

Error Correction Mechanism for Convergence toward the Rational Expectations in the Perspective of a Brief Survey*

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Introduction

Chapter 1: Rational Expectations.

Chapter 2: Convergence to Rational Expectations.

Introduction

The idea of the rational expectations hypothesis (REH) is important theoretical concept in macroeconomic analyses. The theories of convergence toward the rational expectations argue that the mere fact of the efficient utilization of all available information by economic agents, the assumption of identity between agents' expectations and the predictions of a relevant economic theory , and the presumption that agents know the objective probability distribution of outcomes of their concern are insufficient to justify the REH. This paper represents an attempt to introduce an empirically testable convergence mechanism, one of error correcting, toward rational expectations. The paper is organized as follows. In chapter one, we briefly summarize major contributions with implications for the REH, and make a critique of the hypothesis. Chapter two contains four models analyzing learning mechanisms toward rational expectations, and their comparisons. In the end, a further development of the learning mechanism is sketched using a simple error correction model.

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Chapter 1: Rational Expectations

1-3 Major Implications

Muth [1961] introduced the concepts of rationality and rational expectations. He used an agricultural market model represented by a set of equations,

$$\begin{aligned}c_t &= -\beta p_t && \text{(Demand)} \\ P_t &= \gamma p_t^e + u_t && \text{(Supply)} \\ P_t &= c_t && \text{(Market equilibrium)}\end{aligned} \tag{1}$$

Where P_t , c_t , p_t , p_t^e , and u_t are output, consumption, the market price, the expected market price based upon information available through the $(t-1)$ period, and error term with the mean 0 and no serial correlation, respectively (All the variables are measured in terms of deviations from the equilibrium values).

Muth postulates that the average opinion about the future price in an industry is the same as the mathematical expectation derived from the relevant economic model, and called such expectations rational.⁽¹⁾ In other words, this hypothesis claims that rational agents make use of the economic model in which they are modeled. There are two underlying assumptions for the above expectation formation process. (1) The economic system makes use of the available information efficiently. (2) The expectation formation depends specifically on the structure of the economic theory describing the economy.

The equation system (1) exemplifies the concept of rational expectations. By substituting from demand and supply equations into the equilibrium condition, the relationship

$$P_t = \frac{\gamma}{\beta} P_t^e - \frac{1}{\beta} u_t \tag{2}$$

is derived. A conditional expectation of the model is thus

$$E_{t-1} P_t = -\frac{\gamma}{\beta} P_t^e \tag{3}$$

If the aggregate expectation of firms is identical with equation (3), then

(1) Muth rephrased it as follows. The subjective probability distribution of outcomes tends to be identical with the objective probability distribution for the same information set.

$$E_{t-1}p_t = p_t^e. \quad (4)$$

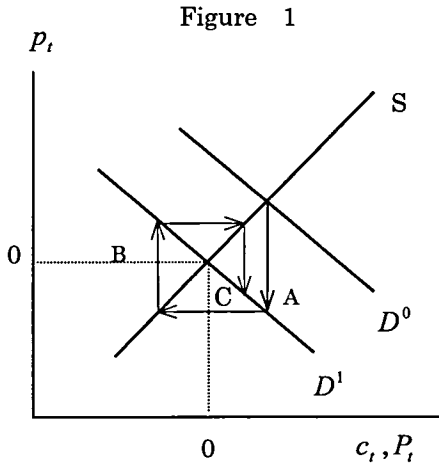
As long as $\frac{\gamma}{\beta} \neq -1$, equations (3) and (4) implies $p_t^e = 0$, or that the price expectation is the same as the equilibrium price. Returning to equation (2), rational expectations $p_t^e = 0$ makes the market price a random walk (white noise)

$$p_t = -\frac{1}{\beta}u_t. \quad (5)$$

The motivation for the use of rational expectations is made clear by comparing it with static and adaptive expectations⁽²⁾ formation scheme (Uzawa and Miyakawa [40]). Suppressing error term from equation system (1), let $p_t^e = p_{t-1}$, then equation (2) turns into

$$p_t = -\frac{\gamma}{\beta}p_{t-1}, \quad (2')$$

so $p_t = p_0 \left(-\frac{\gamma}{\beta}\right)^t$ where p_0 is the initial value of the market price, and $0 < \frac{\gamma}{\beta} < 1$ is assumed.



In figure 1, the demand curve shifts from D^0 to D^1 , and the market price follows the path $A \rightarrow B \rightarrow C \rightarrow \dots$. On the other hand, under an adaptive scheme⁽³⁾

$$p_t^e = \alpha p_{t-1} + (1-\alpha)p_{t-1}^e$$

where $0 < \alpha \leq 1$, equation (2) becomes

$$p_t = -\frac{\gamma}{\beta}[\alpha p_{t-1} + (1-\alpha)p_{t-1}^e]. \quad (2'')$$

(2) See Uzawa and Miyakawa [1982].

(3) See Nerlove [1958] and Muth [1960]. Muth [1960] showed that if a time series is represented as a random walk with a current noise

$$y_t = \varepsilon_t + \sum_{i=1}^{\infty} w_i \varepsilon_{t-i},$$

where ε_t 's are white noises, then an adaptive scheme gives the optimal forecast.

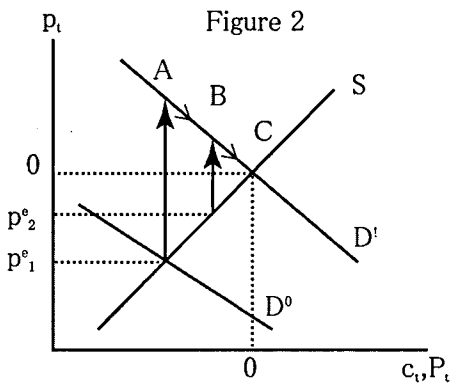
Assuming that equation (1) (without u_t) holds in the period $(t-1)$, p_{t-1}^e is written as

$$p_{t-1}^e = \frac{1}{\gamma} P_{t-1} = -\frac{\beta}{\gamma} p_{t-1}.$$

Therefore, $p_t = p_0 \left[1 - \alpha \left(\frac{\gamma}{\beta} + 1 \right) \right]^t$ obtains where convergence condition is now

$$0 < \alpha \left(\frac{\gamma}{\beta} + 1 \right) < 2.$$

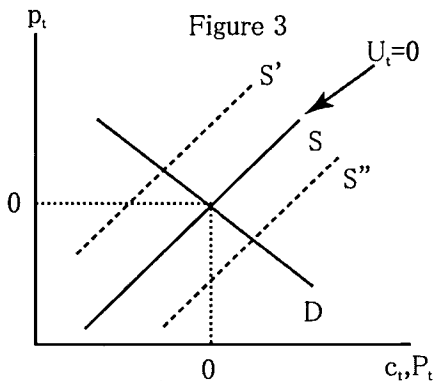
For sufficiently small α , figure 2 shows the dynamic path of the market price



$A \rightarrow B \rightarrow C$ when demand shifts from D^0 to D^1 . These two cases are compared with figure 3 which represents the rational expectations case. In figure 3, it holds that

$$p_t^e = 0$$

which is shown by the intersection between supply and demand curves while the actual supply schedule is fluctuating between S' and S'' due to unpredictable disturbances in production. Figure 1 through figure 3 show that the static and adaptive expectation formation scheme model the behavior of agents who do not change the expectation formation process while facing the continuous and systematic errors in their expectations, and forming rational expectations corrects such errors.



