State Estimation Method for Sound Environment System with Uncertainty and Its Application to Psychological Evaluation

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***Abstract*—The actual sound environment system exhibits various types of linear and non-linear characteristics, and it often contains an uncertainty. Furthermore, the observations in the sound environment are often in the level-quantized form. In this paper, two types of methods for estimating the specific signal for sound environment systems with uncertainty and the quantized observation is proposed by introducing newly a system model of the conditional probability type and moment statistics of fuzzy events. The effectiveness of the proposed theoretical method is confirmed by applying it to the actual problem of psychological evaluation for the sound environment.**

I. Introduction

The internal physical mechanism of actual sound environment system is often difficult to recognize analytically, and it contains uncertainty. Furthermore, the stochastic process observed in the actual phenomenon exhibits complex fluctuation pattern and there are potentially various nonlinear correlations in addition to the linear correlation between input and output time series.

In our previous study, for complex sound environment systems difficult to analyze by using usual structural methods based on the physical mechanism, a nonlinear system model was derived in the expansion series form reflecting various type correlation information from the lower order to the higher order between state variable and observation [1]. The conditional probability density function contains the linear and nonlinear correlations in the expansion coefficients, and these correlations play an important role as the statistical information for the state variable and observation relationship.

On the other hand, it is necessary to pay our attention on the fact that the observation data in the sound environment system are often measured in a level-quantized form and contain fuzziness due to several causes. For example, the human psychological evaluation for loudness can be judged by use of 7 levels from 1.very calm to 7.very noisy [2]. However, each score is affected by the human subjectivity and the border between two neighboring scores are vague [3]. Furthermore, the observation data are often measured in a digital level form at discrete times because various kinds of statistical evaluation (e.g., median, mean, covariance, higher order moments, etc.) for these quantized level data become easier if a digital computer is used. Therefore, in order to evaluate the objective sound environment system, it is desirable to estimate the waveform fluctuation of the specific signal for the system with uncertainty based on the quantized or fuzzy observation data.

As a typical method in the state estimation problem, the Kalman filtering theory and its extended filter are well known [4]. These theories are originally based on the Gaussian property of the state fluctuation form. On the other hand, the actual sound environment systems exhibit complex fluctuation properties and often contain unknown characteristics in the relationship between the state variable and the observation. Thus, it is necessary to improve the previous state estimation methods by taking account of the complexity and uncertainty in the actual systems.

From the above viewpoint, based on the quantized or fuzzy observations, a method for estimating precisely the specific signal for the sound environment system with uncertainty is theoretically proposed in this study. More specifically, first, by adopting an expansion expression of the conditional probability distribution reflecting the information on linear and non-linear correlation between the specific signal and the quantized observation as the system characteristics between them, a method to estimate the time series of the specific signal is theoretically derived. The proposed estimation method can be applied to an actual complex sound environment system with uncertainty by considering the coefficients of conditional probability distribution as unknown parameters and estimating simultaneously these parameters and the specific signal. Next, by introducing fuzzy theory to the uncertainty of the system, the other type of estimation algorithm is derived. The proposed theory is applied to the estimation problem of the psychological evaluation for loudness in sound environment and the effectiveness of the theory is experimentally confirmed.

II. State Estimation of Sound Environment System with Uncertainty

A. Estimation Algorithm by Introducing a Stochastic Model

Consider a complex sound environment system with an uncertainty that cannot be obtained on the basis of the internal physical mechanism of the system. In the observations of actual sound environment system, the sound level data are very often measured in a digital level form at discrete times. This is because some signal processing methods by utilizing a digital computer are indispensable for extracting exactly various quantities for human evaluation based on these quantized level data.

Let x_k and y_k be the input and output signals at a discrete time k for a sound environment system. For example, for the psychological evaluation in sound environment, x_k and y_k denote the physical sound level and human response quantity for it, respectively. It is assumed that there are complex nonlinear relationships between x_k and y_k, which are difficult to find a fundamental relationship between them. Since the system characteristics are unknown, a system model in the form of a conditional probability is adopted. More precisely, attention is focused on the joint probability distribution function $P(x_k, x_{k+1}, y_k)$ reflecting all linear and non-linear correlation information among x_k, x_{k+1} and y_k. Expanding the joint probability distribution function $P(x_k, x_{k+1}, y_k)$ in an orthogonal form based on the product of $P(x_k)$, $P(x_{k+1})$ and $P(y_k)$, the following expression can be derived.

