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Agent-based Simulation Analysis for Network Formation

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***Abstract*—In this paper, we conduct agent-based simulation experiments for network formation analysis. In the published papers, Bala and Goyal (2000) have constructed a mathematical model leading a star network to be strict Nash equilibrium. However, Berninghaus et al. (2007) have conducted the laboratory experiments using human subjects basing on the mathematical model, and the result of the experiments indicates that human subjects do not always make decision just as the mathematical model predicted. In this paper, we propose a simulation model using the adaptive artificial agents to clarify the reason of the deviation from the mathematical predictions.**

I. Introduction

In the network models, each decision maker such as individual, firm, or country in the real world is represented as player and the network indicates the relation between the decision makers. The network models are mathematically defined using the graph model, such that player and a link formed between a pair of players are represented by a node and an edge, respectively. In recent years, a number of mathematical models of network formation focusing on relation between the stability and the efficiency of the networks are reported. In these mathematical models, it is tried to explain some of social phenomena, i.e., stock markets, labor market, and collective actions [1], [2], [8].

Jackson and Wolinsky [8] constructed a mathematical model of network formation such that a link between two players is formed if both of them agree it. Jackson and Wolinsky showed that the model leads the complete, the empty, and the star network to be stable. In the complete network, there exists a link between each pair of players whereas there exists no link in the empty network. In the star network, particular one player, called central player, forms link with all other players, called peripheral players, and there exists no link between any pair of peripheral players. Bala and Goyal [1] constructed two mathematical models of network formation named “one-way flow model” and “two-way flow model”. In both of the models, it is assumed that a link is formed without agreement of both corresponding players, in other words, a link is formed if one player proposes formation of the link. And they indicated that both mathematical models lead a wheel network to be stable. They indicated that the one-way flow model, which only the player who proposes the formation of a link receives utility through the corresponding link, leads a wheel network and the empty network to be stable. They showed that the two-way flow model, which the two players corresponding to one link receive utilities through the link, leads a center-sponsored star network and the empty network to be stable. Here, Bala and Goyal defined center-sponsored and periphery-sponsored star networks. A center-sponsored network is one of star networks such that all formed links are proposed by the central player, and periphery-sponsored star network is one of star networks such that each formed link between each peripheral player and the central player is proposed by each corresponding peripheral player. Additionally, Bala and Goyal indicated that strict Nash equilibrium is stable.

Some studies of laboratory experiments using human subjects to verify the above mentioned mathematical models of network formation are reported. Callander and Plott [3] conducted the laboratory experiments basing on the one-way flow model which is proposed by Bala and Goyal [1], they show that the human subjects formed wheel networks which are predicted strict Nash equilibrium, and the networks are stable. Falk and Kosfeld [5] conducted the laboratory experiments using human subjects to verify the mathematical model of Bala and Goyal. As the results of their experiments, in one-way flow model, wheel networks are formed as stable networks as the prediction of the mathematical model, however, in the two-way flow model, no strict Nash equilibrium, center-sponsored star network, is not formed. Berninghaus et al. [2] suggested a little simpler mathematical model than that of Bala and Goyal. The mathematical model by Berninghaus et al. supposed that each player obtains payoff from other players forming a path shorter than a particular length with him. The prediction of the mathematical model is that a periphery-sponsored star network is the strict Nash equilibrium. Additionally, Berninghaus et al. examined the laboratory experiments using human subjects, the subjects are divided into several groups. As the result of the experiments, in some groups, though periphery-sponsored star networks are formed, some of the subjects in each group deviate from the equilibrium, through some changes of the network structure, and again, a periphery-sponsored star network is formed with central player different from the earlier star network.

In this paper, we focus on the result of the laboratory experiments of Berninghaus et al. [2], and by using an agent-based simulation model, we indicate the reason of that the human subjects deviate from the strict Nash equilibrium in the experiments. As the result, one of the reasons is that human can not always make decision to maximize their utility, but often make decision through trial and error. In this paper, we employ a neural networks as the mechanism of decision making and the genetic algorithms for learning mechanism of the agents.

The rest of this paper is organized as follows: In Section 2 we describe the mathematical model and the laboratory experiments by Berninghaus et al. [2]. In Section 3, we construct an agent-based simulation model for network formation analysis, and in Section 4, we show and analyze the results of the simulation experiments. Finally, we conclude in Section 5.

