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An Interactive Fuzzy Satisficing Method for Fuzzy Random Multiobjective 0-1 Programming Problems through Probability Maximization Using Possibility

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***Abstract*—In this paper, we focus on multiobjective 0-1 programming problems under the situation where stochastic uncertainty and vagueness exist at the same time. We formulate them as fuzzy random multiobjective 0-1 programming problems where coefficients of objective functions are fuzzy random variables. For the formulated problem, we propose an interactive fuzzy satisficing method through probability maximization using of possibility.**

I. Introduction

In the contemporary society, the case that we have to make a decision based on uncertain data or information is increasing. The stochastic programming [5], [2], [4], [21] and the fuzzy programming [24], [14], [20], [19], [18] have developed so far.

In these researches, the randomness and fuzziness have been treated separately. But, in the real decision making problem, there are many situations including two kinds of uncertainty at the same time. For example, we can think the situation that the parameters included in a formulated problem are given by uncertain numbers. As a concept to express such a situation, the concept of fuzzy random variables was proposed [12], [17], [11], [13] and its application to the mathematical programming have been done, e.g., linear programming involving fuzzy random variable coefficients by Wang and Qiao [22], interactive fuzzy random multiobjective mathematical programming by Katagiri et al. [7], [6], [8], [9], [10], fuzzy random multiobjective quadratic programming in portfolio problem by Ammar [1], multi-objective inventory problems under fuzzy random environment by Xu and Liu [23] and the survey of fuzzy stochastic linear programming by Luhandjula [15].

In particular, for multiobjective 0-1 programming problems including fuzzy random variables in the coefficients of objective functions, Katagiri et al. [9] proposed an interactive method based on the expectation optimization model and the variance minimization model using possibility and necessity. In this paper, for fuzzy random multiobjective 0-1 programming problems, we propose an interactive fuzzy satisficing method based on the probability maximization model using possibility.

In section II, we formulate fuzzy random multiobjective 0-1 programming problems. In section III, we introduce fuzzy goals to objective functions in the problems. In section IV, we discuss the formulation through the probability maximization model using possibility and propose an interactive fuzzy satisficing method. In section V, to demonstrate the usefulness of the proposed method, we apply it into an illustrative numerical example. Finally, in section VI, we conclude this paper and refer to further research.

II. Fuzzy Random Multiobjective 0-1 Programming Problems

Fuzzy random variables have been mathematically defined in various ways before now [12], [17], [11], [13]. For example, Kruse and Meyer [11] defined a fuzzy random variable as follows.

Definition 1 (Fuzzy random variable): Let (Ω, B, P) be a probability space, $F(\mathcal{R})$ the set of fuzzy numbers with compact supports and X a measurable mapping $X \to F(\mathcal{R})$. Then X is a fuzzy random variable if and only if given $\omega \in \Omega$, $X_\alpha(\omega)$ is a random interval for any $\alpha \in (0, 1]$, where $X_\alpha(\omega)$ is an α-level set of the fuzzy set $X(\omega)$.
Although there exist some minor differences in several definitions of fuzzy random variables, fuzzy random variables could be roughly understood to be a random variable whose observed values are fuzzy sets. In this paper, we consider the following fuzzy random multiobjective 0-1 programming problem:

$$\left.\begin{array}{ll} \text{minimize} & \tilde{\bar{\boldsymbol{C}}}_l \boldsymbol{x}, \ \ l = 1, 2, \ldots, k \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}, \tag{1}$$

where $\boldsymbol{x}$ is an n dimensional 0-1 decision variable column vector, A is an $m \times n$ coefficient matrix, $\boldsymbol{b}$ is an m dimensional constant column vector. Each element $\tilde{\bar{C}}_{lj}$ of vector $\tilde{\bar{\boldsymbol{C}}}_l$, $l = 1, 2, \ldots, k$ is a fuzzy random variable characterized by the following membership function:

$$\mu_{\tilde{\bar{C}}_{lj}}(\tau) = \begin{cases} L\left(\dfrac{\bar{d}_{lj} - \tau}{\bar{\beta}_{lj}}\right), & \text{if } \tau \leq \bar{d}_{lj} \\ R\left(\dfrac{\tau - \bar{d}_{lj}}{\bar{\gamma}_{lj}}\right), & \text{otherwise}, \end{cases} \tag{2}$$