$$P(x_k, x_{k+1}, y_k) = P(x_k)P(x_{k+1})P(y_k) \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\sum_{t=0}^{\infty} A_{rst}\theta_r^{(1)}(x_k)\theta_s^{(1)}(x_{k+1})\theta_t^{(2)}(y_k) \tag{1}$$

with

$$A_{rst} =< \theta_r^{(1)}(x_k)\theta_s^{(1)}(x_{k+1})\theta_t^{(2)}(y_k) >, \tag{2}$$

where $< >$ denotes the averaging operation on the variables. The linear and non-linear correlation information among x_k, x_{k+1} and y_k is reflected hierarchically in each expansion coefficient A_{rst}. The functions $\theta_r^{(1)}(x_k)$ and $\theta_t^{(2)}(y_k)$ are orthonormal polynomials with the weighting functions $P(x_k)$ and $P(y_k)$ respectively. These orthonormal polynomials can be decomposed by using Schmidt's orthogonalization [5]. From (1), the conditional probability distribution function $P(x_{k+1}|x_k)$ and $P(y_k|x_k)$ are given as

$$P(x_{k+1}|x_k) = P(x_{k+1}) \sum_{r=0}^{R}\sum_{s=0}^{S} A_{rs0}\theta_r^{(1)}(x_k)\theta_s^{(1)}(x_{k+1}). \tag{3}$$

$$P(y_k|x_k) = P(y_k) \sum_{r=0}^{R}\sum_{t=0}^{T} A_{r0t}\theta_r^{(1)}(x_k)\theta_t^{(2)}(y_k). \tag{4}$$

Though (3) and (4) are originally infinite series expansions, finite expansion series are adopted because only finite expansion coefficients are available and the consideration of the expansion coefficients from the first few terms is usually sufficient in practice. Since the objective system contains an unknown structure, the expansion coefficients A_{rs0} and A_{r0t} expressing hierarchically the correlation relationship between x_k, x_{k+1} and x_k, y_k have to be estimated on the basis of the observation y_k. Considering the expansion coefficients A_{rs0} and A_{r0t} as unknown parameter vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\begin{aligned} \mathbf{a} &= (a_1, a_2, \cdots, a_I) = (\mathbf{a}_{(1)}, \mathbf{a}_{(2)}, \cdots, \mathbf{a}_{(S)}), \\ \mathbf{a}_{(s)} &= (A_{1s0}, A_{2s0}, \cdots, A_{Rs0}),\ (s = 1, 2, \cdots, S), \quad (5) \\ \mathbf{b} &= (b_1, b_2, \cdots, b_J) = (\mathbf{b}_{(1)}, \mathbf{b}_{(2)}, \cdots, \mathbf{b}_{(T)}), \\ \mathbf{b}_{(t)} &= (A_{10t}, A_{20t}, \cdots, A_{R0t}),\ (t = 1, 2, \cdots, T), \quad (6) \end{aligned}$$

where $I = (RS)$ and $J = (RT)$ are the number of unknown expansion coefficients to be estimated, the simple dynamical models, $\mathbf{a}_{k+1} = \mathbf{a}_k$ and $\mathbf{b}_{k+1} = \mathbf{b}_k$, are introduced for the simultaneous estimation of the parameters with the specific signal x_k:

To derive an estimation algorithm for the specific signal x_k, attention is focused on Bayes' theorem for the conditional probability distribution [5]. Since the parameter $\mathbf{a}_k$ and $\mathbf{b}_k$ are also unknown, the conditional probability distribution of x_k, $\mathbf{a}_k$ and $\mathbf{b}_k$ is considered.

$$P(x_k, \mathbf{a}_k, \mathbf{b}_k|Y_k) = \frac{P(x_k, \mathbf{a}_k, \mathbf{b}_k, y_k|Y_{k-1})}{P(y_k|Y_{k-1})}, \tag{7}$$

where $Y_k (= \{y_1, y_2, \cdots, y_k\})$ is a set of observation data up to time k. The conditional joint probability distribution $P(x_k, \mathbf{a}_k, \mathbf{b}_k, y_k|Y_{k-1})$ can be generally expanded in a statistical orthogonal expansion series:

$$\begin{aligned} P(x_k, \mathbf{a}_k, \mathbf{b}_k, y_k|Y_{k-1}) &= P_0(x_k|Y_{k-1})P_0(\mathbf{a}_k|Y_{k-1}) \\ P_0(\mathbf{b}_k|Y_{k-1})&P_0(y_k|Y_{k-1}) \sum_{l=0}^{\infty}\sum_{\mathbf{m}=\mathbf{0}}^{\infty}\sum_{\mathbf{n}=\mathbf{0}}^{\infty}\sum_{q=0}^{\infty} B_{l\mathbf{mn}q} \\ &\psi_l^{(1)}(x_k)\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)\psi_q^{(4)}(y_k), \quad (8) \end{aligned}$$

$$B_{l\mathbf{mn}q} =< \psi_l^{(1)}(x_k)\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)\psi_q^{(4)}(y_k)|Y_{k-1} > . \tag{9}$$