Lucas [1972] first applied the Muth rational expectations hypothesis to the macroeconomic model, and derived the neutrality result. His model has a general equilibrium framework in which economic agents decide their behavior through intertemporal utility maximization. Basic assumptions are as follows. (1) The model

economy consists of the two physically separated markets. There are two stochastic disturbances influencing the transactions in each markets, i.e., the allocation of traders and money supply. The former represents the real disturbances due to demand shifts (and relative price change), and the latter the nominal price fluctuations. Information on these disturbances is imperfectly transmitted only through market prices. (2) Based upon Samuelson [1958], the time horizon consists of two periods (the current and the next). In each period, N identical people are born, and live for two periods. The constant population of the economy is thus $2N$ (N of age 0 and N of age 1). Only age 0 person supplies n units of labor, and produces the same units of output. Free disposal and no storage of output are assumed, therefore, the relations $c^0 + c^1 \leq n$, and $c^0 \geq 0$, $c^1 \geq 0$, $n \geq 0$ hold where c^0, c^1 are consumption by young and old generations. (3) There exists fiat money which enters the economy as a transfer to the older generation at the beginning of the period in proportion to the pretransfer holding. No inheritance is assumed. (4) Only possible exchange takes the form of the sales of output by the young to the old in exchange for money. For simplicity of the analysis, it is assumed that the old generation is equally allocated into two markets. The young are allocated in a stochastic way, $\frac{\theta}{2}$ of them going to one, and $1 - \frac{\theta}{2}$ to the other. Once the allocation of agents is made, no switching and no communication between the markets are allowed. In each market, a single market clearing price will prevail. (5) The pretransfer money supply m is known to all agents, but posttransfer quantity $m' = mx$ is unknown until next period where x is a random variable representing a monetary policy instrument. Similarly, θ is an unknown random variable. (6) x' and θ' denote the next period's value of x and θ . x and x' are assumed to be independent, and are assumed to follow the common density f on $(0, \infty)$. θ and θ' are also independent, and have the common density g on $(0, 2)$. (7) The older generation gives no utility to the holding of money, and will supply money inelastically. (8) All members of the younger generation have the common utility function $U(c, n) + EV(c')$ where U is increasing in c , is decreasing in n , is strictly concave, and is twice continuously differentiable. V is increasing, is strictly concave, and is twice continuously differentiable. (9) The young can not buy c' directly. They get cash balance λ in exchange for the goods they produced. (10) Let p' and x' be the next period's price level and the transfer, the next period's balances will then purchase $\frac{x'\lambda}{p}$ units of c' . The conditional distribution function of (x', p') is denoted by $F(x', p' | m, p)$ where p is the current price. Now, under some regularity conditions

on U and V in addition to the above assumptions, the young generation's decision problem is formulated as

$$\max \left[U(c, u) + \int V \left(\frac{x' \lambda}{P} \right) dF(x', p' | m, p) \right]$$

$$\text{s.t. } p(n - c) - \lambda \geq 0.$$

The Kuhn-Tucker conditions are

$$U_c(c, n) - p\mu \leq 0, \quad \text{with equality if } c > 0, \quad (6)$$

$$U_n(c, n) + p\mu \leq 0, \quad \text{with equality if } n > 0, \quad (7)$$

$$p(n - c) - \lambda \geq 0, \quad \text{with equality if } \mu > 0, \quad (8)$$

$$\int V' \left(\frac{x' \lambda}{p'} \right) \frac{x'}{p'} - dF(x', p' | m, p) - \mu \leq 0, \quad \text{with equality if } \lambda > 0 \quad (9)$$

, where μ is a nonnegative multiplier. Equations (6)-(8) are solved for c , n and $p\mu$ as functions of $\frac{\lambda}{p}$ ⁽⁴⁾, in that $p\mu$ is interpreted as the marginal cost of cash holding in terms of foregone utility. Also it follows that n is increasing, and c is decreasing function of $\frac{\lambda}{p}$.

Therefore, $p\mu$ which is denoted as $h\left(\frac{\lambda}{p}\right)$ is shown to be positive, increasing, and continuously differentiable function of $\frac{\lambda}{p}$ ⁽⁵⁾. These relations among c , n , $p\mu = h\left(\frac{\lambda}{p}\right)$, and $\frac{\lambda}{p}$ have fundamental importance in the following arguments. Replacing $p\mu$ by $n\left(\frac{\lambda}{p}\right)$ in equation (9),

(4) For $c > 0$, $n > 0$, equations (6), (7), and (8) give $U_c(c, n) = -U_n(c, n)$ and $c = n - \frac{\lambda}{p}$. Let $\frac{\lambda}{p} = \alpha$, then

$$U_c(n - \alpha, n) = -U_n(n - \alpha, n) \quad \text{and} \quad U_{cc} \left(\frac{\partial n}{\partial \alpha} - 1 \right) + U_{cn} \frac{\partial n}{\partial \alpha} = - \left[U_{nc} \left(\frac{\partial n}{\partial \alpha} - 1 \right) + U_{nn} \frac{\partial n}{\partial \alpha} \right] \quad \text{where } U_{cc},$$

U_{cn} , and U_{nn} are the second partials. Thus, assuming the "noninferiority" conditions $U_{cn} + U_{nn} < 0$

and $U_{cc} + U_{cn} < 0$, the relationship $\frac{\partial n}{\partial \alpha} = \frac{U_{cc} + U_{nc}}{U_{cc} + 2U_{nc} + U_{nn}} > 0$ (The continuity of the 1st partials

are assumed). Similarly, $\frac{\partial c}{\partial \alpha} < 0$ obtains.

(5) By equations (6) and (7), $U_c(c, n) = -U_n(c, n)$ for $c > 0$, $n > 0$. So the relationships $U_{cc} = -U_{cn} < 0$, and $U_{cn} > 0$ follow.

$$h\left(\frac{\lambda}{p}\right)\frac{1}{p} \geq \int V'\left(\frac{x'\lambda}{p}\right)\frac{x'}{p}dF(x', p'|m, p) \quad (10)$$

obtains with equality if $\lambda > 0$. This is implicitly a demand for money function which expresses the current nominal demand for money λ as a function of current and expected prices (p and p').

Lucas analyzes the general equilibrium framework by concentrating on one market due to the assumptions of the identical structure of the two markets and of no communication between them.

The equilibrium condition in the market that receives the $\frac{\theta}{2}$ fraction of the young is characterized by the equality between nominal money demand and supply (In other words, nominal demand and supply of goods are in equilibrium). Total money supply is Nmx (inelastic money supply is assumed), and $\frac{Nmx}{2}$ is supplied in the market.

Money supply per demander is $\frac{\frac{Nmx}{2}}{\frac{\theta N}{2}} = \frac{mx}{\theta}$, thus the equilibrium is represented by

$\lambda = \frac{mx}{\theta}$ Substitution of $\lambda = \frac{mx}{\theta} > 0$ into relationship (10) gives the equilibrium condition

$$h\left(\frac{mx}{\theta p}\right)\frac{1}{p} = \int V'\left(\frac{mxx'}{\theta p'}\right)\frac{x'}{p'}dF(x', p'|m, p) \quad (11)$$

which relates p to p' . Once this condition is solved for p , the equilibrium values of n (employment and output) and c are obtained through the relationship among n , c , and $\frac{\lambda}{p}$. In Lucas' paper, the rational expectations hypothesis is introduced in the derivation of the connection between p and p' . The motion of this economy is characterized by the vector (m, x, θ) , so the price is expressed as a function of (m, x, θ) . He assumes that the "true" conditional (on m) probability distribution of the next period's price $p' = p(m', x', \theta') = p(mx, x', \theta')$ can be derived from the known distribution of x, x', θ' . This is equivalent to saying that the agents' subjective conditional probability distribution of (x, x', θ') is the same as the objective conditional probability distribution. We can thus see the Muth' definition of the REH being adopted in the

context of agent's preference maximization which derives the neutrality result in the subsequent parts.