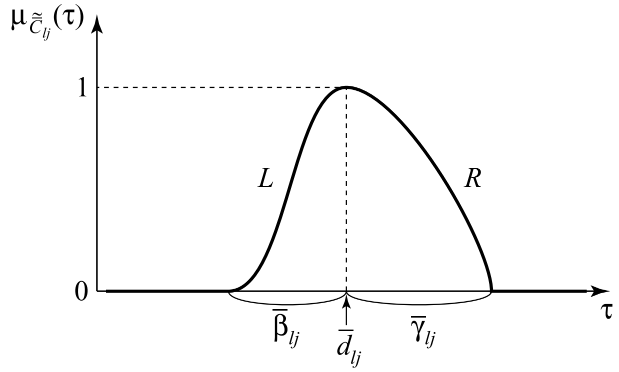

Fig. 1. An example of the membership function $\mu_{\tilde{\tilde{C}}_{lj}}(\cdot)$ of a fuzzy random variable $\tilde{\tilde{C}}_{lj}$.

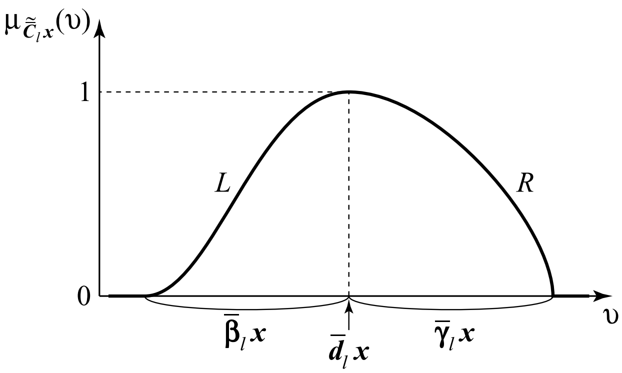

Fig. 2. An example of the membership function $\mu_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\cdot)$ of the lth objective function.

where the function $L(t) = \max\{0, \lambda(t)\}$ is a real-valued continuous function from $[0, \infty)$ to $[0, 1]$, and $\lambda(t)$ is a strictly decreasing continuous function satisfying $\lambda(0) = 1$. Also, $R(t) = \max\{0, \rho(t)\}$ satisfies the same conditions. Furthermore, $\bar{\boldsymbol{d}}_l, \bar{\boldsymbol{\beta}}_l$ and $\bar{\boldsymbol{\gamma}}_l$, $l = 1, 2, \ldots, k$ are n dimensional random variable row vectors defined as $\bar{\boldsymbol{d}}_l = \boldsymbol{d}_l^1 + \bar{t}_l \boldsymbol{d}_l^2$, $\bar{\boldsymbol{\beta}}_l = \boldsymbol{\beta}_l^1 + \bar{t}_l \boldsymbol{\beta}_l^2$ and $\bar{\boldsymbol{\gamma}}_l = \boldsymbol{\gamma}_l^1 + \bar{t}_l \boldsymbol{\gamma}_l^2$ by a random variable $\bar{t}_l$ whose mean is M_l.

Since each coefficient of objective functions is a fuzzy random variable whose observed values are L-R fuzzy numbers, each objective function becomes a fuzzy random variable characterized by the following membership function by the calculation of L-R fuzzy numbers based on the extension principle:

$$\mu_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\upsilon) = \begin{cases} L\left(\dfrac{\bar{\boldsymbol{d}}_l \boldsymbol{x} - \upsilon}{\bar{\boldsymbol{\beta}}_l \boldsymbol{x}}\right), & \text{if } \upsilon \leq \bar{\boldsymbol{d}}_l \boldsymbol{x} \\ R\left(\dfrac{\upsilon - \bar{\boldsymbol{d}}_l \boldsymbol{x}}{\bar{\boldsymbol{\gamma}}_l \boldsymbol{x}}\right), & \text{otherwise.} \end{cases} \quad (3)$$

III. Introduction of Fuzzy Goals

Now, in order to consider the vagueness of the decision maker's judgments as human, we introduce fuzzy goals $\tilde{G}_l$, $l = 1, 2, \ldots, k$ such as "$\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}$ should be substantially less than

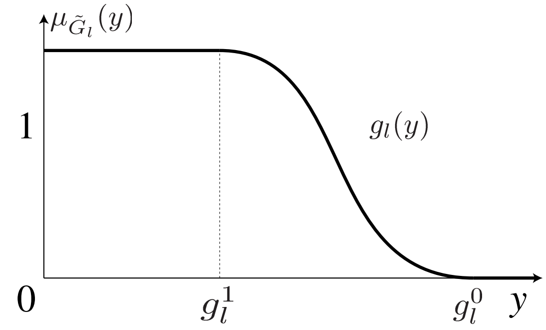

Fig. 3. An example of the membership function $\mu_{\tilde{G}_l}(y)$ of a fuzzy goal $\tilde{G}_l$.