After substituting (8) into (7) and expanding an arbitrary polynomial function $f_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k)$ of x_k, $\mathbf{a}_k$ and $\mathbf{b}_k$ with $((L, \mathbf{M}, \mathbf{N}))$th order in a series expansion form using $\{\psi_l^{(1)}(x_k)\}$, $\{\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)\}$ and $\{\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)\}$:

$$\begin{aligned} &f_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k) \\ &= \sum_{l=0}^{L}\sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{M}}\sum_{\mathbf{n}=\mathbf{0}}^{\mathbf{N}} C_{l\mathbf{mn}}^{L\mathbf{MN}}\psi_l^{(1)}(x_k)\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k), \\ &(C_{l\mathbf{mn}}^{L\mathbf{MN}}; appropriate\ constants), \quad (10) \end{aligned}$$

by taking the conditional expectation of the function $f_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k)$ and using the orthonormal condition for the functions $\psi_l^{(1)}(x_k)$, $\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)$ and $\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)$, the estimate of the function $f_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k)$ can be derived as follows:

$$\begin{aligned} \hat{f}_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k) &=< f_{L,\mathbf{M},\mathbf{N}}(x_k, \mathbf{a}_k, \mathbf{b}_k)|Y_k > \\ &= \frac{\sum_{l=0}^{L}\sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{M}}\sum_{\mathbf{n}=\mathbf{0}}^{\mathbf{N}}\sum_{q=0}^{\infty} C_{l\mathbf{mn}}^{L\mathbf{MN}} B_{l\mathbf{mn}q}\psi_q^{(4)}(y_k)}{\sum_{q=0}^{\infty} B_{\mathbf{000}q}\psi_q^{(4)}(y_k)}. \quad (11) \end{aligned}$$

The four functions $\psi_l^{(1)}(x_k)$, $\psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k)$, $\psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)$ and $\psi_q^{(4)}(y_k)$ are orthonormal polynomials of degrees l, $\mathbf{m} = (m_1, m_2, \cdots, m_I)$, $\mathbf{n} = (n_1, n_2, \cdots, n_J)$ and q with weighting functions $P_0(x_k|Y_{k-1})$, $P_0(\mathbf{a}_k|Y_{k-1})$, $P_0(\mathbf{b}_k|Y_{k-1})$ and $P_0(y_k|Y_{k-1})$, which can be chosen as the probability functions describing the dominant parts of the actual fluctuation or as the well-known standard probability distributions.

As an example of standard probability functions for the specific signal and the parameter, consider the Gaussian distribution:

$$\begin{aligned} P_0(x_k|Y_{k-1}) &= N(x_k; x_k^*, \Gamma_{x_k}), & (12)\\ P_0(\mathbf{a}_k|Y_{k-1}) &= \prod_{i=1}^{I} N(a_{i,k}; a_{i,k}^*, \Gamma_{a_{i,k}}), & (13)\\ P_0(\mathbf{b}_k|Y_{k-1}) &= \prod_{j=1}^{J} N(b_{j,k}; b_{j,k}^*, \Gamma_{b_{j,k}}) & (14) \end{aligned}$$

with

$$\begin{gathered} N(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\{-\frac{(x-\mu)^2}{2\sigma^2}\},\\ x_k^* =< x_k|Y_{k-1} >, \Gamma_{x_k} =< (x_k - x_k^*)^2|Y_{k-1} >,\\ a_{i,k}^* =< a_{i,k}|Y_{k-1} >, \Gamma_{a_{i,k}} =< (a_{i,k} - a_{i,k}^*)^2|Y_{k-1} >,\\ b_{j,k}^* =< b_{j,k}|Y_{k-1} >, \Gamma_{b_{j,k}} =< (b_{j,k} - b_{j,k}^*)^2|Y_{k-1} > . \quad (15) \end{gathered}$$

Furthermore, as the fundamental probability function on the level-quantized observation, the generalized binomial distribution [6] with level difference interval h_y can be chosen:

$$P_0(y_k|Y_{k-1}) = B(y_k; N_{y_k}, M_y, p_{y_k}, h_y) \quad (16)$$

with

$$\begin{aligned} B(y;N,M,p,h) &= \frac{(\frac{N-M}{h})!}{(\frac{y-M}{h})!(\frac{N-y}{h})!} p^{\frac{y-M}{h}} (1-p)^{\frac{N-y}{h}},\\ p_{y_k} &= \frac{y_k^* - M_y}{N_{y_k} - M_y},\ y_k^* =< y_k|Y_{k-1} >,\\ N_{y_k} &= \frac{(y_k^* - M_y)h_y y_k^* - y_M \Omega_{y_k}}{(y_k^* - M_y)h_y - \Omega_{y_k}},\\ \Omega_{y_k} &= < (y_k - y_k^*)^2|Y_{k-1} >, \quad (17) \end{aligned}$$