II. Mathematical Models and Laboratory Experiments of Network Formation

We describe the mathematical model and the laboratory experiments by Berninghaus et al. [2].

A. Mathematical model of network formation

Let $N = \{1, 2, ..., n\}$ be a set of players and $\overrightarrow{ij}$ be a link which is formed by a proposal of player $i \in N$ and acceptance of player $j \in N$, and ij represent $\overrightarrow{ij}$ or $\overrightarrow{ji}$. Here, $\overrightarrow{ij}$ is called an *active link* of player i and a *passive link* of player j. Let L be a set of links, here, a network is defined as a set of all players and a set of links, $g = (N, L)$.

For a set of links $L^c = \{\overrightarrow{ij} \mid \forall i, j \in N, i \neq j\}$, a network $g^c = (N, L^c)$ is called a *complete network*, and $g^\phi = (N, \phi)$ is called an *empty network*. A complete network and an empty network with six players are shown in Fig. 1.

Fig. 1. A complete network and an empty network

For player $i \in N$ and a set of links $L^s = \{ij \mid \forall j \in N \setminus \{i\}\}$, $g^s = (N, L^s)$ is called a *star network*. For player i and a set of links $L^w = \{i_2 i_1, \ldots, i_n i_{n-1}, \ldots, i_1 i_n\}$, $g^w = (N, L^w)$ is called a *wheel network*. Especially, for player i and a set of links $L^s_c = \{\overrightarrow{ij} \mid \forall j \in N \setminus \{i\}\}$, $g^s_c = (N, L^s_c)$, a network $g = (N, L^s_c)$ is called a *center-sponsored star network*. For a set of links $L^s_p = \{\overrightarrow{ji} \mid \forall j \in N \setminus \{i\}\}$, $g^s_p = (N, L^s_p)$, a network $g = (N, L^s_p)$ is called a *periphery-sponsored star network*. A center-sponsored star network and a periphery-sponsored star network are shown in Fig. 2.

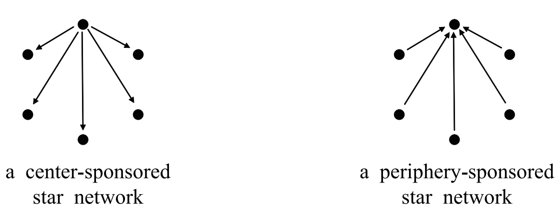

Fig. 2. Star networks

For player $i \in N$, a function Π_i which associates a real value for a network $g = (N, L)$ is called a *utility function*. Let $L_{-i} = \{\overrightarrow{jk} \mid \overrightarrow{jk} \in L, j, k \in N, j \neq i\}$ be a set of links which all links which are formed by player i in network $g = (N, L)$ are deleted. The set of links L^*_i is *best response* of player i for L^*_{-i} if and only if

$$\Pi_i(g^* = (N, L^*_i)) \geq \Pi_i(g_{-i} = (N, L_i \cup L^*_{-i})), \forall i \in N \quad (1)$$

holds. Here, let $BR_i(g_{-i})$ be the set of best responses of player i for network g_{-i}.

Definition A network $g = (N, L)$ is a *Nash equilibrium network* if $g_i \in BR_i(g_{-i})$ for all $i \in N$, i.e., all players are playing a Nash equilibrium in a Nash equilibrium network. A *strict* Nash equilibrium network is one where each player gets a strictly higher payoff with his current strategy than he would with any other strategy.

Let $P(i)$ be the number of passive links of player i, and we focus on networks in which there is a unique player i^* who forms the largest number of active links in all players. In other words, $P(i^*) > P(j)$ holds.

Let $d(g; g^s_p)$ a *distance measure* [2] of network g for a periphery-sponsored star network. It indicates a constructive distinction between a network $g = (N, L)$ and a periphery-sponsored star network $g^s_p = (N, L^s_p)$. $d(g; g^s_p)$ is calculated as following equation.

$$d(g; g^s_p) = \begin{cases} |P(i^*) - \max_{j \neq i^*}\{P(j)\} - (n-1)| \\ \qquad : \text{if } \exists i^* \text{ and } P(i^*) > \frac{n-1}{2}, \\ n-1 : \text{otherwise.} \end{cases} \quad (2)$$

If the difference between the maximum and the second highest number of passive links is small, then it indicates that a network g is far from a periphery-sponsored star network. If $(g; g^s_p) = 0$, it indicates that the network g induces a periphery-sponsored star network.

