# An Interactive Fuzzy Satisficing Method for Multiobjective Stochastic Integer Programming Problems through Simple Recourse Model

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*Abstract*—Two major approaches to deal with randomness or impression involved in mathematical programming problems have been developed. The one is called stochastic programming, and the other is called fuzzy programming. In this paper, we focus on multiobjective integer programming problems involving random variable coefficients in constraints. Using the concept of simple recourse, such multiobjective stochastic integer programming problems are transformed into deterministic ones. As a fusion of stochastic programming and fuzzy one, after introducing fuzzy goals to reflect the ambiguity of the decision maker's judgments for objective functions, we propose an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker by updating the reference membership levels.

## I. INTRODUCTION

In constructing mathematical models of actual decision making situation in the real world, we often need to reflect the randomness or the imprecision involved in the situation since we cannot always know exact values of all parameters.

Stochastic programming based on the probability theory, has been developed in various ways [2], [25], e.g., two stage problem or recourse model [6], [23], chance constrained programming [4], [5], [9]. In particular, for multiobjective stochastic linear programming problems, Stancu-Minasian [20] considered the minimum risk approach, Teghem et al. [21] and Urli et al. [22] proposed interactive methods. Furthermore, efficient solution concepts for them and their relations have been discussed by Caballero et al. [3].

On the other hand, fuzzy mathematical programming representing the ambiguity in decision making situations by fuzzy concepts has attracted attention of many researchers [13], [15]. Fuzzy multiobjective linear programming, first proposed by Zimmermann [26], has been rapidly developed [11], [18], [19].

As a hybrid of the stochastic approach and the fuzzy one, Wang et al. [24] dealt with mathematical programming problems with fuzzy random variables and Liu et al. [10] studied chance constrained programming involving fuzzy parameters and many researches about this issue have been reported [12], [14]. In particular, for multiobjective stochastic linear programming problems, Hulsurkar et al. [8] discussed an approach based on fuzzy programming. However, in their method, since membership functions for the objective functions are supposed to be aggregated by minimum operator or product operator, obtained solutions may not sufficiently reflect the decision maker's preference. To overcome this drawback, Sakawa et al. [16], [17] showed that satisficing solutions to multiobjective stochastic linear programming problems sufficiently reflecting the decision maker's preference can be derived through the interactive fuzzy satisficing method based on chance constrained programming models. In these existing methods for multiobjective stochastic linear programming problems [8], [16], [17], constraints including random variables are reduced to chance constrained conditions which mean that the constraints need to be satisfied with a certain probability (satisficing level). Then, the loss or cost caused by the violation of constraints for observed values is not reflected in the formulation and solution.

Under these circumstances, in this paper, focusing on the simple recourse model to consider the penalty reflecting on the degree of violation of constraints for observed values [7], we transform a multiobjective stochastic integer programming problems into equivalent deterministic multiobjective integer programming problems. After introducing fuzzy goals to reflect the ambiguous judgment of the decision maker on objective functions, we propose an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker by updating the reference membership levels.

## II. MULTIOBJECTIVE STOCHASTIC INTEGER PROGRAMMING PROBLEM

In this paper, we deal with multiobjective integer programming problems involving random variable coefficients in the right-hand side of constraints formulated as:

$$\left. \begin{array}{ll} \text{minimize} & z_l(\boldsymbol{x}) = \boldsymbol{c}_l\boldsymbol{x}, \ \ l = 1, 2, \ldots, k \\ \text{subject to} & A\boldsymbol{x} = \boldsymbol{b}(\omega) \\ & x_j \in \{0, 1, \ldots, \nu_j\}, \ \ j = 1, 2, \ldots, n \end{array} \right\} \quad (1)$$

where $\boldsymbol{x}$ is an $n$ dimensional integer decision variable column vector, $\boldsymbol{c}_l$, $l = 1, 2, \ldots, k$ are $n$ dimensional coefficient row vectors, $A$ is an $m \times n$ coefficient matrix, and $\boldsymbol{b}(\omega)$ is an $m$ dimensional random variable column vector.

We are often faced with optimization problems involving randomness like (1). For instance, in a company producing $m$ products by $n$ processes, there may exist a multiobjective optimization problem that the decision maker hopes to minimize the production cost and minimize the amount

of wastes simultaneously under the situation that for each decision variable $x_j$ representing the discrete production level for the $j$ th process, $j = 1, 2, \ldots, n$, the unit production cost coefficient $c_{1j}$ or the unit waste amount coefficients $c_{2j}$ and unit production amount coefficients $a_{ij}$ of the $i$ th product, $i = 1, 2, \ldots, m$, are known while each demand $b_i$ for the $i$ th product, $i = 1, 2, \ldots, m$ varies randomly.