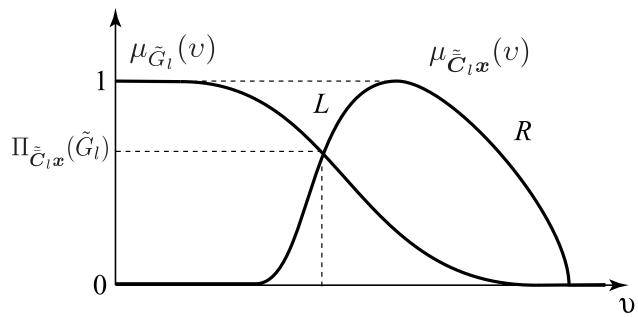

Fig. 4. The degree of possibility $\Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l)$.

or equal to a certain value" characterized by the following membership function for each objective function:

$$\mu_{\tilde{G}_l}(y) = \begin{cases} 1, & \text{if } y < g_l^1 \\ g_l(y), & \text{if } g_l^1 \leq y \leq g_l^0 \\ 0, & \text{if } y > g_l^0, \end{cases} \quad (4)$$

where $g_l(\cdot)$ is a strictly decreasing function.

IV. Probability Maximization Model Using Possibility

If we regard $\mu_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\cdot)$ as a possibility distribution, the degree $\Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l)$ of the possibility satisfying the fuzzy goal $\tilde{G}_l$ under the distribution is given by the follows using the possibility:

$$\Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l) = \sup_{\upsilon} \min\left\{\mu_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\upsilon), \mu_{\tilde{G}_l}(\upsilon)\right\}. \quad (5)$$

In this research, we consider the following problem to maximize the degree of possibility that each fuzzy goal is fulfilled in place of (1):

$$\left.\begin{array}{ll} \text{maximize} & \Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l), \ l = 1, 2, \ldots, k \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \quad (6)$$

Since possibilities $\Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l)$ in (6) vary at random because of the randomness of $\bar{\boldsymbol{d}}_l$, $\bar{\boldsymbol{\beta}}_l$ and $\bar{\boldsymbol{\gamma}}_l$, problem (6) is a stochastic multiobjective 0-1 programming problem. Here, the maximization of $\Pi_{\tilde{\tilde{\boldsymbol{C}}}_l \boldsymbol{x}}(\tilde{G}_l)$ in (6) is replaced with the maximization of

$\Pr\left[\Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l) \geq h_l\right]$ based on the probability maximization model to maximize the probability that $\Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l)$ is greater than or equal to a certain permissible level h_l:

$$\left.\begin{array}{ll} \text{maximize} & \Pr\left[\Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l) \geq h_l\right],\ l = 1, 2, \ldots, k \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \tag{7}$$

For any elementary event, inequalities $\Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l) \geq h_l$, $l = 1, 2, \ldots, k$ can be transformed as:

$$\begin{aligned} & \Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l) \geq h_l \\ \Leftrightarrow\ & \sup_{v} \min \left\{\mu_{\tilde{\bar{C}}_l \boldsymbol{x}}(v),\ \mu_{\tilde{G}_l}(v)\right\} \geq h_l \\ \Leftrightarrow\ & \exists v : \mu_{\tilde{\bar{C}}_l \boldsymbol{x}}(v) \geq h_l,\ \mu_{\tilde{G}_l}(v) \geq h_l \\ \Leftrightarrow\ & \exists v : L\left(\frac{\bar{\boldsymbol{d}}_l \boldsymbol{x} - v}{\bar{\boldsymbol{\beta}}_l \boldsymbol{x}}\right) \geq h_l,\quad R\left(\frac{v - \bar{\boldsymbol{d}}_l \boldsymbol{x}}{\bar{\boldsymbol{\gamma}}_l \boldsymbol{x}}\right) \geq h_l, \\ & \quad \mu_{\tilde{G}_l}(v) \geq h_l \\ \Leftrightarrow\ & \exists v : \{\bar{\boldsymbol{d}}_l - L^*(h_l)\bar{\boldsymbol{\beta}}_l\}\boldsymbol{x} \leq v \leq \{\bar{\boldsymbol{d}}_l + R^*(h_l)\bar{\boldsymbol{\gamma}}_l\}\boldsymbol{x}, \\ & \quad v \leq \mu^*_{\tilde{G}_l}(h_l) \\ \Leftrightarrow\ & \{\bar{\boldsymbol{d}}_l - L^*(h_l)\bar{\boldsymbol{\beta}}_l\}\boldsymbol{x} \leq \mu^*_{\tilde{G}_l}(h_l) \end{aligned}$$