An age 0 agent takes the expectation of equation (11) with respect to the conditional (on $p(m, x, \theta)$) probability distribution of (x, x', θ') which is denoted as $G(x, x', \theta' | p(m, x, \theta))$. Substituting these notations into equation (11), we get

$$h \left[\frac{mx}{\theta p(m, x, \theta)} \right] \frac{1}{p(m, x, \theta)} = \int V' \left[\frac{mxx'}{\theta p(m\xi, x', \theta')} \right] \frac{x'}{p(m\xi, x', \theta')} dG(\xi, x', \theta' | p(m, x, \theta)) \quad (12)$$

, where ξ represents information obtained on x ($x = \xi$ is assumed later). This gives the definition of the equilibrium price that is a continuous, nonnegative function $P(\cdot)$ of (m, x, θ) which satisfies equation (12).

Introducing a change of variables and additional notations

$$p(m, x, \theta) = m\phi\left(\frac{x}{\theta}\right), \quad z = \frac{x}{\theta}, \quad z' = \frac{x'}{\theta'}, \quad H(z, \theta): \text{ the joint density of } z \text{ and } \theta,$$

$\bar{H}(z, \theta)$: conditional (on z) density of θ , equation (12) becomes

$$h \left[\frac{z}{\phi(z)} \right] \frac{z}{\phi(z)} = \int V' \left[\frac{\theta' z'}{\theta \phi(z)} \right] \frac{\theta' z'}{\theta \phi(z')} \bar{H}(z, \theta) H(z', \theta') d\theta dz' d\theta' \quad (13)$$

where independence among

$\theta - \theta', x - x', z - z', \theta - x, \theta' - x' \dots$, the probability relation

$$g_0(\theta, z', \theta' | z) = g_1(\theta | z) g_2(z', \theta' | \theta, z) = g_1(\theta | z) g_2(z', \theta')$$

(g_i 's are densities), and the assumption $\xi = x$ were used. The existence of an equilibrium price (or $\phi(z)$) which satisfies equations (12) and (13) is shown from the contraction fixed point theorem⁽⁶⁾. Based upon this result, he goes on to the derivation of the three key theorems. "Theorem 2": Let $\theta = 1$ with probability 1, and y^* be the unique solution to $h(y) = V'(y)$, then $p(m, x, \theta) = \frac{mx}{y^*}$ ⁽⁷⁾ is a unique solution to equation (12).

"Theorem 3": Let $x = 1$ with probability 1, then equation (12) has a unique solution $p(m, x, \theta) = m\phi\left(\frac{1}{\theta}\right)$ ⁽⁸⁾ where ϕ has an elasticity $0 < \frac{z\phi'(z)}{\phi(z)} < 1$. "Theorem 4": Under

(6) See Apostol [2].

(7) This shows that if the disturbance is known to be purely nominal, then no real effect will result.

y^* corresponds to $\left(\frac{\lambda}{p}\right)^*$ which decides n^* and c^* .

(8) Thus, if the nature of the disturbances is purely real, the change in the price will have the real effects on n and c through the change in $\frac{\lambda}{p(m, x, \theta)}$.

some other additional assumptions, equation (12) has a unique solution $p(m, x, \theta) = m\phi\left(\frac{x}{\theta}\right)$ where the ϕ 's elasticity is between 0 and 1.

Now, comparing theorem 2 with theorem 4, he derives the neutrality of money which is represented in the way that "monetary changes have real consequences only because agents can not discriminate perfectly between real and monetary demand shifts". This statement can be rephrased as follows. If there does not exist real disturbances such as random changes in population, a monetary change can have no real effects under the REH, and because real disturbances are purely random, the monetary policy can not realize the expected effects on real variables.

In comparison with this result, the Friedmanian k-percent money supply rule⁽⁹⁾ is discussed, and he shows that this rule actually satisfies the Pareto optimality in the economy under consideration.

Lucas [1976] made use of the REH to criticize the econometric evaluation of policy effects. In summary, he points out that the central assumption of the econometric policy evaluation is the invariability of functional form and parameter values of agents' behavioral equations under arbitrary changes of policy variables, and argues that rational agents will take account of policy changes in advance, and will consequently change their decision variables. Therefore, the effects of alternative policy rules can not be compared based upon current econometric models regardless of the forecasting performance of these models.

On the basis of Lucas' analysis, Sargent and Wallace [1975] [1976] derived the drastic result of policy ineffectiveness under the REH.

Sargent and Wallace [1975] derived the (monetary) policy ineffectiveness from the following model:

Lucas supply function

$$y_t = a_1 k_{t-1} + a_2 (p_t - p_{t-1}^*) + u_{1t}, \quad a_i > 0, \quad i = 1, 2. \quad (14)$$

Demand function

(9) Friedman [1968] insisted that monetary policy does not affect real variables in the long run through agents' expectation formation, and actual monetary policy has been taken "too late and too far" and has caused the economic disorder. His recommendation was to fix the money growth rate to a certain percentage to produce moderate inflation or deflation rather than the wide and erratic fluctuations. Friedman [1977] went on further to say that the wide variation in the actual and anticipated inflation created by discretionary monetary policies lowers economic efficiency (price signal jammed, etc.), and explains the upward sloping Phillips Curve as shown by the data in the recent periods.

$$y_t = b_1 k_{t-1} + b_2 [r_t - ({}_{t+1}p_{t-1}^* - {}_t p_{t-1}^*)] + b_3 z_t + u_{2t}, \quad b_1 > 0, b_2 < 0. \quad (15)$$

Portfolio balance equation

$$m_t = p_t + c_1 y_t + c_2 r_t + u_{3t}, \quad c_1 > 0, c_2 < 0. \quad (16)$$

Determination of productive capacity

$$k_t = d_1 k_{t-1} + d_2 [r_t - ({}_{t+1}p_{t-1}^* - {}_t p_{t-1}^*)] + d_3 z_t + u_{4t}, \quad d_2 < 0. \quad (17)$$

Evolution of exogenous variables

$$z_t = \sum_{j=1}^q \rho_j z_{t-j} + \xi_t \quad (18)$$

$$u_{it} = \sum_{j=1}^q \rho_{ij} u_{i,t-j} + \xi_{it},$$

where y_t , p_t , and m_t are the logs of output, the price level, and the money supply; r_t is the nominal interest rate (not in the log); z_t is the vector of exogenous variables (government expenditure, tax rate, etc.); ${}_{t+i}p_{t-j}^*$ is the public's expectation formed at the end of $t-j$ of the price level to prevail in the period of $t+i$; k_t is a measure of productive capacity (capital, labor, etc.); ξ_t 's are white noises, and u 's are random errors which are generated by equation (18).

The monetary authority has two choices for the stabilization policy; (1) A deterministic feedback interest rate rule

$$r_t = G\theta_{t-1}^* \quad (19)$$

Where θ_{t-1}^* is the vector of all of the current and the past endogenous and exogenous variables at the end of the period $t-1$, and G is a conformable parameter vector. (2) A deterministic money supply feedback rule

$$m_t = H\theta_{t-1}^* \quad (20)$$

where H is the counterpart of G in equation (19).

The authority chooses G or H to minimize a certain type of (quadratic) loss function. They compare the policy implications for the autoregressive and the rational expectation versions of the price level.

In the autoregressive version, the two distributed-lag schemes are assumed:

$$p_t^* = \sum_{i=0}^q v_{1i} p_{t-i} \quad (21)$$

$$p_t^* = \sum_{i=0}^q v_{2i} p_{t-i} \quad (22)$$

where v_{1i} and v_{2i} are fixed coefficients.