Since (1) contains random variable coefficients, we cannot directly apply solution methods or solution concepts for ordinary mathematical programming problems to it. If the decision maker wishes to take the cost of the shortage or surplus of products caused by the randomness of demand into account, recourse models to consider the penalty depending on the degree of violation of constraints for observed values seem more desirable than chance constrained condition programming models [4], [5], [9] where chance constrained conditions mean that the constraints need to be satisfied with a certain probability (satisficing level). In this paper, we adopt the simple recourse model [7] which would be the most fundamental and practical among recourse models for situation that the shortage or surplus of products can be directly compensated by purchase of equivalent alternative products or the disposal of products.

## III. MULTIOBJECTIVE INTEGER SIMPLE RECOURSE PROBLEMS

In problem (1), we assume that the decision maker must make a decision before he knows observed values of random variables. In recourse approaches, the penalty of violation of constraints is incorporated into objective functions in order to consider the loss caused by randomness.

To be more specific, denoting the difference between $A\boldsymbol{x}$ and $\boldsymbol{b}(\omega)$ by two random vectors $\boldsymbol{y}^+ = (y_1^+, y_2^+, \ldots, y_m^+)^T$ and $\boldsymbol{y}^- = (y_1^-, y_2^-, \ldots, y_m^-)^T$, (1) can be reformulated as the following multiobjective integer simple recourse problem.

$$
\left.
\begin{array}{ll}
\text{minimize} & w_l(\boldsymbol{x}) = \boldsymbol{c}_l + R_l(\boldsymbol{x}), \ \ l = 1, 2, \ldots, k \\
\text{subject to} & A\boldsymbol{x} + \boldsymbol{y}^+ - \boldsymbol{y}^- = \boldsymbol{b}(\omega) \\
& x_j \in \{0, 1, \ldots, \nu_j\}, \ \ j = 1, 2, \ldots, n \\
& \boldsymbol{y}^+ \geq \boldsymbol{0}, \ \ \boldsymbol{y}^- \geq \boldsymbol{0}
\end{array}
\right\} \quad (2)
$$

In (2),

$$
R_l(\boldsymbol{x}) = \mathrm{E}\left\{ \min_{\boldsymbol{y}^+, \boldsymbol{y}^-} (\boldsymbol{p}_l\boldsymbol{y}^+ + \boldsymbol{q}_l\boldsymbol{y}^-) \right\} \quad (3)
$$

is called the expectation of a recourse for the $l$ th objective, where $\boldsymbol{p}_l$ and $\boldsymbol{q}_l$ are constant row vectors. Since each element of $\boldsymbol{y}^+ = (y_1^+, y_2^+, \ldots, y_m^+)^T$ means the shortage of each product and each element of $\boldsymbol{y}^- = (y_1^-, y_2^-, \ldots, y_m^-)^T$ means the surplus of each product, each element of $\boldsymbol{p}_l$ is regarded as the unit cost to compensate the shortage of each product and each element of $\boldsymbol{q}_l$ is regarded as the unit cost to dispose the surplus of each products. For $\boldsymbol{p}_l$ and $\boldsymbol{q}_l$, the assumption $\boldsymbol{p}_l + \boldsymbol{q}_l \geq \boldsymbol{0}$ seems natural because we could improve the objective function value infinitely by increasing $y_i^+$ and $y_i^-$ infinitely if $p_{li} + q_{li} < 0$ for some $i$.

From the assumption, complementary relations

$$
\hat{y}_i^+ > 0 \to \hat{y}_i^- = 0, \ \ \hat{y}_i^- > 0 \to \hat{y}_i^+ = 0, \ \ i = 1, 2, \ldots, m
$$

hold for optimal recourse variable vector $\hat{\boldsymbol{y}}^+, \hat{\boldsymbol{y}}^-$. Then, the following equations

$$
\hat{y}_i^+ = b_i^o - \sum_{j=1}^n a_{ij}x_j, \ \ \hat{y}_i^- = 0, \ \text{if } b_i^o \geq \sum_{j=1}^n a_{ij}x_j
$$

$$
\hat{y}_i^+ = 0, \ \ \hat{y}_i^- = \sum_{j=1}^n a_{ij}x_j - b_i^o \ \text{if } b_i^o < \sum_{j=1}^n a_{ij}x_j
$$

are led for $i = 1, 2, \ldots, m$ , where $b_i^o$ is the observed value of $b_i(\omega)$.