where $L^*(\cdot), R^*(\cdot)$ and $\mu^*_{\tilde{G}_l}(\cdot)$ are pseudo-inverse functions defined as $L^*(h_l) = \sup\{r \mid L(r) \geq h_l\}$, $R^*(h_l) = \sup\{r \mid R(r) \geq h_l\}$, $\mu^*_{\tilde{G}_l}(h_l) = \sup\{r \mid \mu_{\tilde{G}_l}(r) \geq h_l\}$, $0 < h_l \leq 1$.

In addition, if we assume $\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x} > 0$, $l = 1, 2, \ldots, k$ for all $\boldsymbol{x} \in \{\boldsymbol{x} \in \{0, 1\}^n \mid A\boldsymbol{x} \leq \boldsymbol{b}\}$ and we denote the distribution function of the random variable $\bar{t}_l$ by $T_l(\cdot)$, we obtain

$$\begin{aligned} & \Pr\left[\Pi_{\tilde{\bar{C}}_l \boldsymbol{x}}(\tilde{G}_l) \geq h_l\right] \\ =\ & \Pr\left[\{\bar{\boldsymbol{d}}_l - L^*(h_l)\bar{\boldsymbol{\beta}}_l\}\boldsymbol{x} \leq \mu^*_{\tilde{G}_l}(h_l)\right] \\ =\ & \Pr\left[\{(\boldsymbol{d}^1_l + \bar{t}_l \boldsymbol{d}^2_l) - L^*(h_l)(\boldsymbol{\beta}^1_l + \bar{t}_l \boldsymbol{\beta}^2_l)\}\boldsymbol{x} \leq \mu^*_{\tilde{G}_l}(h_l)\right] \\ =\ & \Pr\left[\bar{t}_l \leq \frac{\{L^*(h_l)\boldsymbol{\beta}^1_l - \boldsymbol{d}^1_l\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x}}\right] \\ =\ & T_l\left(\frac{\{L^*(h_l)\boldsymbol{\beta}^1_l - \boldsymbol{d}^1_l\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x}}\right). \end{aligned}$$

Then, problem (7) is transformed into the following equivalent deterministic multiobjective 0-1 programming problem:

$$\left.\begin{array}{ll} \text{maximize} & p_l(\boldsymbol{x}) = \\ & \quad T_l\left(\dfrac{\{L^*(h_l)\boldsymbol{\beta}^1_l - \boldsymbol{d}^1_l\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x}}\right), \\ & \qquad l = 1, 2, \ldots, k \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \tag{8}$$

We introduce fuzzy goals like "$p_l(\boldsymbol{x})$ should be substantially greater than or equal to a certain value" to consider the vagueness of the decision maker's judgments on $p_l(\boldsymbol{x})$ in (8). Then, problem (8) is reformulated as the following problem:

$$\left.\begin{array}{ll} \text{maximize} & (\mu_1(p_1(\boldsymbol{x})), \ldots, \mu_k(p_k(\boldsymbol{x}))) \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \tag{9}$$

In order to derive a satisficing solution to (9), we develop an interactive fuzzy satisficing method that the decision maker interactively updates the reference membership levels $\bar{\mu}_l$, $l = 1, 2, \ldots, k$ reflecting his aspiration level to each fuzzy goal considering the optimal solution to the following minimax problem

$$\left.\begin{array}{ll} \text{minimize} & \max_{l=1,\ldots,k} \{\bar{\mu}_l - \mu_l(p_l(\boldsymbol{x}))\} \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \tag{10}$$

Introducing an auxiliary variable v, (10) is rewritten as:

$$\left.\begin{array}{ll} \text{minimize} & v \\ \text{subject to} & \bar{\mu}_l - \mu_l(p_l(\boldsymbol{x})) \leq v,\ l = 1, 2, \ldots, k \\ & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}, \tag{11}$$

equivalently,

$$\left.\begin{array}{ll} \text{minimize} & v \\ \text{subject to} & p_l(\boldsymbol{x}) \geq \mu^*_l(\bar{\mu}_l - v),\ l = 1, 2, \ldots, k \\ & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\} \tag{12}$$

where $\mu^*_l(\cdot)$ is a pseudo-inverse function defined as $\mu^*_l(s) = \inf\{r \mid \mu_l(r) \geq s\}$, $0 < s \leq 1$.