Suppose the authority is following the money supply rule (20). Equations (14)-(18), (21), and (22) have a reduced form difference equation

$$Y_{1t} = \sum_{i=1}^q A_i Y_{1t-i} + \sum_{i=0}^q B_i m_{t-i} + \phi_{1t}, \quad (23)$$

where Y_{1t} is a vector ($y_t, p_t, r_t, k_{t-1}, z_t$), and ϕ_{1t} is a random vector consisting of ξ 's in equation (18). The A_i 's are conformable vector, and B_i 's are scalars both of which depend on the parameters of equations (14)-(18), and (22). The authorities decide H of equation (20) to minimize the loss function subject to equation (23).

This process is examined for the one period reduced forms for y_t and p_t ; From equations (14), (15), and (16),

$$p_t = J_0 ({}_t p_{t-1}^*) + J_1 ({}_{t+1} p_{t-1}^*) + J_2 m_t + X_t \quad (24)$$

is obtained where

$$J_0 = [a_2(1 + b_2 c_2^{-1}) + b_2] / [a_2(1 + b_2 c_2^{-1}) + b_2 c_2^{-1}] < 1,$$

$$J_1 = (1 - J_0) / (1 - c_2^{-1}),$$

$$J_2 = -c_2^{-1} J_1,$$

and X_t is a linear function of k_{t-1} , z_t , and u_{it} 's. Substituting from equation (24) into equation (14) gives the one-period reduced form of y_t , and thus it turns out that the monetary policy rule (20) affects real output under the autoregressive expectation scheme.

Now, the rational expectation of the price level is defined as

$${}_{t+i} p_{t-j}^* = E_{t-j} p_{t+i} \quad (25)$$

where $E_{t-j} p_{t+i}$ is the mathematical expectation of p_{t+i} conditional on all information available through the end of period $t-j$ represented by θ_{t-j}^* .

With equation (25) assumed, equations (14), (15), and (16) give the reduced form

$$p_t = J_0 E_{t-1} p_t + J_1 E_{t-1} p_{t+1} + J_2 m_t + X_t. \quad (26)$$

Taking expectation $E_{t-1} p_t$ of equation (26), and subtracting it from equation (26) gives

$$\begin{aligned} p_t - E_{t-1} p_t &= J_2 (m_t - E_{t-1} m_t) \\ &+ X_t - E_{t-1} X_t = X_t - E_{t-1} X_t \end{aligned} \quad (\text{due to equation (20)}), \quad (27)$$

where $X_t - E_{t-1} X_t$ contains only the exogenous innovations in equation (18). Therefore, the expectation error $p_t - E_{t-1} p_t$ is independent of the money supply rule implying that (after substituting from equations (25) and (27) into equation (14)) the monetary policy is ineffective in the determination of output:

$$y_t = a_1 k_{t-1} + a_2 (X_t - E_{t-1} X_t) + u_t. \quad (28)$$

Further, substituting from equation (28) into equation (15), the real rate of interest also turns out to be independent of the monetary policy.

Sargent and Wallace [1976] argues that the following three conditions are necessary for the monetary policy to be taken in a discretionary manner. (a) The economic structure is of the extensive simultaneity, (b) The effects of shocks on the endogenous variables are serially correlated, and therefore predictable, (c) The structure of these lag effects is constant over time, and does not depend on policy actions taken by the monetary authorities.⁽¹⁰⁾

They refute the condition (c), and show that when the agents' expectations are rational in the Muth sense, the monetary policy with a feedback rule does not affect the real variable (unemployment here) at all. The structural equations of their analysis are the Phillips Curve conformable with the natural rate hypothesis and the price equation reflecting "Okun's Law".

$$p_t - p_{t-1} = \phi_0 + \phi_1 u_t + p_t^* - p_{t-1} + \varepsilon_t, \quad \phi_1 < 0, \quad (29)$$

or

$$p_t - p_{t-1}^* = \phi_0 + \phi_1 u_t + \varepsilon_t, \quad (30)$$

and

$$p_t = a m_t + b x_t + c u_t, \quad c < 0, \quad (31)$$

where u_t is the unemployment rate, p_t the log of the price level, p_t^* the log of the agents' price expectation formed as of the period $t-1$, m_t the log of the money supply, x_t a vector of exogenous variables, and ε_t a random error.

(10) The condition (c) hits on the Lucas critique. The policy ineffectiveness analysis of Sargent-Wallace [1976] centers on this assumption.

When agents form expectations rationally

$$p_t^* = E_{t-1} p_t, \quad (32)$$

then,

$$u_t = [a(m_t - E_{t-1} m_t) + b(x_t - E_{t-1} x_t) - \varepsilon_t] / (\phi_1 - c) - \frac{\phi_0}{\phi_1} \quad (33)$$

follows.

Now suppose the monetary authorities use the feedback policy rule

$$m_t = G\theta_{t-1} + \eta_t \quad (34)$$

where θ_{t-1} and G are conformable vectors representing a set of variables observed in the periods $t-1$ and earlier, and a list of policy coefficients, respectively. η_t is an independent error with $E[\eta_t | \theta_{t-1}] = 0$. Agents are assumed to form rational expectations on m_t . Thus

$$E_{t-1} m_t = E m_t | \theta_{t-1}. \quad (35)$$

Substituting from equation (20) into equation (18),

$$u_t = [a\eta_t + b(x_t - E_{t-1} x_t) - \varepsilon_t] / (\phi_1 - c) - \frac{\phi_0}{\phi_1} \quad (36)$$

is obtained where G does not appear, therefore it turns out that unemployment is independent of the discretionary choice of G . The policy implication they derive from this conclusion is that the feedback rule under the REH is identical with setting $G = 0$ (avoiding the price fluctuation caused by G in equation (31)), and thus the k-percent rule is provided with a theoretical foundation.

1-3 Rational expectations with rigidities.

Fischer [1977] and Taylor [1980] took into account the institutional aspect (the wage contract), and showed that there is room for an active monetary policy to influence the real variables even under the REH.⁽¹¹⁾ In line with the representation that the Sargent-Wallace result of neutrality holds in a one period contract case, Fisher [1977] introduces the two period nonindexed labor contracts⁽¹²⁾, i.e., (1) All labor contracts are drawn up at the end of the period t , specify nominal wages for the periods $(t+1)$ and

(11) Comparing with other REH models which are characterized as having the equilibrium framework, their analysis gives conspicuous examples of the disequilibrium approach with the REH.

(12) Indexed contracts which duplicate the effects of one period contracts are shown to make monetary policy lose its effectiveness, but are regarded as unrealistic in general mainly due to the difficulty of calculating their terms.

($t+2$) and are subject to

$${}_{t-i}W_t = {}_{t-i}P_t, \quad i = 1, 2 \quad (37)$$

to keep constancy of the real wage where ${}_{t-i}W_t$ and ${}_{t-i}P_t$ are the logs of the nominal wage paid in t as specified in contracts drawn up at the period ($t-i$), and the expected price formed at the end of the period ($t-i$). (2) The aggregate supply curve is

$$Y_t^s = \frac{1}{2} \sum_{i=1}^2 (P_t - {}_{t-i}W_t) + u_t = -\frac{1}{2} \sum_{i=1}^2 (P_t - {}_{t-i}P_t) + u_t \quad (38)$$

where u_t is a random error. (3) The velocity equation

$$Y_t = M_t - P_t - v_t \quad (39)$$

is assumed where Y_t , M_t , and v_t are the logs of output, money supply, and error term, respectively. (4) u_t and v_t are assumed to follow the first order Markov process

$$u_t = \rho_1 u_{t-1} + \varepsilon_t, \quad |\rho_1| < 1 \quad (40)$$

$$v_t = \rho_2 v_{t-1} + \eta_t, \quad |\rho_2| < 1 \quad (41)$$

where ε_t and η_t are white noises. (5) The monetary rule is

$$M_t = \sum_{i=1}^{\infty} a_i u_{t-i} + \sum_{i=1}^{\infty} b_i v_{t-i} \quad (42)$$