If $b_i(\omega), \ i = 1, 2, \ldots, m$ are mutually independent, the expectation of the recourse $\mathrm{E}\left\{ \min\limits_{\boldsymbol{y}^+, \boldsymbol{y}^-} (\boldsymbol{p}_l\boldsymbol{y}^+ + \boldsymbol{q}_l\boldsymbol{y}^-) \right\}$ can be calculated as:

$$
\begin{aligned}
& \mathrm{E}\left\{ \min_{\boldsymbol{y}^+, \boldsymbol{y}^-} (\boldsymbol{p}_l\boldsymbol{y}^+ + \boldsymbol{q}_l\boldsymbol{y}^-) \right\} \\
= \ & \mathrm{E}\left\{ \boldsymbol{p}_l\hat{\boldsymbol{y}}^+ + \boldsymbol{q}_l\hat{\boldsymbol{y}}^- \right\} = \sum_{i=1}^m \mathrm{E}\left\{ p_{li}\hat{y}_i^+ + q_{li}\hat{y}_i^- \right\} \\
= \ & \sum_{i=1}^m p_{li}\mathrm{E}\{\hat{y}_i^+\} + \sum_{i=1}^m q_{li}\mathrm{E}\{\hat{y}_i^-\} \\
= \ & \sum_{i=1}^m p_{li} \int_{\sum_{j=1}^n a_{ij}x_j}^{+\infty} \left( b_i - \sum_{j=1}^n a_{ij}x_j \right) dF_i(b_i) \\
& + \sum_{i=1}^m q_{li} \int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j} \left( \sum_{j=1}^n a_{ij}x_j - b_i \right) dF_i(b_i) \\
= \ & \sum_{i=1}^m p_{li}\mathrm{E}\{b_i\} - \sum_{i=1}^m (p_{li} + q_{li}) \int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j} b_i dF_i(b_i) \\
& - \sum_{i=1}^m p_{li} \sum_{j=1}^n a_{ij}x_j \\
& + \sum_{i=1}^m (p_{li} + q_{li}) \left( \sum_{j=1}^n a_{ij}x_j \right) F_i\left( \sum_{j=1}^n a_{ij}x_j \right)
\end{aligned}
$$

where $F_i(\cdot)$ is the probability distribution function of $b_i(\omega)$. Then, (2) is equivalent to the following problem.

$$
\left.
\begin{array}{ll}
\text{minimize} & Z_l(\boldsymbol{x}), \ l = 1, 2, \ldots, k \\
\text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\}, \ j = 1, 2, \ldots, n
\end{array}
\right\} \quad (4)
$$

where

$$
\begin{aligned}
Z_l(\boldsymbol{x}) = \ & \sum_{i=1}^m p_{li}\mathrm{E}\{b_i\} + \sum_{j=1}^n \left( c_{lj} - \sum_{i=1}^m a_{ij}p_{li} \right) x_j \\
& \sum_{i=1}^m (p_{li} + q_{li}) \left\{ \left( \sum_{j=1}^n a_{ij}x_j \right) F_i\left( \sum_{j=1}^n a_{ij}x_j \right) \right. \\
& \left. - \int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j} b_i dF_i(b_i) \right\},
\end{aligned}
$$

and let $\boldsymbol{X} = \{\boldsymbol{x} \mid x_j \in \{0, 1, \ldots, \nu_j,\}, \ j = 1, 2, \ldots, n\}$

In general, there rarely exists a complete optimal solution that simultaneously optimizes all objective functions for a multiobjective programming problem.

As a reasonable solution concept for (4), we define the following $R$-Pareto optimal solution.

**Definition 3.1.** ($R$-Pareto optimal solution). $\boldsymbol{x}^* \in \boldsymbol{X}$ *is said to be an $R$-Pareto optimal solution if there does not exist another $\boldsymbol{x} \in \boldsymbol{X}$ such that $Z_l(\boldsymbol{x}) \leq Z_l(\boldsymbol{x}^*)$ for any $l \in \{1, 2, \ldots, k\}$ and $Z_j(\boldsymbol{x}) < Z_j(\boldsymbol{x}^*)$ for at least one $j \in \{1, 2, \ldots, k\}$.*