In a network g, let the neighbors of player i denote as follows:

1) *active neighbors* of i:
$N^a_i(g) = \{j \mid \overrightarrow{ij} \in L, j \in N, j \neq i\}$

2) *passive neighbors* of i:
$N^p_i(g) = \{j \mid \overrightarrow{ji} \in L, j \in N, j \neq i\}$

3) *indirect neighbors* of i:
$N^{ind}_i(g) = \{k \mid \overrightarrow{ij} \in L \text{ and } jk \in L, j \in N, j \neq i\}$

Let $N_i(g)$ denote the set of neighbors of player i.

$$N_i(g) := N^a_i(g) \cup N^p_i(g) \cup N^{ind}_i(g) \quad (3)$$

Here, let $|N_i(g)|$ and $|N^a_i(g)|$ be the cardinality of $N_i(g)$ and $N^a_i(g)$, respectively. All links are supposed to be symmetry for all available link cost, and represented as $c\ (> 0)$. The amount of benefit which player i obtains from one of neighbors is $a\ (> 0)$. The player's utility is represented as follows:

$$\Pi_i(g) := a\ |N_i(g)| - c\ |N^a_i(g)| \quad (4)$$

Berninghaus et al. gave sufficient conditions for a periphery-sponsored star for strict Nash equilibrium network.

Proposition 1 (Berninghaus et al.) If inequalities $c < (n-1)a$ and $n > 3$ hold, a periphery-sponsored star network is a strict Nash equilibrium network.

B. Laboratory experiments

Berninghaus et al. [2] conducted two kinds of laboratory experiments using human subjects. These experiments are called *discrete time* experiments and *continuous time* experiments, respectively. In the discrete time experiments, the subjects make decisions about network formation at each sequential periods. However, in the continuous time experiments, they can change their strategy at convenient moments for each subject. In both kinds of experiments, the subjects select a strategy whether to form, to delete a link between another subject, or do nothing. They can form or delete a link unilaterally.

In discrete time experiments, the human subjects are divided into ten groups and each group consists of six human subjects. At each period, a human subject chooses other human subjects whom he wants to form active link, and each human subject makes decisions once, the network structure is modified. In this network, human subjects obtain information about their payoff a, link cost c, their current utilities and the current network structure. The initial networks of all groups are the complete networks, and let values of parameters be $(a,c) = (3,2)$. Then, at the first period, utility of each human subject is 5.

In continuous time experiments, the human subjects are divided into eight groups. They form or delete their links at any time, and the human subjects receive the information of networks five times per second. The current utility is computed every fifth of a second and informed human subjects with their information about their payoff, link cost and the current network structure. Let the values of parameters of payoff and link cost be $(a,c) = (3,2)$. For example, if there are two players and a link is formed between them, then the utility per minute of player who forms an active link is $a-c = 1$ and the utility per minute of another player is $a = 3$. In the laboratory experiments by Berninghaus et al., utility of each human subject is accumulated over 30 min and paid out after each experiment is finished.

The experimental results are summarized as follows: In both kinds of experiments, in almost of groups periphery-sponsored star networks are formed such as predicted by the mathematical model of Bala and Goyal. In discrete time experiments, some of the groups which form a strict Nash equilibrium network deviate from it after some periods, and the third, some groups deviate from the strict Nash equilibrium network to form periphery-sponsored star networks with a different central player in continuous time experiments. We briefly summarize the characteristic features of experimental results in the discrete time and in the continuous time by Berninghaus et al. as follows.

Result of the experiments (discrete time) In 3 groups, strict Nash equilibrium networks, periphery-sponsored star networks, are formed. In two groups of the three groups in which strict Nash equilibrium networks are formed, some of the human subjects deviate from the strict Nash equilibrium networks, and in one group, the human subjects does not deviate from it.

Result of the experiments (continuous time) In 7 groups, at least one strict Nash equilibrium network, periphery-sponsored star network, is formed. In all groups except one group in which strict Nash equilibrium networks are formed, some of the human subjects deviate from the strict Nash equilibrium networks and form the other periphery-sponsored star networks with the different central players.