where M_y is the minimum level of the observation. The orthonormal polynomials with four weighting probability distributions in (12)-(14) and (16) can be determined as

$$\begin{aligned} \psi_l^{(1)}(x_k) &= \frac{1}{\sqrt{l!}} H_l(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}), & (18)\\ \psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k) &= \prod_{i=1}^{I} \frac{1}{\sqrt{m_i!}} H_{m_i}(\frac{a_{i,k} - a_{i,k}^*}{\sqrt{\Gamma_{a_{i,k}}}}), & (19)\\ \psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k) &= \prod_{j=1}^{J} \frac{1}{\sqrt{n_j!}} H_{n_j}(\frac{b_{i,k} - b_{i,k}^*}{\sqrt{\Gamma_{b_{i,k}}}}), & (20)\\ \psi_q^{(4)}(y_k) &= Bp_q(y_k; N_{y_k}, M_y, p_{y_k}, h_y) & (21) \end{aligned}$$

with

$$Bp_q(y;N,M,p,h) = \{(\frac{N-M}{h})^{(q)} q!\}^{-1/2} (\frac{1-p}{p})^{q/2}$$
$$\frac{1}{h^q} \sum_{j=0}^{q} {}_qC_j (-1)^{q-j} (\frac{p}{1-p})^{q-j} (N-y)^{(q-j)} (y-M)^{(j)}, \quad (22)$$

where $H_l(\ )$ denotes the Hermite polynomial with lth order [7], and $y^{(j)}$ is the jth order factorial function defined by [6]

$$\begin{aligned} y^{(q)} &= y(y-h_y)(y-2h_y)\cdots(y-(q-1)h_y),\\ y^{(0)} &= 1. \quad (23) \end{aligned}$$

Using the property of conditional expectation and (3)(4), the two variables y_k^* and Ω_{y_k} in (17) can be expressed in functional forms on predictions of x_k, $\mathbf{a}_k$ and $\mathbf{b}_k$ at a discrete time $k-1$ (i.e. the expectation value of arbitrary functions of x_k, $\mathbf{a}_k$ and $\mathbf{b}_k$ conditioned by Y_{k-1}), as follows:

$$\begin{aligned} y_k^* &=< \int y_k P(y_k|x_k) dy_k | Y_{k-1} >\\ &=< \sum_{r=0}^{\infty} \sum_{t=0}^{1} d_{1t} A_{r0t} \theta_r^{(1)}(x_k) | Y_{k-1} >\\ &=< \sum_{r=0}^{\infty} \sum_{t=0}^{1} d_{1t} A_{r0t} \int \theta_r^{(1)}(x_k) P(x_k|x_{k-1}) dx_k | Y_{k-1} >\\ &= \sum_{r=0}^{\infty} \sum_{t=0}^{1} d_{1t} < A_{r0t} \mathbf{A}_{(r),k} \Theta(x_{k-1}) | Y_{k-1} >, \quad (24)\\ \Omega_k &=< \int (y_k - y_k^*)^2 P(y_k|x_k) dy_k | Y_{k-1} >\\ &= \sum_{r=0}^{\infty} \sum_{t=0}^{2} d_{2t} < A_{r0t} \mathbf{A}_{(r),k} \Theta(x_{k-1}) | Y_{k-1} > \quad (25) \end{aligned}$$

with

$$\begin{aligned} \mathbf{A}_{(r),k} &= (0, \mathbf{a}_{(r),k}), (r = 1, 2, \cdots),\\ \mathbf{A}_{(0),k} &= (1, 0, 0, \cdots, 0),\\ \Theta(x_k) &= (\theta_0^{(1)}(x_k), \theta_1^{(1)}(x_k), \cdots, \theta_R^{(1)}(x_k))', \quad (26) \end{aligned}$$

where $'$ denotes the transpose of a matrix. The coefficients d_{1s} and d_{2s} in (24) and (25) are determined in advance by expanding y_k and $(y_k - y_k^*)^2$ in the following orthogonal series forms:

$$y_k = \sum_{i=0}^{1} d_{1i} \theta_i^{(2)}(y_k), (y_k - y_k^*)^2 = \sum_{i=0}^{2} d_{2i} \theta_i^{(2)}(y_k). \quad (27)$$