Then, problem (12) can be rewritten as:

$$\left.\begin{array}{ll} \text{minimize} & v \\ \text{subject to} & \dfrac{\{L^*(h_l)\boldsymbol{\beta}^1_l - \boldsymbol{d}^1_l\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x}} \\ & \qquad \geq T^*_l(\mu^*_l(\bar{\mu}_l - v)), \\ & \qquad l = 1, 2, \ldots, k \\ & A\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\} \tag{13}$$

where $T^*_l(\cdot)$ is a pseudo-inverse function defined as $T^*_l(s) = \inf\{r \mid T_l(r) \geq s\}$, $0 < s \leq 1$.

We consider the method to obtain the solution for the problem (13) using branch-and-bound method. When we solve by branch-and-bound method, we consider the following continuous relaxed problem:

$$\left.\begin{array}{ll} \text{minimize} & v \\ \text{subject to} & \dfrac{\{L^*(h_l)\boldsymbol{\beta}^1_l - \boldsymbol{d}^1_l\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}^2_l - L^*(h_l)\boldsymbol{\beta}^2_l\}\boldsymbol{x}} \\ & \qquad \geq T^*_l(\mu^*_l(\bar{\mu}_l - v)), \\ & \qquad l = 1, 2, \ldots, k \\ & A\boldsymbol{x} \leq \boldsymbol{b} \\ & 0 \leq x_j \leq 1,\ j = 1, 2, \ldots, n \\ & \boldsymbol{x} \in R^n \end{array}\right\} \tag{14}$$

Here, it is equivalent to obtaining minimum v existing feasible solutions to obtain minimum v of the problem. It is equivalent to obtaining minimum v where an executable solution exists to obtain minimum v of problem(14). Note that the following inequalities hold

$$\bar{\mu}_{\max} - 1 \le v \le \bar{\mu}_{\max}$$

where $\bar{\mu}_{\max}$ is the maximal value of all $\bar{\mu}_l,\ l = 1, 2, \ldots, k$.

Obtaining the optimal value of v to problem (14) is equivalent to finding the minimum of v so that the set of feasible solutions to (14) is not empty. Although (14) is a nonlinear programming problem, we can easily find the minimum of v by the algorithm based on the bisection method and the simplex method since the constraints of (14) are linear if v is fixed.

After the minimum value v^* of v is obtained, in order to determine $\boldsymbol{x}^*$ corresponding to v^* uniquely, we substitute v^* for the constrains of problem (14) and solve the following linear fractional programming problem:

$$\left.\begin{array}{ll} \text{minimize} & \dfrac{-\{L^*(h_1)\boldsymbol{\beta}_1^1 - \boldsymbol{d}_1^1\}\boldsymbol{x} - \mu^*_{\tilde{G}_1}(h_1)}{\{\boldsymbol{d}_1^2 - L^*(h_1)\boldsymbol{\beta}_1^2\}\boldsymbol{x}} \\ \text{subject to} & \dfrac{\{L^*(h_l)\boldsymbol{\beta}_l^1 - \boldsymbol{d}_l^1\}\boldsymbol{x} + \mu^*_{\tilde{G}_l}(h_l)}{\{\boldsymbol{d}_l^2 - L^*(h_l)\boldsymbol{\beta}_l^2\}\boldsymbol{x}} \\ & \qquad \ge T_l^*(\mu_l^*(\bar{\mu}_l - v^*)), \\ & \qquad\qquad l = 1, \ldots, k \\ & A\boldsymbol{x} \le \boldsymbol{b} \\ & 0 \le x_j \le 1,\ \ j = 1, 2, \ldots, n \\ & \boldsymbol{x} \in R^n \end{array}\right\}. \tag{15}$$

Since (15) is a linear fractional programming problem, using the variable transformation by Charnes and Cooper [3]

$$t = \frac{1}{\{\boldsymbol{d}_1^2 - L^*(h_1)\boldsymbol{\beta}_1^2\}\boldsymbol{x}}, \quad \boldsymbol{y} = t \cdot \boldsymbol{x}, t > 0$$

and letting $\tau_l = T_l^*(\mu_l^*(\bar{\mu}_l - v^*))$, (15) is transformed into the following equivalent linear programming problem:

