tokyo-special-ward

: 3244 samples in 1975 and '80, 3245 samples in '85 and '90, and 3235 samples in '95

Appended Table 2 Relation between city's rank and city's size

(cities)

sample	1975 (S50)			1980 (S55)			1985 (S60)			1990 (H02)			1995 (H07)		
size	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value
5															
6	Contents in this place are the same as Appended Table 1.														
...															
150															
200	1.302	0.981	100.8	1.311	0.983	108.1	1.319	0.983	108.2	1.260	0.986	144.1	1.335	0.985	115.7
300	1.235	0.983	132.9	1.239	0.984	137.5	1.250	0.985	138.7	1.260	0.986	144.1	1.275	0.987	151.3
350	1.222	0.985	152.9	1.226	0.986	157.7	1.230	0.986	155.5	1.235	0.986	157.3	1.248	0.987	163.0
400	1.214	0.987	173.8	1.216	0.988	178.0	1.216	0.987	173.6	1.219	0.987	174.2	1.231	0.988	179.3
450	1.207	0.988	194.3	1.206	0.989	197.3	1.207	0.988	192.4	1.205	0.988	189.4	1.216	0.988	193.9
500	1.200	0.989	213.3	1.195	0.989	212.9	1.191	0.988	203.1	1.185	0.987	195.0	1.194	0.987	196.0
550	1.192	0.990	231.3	1.182	0.989	224.3	1.173	0.987	207.8	1.161	0.985	193.2	1.167	0.985	189.1
600	1.183	0.990	245.6	1.166	0.989	227.7	1.151	0.986	205.0	1.135	0.983	185.9	1.137	0.982	178.7
610	1.181	0.990	247.7	1.162	0.988	227.4	1.147	0.986	203.6	1.128	0.982	182.8	1.130	0.980	174.4
620	1.178	0.990	248.7	1.158	0.988	225.9	1.141	0.985	200.8	1.122	0.981	179.2	1.122	0.979	170.3
630	1.174	0.990	247.1	1.152	0.987	221.4	1.135	0.984	196.7	1.114	0.980	174.5	1.113	0.978	165.4
640	1.170	0.989	243.0	1.146	0.986	214.0	1.127	0.983	190.2	1.105	0.978	168.7	1.104	0.976	159.9
650	1.163	0.988	234.6	1.137	0.984	202.8	1.117	0.980	180.1	1.093	0.975	159.1	1.092	0.973	152.8
*655	1.154	0.985	206.7	1.120	0.977	168.2	1.104	0.975	161.0	1.084	0.972	150.7	1.085	0.971	148.0
660													1.077	0.969	142.6
*664													1.063	0.962	129.6

city's size (S): the number of population in a city

city's rank (R): the ranking of the city's size

regression equation: $Y=-aX+b$, where $X=\log S$, $Y=\log R$, $b=\log A$, $(R=AS^{-a})$

elas. (a): the elasticity of city's rank to city's size: Pareto exponent

: the absolute value of the coefficient of X in the regression equation

deter.: the coefficient of determination adjusted for the degree of freedom

sample size: the number of samples (cities) taken in order of city's rank

*: all samples: all of cities, including Tokyo-special-ward

: 655 samples in 1975, '80 and '90, 656 samples in '85, and 664 samples in '95

Appended Table 3 Relation between city's rank and city's size

(areas)

sample size	1975 (S50)			1980 (S55)			1985 (S60)			1990 (H02)			1995 (H07)		
	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-valuee
5	0.632	0.970	11.4	0.637	0.973	12.0	0.635	0.973	12.1	0.629	0.971	11.6	0.620	0.976	12.7
6	0.668	0.963	11.4	0.670	0.967	12.2	0.668	0.968	12.3	0.663	0.965	11.8	0.657	0.964	11.6
7	0.699	0.958	11.7	0.699	0.964	12.6	0.697	0.965	12.8	0.693	0.961	12.2	0.690	0.955	11.3
8	0.726	0.956	12.4	0.729	0.957	12.4	0.729	0.954	12.0	0.725	0.951	11.6	0.722	0.946	11.1
9	0.754	0.948	12.1	0.758	0.948	12.1	0.758	0.945	11.8	0.755	0.942	11.4	0.752	0.935	10.8
10	0.781	0.941	12.1	0.783	0.944	12.4	0.781	0.944	12.3	0.777	0.942	12.1	0.774	0.937	11.6
20	0.949	0.934	16.4	0.946	0.937	16.9	0.941	0.938	16.9	0.934	0.935	16.6	0.931	0.932	16.2
30	1.072	0.917	17.9	1.065	0.923	18.6	1.057	0.926	19.0	1.045	0.928	19.4	1.040	0.928	19.4
40	1.132	0.932	23.2	1.120	0.938	24.2	1.111	0.940	24.8	1.100	0.941	25.0	1.098	0.940	24.8
50	1.166	0.945	29.0	1.158	0.948	29.9	1.149	0.949	30.3	1.137	0.950	30.6	1.136	0.950	30.4
55	1.181	0.949	31.7	1.172	0.952	32.7	1.163	0.953	33.2	1.150	0.954	33.6	1.147	0.954	33.5
60	1.191	0.953	34.7	1.182	0.956	35.7	1.172	0.957	36.2	1.158	0.958	36.7	1.156	0.958	36.6
65	1.190	0.957	37.8	1.179	0.959	38.9	1.168	0.961	39.5	1.154	0.962	40.0	1.154	0.961	39.9
70	1.181	0.960	40.8	1.168	0.962	41.8	1.156	0.963	42.3	1.141	0.964	42.7	1.142	0.963	42.7
80	1.164	0.965	46.6	1.148	0.966	47.3	1.133	0.966	47.6	1.115	0.966	47.6	1.113	0.966	47.3
90	1.133	0.965	49.8	1.116	0.966	50.3	1.102	0.966	50.6	1.085	0.967	50.9	1.080	0.966	50.1
100	1.088	0.961	49.5	1.070	0.961	49.5	1.052	0.960	49.0	1.033	0.959	48.4	1.021	0.956	46.4
110	0.988	0.934	39.2	0.971	0.933	39.1	0.955	0.932	38.8	0.937	0.932	38.5	0.923	0.927	37.1