Furthermore, using (3) and (4), and the orthonormal condition of $\theta_r^{(1)}(x_k)$ and $\theta_t^{(2)}(y_k)$, each expansion coefficient $B_{l\mathbf{mn}q}$ defined by (9) can be obtained through the similar calculation process to (24) and (25), as follows:

$$B_{l\mathbf{mn}q} = \sum_{r=0}^{\infty} \sum_{t=0}^{q} \sum_{j=0}^{l+r} d_{qt} w_{l+r,j} < \psi_{\mathbf{m}}^{(2)}(\mathbf{a}_k) \psi_{\mathbf{n}}^{(3)}(\mathbf{b}_k)$$
$$A_{r0t} \mathbf{A}_{(j),k} \Theta(x_{k-1}) | Y_{k-1} >, \quad (28)$$

where d_{qi} and $w_{l+r,j}$ are appropriate coefficients satisfying the following equalities:

$$\psi_q^{(4)}(y_k) = \sum_{i=0}^{q} d_{qi}\theta_i^{(2)}(y_k),$$
$$\psi_l^{(1)}(x_k)\theta_r^{(1)}(x_k) = \sum_{j=0}^{l+r} w_{l+r,j}\theta_j^{(1)}(x_k). \quad (29)$$

Furthermore, by substituting the dynamical models of $\mathbf{a}_k$ and $\mathbf{b}_k$ into (24), (25) and (28), the parameters y_k^*,Ω_{y_k} and the expansion coefficient $B_{l\mathbf{mn}q}$ can be given in functional forms on estimations of x_{k-1}, $\mathbf{a}_{k-1}$ and $\mathbf{b}_{k-1}$. Therefore, the recurrence estimation of the specific signal can be achieved.

B. Estimation Algorithm by Introducing a Fuzzy Theory

In the observations of actual sound environment system, the sound level data often contain fuzziness due to human subjectivity in noise evaluation, confidence limitations in sensing devices, and quantizing errors in digital observations, etc. Let z_k be fuzzy observation obtained from y_k. For example, for the psychological evaluation in sound environment, x_k and y_k denote respectively the physical sound level and human response quantity for it. Since the system characteristics are unknown, the observation model in a form of a conditional probability in (4) is adopted. Furthermore, z_k expresses the loudness scores (1.very calm, 2.calm, 3.mostly calm, 4.little noisy, 5.noisy, 6.faily noisy, 7.very noisy) taking the individual and psychological situation into consideration for y_k. The fuzziness of z_k is characterized by the membership function $\mu_{z_k}(y_k)$. As the membership function, a Gaussian type function:

$$\mu_{z_k}(y_k) = \exp\{-\alpha(y_k - z_k)^2\}, \quad (30)$$

where $\alpha(>0)$ is a parameter, is adopted from the viewpoint of mathematical analysis. Though the parameter α in (30) can be generally given based on the prior information (or, through trial and error), it can be regarded as unknown parameter and estimated simultaneously with the specific signal x_k and the parameter $\mathbf{b}_k$. First, a simple dynamical model for the parameter, $\alpha_{k+1} = \alpha_k$, is naturally introduced.

Next, as the similar manner to (7), by paying our attention to the conditional joint probability density function of x_k, $\mathbf{b}_k$ and α_k, the following expression is obtained.

$$\begin{aligned} P(x_k, \mathbf{b}_k, \alpha_k|Z_k) &= \frac{P(x_k, \mathbf{b}_k, \alpha_k, z_k|Z_{k-1})}{P(z_k|Z_{k-1})} \\ &= P_0(x_k|Z_{k-1})P_0(\mathbf{b}_k|Z_{k-1})P_0(\alpha_k|Z_{k-1}) \\ &\sum_{l=0}^{\infty}\sum_{\mathbf{m}=\mathbf{0}}^{\infty}\sum_{n=0}^{\infty}\sum_{q=0}^{\infty} D_{l\mathbf{m}nr}\psi_l^{(1)}(x_k)\psi_{\mathbf{m}}^{(3)}(\mathbf{b}_k) \\ &\psi_n^{(5)}(\alpha_k)\psi_q^{(6)}(z_k)/\sum_{r=0}^{\infty} D_{\mathbf{000}q}\psi_q^{(6)}(z_k) \end{aligned} \quad (31)$$

with

$$D_{l\mathbf{m}nq} =< \psi_l^{(1)}(x_k)\psi_{\mathbf{m}}^{(3)}(\mathbf{b}_k)\psi_n^{(5)}(\alpha_k)\psi_r^{(6)}(z_k)|Z_{k-1} >, \quad (32)$$

where $Z_k (= \{z_1, \cdots, z_k\})$ is a set of fuzzy observation data. The two functions $\psi_n^{(5)}(\alpha_k)$ and $\psi_q^{(6)}(z_k)$ denote the orthonormal polynomials of degrees n and q, with the fundamental probability density functions $P_0(\alpha_k|Z_{k-1})$ and $P_0(z_k|Z_{k-1})$ of α_k and z_k as weighting functions. Based on (31), through the similar calculation process to (11), the estimate of an arbitrary polynomial function $f_{L,\mathbf{M},N}(x_k, \mathbf{b}_k, \alpha_k)$ of x_k, $\mathbf{b}_k$ and α_k of $(L, \mathbf{M}, N)$th order can be derived, as follows:

$$\begin{aligned} \hat{f}_{L,\mathbf{M},N}(x_k, \mathbf{b}_k, \alpha_k) &=< f_{L,\mathbf{M},N}(x_k, \mathbf{b}_k, \alpha_k)|Z_k > \\ &= \frac{\sum_{l=0}^{L}\sum_{\mathbf{m}=\mathbf{0}}^{\mathbf{M}}\sum_{n=0}^{N}\sum_{q=0}^{\infty} E_{l\mathbf{m}n}^{L\mathbf{M}N} D_{l\mathbf{m}nq}\psi_q^{(6)}(z_k)}{\sum_{q=0}^{\infty} D_{\mathbf{000}q}\psi_q^{(6)}(z_k)}. \end{aligned} \quad (33)$$

All the coefficients $E_{l\mathbf{m}n}^{L\mathbf{M}N}$ are appropriate constants in the case when the function $f_{L,\mathbf{M},N}(x_k, \mathbf{b}_k, \alpha_k)$ is expressed in a series expansion form similar to (10) using $\{\psi_l^{(1)}(x_k)\}$, $\{\psi_{\mathbf{m}}^{(3)}(\mathbf{b}_k)\}$ and $\{\psi_n^{(5)}(\alpha_k)\}$.

As a concrete example of the fundamental probability density functions for the parameter α_k and z_k, the Gaussian distribution and the generalized binomial distribution are adopted respectively:

$$P_0(\alpha_k|Z_{k-1}) = N(\alpha_k; \alpha_k^*, \Gamma_{\alpha_k}), \quad (34)$$
$$P_0(z_k|Z_{k-1}) = B(z_k; N_{z_k}, M_z, p_{z_k}, h_z) \quad (35)$$

with

$$\begin{aligned} \alpha_k^* &= < \alpha_k|Z_{k-1} >, \Gamma_{\alpha_k} =< (\alpha_k - \alpha_k^*)^2|Z_{k-1} >, \\ p_{z_k} &= \frac{z_k^* - M_z}{N_{z_k} - M_z}, \ z_k^* =< z_k|Z_{k-1} >, \\ N_{z_k} &= \frac{(z_k^* - M_z)h_z z_k^* - M_z\Omega_{z_k}}{(z_k^* - M_z)h_z - \Omega_{z_k}}, \\ \Omega_{z_k} &= < (z_k - z_k^*)^2|Z_{k-1} > . \end{aligned} \quad (36)$$

Then, the orthonormal polynomials with two weighting probability density functions in (34) and (35) can be given as

$$\psi_n^{(5)}(\alpha_k) = \frac{1}{\sqrt{n!}}H_n(\frac{\alpha_k - \alpha_k^*}{\sqrt{\Gamma_{\alpha_k}}}), \quad (37)$$
$$\psi_q^{(6)}(z_k) = Bp_q(z_k; N_{z_k}, M_z, p_{z_k}, h_z). \quad (38)$$

After applying moment statistics of fuzzy events which are generalization of mean and variance of fuzzy events [8], by applying (4), the two variables z_k^* and Ω_{z_k} in (36) and the expansion coefficient $D_{l\mathbf{m}nq}$ are expressed in concrete forms, as follows:

$$z_k^* = \frac{\sum_{t=0}^{T} < e_t\mathbf{B}_{(t),k}\Theta(x_k)|Z_{k-1} >}{\sum_{t=0}^{T} < h_t\mathbf{B}_{(t),k}\Theta(x_k)|Z_{k-1} >}, \quad (39)$$

$$\Omega_{z_k} = \frac{\sum_{t=0}^{T} < f_t \mathbf{B}_{(t),k} \Theta(x_k)|Z_{k-1} >}{\sum_{t=0}^{T} < h_t \mathbf{B}_{(t),k} \Theta(x_k)|Z_{k-1} >}, \quad (40)$$

$$\begin{aligned} D_{l\mathbf{m}nq} &= \sum_{t=0}^{T} < g_t \psi_l^{(1)}(x_k) \psi_{\mathbf{m}}^{(3)}(\mathbf{b}_k) \psi_n^{(4)}(\alpha_k) \mathbf{B}_{(t),k} \\ &\quad \Theta(x_k)|Z_{k-1} > \\ &\quad / \sum_{t=0}^{T} < h_t \mathbf{B}_{(t),k} \Theta(x_k)|Z_{k-1} > \end{aligned} \quad (41)$$