Now using the REH and the relation $E_{t-2}({}_{t-1}P_t) = {}_{t-2}P_t$ ⁽¹³⁾

those equations produce

$$Y_t = \frac{1}{2}(\varepsilon_t - \eta_t) + \frac{1}{3}[\varepsilon_{t-1}(a_1 + 2\rho_1) + \eta_{t-1}(b_1 - \rho_2)] + \rho_1^2 u_{t-2} \quad (43)$$

as output. Essentially, this arises from the monetary action in the interim based on the information of ε_{t-1} and η_{t-1} . The monetary authorities react to these new disturbances and can change the real wage⁽¹⁴⁾ (and therefore the real output level). Thus, corresponding to the Sargent and Wallace [1976]'s optimal feedback rule, the output variance minimizing values of a_1 and b_1 are as follows.

$$\begin{aligned} a_1 &= -2\rho_1 \\ b_1 &= \rho_2 \end{aligned} \quad (44)$$

Taylor [1982] derives serial correlation (business cycles) of real variables under

$$\begin{aligned} (13) \quad E_{t-2}({}_{t-1}P_t) &= \iint p_t g_1(p_t) dp_t \Big| g_2(p_t) dp_t \\ &= \iint p_t g_2(p_t) dp_t \Big| g_1(p_t) dp_t \\ &= p_t \int g_1(p_t) dp_t = p_t \end{aligned}$$

where g_1 and g_2 are conditional densities with information set available through the end of $t-1$ and $t-2$.

(14) P_t can be shown to be a function of M_t .

“staggered” contract with the REH, and shows that the stabilization policy is a nontrivial problem. He assumes that wage contracts are staggered. When all wage contracts are N periods long, $1/N$ fraction of all firms draws up their contracts. Wage rates in the current contracts reflect previous and future rates (weights are assumed to be proportional to the number of overlapping periods), and are sensitive to the state of the labor market.

Thus, a wage setting procedure is

$$x_t = \sum_{s=1}^{N-1} b_s x_{t-s} + \sum_{s=0}^{N-1} b_s \hat{x}_{t+s} + \frac{h}{n} \sum_{s=0}^{N-1} \hat{e}_{t+s} + \varepsilon_t \quad (45)$$

where x_t , \hat{x}_{t+s} , \hat{e}_t , ε_t , and b_s are the logs of the nominal and of the expected ($E(\cdot | I_{t-1})$ where I_{t-1} is the information set representing information available through the period of $t-1$) wages, a measure of excess demand for labor, a white noise

error, and the symmetric weights of past and future wages ($b_s = \frac{1-s}{N-1}$ changes

proportionately to the number of overlapping periods⁽¹⁵⁾), respectively. Other conditions of his model are the following equations:

$$y_t + p_t = m_t + v_t \quad (46)$$

$$p_t = \frac{1}{N} \sum_{i=0}^{i=n-1} x_{t-i} \quad (47)$$

$$e_t = g_2 y_t \quad (48)$$

$$m_t = g_3 p_t \quad (49)$$

where

p_t = log of price level, y_t = log of deviation of real output from full-employment output, m_t = log of deviation of nominal money supply from its full-employment level, v_t = stochastic velocity shock.

Now Substituting from equation (48) into equation (45), and equation (49) into equation (46),

$$x_t = \sum_{s=1}^{N-1} b_s x_{t-s} + \sum_{s=1}^{N-1} b_s \hat{x}_{t+s} + \frac{\gamma}{N} \sum_{s=0}^{N-1} \hat{y}_{t+s} + \varepsilon_t \quad (50)$$

$$y_t = -\beta p_t + v_t \quad (51)$$

(15) Weights of the past and future (expected) wages sum up to one:

$$\begin{aligned} 2 \sum_{s=1}^{N-1} b_s &= \frac{2}{N-1} \left[(N-1) - \frac{1}{N} \sum_{s=1}^{N-1} s \right] = \frac{2}{N-1} \left[(N-1) - \frac{1}{N} \frac{N(N-1)}{2} \right] \\ &= 2 \left(1 - \frac{1}{2} \right) = 1. \end{aligned}$$

obtains where $\gamma = hg_2$ and $\beta = (1 - g_3)$.

From equations (47) and (51) (taking the conditional expectation),

$$\hat{y}_{t+s} = -\beta \hat{p}_{t+s} = -\frac{\beta}{N} \sum_{i=0}^n \hat{x}_{t+s-i}, \quad (52)$$

and substituting, from equation (52) into equation (50),

$$x_t = \sum_{s=1}^n b_s \hat{x}_{t-s} + \sum_{s=1}^n b_s \hat{x}_{t+s} - \frac{\beta\gamma}{N_2} \sum_{s=0}^n \sum_{i=0}^n \hat{x}_{t+s-i} + \varepsilon_t \quad (50')$$

obtains where $n = N - 1$. This is further rearranged to

$$\sum_{s=1}^n b_s \hat{x}_{t-s} - c \hat{x}_t + \sum_{s=1}^n b_s \hat{x}_{t+s} = 0 \quad (53)$$

where $c = \frac{N + \beta\gamma}{N - n\beta\gamma}$. While denoting the lag polynomials in equation (53) by

$$B(L) = \sum_{s=-n}^n b_s L^s \quad (54)$$

equation (53) can be expressed as

$$B(L) = \hat{x}_t = 0. \quad (55)$$

Applying the factorization theorem,

$$B(L) = \lambda A(L)A(L^{-1}) \quad (56)$$

where λ is a normalization constant, and

$$A(L) = \sum_{s=0}^n \alpha_s L^s \quad \text{with } \alpha_0 = 1, \quad (54')$$

the rational expectations reduced-form stochastic difference equation is

$$A(L)x_t = \varepsilon_t \quad (57)$$

But from equation (47),

$$p_t = D(L)x_t \quad (58)$$

holds where $D(L) = \frac{1}{N} \sum_{s=0}^n L^s$. Using equations (57) and (58),

$A(L)x_t = \varepsilon_t$, and $A(L)[D(L)]^{-1} p_t = \varepsilon_t$, thus

$$A(L)p_t = D(L)\varepsilon_t \quad (59)$$

follows.

This gives the behavior of output (and so unemployment) through equation (51).

Taylor[1980] shows this derivation for the case of $N = 2$: Equation (59) gives

$$P_t = a_1 p_{t-1} + \frac{\varepsilon_t}{2} + \frac{\varepsilon_{t-1}}{2}, \quad 0 \leq a_1 \leq 1 \quad (60)$$

where $a_1 = -\alpha_1$ in (54'). Changing this ARMA (1,1) process to the moving average

representation,

$$p_t = \frac{1}{2}[\varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \dots] \quad (61)$$

holds where $\psi_i = a_1^{i-1}(1 + a_1)$, $i = 1, 2, \dots$. Now equation (61) gives the moving average representation for real output

$$y_t = v_t - \frac{\beta}{2}[\varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \dots]. \quad (62)$$

In equation (62), two things are observed. (1) If the wage contracts are staggered, the occurrence of serial correlations (business cycle) of output and employment is theoretically explained. (2) The monetary policy variable g_3 in $\beta = 1 - g_3$ affects the real variables unless $g_3 = 1$ which is characterized as the perfectly accommodative monetary control.

1-3 Other works

In Lucas [1973], he introduces the “Lucas supply function”, and gives an empirical implication to the natural rate hypothesis based upon the data from sixteen countries.