city's size (S): the number of population in an area

city's rank (R): the ranking of the area's size

regression equation: $Y=-aX+b$, where $X=\log S$, $Y=\log R$, $b=\log A$, ($R=AS^{-a}$)

elas. (a): the elasticity of city's rank to city's size: Pareto exponent

: the absolute value of the coefficient of X in the regression equation

deter.: the coefficient of determination adjusted for the degree of freedom

sample size: the number of samples (areas) taken in order of city's rank

Appended Table 4 Relation between city's rank and city's size

(prefectures)

sample size	1975(S50)			1980(S55)			1985(S60)			1990(H02)			1995(H07)		
	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value	elas. (a)	deter.	t-value
5	1.894	0.958	9.6	2.190	0.996	30.4	2.328	0.998	42.5	2.569	0.981	14.3	2.796	0.955	9.3
6	2.002	0.954	10.2	2.280	0.989	21.4	2.418	0.992	24.5	2.521	0.986	18.5	2.656	0.963	11.4
7	2.094	0.952	11.0	2.358	0.985	20.0	2.448	0.993	29.3	2.554	0.988	22.7	2.682	0.972	14.4
8	2.122	0.961	13.2	2.376	0.988	24.1	2.499	0.992	29.2	2.606	0.988	24.4	2.707	0.977	17.3
9	2.152	0.967	15.4	2.426	0.987	24.5	2.487	0.993	34.2	2.547	0.988	25.7	2.624	0.977	18.5
10	2.035	0.960	14.8	2.159	0.951	13.2	2.171	0.946	12.6	2.188	0.933	11.3	2.211	0.915	9.9
11	1.805	0.925	11.1	1.787	0.881	8.7	1.808	0.881	8.7	1.789	0.862	8.0	1.813	0.849	7.6
12	1.663	0.911	10.6	1.640	0.873	8.7	1.666	0.874	8.8	1.658	0.860	8.3	1.663	0.845	7.8
13	1.605	0.915	11.4	1.573	0.879	9.4	1.582	0.877	9.3	1.553	0.860	8.6	1.543	0.844	8.1
14	1.576	0.921	12.4	1.539	0.889	10.2	1.534	0.885	10.0	1.497	0.868	9.3	1.484	0.853	8.7
15	1.519	0.921	12.8	1.474	0.890	10.7	1.468	0.886	10.5	1.441	0.873	9.9	1.434	0.861	9.4
16	1.487	0.925	13.6	1.439	0.896	11.4	1.434	0.892	11.2	1.407	0.881	10.6	1.399	0.869	10.0
17	1.467	0.929	14.5	1.412	0.902	12.2	1.414	0.900	12.0	1.387	0.889	11.4	1.378	0.878	10.8
18	1.452	0.934	15.5	1.396	0.908	13.0	1.402	0.907	12.9	1.376	0.897	12.2	1.368	0.887	11.6
19	1.445	0.938	16.6	1.389	0.915	13.9	1.389	0.913	13.7	1.366	0.904	13.0	1.358	0.895	12.4
20	1.439	0.942	17.6	1.387	0.920	14.8	1.385	0.918	14.7	1.361	0.910	13.9	1.354	0.902	13.2
25	1.445	0.957	23.1	1.405	0.940	19.4	1.383	0.939	19.3	1.357	0.933	18.2	1.344	0.926	17.4
30	1.438	0.965	28.3	1.395	0.951	23.8	1.366	0.951	23.6	1.339	0.946	22.6	1.324	0.941	21.5
35	1.386	0.966	31.2	1.356	0.957	27.6	1.334	0.957	27.6	1.311	0.954	26.4	1.299	0.950	25.3
40	1.359	0.969	35.1	1.337	0.963	31.7	1.317	0.963	31.8	1.295	0.960	30.5	1.283	0.956	29.3
45	1.300	0.965	35.1	1.278	0.959	32.2	1.256	0.959	32.0	1.236	0.957	31.1	1.224	0.954	30.1
47	1.265	0.959	32.9	1.243	0.953	30.7	1.222	0.953	30.7	1.203	0.951	30.0	1.190	0.948	29.1

city's size (S): the number of population in a prefecture

city's rank (R): the ranking of the city's size

regression equation: $Y=-aX+b$, where $X=\log S$, $Y=\log R$, $b=\log A$, $(R=AS^{-a})$

elas. (a): the elasticity of city's rank to city's size: Pareto exponent

: the absolute value of the coefficient of X in the regression equation

deter.: the coefficient of determination adjusted for the degree of freedom

sample size: the number of samples (prefectures) taken in order of city's rank