with

$$\begin{aligned} \mathbf{B}_{(t),k} &= (0, \mathbf{b}_{(t),k}), (t = 1, 2, \cdots), \\ \mathbf{B}_{(0),k} &= (1, 0, 0, \cdots, 0), \end{aligned} \quad (42)$$

where e_t, f_t, g_t and h_t are expansion coefficients satisfying the following relations:

$$\mu_{z_k}(y_k) y_k = \sum_{i=0}^{\infty} e_i \theta_i^{(2)}(y_k), \quad (43)$$

$$\mu_{z_k}(y_k)(y_k - z_k^*)^2 = \sum_{i=0}^{\infty} f_i \theta_i^{(2)}(y_k), \quad (44)$$

$$\mu_{z_k}(y_k) \psi_r^{(6)}(y_k) = \sum_{i=0}^{\infty} g_i \theta_i^{(2)}(y_k), \quad (45)$$

$$\mu_{z_k}(y_k) = \sum_{i=0}^{\infty} h_i \theta_i^{(2)}(y_k). \quad (46)$$

The expansion coefficient $D_{l\mathbf{m}nq}$ in (41) can be given by the predictions of x_k, $\mathbf{b}_k$ and α_k. Furthermore, by introducing the following simple system model instead of (3),

$$x_{k+1} = F x_k + G u_k, \quad (47)$$

where u_k is the random input with mean 0 and variance σ_u^2 and F, G are system parameters, the prediction algorithm for an arbitrary polynomial function can be given in the form of estimates for the polynomial functions of x_k, $\mathbf{b}_k$ and α_k. Therefore, by combining the estimation algorithm of (33) with the prediction algorithm, the recurrence estimation of the specific signal can be obtained.

III. Application to Psychological Evaluation for Loudness

To find the quantitative relationship between the loudness for human and the physical sound level for environmental noise is important from the viewpoint of noise assessment. Especially, in the evaluation for a regional sound environment, the investigation based on questionnaires to the regional inhabitants is often given when the experimental measurement at every instantaneous time and at every point in the whole area of the region is difficult. Therefore, it is very important to estimate the sound level based on the loudness data. It has been reported that the loudness based on the human sensitivity can be distinguished each other from 7 loudness scores, for instance, 1.very calm, 2.calm, 3.mostly calm, 4.little noisy, 5.noisy, 6.fairly noisy, 7.very noisy, in the psychological acoustics [2]. After recording the road traffic noise by use of a sound level meter and a data recorder, by replaying the recorded tape through amplifier and loudspeaker in a laboratory room, 6 female subjects (A, B, $\cdots$, F) aged of 22-24 with normal hearing ability judged one score among 7 loudness scores (i.e., 1, 2, $\cdots$, 7) at every 5 [sec.], according to their impressions for the loudness at each moment using 7 categories from very calm to very noisy. The mean and standard deviation of the road traffic noise were 71.4 [dB(A)] and 7.23 [dB(A)], respectively. Furthermore, the mean and standard deviation for the loudness scores of each subject, and the correlation coefficients between the road traffic noise levels and the loudness scores are shown in Table 1.

The state estimation method proposed in Section II.A was applied to an estimation of the time series x_k for sound level of a road traffic noise based on the successive judgments y_k on loudness scores. Figure 1 shows one of the estimated results of the waveform fluctuation of the sound level based on the loudness score by a subject. In this figure, the horizontal axis shows the discrete time k, of the estimation process, and the vertical axis represents the sound level (A-weighted sound pressure level). The finite numbers of expansion coefficients $B_{l\mathbf{mn}q} (q \leq 2)$ in the proposed estimation algorithm (11) employing the system models of conditional probability type (3) and (4) with $R = S = T = 2$ were used in this estimation. In principle, it is expected that the successive addition of higher expansion terms reflecting higher order statistics in the proposed algorithm moves the theoretical estimation closer to the true values. However, higher order statistics based on the finite numbers of observed sample data give us unstable information with less reliability. It remains as one of the future problems to derive a method for determining an optimal order for the conditional probability distribution in expansion series form like (3) and (4).

One of the estimated results by applying the algorithm proposed in Section II.B is shown in Fig.2. For comparison, the estimated results by the previously reported method [1] and the extended Kalman Filter [9] are shown in Fig.3. The estimated results of the parameter α of membership function in (30) are shown in Table 2. The root mean squared error of the estimation is shown in Table 3. It is obvious that the proposed methods show more accurate estimations than the results based on the previous estimation method and the extended Kalman filter. By comparing Table 3 with Table 1, it can be found that the more accurate estimation results are obtained in cases with the larger values of the correlation coefficient between the sound levels and the loudness scores.