He assumes that the log of supply in a certain market (z) at time t ($y_t(z)$) consists of the log of a normal (or secular, depending on capital accumulation and population change) component (y_m) and the log of a cyclical component in z ($y_{ct}(z)$)

$$y_t(z) = y_m + y_{ct}(z) \quad (63)$$

, where $y_m = \alpha + \beta t$ ⁽¹⁶⁾, and

$$y_{ct} = \gamma[p_t(z) - E(p_t | I_t(z))] + \lambda y_{c,t-1}(z) \quad (64)$$

are assumed. $p_t(z)$ is the actual price in z in the period t , and $E(p_t | I_t(z))$ the conditional mean of the aggregate price level based on $I_t(z)$ that is information available in z in the period t . The prior distribution of p_t is assumed to be normal with the mean \bar{p}_t and the variance σ^2 . $p_t(z)$ is related to p_t by

$$p_t(z) = p_t + z \quad (65)$$

(16) This equation has a role only in the derivation of equilibrium output.

(17) $|\lambda| < 1$ is assumed. All variables are in the logarithmic term.

, where z is normal with the mean 0 and the variance τ^2 , and independent of p_t .

From these assumptions, the optimal (unbiased, minimum variance) forecast of p_t , ($E(p_t | I_t(z))$) is derived as

$$E(p_t | I_t(z)) = (1 - \theta)p_t(z) + \theta \bar{p}_t \quad (66)$$

where $\theta = \frac{\tau^2}{\sigma^2 + \tau^2}$ while the variance of p_t is $\theta\sigma^2$.⁽¹⁸⁾ Combining equations (63),

(64), and (66) gives the market supply function

$$y_t(z) = y_{nt} + \theta\gamma(p_t(z) - \bar{p}_t) + \lambda y_{c,t-1}(z) \quad (67)$$

Averaging over markets,

$$y_t = y_{nt} + \theta\gamma(p_t - \bar{p}_t) + \lambda(y_{t-1} - y_{n,t-1}) \quad (68)$$

obtains. With a demand function given as

$$y_t + p_t = x_t \quad (69)$$

where x_t is the log of nominal output, and Δx_t is assumed to be independent, normal variable with the mean δ and the variance σ_x^2 ,⁽¹⁹⁾ the conjectured price solution is

$$p_t = \Pi_0 + \Pi_1 x_t + \Pi_2 x_{t-1} + \Pi_3 x_{t-2} + \dots + \eta_1 y_{t-1} + \eta_2 y_{t-2} + \dots + \xi_0 y_{nt}. \quad (70)$$

$$(18) \quad P_t = \bar{p}_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

$$P_t(z) = P_t + z_t, \quad z_t \sim N(0, \tau^2)$$

Also, the information set contains $P_t(z)$ and \bar{p}_t . Let linear combination of \bar{p}_t and

$p_t(z)$ form an optimal forecast of P_t^*

$$P_t^* = \alpha \bar{p}_t + \beta p_t(z),$$

then $P_t^* = (\alpha + \beta)P_t - \alpha\varepsilon_t + \beta z_t$, and the forecast error is

$$P_t^* - p_t = (\alpha + \beta - 1)p_t - \alpha\varepsilon_t + \beta z_t.$$

Unbiasedness requires $E(P_t^* - p_t) = 0$, and thus $\alpha + \beta = 1$, so

$P_t^* = \alpha \bar{p}_t + (1 - \alpha)p_t(z)$. Minimum variance requirement is that

$\text{var}(P_t^* - p_t) = \alpha^2 \sigma^2 + (1 - \alpha)^2 \tau^2$, and

$$\frac{\partial \text{var}(P_t^* - p_t)}{\partial \alpha} = 2\alpha\sigma^2 - 2(1 - \alpha)\tau^2 = 0, \text{ thus } \alpha = \frac{\tau^2}{\sigma^2 + \tau^2}.$$

Setting this α equal to θ in the text, $\theta = \frac{\tau^2}{\sigma^2 + \tau^2}$ obtains while

$$E[p_t - ((1 - \theta)p_t(z) + \theta \bar{p}_t)]^2 = \theta^2 \sigma^2 + (1 - \theta)^2 \tau^2 = \theta \sigma^2 \text{ holds.}$$

(19) By assuming unit elasticity for x_t with respect to p_t , x_t is regarded as exogenous in the sense that the change in the slope of supply function does not alter x_t .

Using $E(\Delta x_t) = \delta$,

$$p_t = \Pi_0 + \Pi_1(x_{t-1} + \delta) + \Pi_2 x_{t-1} + \Pi_3 x_{t-2} + \dots + \eta_1 y_{t-1} + \eta_2 y_{t-2} + \dots + \xi_0 y_{nt} \quad (71)$$

obtains. Subtracting equation (57) from equation (56), and using equations (68) and (69) gives the identity

$$y_{nt} = \theta \gamma \Pi_1 (\Delta x - \delta) + \lambda (y_{t-1} - y_{n,t-1}) = -\Pi_0 + (1 - \Pi_1)x_t - \Pi_2 x_{t-1} - \Pi_3 x_{t-2} - \dots - \eta_1 y_{t-1} - \eta_2 y_{t-2} - \dots - \xi_0 y_{nt}.$$

Solving this by the undetermined parameter method,

$$\Delta p_t = -\beta + (1 - \Pi)\Delta x_t + \Pi \Delta x_{t-1} - \lambda \Delta y_{c,t-1} \quad (72)$$

, and similarly (using $y_{nt} = \alpha + \beta t$),

$$y_{ct} = -\Pi \delta + \Pi \Delta x_t + \lambda y_{c,t-1} \quad (73)$$

obtains where

$$\Pi = \frac{\theta \gamma}{1 + \theta \gamma}. \quad (74)$$

Substituting from $\theta = \frac{\tau^2}{\sigma^2 + \tau^2}$ into equation (74),

$$\Pi = \frac{\tau^2 \gamma}{\sigma^2 + \tau^2 (1 + \gamma)}. \quad (74')$$

But from equation (57), the conditional variance of $p_t(\sigma^2)$ is identical with

$$(1 - \Pi)^2 \sigma_x^2$$

$$\Pi = \frac{\tau^2 \gamma}{(1 - \Pi)^2 \sigma_x^2 + \tau^2 (1 + \gamma)} \quad (75)$$

which is negatively related to σ_x^2 for fixed τ^2 and γ . His empirical finding is that the larger the variance of the change in nominal GNP (Δx_t), the smaller the estimated Π in equation (73). From this result, it is evident that the slope of the Phillips Curve tends to be "less favorable" when the aggregate demand policies which changes the inflation rate are volatile. Not to mention, this result is consistent with the natural rate hypothesis and the k-percent money supply rule.

Lucas [1977] explains business cycles in a general equilibrium setting. His approach is characterized as the general equilibrium framework with imperfect information. Sustained inflation or deflation can not continue to affect unemployment

and real output (homogeneity of degree 0 for supply and demand function). However, when agents make decisions optimally, and they can not discriminate between general and relative price movements,⁽²⁰⁾ a temporary general price change can induce business cycles, i.e., when agents observe the increase in their commodity price (local), they misunderstand this increase as a relative price change, and raise production levels (future capacity effect postpone agents' recognition of their mistake through dampening the general price level, so that the period of business cycle is prolonged).

He then introduces the money balances as a major cause of business cycles. If there were a fixed relation between the short-term general price movements and the lagged movements in some known monetary aggregates, agents could easily avoid the discrepancies described above. But in reality, a discretionary monetary policy would make it impossible for agents to get such information on the general price level through observing the magnitude of monetary aggregates. Rather sizable and unsystematic shifts in monetary aggregates add an additional noise to the agents' expectation formation process, and create comovements among prices, unemployment, real output, etc.