$$\left.\begin{array}{ll} \text{minimize} & -\{L^*(h_1)\boldsymbol{\beta}_1^1 - \boldsymbol{d}_1^1\}\boldsymbol{y} - \mu^*_{\tilde{G}_1}(h_1) \cdot t \\ \text{subject to} & [\tau_l\{\boldsymbol{d}_l^2 - L^*(h_l)\boldsymbol{\beta}_l^2\} \\ & \quad +\{\boldsymbol{d}_l^1 - L^*(h_l)\boldsymbol{\beta}_l^1\}]\boldsymbol{y} \\ & \qquad -\mu^*_{\tilde{G}_l}(h_l) \cdot t \le 0, \\ & \qquad\qquad l = 1, \ldots, k \\ & \{\boldsymbol{d}_1^2 - L^*(h_1)\boldsymbol{\beta}_1^2\}\boldsymbol{y} = 1 \\ & A\boldsymbol{y} - t \cdot \boldsymbol{b} \le 0 \\ & 0 \le y_j \le t,\ \ j = 1, 2, \ldots, n \\ & t \ge 0 \end{array}\right\}. \tag{16}$$

[Interactive Fuzzy Satisficing Method]

Step 1: In order to specify membership functions $\mu_{\tilde{G}_l}(\cdot)$ of fuzzy goals G_l for objective functions, the following optimization problems to minimize and maximize the expectation of each objective function are solved.

$$\left.\begin{array}{ll} \text{minimize} & (\boldsymbol{d}_l^1 + M_l \cdot \boldsymbol{d}_l^2)\boldsymbol{x} \\ \text{subject to} & A\boldsymbol{x} \ \le \ \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\},\ l = 1, 2, \ldots, k \tag{17}$$

$$\left.\begin{array}{ll} \text{maximize} & (\boldsymbol{d}_l^1 + M_l \cdot \boldsymbol{d}_l^2)\boldsymbol{x} \\ \text{subject to} & A\boldsymbol{x} \ \le \ \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\},\ l = 1, 2, \ldots, k \tag{18}$$

Since these problems are linear 0-1 programming problems, they can be solved by the branch and bound method using linear programming. On the basis of optimal values to these problems, ask the decision maker to determine the membership functions $\mu_{\tilde{G}_l}(\cdot)$ and permissible levels $h_l,\ l = 1, 2, \ldots, k$.

Step 2: In order to specify membership functions $\mu_l(\cdot)$ of fuzzy goals for $p_l(\cdot)$, the following optimization problems to minimize and maximize each of $p_l(\cdot)$ are solved.

$$\left.\begin{array}{ll} \text{minimize} & p_l(\boldsymbol{x}) \\ \text{subject to} & A\boldsymbol{x} \le \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}, l = 1, 2, \ldots, k \tag{19}$$

$$\left.\begin{array}{ll} \text{maximize} & p_l(\boldsymbol{x}) \\ \text{subject to} & A\boldsymbol{x} \le \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}, l = 1, 2, \ldots, k \tag{20}$$

Since these problems are reduced to linear fractional 0-1 programming problems, they can be solved by the branch and bound method using the variable transformation by Charnes and Cooper [3] and linear programming. On the basis of optimal values to these problems, ask the decision maker to determine the membership functions $\mu_l(\cdot),\ l = 1, 2, \ldots, k$.

Step 3: Set the initial reference membership levels $\bar{\mu}_l,\ l = 1, 2, \ldots, k$ to 1.0.

Step 4: For the reference membership levels $\bar{\mu}_l,\ l = 1, 2, \ldots, k$, solve the corresponding minimax problem:

$$\left.\begin{array}{ll} \text{minimize} & \max\limits_{l=1,\ldots,k}\{\bar{\mu}_l - \mu_l(p_l(\boldsymbol{x}))\} \\ \text{subject to} & A\boldsymbol{x} \le \boldsymbol{b} \\ & \boldsymbol{x} \in \{0, 1\}^n \end{array}\right\}. \tag{21}$$

This problem can be solved by the branch and bound method using the bisection method, the variable transformation by Charnes and Cooper [3] and linear programming.

Step 5: If the decision maker is satisfied with the current solution obtained in step 4, the algorithm is terminated. Otherwise, update the reference membership levels $\bar{\mu}_l,\ l = 1, 2, \ldots, k$ and return to Step 4.

In Step 4, we can use a branch-and-bound method based on bisection method and simplex method to solve the minimax problem (13).