## IV. AN INTERACTIVE FUZZY SATISFICING METHOD

In order to consider imprecise nature of the decision maker's judgment for each objective function $Z_l(\boldsymbol{x})$ in (4), we introduce fuzzy goals such as "$Z_l(\boldsymbol{x})$ should be substantially less than or equal to a certain value." Then, (4) can be rewritten as:

$$\left. \begin{array}{ll} \text{maximize} & \mu_l(Z_l(\boldsymbol{x})), \ l = 1, 2, \ldots, k \\ \text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\}, \ j = 1, 2, \ldots, n \end{array} \right\} \quad (5)$$

where $\mu_l(\cdot)$ is a membership function to quantify the fuzzy goal for the $l$ th objective function in (4). To be more specific, if the decision maker feels that $Z_l(\boldsymbol{x})$ should be less than or equal to at least $Z_{l,0}$ and $Z_l(\boldsymbol{x}) \leq Z_{l,1}(< Z_{l,0})$ is satisfactory, the shape of a typical membership function is shown in Fig. 1.
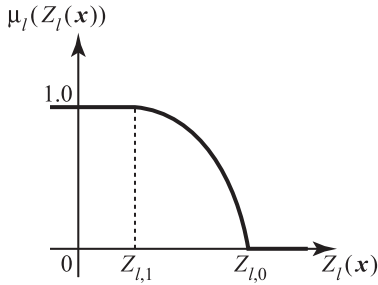


Fig. 1.   An example of a membership function $\mu_l(Z_l(\boldsymbol{x}))$

Since (5) is regarded as a fuzzy multiobjective decision making problem, there rarely exists a complete optimal solution that simultaneously optimizes all membership functions.

As a reasonable solution concept for such fuzzy multiobjective decision making problems, Sakawa et al. [18] defined M-Pareto optimality on the basis of membership function values by directly extending the Pareto optimality for multiobjective programming problems.

**Definition 4.1.** (M-Pareto optimal solution). $\boldsymbol{x}^* \in X$, *where $X$ is the feasible region of the problem, is said to be an M-Pareto optimal solution if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_l(z_l(\boldsymbol{x})) \geq \mu_l(z_l(\boldsymbol{x}^*))$ for any $l \in \{1, 2, \ldots, k\}$ and $\mu_j(z_j(\boldsymbol{x})) > \mu_j(z_j(\boldsymbol{x}^*))$ for at least one $j \in \{1, 2, \ldots, k\}$ where $z_l(\cdot)$s stand for objective functions.*

Based on the concept of $R$-Pareto optimal solution and that of M-Pareto optimal solution, we now define M-$R$-Pareto optimal solution.

**Definition 4.2.** (M-$R$-Pareto optimal solution). $\boldsymbol{x}^* \in \boldsymbol{X}$ *is said to be an M-$R$-Pareto optimal solution to (5) if and only if there does not exist another $\boldsymbol{x} \in \boldsymbol{X}$ such that $\mu_l(Z_l(\boldsymbol{x})) \geq \mu_l(Z_l(\boldsymbol{x}^*))$ for all $l \in \{1, 2, \ldots, k\}$ and $\mu_j(Z_j(\boldsymbol{x})) > \mu_j(Z_j(\boldsymbol{x}^*))$ for at least one $j \in \{1, 2, \ldots, k\}$ in (5).*

Introducing an aggregation function $\mu_D(\boldsymbol{x})$ for $k$ membership functions in (5), problem (5) can be rewritten as:

$$\left. \begin{array}{ll} \text{maximize} & \mu_D(\boldsymbol{x}) \\ \text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\} \ j = 1, 2, \ldots, n \end{array} \right\} \quad (6)$$

The aggregation function $\mu_D(\boldsymbol{x})$ represents the degree of satisfaction or preference of the decision maker for whole of $k$ fuzzy goals.

Following conventional fuzzy approaches, as the aggregation function, Hulsurkar et al. [8] adopted the minimum operator [1] define by

$$\mu_D(\boldsymbol{x}) = \min_{l=1,2,\ldots,k} \mu_l(Z_l(\boldsymbol{x})) \quad (7)$$

and the product operator [26] defined by

$$\mu_D(\boldsymbol{x}) = \prod_{l=1}^{k} \{\mu_l(Z_l(\boldsymbol{x}))\}. \quad (8)$$

Although these operators are widely used as an aggregation function, the usefulness of the minimum operator or the product operator is limited since the preference of the decision maker is not always well expressed by them in general decision situations. It would be desirable to identify an appropriate aggregation function which well represents the decision maker's preference, but it is rarely possible to identify such the aggregation function explicitly and exactly. As an alternative, interactive methods which derive the local information of the decision maker's preference through interactions and find a satisficing solution for the decision maker without the explicit identification of the aggregation function seem promising to (5). In this paper, we develop an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker through interaction proposed by Sakawa et al. [18]. In their method, in order to derive a satisficing solution, the decision maker interactively updates aspiration levels of achievement for membership values of all fuzzy goals, called reference membership levels, until he is satisfied [18].