The policy implication derived from these arguments is to adopt monetary stability to reduce the volatility in real, aggregate variables, and to increase welfare.

Barro [4] derives many similar results to Lucas [1972] [1973]. (1) When agents can not discriminate between the nominal and real disturbances, a monetary policy can affect real output. (2) When the variance of money growth gets larger, agents attribute a larger fraction of price change to monetary forces, and output responsiveness to a monetary disturbance is reduced. Thus, the slope of the Phillips Curve gets steeper. (3) The k -percent money supply rule without feedback is socially optimal.

In addition to these results, he analyzes some cases with superior information for the monetary authority. The results are as follows. (1) When the authorities do not possess superior information, the monetary policy with feedback is inpotent in the determination of real output. (2) When the authority has superior information, countercyclical policies can be beneficial. But this effect can be achieved by providing people with the information (if the provision cost is neglected).

1-3 General Characterization.

(20) Information is imperfect in this sense. Although agents form their expectations on the general price level rationally in the Muth sense, temporary shocks create this difference. But if agents observe sustained movements in the general price level, then it is supposed to be recognized under the REH in the sense that agents are not "fooled" continuously and systematically.

As we have seen in this chapter, the REH gives a powerful technical tool to introduce the expectation formation mechanism into an economic analysis. In that, it was insisted that the REH is a natural assumption in the sense that it avoids the systematic errors observed in the conventional expectation formation models (static or adaptive models), and economists are obviously not interested in the analysis based on irrational postulates. However, the introduction of the REH caused a wide spread controversy on several factors such as its plausibility, the relation of its basic property with the decentralized market system, its policy implication, its relation with the well known statement of classical economics, etc. In this section, we examine these arguments to characterize the general implication of introducing the REH.

McCallum [1980] regards the REH as the “natural” hypothesis to use in the neoclassical scheme because of its plausible postulates that agents purposefully collect and utilize information, so that they can avoid making systematic errors.

Barro and Fischer [1976] shows a list of reactions to the REH, and gives their interpretations. (1) Individuals can not make the necessary calculation (to conform with the REH) since even most economists can not do. \leftrightarrow Most individuals have never seen a Lagrangean, but microeconomic analysis of individual behavior is not rejected due to this reason. (2) The REH neglects uncertainty over the agents’ model selection. \leftrightarrow It is better than a rule of thumb that are not based upon the expectations from the relevant economic model. The alternatives are accompanied with agents making systematic mistakes which is not a theoretically desirable property (common to McCallum’s). (3) The REH models always use the equilibrium approach, but there exist short-run price or wage rigidities, and this gives room for a countercyclical policy. Therefore, the policy ineffectiveness results of the REH models are not acceptable. \leftrightarrow The REH does not contradict the disequilibrium framework. It can be incorporated into a long-term labor contract setting to allow for the effectiveness of stabilizing policies.

These views seem to be representative examples that support the REH. However, there have been more critical responses. They relate the REH to the working of decentralized market economy, and clarify the basic implications of its application.

Most of the critical responses seem to refer to the general characterization of the market economy by Hayek [1945]. (1) There exists a quantity of information which is beyond any single mind. The problem is how to extend the span of utilization of resources (information) beyond the span of the control of any one mind (p 527). (2) The knowledge we need never exists in a concentrated or integrated form (but rather

dose as dispersed bits of incomplete and frequently contradictory knowledge) (p 519). (3) Economic problems arise always and only in consequence of changes, so ignorance of changes or day-to-day adjustments implies that economic problems have become less important (p 523). (4) The assumption of full knowledge being known to a single mind dismisses the economic problem, and disregards everything that is important and significant in the real world.

In accordance with these views, Uzawa and Miyazawa [1982] says that the REH assumes that the economic agents have the knowledge on the policy rules, and on the slope of demand and supply curves in the market to which they belong (therefore, the equilibrium of the goods and money markets). This type of agents are idiosyncratic in the context of usual utility and production theory because it suffices for agents to maximize utility and profits to make demand and supply decisions according to their tastes and technologies in response to price signals given by the markets. In contrast to these conventional agents, "rational" agents must recognize other agents' tastes and technologies, and aim at hitting on the market equilibrium. The process of getting knowledges on the market structure and calculating the equilibrium values includes not only collecting information on economic variables, but also collecting knowledges on the political, cultural, and social backgrounds of agents who participate the markets. Therefore, they conclude that the emergence and acceptance of the REH implies the extended application of the concept of "homoecomics" to the whole society context that is hardly justifiable. Uzawa [1979] also raises the same issues, and says that the REH implies the negation of a decentralization (from the standpoint of households and firms) in the market economy.

Yoshikawa [1981]'s criticism is that "rational" agents are considered to achieve the subjective optimization, and the REH is thus often said to have solid microfoundation. However, the optimization per se is an empty concept, and the specification of the constraint conditions under which the optimization is made is of realistic importance.⁽²¹⁾

In the context of policy ineffectiveness arguments, the REH implies that foreseen policy changes do not alter the agents' constraints.⁽²²⁾ In Keynesian economics, however, only when the policy changes are foreseen or recognized, real variables are influenced (due to the reduction in the demand for goods for firms, and the restricted liquidities for household, for instance).

Taylor [1983] casts practical doubts on the REH. (1) The costs of adopting

(21) See Solow [1980].

(22) Buiter [1980] points out that under competitive Walrasian equilibrium, fiscal policies, in general, can affect real variables.

sophisticated forecasting procedures tend to exceed benefits from the agents' point of view (the social benefits might be sufficiently large). (2) The consensus on the model selection is hard to achieve (the lack of consensus among economists is widely observed).

Frydman and Phelps [1983]'s socio-economic arguments are similar to Uzawa and Miyakawa [1982], and Uzawa [1979]. But, they evaluate the main contribution of the REH as pointing toward the macrotheoretic foundation of micro and macro economic behavior due to the fact that it postulates that "the agents modeled themselves form models of their surrounding world".

In responding to these views, the implications of the REH will be summarized as follows. To Uzawa and Miyakawa [1982] , Uzawa [1979], and Frydman and Phelps [1983], the basic concept of a decentralized market economy is kept in the models with the REH. The price signal argument is based on the model with the infinite adjustment speed. In reality information on prices is used to form the expectation about the future economic circumstances in which agents must decide their behavior. The construction that the market prices (equilibrium or disequilibrium implied) are the signals for agents to which they should conform is based on the idea that the prices will be prevailing when agents act according to their decision. However, this is actually a simplified and imaginary process. Decision making processes are essentially of the intertemporal nature. Agents thus have to base their decisions on the expectations of future events. We can talk about the expectation formation mechanism within the context of decentralized market economy.

Taylor [1983]'s points can be conceptually overcome as he indicates himself. However, those practical criticism are thought of as real importance. The validity of the REH as a useful conceptual device will depend on whether it abstracts from important aspects of agents behavior when the theory that is based on the REH tries to analyze a specific phenomenon. In Muth's original paper, he dealt with an isolated market where the frequent and detailed exchange of information among agents is plausible, and the sufficient volume of information to form rational expectations will be comparatively small, thus the REH might be appropriate to describe the agents' behavior. If the analysis involves complicated interaction among a large number of markets, and focuses on macroeconomic behavior, then there will be room to doubt the plausibility of the hypothesis. However, if the hypothesis is used to describe an average agents' behavior (while accepting the differences among them), it might turn out to be a meaningful device. The analysis of convergence to rational expectations examines this problem.

Chapter 2 : Convergence to Rational Expectations.