V. Numerical Example

To demonstrate the effectiveness of the proposed interactive satisficing method for fuzzy random multiobjective 0-1 programming problem, we consider the following problem (22) as a numerical example:

$$\left.\begin{array}{ll} \text{minimize} & \tilde{\bar{C}}_{11}x_1+\tilde{\bar{C}}_{12}x_2+\tilde{\bar{C}}_{13}x_3+\tilde{\bar{C}}_{14}x_4 \\ & \quad +\tilde{\bar{C}}_{15}x_5+\tilde{\bar{C}}_{16}x_6+\tilde{\bar{C}}_{17}x_7 \\ & \quad\quad +\tilde{\bar{C}}_{18}x_8+\tilde{\bar{C}}_{19}x_9 \\ & \quad\quad\quad +\tilde{\bar{C}}_{110}x_{10}, \\ \text{minimize} & \tilde{\bar{C}}_{21}x_1+\tilde{\bar{C}}_{22}x_2+\tilde{\bar{C}}_{23}x_3+\tilde{\bar{C}}_{24}x_4 \\ & \quad +\tilde{\bar{C}}_{25}x_5+\tilde{\bar{C}}_{26}x_6+\tilde{\bar{C}}_{27}x_7 \\ & \quad\quad +\tilde{\bar{C}}_{28}x_8+\tilde{\bar{C}}_{29}x_9 \\ & \quad\quad\quad +\tilde{\bar{C}}_{210}x_{10}, \\ \text{minimize} & \tilde{\bar{C}}_{31}x_1+\tilde{\bar{C}}_{32}x_2+\tilde{\bar{C}}_{33}x_3+\tilde{\bar{C}}_{34}x_4 \\ & \quad +\tilde{\bar{C}}_{35}x_5+\tilde{\bar{C}}_{36}x_6+\tilde{\bar{C}}_{37}x_7 \\ & \quad\quad +\tilde{\bar{C}}_{38}x_8+\tilde{\bar{C}}_{39}x_9 \\ & \quad\quad\quad +\tilde{\bar{C}}_{310}x_{10}, \\ \text{subject to} & x_1+x_2+x_3+0.3x_4+0.3x_5+x_6 \\ & \quad +2x_7+x_8+0.1x_9+1.5x_{10} \\ & \quad\quad \leq 5, \\ & 200x_1+600x_2+300x_3+200x_4 \\ & \quad +200x_5+2000x_6+1000x_7 \\ & \quad\quad +700x_8+100x_9 \\ & \quad\quad\quad +700x_{10}\leq 2500, \\ & x_1+x_2+x_3+x_4+x_5+x_6+x_7 \\ & \quad +x_8+x_9+x_{10}\leq 6, \\ & x_1+x_2+x_3+x_4+x_5+x_6+x_7 \\ & \quad +x_8+x_9+x_{10}\geq 1, \\ & x_j\in\{0,1\},\quad j=1,\ldots,10 \end{array}\right\} \quad (22)$$

In the problem (22), each parameters for each objective function is given as the following numbers:

$\boldsymbol{d}_1^1=(-2,-3,-2,-1,-1.5,-2.5,-4,-3,-0.5,-3)$,
$\boldsymbol{d}_2^1=(6,5,3,2,4,8,12,-5,-3,-5)$,
$\boldsymbol{d}_3^1=(5,6,4,3,3,6,10,6,1,7)$,
$\boldsymbol{d}_1^2=(0.3,0.5,0.2,0.1,0.2,0.5,0.8,0.7,0.05,0.8)$,
$\boldsymbol{d}_2^2=(0.4,0.5,0.2,0.2,0.3,0.6,1.1,0.4,0.2,0.4)$,
$\boldsymbol{d}_3^2=(0.6,0.7,0.2,0.3,0.1,0.5,1.0,0.5,0.01,0.3)$,
$\boldsymbol{\beta}_1^1=(0.1,0.2,0.1,0.1,0.1,0.2,0.3,0.2,0.1,0.1)$,
$\boldsymbol{\beta}_2^1=(0.1,0.2,0.1,0.2,0.2,0.3,0.4,0.2,0.2,0.2)$,
$\boldsymbol{\beta}_3^1=(0.2,0.2,0.1,0.1,0.1,0.2,0.1,0.1,0.1,0.1)$,
$\boldsymbol{\beta}_1^2=(0.01,0.02,0.01,0.01,0.01,0.02,0.03,0.02,0.01,0.01)$,
$\boldsymbol{\beta}_2^2=(0.01,0.02,0.01,0.02,0.02,0.03,0.04,0.02,0.02,0.02)$,
$\boldsymbol{\beta}_3^2=(0.02,0.02,0.01,0.01,0.01,0.02,0.01,0.01,0.01,0.01)$,
$\boldsymbol{\gamma}_1^1=(0.2,0.4,0.2,0.2,0.2,0.4,0.6,0.4,0.2,0.2)$,
$\boldsymbol{\gamma}_2^1=(0.2,0.4,0.2,0.4,0.4,0.6,0.8,0.4,0.4,0.4)$,
$\boldsymbol{\gamma}_3^1=(0.4,0.4,0.2,0.2,0.2,0.4,0.2,0.2,0.2,0.2)$,
$\boldsymbol{\gamma}_1^2=(0.02,0.04,0.02,0.02,0.02,0.04,0.06,0.04,0.02,0.02)$,
$\boldsymbol{\gamma}_2^2=(0.02,0.04,0.02,0.04,0.04,0.06,0.08,0.04,0.04,0.04)$,
$\boldsymbol{\gamma}_3^2=(0.04,0.04,0.02,0.02,0.02,0.04,0.02,0.02,0.02,0.02)$,

