Yet another representation of SO(3) by spherical functions for pose estimation

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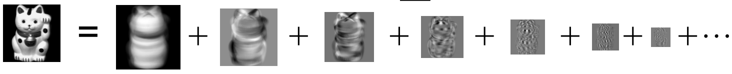
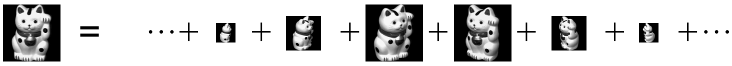
1DOF examples of pose representation

Approx. by Eigenimages $\boldsymbol{x}=\sum c_i \boldsymbol{e}_i$



Approx. by training images $\boldsymbol{x}=\sum b_i \boldsymbol{x}_i$



Training methods $\boldsymbol{p}_i = F \boldsymbol{x}_i$ Linear Regression EbC CCA Manifold learning

Pose should be Continuous!

$$\boldsymbol{p}=F\boldsymbol{x}=\sum b_i F \boldsymbol{x}_i=\sum b_i \boldsymbol{p}_i$$

Estimate of pose can be approx. by poses of training images.

Pose should be Bijective! (one-to-one)

representation	training	
angles	$10=F\boldsymbol{x}_{10},\dots$	✗ 180° ≃ ⋯+ 330° + 340° + 350° + 10° + 20° + 30° +⋯ Discontinuous at 360°: NG
sin,cos	$\begin{pmatrix}\sin(10^\circ)\\ \cos(10^\circ)\end{pmatrix}=F\boldsymbol{x}_{10},\dots$	○ sin(0°) cos(0°) ≃ ⋯+ sin(330°) + sin(340°) + sin(350°) + sin(10°) + sin(20°) + sin(30°) +⋯ ; ⋯+ cos(330°) + cos(340°) + cos(350°) + cos(10°) + cos(20°) + cos(30°) +⋯ Continuous : GOOD!

$\boldsymbol{p}_1 \neq \boldsymbol{p}_2$

$\boldsymbol{p}_1 = F\boldsymbol{x}_i$

$\boldsymbol{p}_2 = F\boldsymbol{x}_i$

Impossible! To satisfy Both equs.

3DOF Pose Representations: comparison

	Fixed angles (roll-pitch-yaw)	Euler angles	Angle-axis (Exponential map)	Unit quaternions	Rotation Matrix	Spherical representation
Continuity	✗	✗	⚠	✓	✓	✓
Bijection (one-to-one)	✗	✗	✗	✗	✓	✓
Higher Freq.	✗	✗	✗	✗	✗	✓
	Due to gimbal lock		Rotation angles × (sin,cos) ○	q and -q are the same pose		Proposed method

Rotation matrix and Spherical representation Are suit for pose representation

can be used to expanding an image as a function on SO(3)!

$$\boldsymbol{x}=\sum c_{l_1 l_2 l_3} Y_{l_1 l_2 l_3}(\theta_1', \theta_2', \theta_3')$$

A spherical function representation of SO(3)

$$Y_{l_1 l_2 l_3}(\theta_1,\theta_2,\theta_3)=\sqrt{b_{l_1 l_2 l_3}}\, C_{l_1}^{1,l_2}(\cos\theta_1)\, P_{l_2}^{l_3}(\cos\theta_2)\, e^{-l_3\theta_3 i}$$

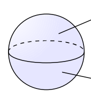
$l_1 \in 2\mathbb{Z}$, $l_2, l_3 \in \mathbb{Z}$, $l_1 \geq l_2 \geq |l_3|$, $0 \leq \theta_1 \leq \frac{\pi}{2}$, $0 \leq \theta_2 \leq \pi$, $0 \leq \theta_3 \leq 2\pi$

Polar coordinates of a point on S^3 — Associated Gegenbauer function — associated Legendre function — Complex numbers on the unit circle

Spherical functions: Orthonormal basis on S^n

- n=1: complex Fourier basis
- n=2: spherical harmonics
- n=3: spherical functions on S^3

Even functions of spherical functions on S^3: Spherical functions on SO(3)



The lowest Frequency ($2l_1=2$)

$Y_{2,0,0}$

$Y_{2,1,0}, Y_{2,1,\pm1}$

$Y_{2,2,0}, Y_{2,2,\pm1}$

$Y_{2,2,\pm2}$

Real num. Omit coeff. One-to-one

Higher Freq. ($2l_1>2$)

Correspond to $R^2, R^3, \dots$ Many-to-one

Unit quaternions q_1, q_2, q_3, q_4

Two-to-one Two-to-one

$$R=\begin{pmatrix} q_1^2+q_2^2-q_3^2-q_4^2 & 2q_2q_3-2q_1q_4 & 2q_2q_4+2q_1q_3 \\ 2q_2q_3+2q_1q_4 & q_1^2-q_2^2+q_3^2-q_4^2 & 2q_3q_4-2q_1q_2 \\ 2q_2q_4-2q_1q_3 & 2q_3q_4+2q_1q_2 & q_1^2-q_2^2-q_3^2+q_4^2 \end{pmatrix}$$

Spherical representation

Y_0, Y_1, Y_2, Y_3

Y_4, Y_5, Y_6, Y_7, Y_8

One-to-one

Rotation matrix

$$R=\begin{pmatrix} \frac{Y_0+2Y_4}{3} & 2Y_5-2Y_3 & 2Y_6+2Y_2 \\ 2Y_5+2Y_3 & \frac{Y_0-Y_4+3Y_7}{3} & 2Y_8-2Y_1 \\ 2Y_6-2Y_2 & 2Y_8+2Y_1 & \frac{Y_0-Y_4-3Y_7}{3} \end{pmatrix}$$

Applications:

Analysis of a function f(R) of a rotation matrix

$f(R)=|x'-(Rx+t)|^2$

Expansion of an image x(R) of a rotation matrix

$x(R)=\sum c\, Y(R)$