III. Agent-based Simulation Model

In this paper, we propose a simulation model (A) corresponds to the discrete time experiments by Berninghaus et al. and a simulation model (B) corresponds to the continuous time experiments. Multiple agents who are artificial and adaptive, as alternatives to players, make decisions based on neural networks composed of three layers, i.e., input layers, the hidden layers and output layers [7]. Then, player i accept the decision making of the agent which is selected with roulette selection in agents $A_i = \{i_1, i_2, \ldots, i_m\}, i = 1, 2, \ldots, n$ correspond to i. Therefore, player basically accept the decision making of the agent which has the maximum value of the fitness, while he accept the decision making of the other agent. Each agent $i_k, k = 1, 2, \ldots, m$ has the weights and thresholds of neural networks as the gene information, and they learn the neural networks by genetic algorithms [4], [6], [9].

The outline of simulation model is as follows:

- **Step 1** Generate a set of agents who have the weights(w_1, w_2) and thresholds(θ_1, θ_2) of neural network as the gene information for one player, and give $w_1, w_2, \theta_1, \theta_2$ the real value in $[-1, 1]$ randomly.
- **Step 2** $i = 1$.
- **Step 3** Player i makes a decision based on the agents correspond to i.
- **Step 4** Agent i_k makes a decision based on neural networks.
- **Step 5** Apply genetic algorithms to gene, and repeat reproduction by roulette selection, one-point crossover, mutation and elitist preserving selection until the final generation.
- **Step 6** If $i < n$, then let $i := i+1$ and return to Step 3. Otherwise, go to Step 7.
- **Step 7** Repeat from Step 2 to Step 6 prescribed times.

Let r be discount rate for the past payoffs of each player, and $g(t)$ be network at t-th period. Then the value of the fitness of the agent i_k at t-th period is calculated as follows by using the payoffs which player i obtains during previous P_f terms. Note that 1 term corresponds to $t = n$ periods.

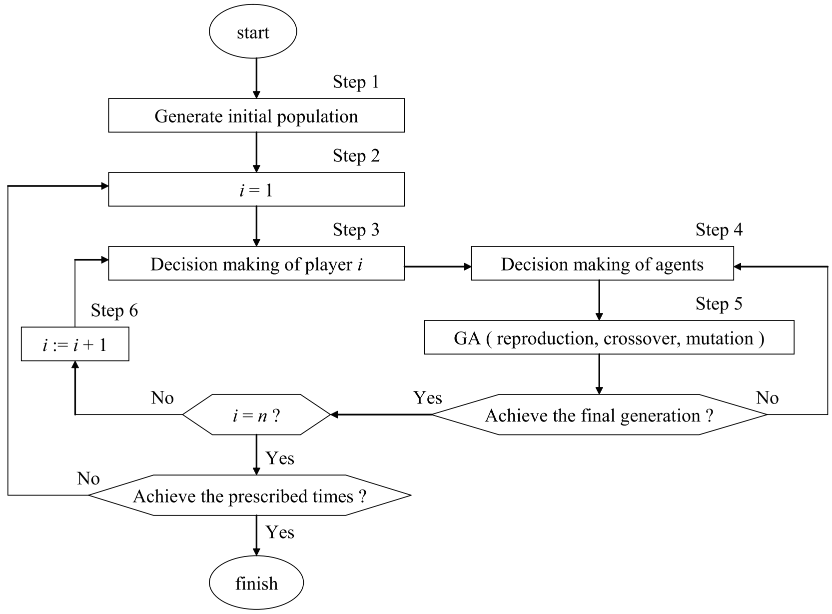

Fig. 3. The outline of simulation model

$$f_{i_k}(g(t)) = \sum_{\tau=0}^{P_f \times n} r^{\tau} \Pi_i(g(t+1-\tau)), \tag{5}$$

At one term, n players make decisions in the given random set order, and m agents correspond to each player make decisions based on the neural network shown in Fig. 4. After one player makes decision, the network is modified, and players repeat their decision-making T terms. The neural network used by the agents and the gene information are shown in Fig. 4.