To be more specific, for the decision maker's reference membership levels $\bar{\mu}_l$, $l = 1, 2, \ldots, k$, the following augmented minimax problem is repeatedly solved.

$$\left. \begin{array}{ll} \text{minimize} & \max_{l=1,\ldots,k} [\bar{\mu}_l - \mu_l(Z_l(\boldsymbol{x})) \\ & \quad + \rho \sum_{i=1}^{k} (\bar{\mu}_i - \mu_i(Z_i(\boldsymbol{x})))] \\ \text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\}, \ j = 1, 2, \ldots, n \end{array} \right\} \quad (9)$$

In (9), $\rho$ is a sufficiently small positive number and the corresponding optimal solution to (9) is nearest to the requirements in the augmented minimax sense or better than them if the reference membership levels are attainable.

The relationship between an optimal solution to (9) and the M-$R$-Pareto optimality can be characterized by the following theorems.

**Theorem 4.1.** *If $\boldsymbol{x}^* \in \boldsymbol{X}$ is an optimal solution to (9) for some $\bar{\mu}_l$, $l = 1, 2, \ldots, k$, then $\boldsymbol{x}^*$ is an M-$R$-Pareto optimal solution to (5).*

**Theorem 4.2.** *If $\boldsymbol{x}^* \in \boldsymbol{X}$ is an M-R-Pareto optimal solution to (5), then there exists $\bar{\mu}_l$, $l = 1, 2, \ldots, k$ such that $\boldsymbol{x}^*$ is an optimal solution to (9).*

We now summarize the interactive algorithm.

Interactive fuzzy satisficing method

**Step 1:** Calculate individual minima $Z_{l,\min}$ of objective functions $Z_l(\boldsymbol{x})$, $l = 1, 2, ..., k$, in (5) by solving the following problems.

$$\left. \begin{array}{ll} \text{minimize} & Z_l(\boldsymbol{x}) \\ \text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\}, \ j = 1, 2, \ldots, n \end{array} \right\} \quad (10)$$

Then, calculate $Z_{l,\min}$ from optimal solutions $\boldsymbol{x}_{\min}^l$, $l = 1, 2, \ldots, k$ to (10). Go to step 2.

**Step 2:** Ask the decision maker to subjectively determine membership functions $\mu_l(Z_l(\boldsymbol{x}))$ for objective functions, based on minimal values $Z_{l,\min}$ calculated in step 1. Go to step 3.

**Step 3:** Ask the decision maker to set the initial reference membership levels (they are often set as $\bar{\mu}_l = 1$, $l = 1, 2, \ldots, k$). Go to step 4.

**Step 4:** Solve the following minimax problem for given reference membership levels $\bar{\mu}_l$, $l = 1, 2, \ldots, k$.

$$\left. \begin{array}{ll} \text{minimize} & \max\limits_{l=1,\ldots,k} [\bar{\mu}_l - \mu_l(Z_l(\boldsymbol{x})) \\ & \qquad + \rho \sum\limits_{i=1}^{k} (\bar{\mu}_i - \mu_i(Z_i(\boldsymbol{x})))] \\ \text{subject to} & x_j \in \{0, 1, \ldots, \nu_j\}, \ j = 1, 2, \ldots, n \end{array} \right\} \quad (11)$$

Then, calculate membership function values $\mu_l(Z_l(\boldsymbol{x}^*))$, $l = 1, 2, \ldots, k$ corresponding to the optimal solution $\boldsymbol{x}^*$ to (11), which is guaranteed to be an M-$R$-Pareto optimal solution to (5). Go to step 5.

**Step 5:** If the decision maker is satisfied with $\mu_l(Z_l(\boldsymbol{x}^*))$, $l = 1, 2, \ldots, k$ obtained in step 4, stop. Otherwise, ask the decision maker to update the reference membership levels $\bar{\mu}_l$, $l = 1, 2, \ldots, k$ in consideration of the current membership function values $\mu_l(Z_l(\boldsymbol{x}^*))$. Go to step 4.

## V. CONCLUSIONS

In this paper, we focused on multiobjective integer programming problems involving random variable coefficients. After we reformulated them as multiobjective simple recourse problems based on the concept of simple recourse, we introduced fuzzy goals for objective functions to consider the ambigious or fuzzy judgments of the decision maker. Then, we proposed an interactive fuzzy satisficing method as a fusion of stochastic approach and fuzzy one to derive a satisficing solution for the decision maker.

As future problems, we are going to consider an illustrative numerical example and show the efficiency of the proposed method.

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