and random variables $\bar{t}_l$, $l=1,2,3$ are assumed to be Gaussian random variables with mean 0 and variance 5^2.

TABLE I
PROCESS OF INTERACTION

	1st	2nd	3rd
$\bar{\mu}_1$	1.0	1.0	1.0
$\bar{\mu}_2$	1.0	1.0	0.8
$\bar{\mu}_3$	1.0	0.7	0.7
$\mu_1(p_1(\boldsymbol{x}))$	0.506	0.664	0.608
$\mu_2(p_2(\boldsymbol{x}))$	0.575	0.618	0.417
$\mu_3(p_3(\boldsymbol{x}))$	0.539	0.298	0.329
x_1	0	0	1
x_2	1	1	0
x_3	0	0	0
x_4	0	1	0
x_5	0	0	1
x_6	0	0	0
x_7	0	0	0
x_8	1	1	1
x_9	0	1	1
x_{10}	0	0	0

For this numerical example, we apply the interactive fuzzy satisficing method proposed in the previous section and the result is summarized in Table I.

After solving all of (17) and (18) by the branch and bound method using and linear programming, ask the decision maker to determine the membership function $\mu_{\tilde{G}_l}(\cdot)$ for each objective function $\tilde{\bar{\boldsymbol{C}}}_l\boldsymbol{x}$ in (1) and permissible levels $h_1=0.6$, $h_2=0.6$, $h_3=0.6$.

For these permissible levels, all of (19) and (20) are solved by the branch and bound method using the variable transformation [3] and linear programming, ask the decision maker to determine the membership function $\mu_l(\cdot)$ for each objective function $p_l(\cdot)$ in (8).

Then, the initial reference membership levels $\bar{\mu}_l$, $l=1,2,3$ are set to 1.0 and the corresponding minimax problem (21) is solved. The result is shown in the second column of Table I. Since the decision maker prefers to improve $\mu_1(p_1(\boldsymbol{x}))$ at the sacrifice of $\mu_3(p_3(\boldsymbol{x}))$, he updates the reference membership levels to $\bar{\mu}_1=1.0$, $\bar{\mu}_2=1.0$, $\bar{\mu}_3=0.7$.

Again the minimax problem for the updated reference membership levels is solved and the result is shown in the third column of Table I. Since the decision maker feels that $\mu_3(p_3(\boldsymbol{x}))$ is too low, he updates the reference membership levels to $\bar{\mu}_1=1.0$, $\bar{\mu}_2=0.8$, $\bar{\mu}_3=0.7$ to enlarge $\mu_3(p_3(\boldsymbol{x}))$ even if $\mu_2(p_2(\boldsymbol{x}))$ decreases.

After the corresponding minimax problem is solved, the result is obtained shown in the fourth column of Table I. Since the decision maker is satisfied with the result, the algorithm is terminated.

VI. Conclusion

In this paper, we focused on multiobjective 0-1 programming problems whose coefficients of objective functions are fuzzy random variables. After introducing fuzzy goals for objective functions to reflect the vagueness of the decision maker's judgment as human, we regarded the minimization of objective functions as the maximization the degree of possibility that each objective function fulfills the corresponding fuzzy goal. Since the degree of possibility is a random variable, we adopted the probability maximization model as a decision making model. Then, we reduced the fuzzy random multiobjective 0-1 programming problem to a deterministic multiobjective 0-1 programming problem and discussed an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker. In the discussion, we showed that all problems in the proposed interactive method can be solved by the branch and bound based on linear programming. In the future, we will discuss the case based on the degree of necessity and other stochastic programming models, and consider fuzzy random multiobjective integer programming problems.

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