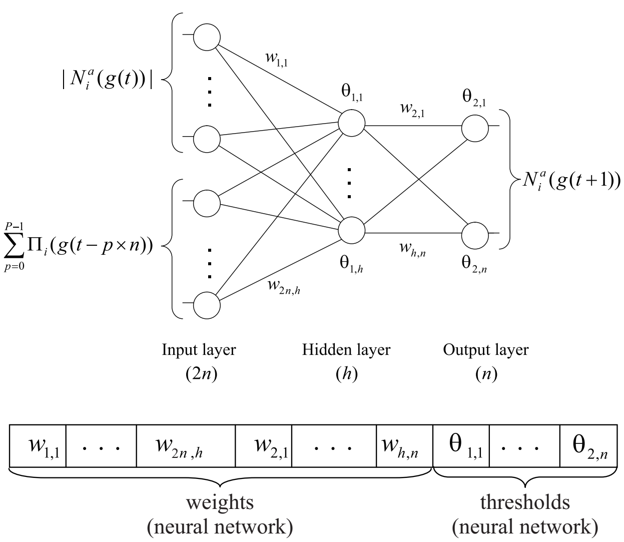

Fig. 4. Neural network used in simulation model and gene information

The information of the networks for past P periods as follows is given to the input layers of neural network:

1) Number of active links of all players at the previous period: $|N_i^a(g(t))|, \;\; i = 1, 2, \ldots, n$
2) Cumulative payoffs of all players for P periods as frequency as n periods: $\sum_{p=0}^{P-1} \Pi_i(g(t - p \times n)), \;\; i = 1, 2, \ldots, n$

The output values from each node of the output layers correspond to n players, if each output value is larger than the threshold φ then the player form a link, and if not then he delete a link. Each player makes a decision based on the agent which corresponds to him and is selected with roulette selection. The transfer function of the hidden layers and the output layers of neural network be the sigmoid function ($f(x) = \frac{1}{1+e^{-x}}$).

In simulation models (A) and (B), players make decisions based on neural network shown in Fig. 4. In the simulation model (A) players make decisions at every periods in the previous set order, while in the simulation model (B) one player is selected from n players at every periods and he makes a decision.

IV. Results of the Simulation Experiments

In this paper, the number of players is $n = 6$, and the information values and link cost are $(a, c) = (3, 2)$ according to the laboratory experiments by Berninghaus et al. [2]. The values of rest parameters which are used in our simulation experiments are shown in Table I.

We conduct the previous experiments about the number of agents and the number of final generation, and we use the parameters in Table I which shows the similar results to the results of laboratory experiments by Berninghaus et al. in a number of ways.

A. Result of the experiments in simulation model (A)

In the simulation model (A), the experiments are conducted with 15 terms and 10 groups according to the laboratory

TABLE I
THE VALUES OF PARAMETERS

simulation model	(A)	(B)
number of agents	$m = 10$	$m = 50$
number of final generation	$f_g = 5$	$f_g = 30$
discount rate for past payoff	$r = 0.99$	
crossover probability	$p_c = 0.8$	
mutation probability	$p_m = 0.01$	
generation gap	$G = 0.5$	

experiments of Berninghaus et al. [2]. In some groups in our experiments, periphery-sponsored star networks are formed as the laboratory experiments by Berninghaus et al. Here, a periphery-sponsored star network is the strict Nash equilibrium network. However, in our experiments, a part of players in some of the groups deviate from the equilibrium. The results of the simulation (A) are shown in Fig. 5.

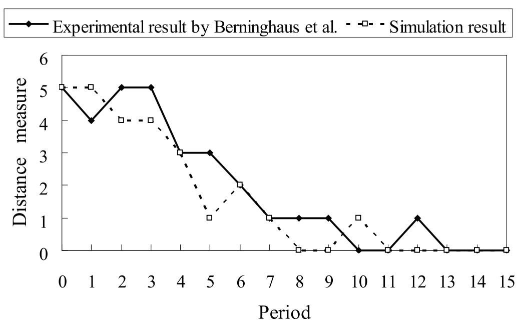

Fig. 5. The distance measure for a periphery-sponsored star network in simulation model (A)

From Fig. 5, the distance measure converges to 0 as each experiment runs, it indicates that a periphery-sponsored star network which is the strict Nash equilibrium network is formed as the result of the laboratory experiments by Berninghaus et al. Some players sometimes deviate from the equilibrium, but the equilibrium is formed again.

B. Result of the experiments in simulation model (B)

In simulation model (B), the experiments are conducted with 10000 terms with 10 groups. Here, let P be the number of periods which each agent obtains as the information of the past network structures, and let P_f be the number of terms which is applied for calculation of the fitness of the agents. As described later, the values of P and P_f indicate that how many periods the players use for decision making. In other words, they indicate that how many indicators are applied to neural networks. If the value of P and P_f are large, the players are interpreted as that have long-term view for decision making.