IV. Conclusions

In this paper, based on the quantized or fuzzy observation data, a new method for estimating the specific signal for sound environment systems with uncertainty has been propoesd. The proposed estimation method has been realized by introducing a system model of conditional probability type and a

Table 1 Statistics of loudness scores and correlation coefficients between the sound level and the loudness scores.

Subject	A	B	C	D	E	F
Mean	4.18	4.46	4.19	4.20	4.93	5.05
Standard Deviation	1.11	1.06	0.781	0.784	0.712	0.915
Correlation Coefficient	0.891	0.823	0.811	0.796	0.826	0.870

fuzzy theory. The proposed method has been applied to the estimation of an actual sound environment, and it has been experimentally verified that better results have been obtained as compared with the results by use of the previous method and the extended Kalman filter.

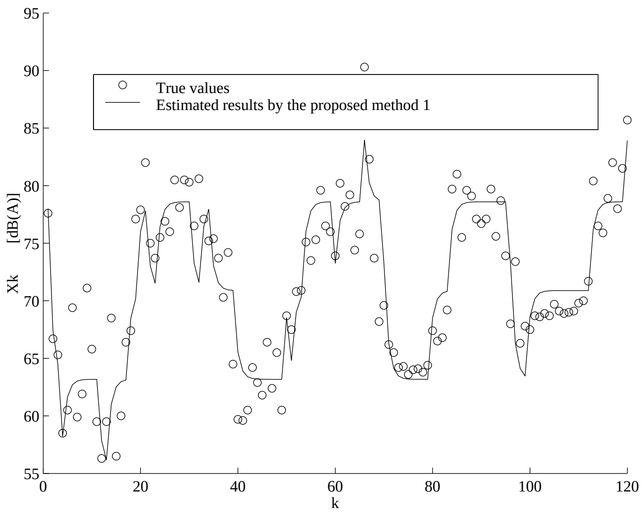

Fig. 1. Estimation results of the sound level by use of the method 1.

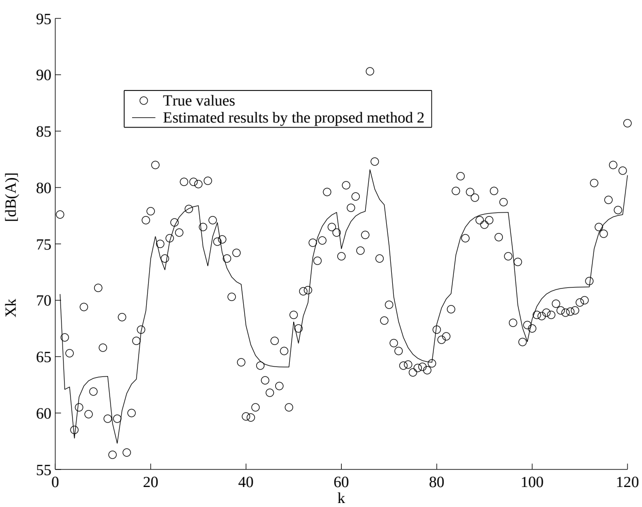

Fig. 2. Estimation results of the sound level by use of the method 2.

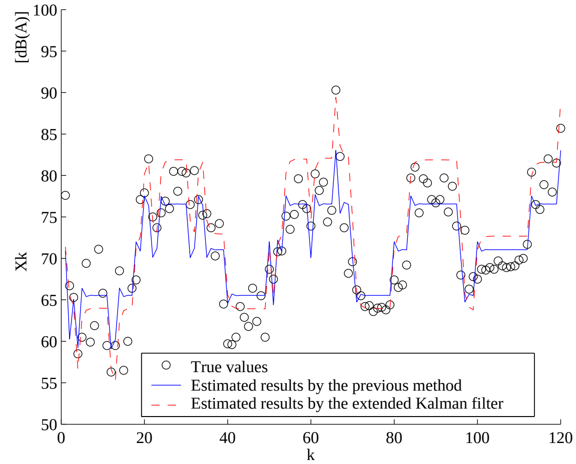

Fig. 3. Estimation results of the sound level by use of the previous method and the extended Kalman filter.

Table 2 Estimated results of the parameter of membership function.

Subject	A	B	C	D	E	F
Estimates	0.161	0.201	0.483	0.231	0.241	0.245

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Table 3 Root mean squared error of the estimation in [dB(A)].

Subject	A	B	C	D	E	F
Method 1 in Section II.A	3.13	3.67	3.88	3.97	3.61	3.16
Method 2 in Section II.B	3.15	3.93	4.32	4.13	3.82	3.44
Previous Method	3.94	4.89	4.56	4.28	3.91	3.59
Extended Kalman Filter	5.04	7.53	16.6	7.99	5.46	4.17