We compare the simulation experiments with $(P, P_f) = (1, 1)$ which is interpreted as the players have short-term view for decision making and with $(P, P_f) = (10, 10)$ which is interpreted as the players have long-term view for decision making. In Table II shows the number of groups where the periphery-sponsored star networks with the number of the central players have been formed in 10 groups.

The agents make decisions basing on the information of networks for past P periods. From Table II, in some of the groups, several kinds of periphery-sponsored star networks which has different central players are formed, i.e., change of the central players are observed in some groups. The players accept the decision making of the agent which is selected with roulette selection, therefore in the simulation result with $(P, P_f) = (1, 1)$ the central players probabilistically change, but the variance of the number of the central players is small. However the simulation result with $(P, P_f) = (10, 10)$, the variance of the number of the central players is large and the number of the central players is uniform. Thus, long-term information of past network structures leads change of the central players.

A process of the change of the central players are shown in Fig. 6.

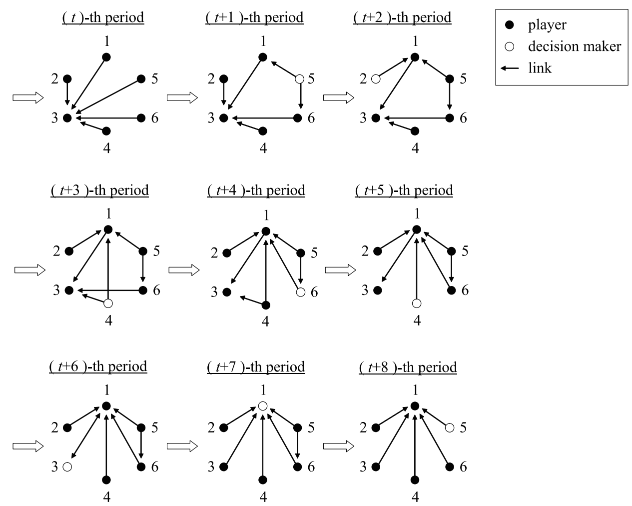

Fig. 6. A process of network formation (example)

In Fig. 6, the black circles, the white circle and the arrows indicate players, player making decision and links, respectively. The numbers written near the circles show player's number and the current period is shown at the upper in Fig. 6. At t-th period, the strict Nash equilibrium network with central player 3 is formed and the network formation transits from t-th period to $(t+8)$-th period.

From Fig. 6, at $(t+1)$-th period, peripheral player 5 deviates from the best response, i.e., delete a link with central player 3 and form a link with player 1 and 6. Then other peripheral player 2 approve player 5 and form a link with player 1 at $(t+2)$-th period. Finally many players form links with one player who is not a central player and the central player of the strict Nash equilibrium changes from player 3 to player 1. Thus, coordinative behavior among players leads a change of the central player.

There exists some human subjects who deviate from the strict Nash equilibrium network in the laboratory experiments by Berninghaus et al. The result of our experiments indicates two reasons of the deviation from the equilibrium of the

TABLE II
NUMBER OF THE CENTRAL PLAYERS IN THE PERIPHERY-SPONSORED STAR NETWORKS IN SIMULATION MODEL (B)

Number of the central players	Simulation result		Experimental result
	$(P,P_f)=(1,1)$	$(P,P_f)=(10,10)$	by Berninghaus et al. [2]
0	0	0	1
1	3	2	1
2	0	2	2
3	5	1	2
4	2	3	1
5	0	1	0
6	0	1	1
Mean	2.6	3.2	2.6
Variance	1.38	2.84	3.41

human subjects. The first is that the decision making of human subjects bases on trial and error mechanism. The second is that the human subjects refer to the long-term information of network structures and the other players' utilities when they make decisions.

V. CONCLUSION AND REMARKS

By the mathematical model of network formation by Bala and Goyal [1], the strict Nash equilibrium is stable. However, in the laboratory experiments using human subjects by Berninghaus et al. [2], after the formation of the strict Nash equilibrium human subjects deviate from it, and that contradict the result of the mathematical model.

In this paper, we provide that one of the reason of the deviation from the strict Nash equilibrium in the laboratory experiments using human subjects by simulation analysis using artificial adaptive agents. It indicates that human have the mechanism of decision making by trial and error and with a long-term view.

Finally, we propose future works for this paper. To conduct some kinds of simulation experiments of network formation with other several conditions, for example the information values and link cost are asymmetry for all players. A new mathematical model which can explain the behavior of the human subjects is one of the works.

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