2-1 Convergence As a Learning Process

Friedman [1979], Decanio [1979], Cyert and Degroot [1974], Taylor [1975] paid attention to the sufficient volume of information to form rational expectations, and introduced the idea of the learning process in the context of the REH.⁽²³⁾

Friedman [1979] starts with the common sense concept of rationality. If agents do their best under given circumstances, their decision is perfectly rational. The REH requires more than that. Agents must know the functional forms and the parameter values of the relevant economic model to form rational expectations (His concern is only about this type of rational expectation formation. He intends to reexamine Sargent and Wallace's derivation([1975] · [1976]) of classical neutrality results from the standpoint of the learning process. Later, we will see Cyert and Degroot's variation of the definition of the REH).

Thus, agents need to collect the sufficient volume of information the acquisition of which is hardly evident.

Suppose now agents believe the following equation

$$y_{\tau} = x'_{\tau} \beta + u_{\tau} \tag{76}$$

to be true generating function for some variable y_{τ} where β is a vector of fixed coefficients, u_{τ} a white noise error, and x_{τ} a vector of predetermined variables. As of the period $t-1$, let agents have the current and past observations (y_{τ}, x_{τ}) for periods $\tau = t-1, t-2, \dots, T_0$, and want to form the best (in the sense of the minimum variance) forecast of future values of $y_t, y_{t+1}, y_{t+2}, \dots$ conditional on predetermined values of $x_t, x_{t+1}, x_{t+2}, \dots$.

Such expectations are

$$E_{t-1}(y_{\tau}) = x'_{\tau} b_{t-1}, \quad \tau = t, t+1, t+2, \dots \tag{77}$$

, where b_{t-1} is the least-squares estimator of β conditional on all available observations through $t-1$, and E_{t-1} is the conditional expectations operator. b_{t-1} is expressed as

(23) Bray [1982] analyzes the convergence process in the case with the two types of agents, i.e., the informed and the uninformed agents. She concludes that the REE (rational expectations equilibrium) is a long-run concept. Her contribution is in a sophisticated investigation of the conditions under which convergence is achieved. However, the results there are similar to the other papers that we will examine. The assumption that informed agents are originally in the REE, and only uninformed agents' learning is concerned seems not to be of essential importance. Results in other papers can be applied to uninformed agents' learning.

$$b_{t-1} = (X'_{t-1} X_{t-1})^{-1} X'_{t-1} y_{t-1} \quad (78)$$

where

$$X_{t-1} = \begin{bmatrix} x'_{t-1} \\ x'_{t-2} \\ \vdots \\ x'_{T_0+1} \\ x'_{T_0} \end{bmatrix} \quad Y_{t-1} = \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{T_0+1} \\ y_{T_0} \end{bmatrix}$$

As time goes on, agents will revise their expectations due to greater information available. The change in their expectation generation process is described as

$$E_t(y_\tau) = x'_\tau b_\tau, \quad \tau = t+1, t+2, \dots \quad (80)$$

where

$$b_t = (X'_t X_t)^{-1} X'_t y_t \quad (81)$$

, and X_t and y_t now include the current observations. This procedure is fully optimal in the sense of the Muth's first condition of full exploitation of all available information. If the expectations formed in the system of equation (76) through equation (81) are Muth rational, they must satisfy the error-orthogonality property (as in Muth [1961] and Sargent and Wallace [1975], [1976]). From equation (76) through equation (80), the expectation errors are written as

$$y_\tau - E_t(y_\tau) = x'_\tau (\beta - b_t) + u_\tau, \quad \tau = t+1, t+2, \dots \quad (82)$$

which satisfies the required property only when $b_t = \beta$ holds. He shows the two arguments from which the REH is unlikely: (1) misspecification, and (2) limited observation.

Misspecification arises in two instances. 1) when the functional form is wrong, and 2) when some relevant variables were erroneously omitted. In either case, agents are not necessarily supposed to find their errors as time passes,⁽²⁴⁾ thus the error-orthogonality property (therefore, the REH) is vitiated.

As for the second reason for non convergence, he argues that b_t is a consistent estimator of β from the property of error term in equation (76), however, the number of observations will be in general always insufficient to make b_t converge to β for the following reasons. (1) The agents' model always suffer from misspecification, thus it can be good an approximation to the true model only over finite time periods. (2) Due to the development in the data processing technique, old observations become

(24) It might be thought that agents will avoid making consistent errors, and will discover misspecification. However, when we think of economic theories that are changing over time (this corresponds to the change in the true structure that is generating observations in the REH context), the agents' correction of specifications may not be promising. Friedman's allegation is thus justifiable.

inconsistent with recent observations. (3) Even if the same method of data collection is utilized, the data generating process has been changing over time, and old observations are thus no longer useful to form expectations about the future.

From these characterization of the agents' learning process, he concludes that the REH is of the long-run equilibrium nature, but convergence of the learning to rational expectations is unlikely.

In the remaining part of his paper, he supports the adaptive mechanism as a realistic expectation formation process followed by economic agents (based upon his preceding arguments). He showed the following theorem that when x_t series is stationary, then the following relation holds through the iterative regression

$$b_t = b_{t-1} + \gamma_t (y_t - E_{t-1}(y_t)) \quad (83)$$

$$E_t(y_\tau) - E_{t-1}(y_\tau) = \gamma_{t\tau} (y_t - E_{t-1}(y_t)), \quad \tau = t+1, t+2, \dots \quad (84)$$

where

$$\gamma_t = \frac{(X'_{t-1} X_{t-1})^{-1} x_t}{1 + x'_t (X'_{t-1} X_{t-1})^{-1} x_t} \quad (85)$$

$$\gamma_{t\tau} = x'_\tau \gamma_t, \quad \tau = t+1, t+2, \dots \quad (86)$$

, and $\gamma_{t\tau}$ varies over time around some decreasing value

$$\gamma_t^* = \bar{x}' \gamma_t \quad (87)$$

while \bar{x} is the mean of x_t .

Therefore, it is argued that the adaptive expectation model turns out to be good an approximation to agent's expectation formation mechanism if x_t is stationary. However, it must be noted that even if $\gamma_{t\tau}$ goes to zero, and thus the relation $E_t(y_\tau) = E_{t-1}(y_\tau)$ tends to hold as time goes on (implying that observations through the period $t-1$ are approximately sufficient to make optimal expectations for the future values of y_t in the sense that observations in the next period do not greatly improve the previous expectation), this does not mean that agents expectations are Muth rational. Equation (83) tells that the key error-orthogonality property is not satisfied.

Decanio [1979] starts with extending the Muth's original model to include arbitrary lag structures for exogenous variable(s), a supply variable, and disturbances. Unlike Friedman's model, he explicitly introduces the structural change in the expectation formation function which was initially and arbitrarily conceived by agents based on full utilization of all available information, and derives the result that while observing the actual evolution of the market price, the error modification process does not necessarily leads to rational expectations.

His model is

$$q_t = -\beta p_t + \delta(L)x_t + w_t \quad (\text{Demand}) \quad (88)$$

$$q_t = \gamma p_t^e + v(L)x_t + \mu(L)q_t + v_t \quad (\text{Supply}) \quad (89)$$

while the market clearing condition is also assumed. Here q_t , p_t , p_t^e are quantity, price, and price expectation in the period t measured from their equilibrium values. x_t is an exogenous variable. $\delta(L)$, $v(L)$, and $\mu(L)$ are the lag polynomials of arbitrary orders:

$$\delta(L) = \sum_{i=0}^{\infty} \delta_i L^i, \quad v(L) = \sum_{i=0}^{\infty} v_i L^i, \quad \mu(L) = \sum_{i=0}^{\infty} \mu_i L^i \quad (90)$$

where $\mu(L)$ represents supply adjustment cost (making current supply depends only on the past levels, so $\mu_0 = 0$ is assumed).

By the market clearing condition, the relationship

$$p_t = -\left(\frac{\gamma}{\beta}\right)p_t^e - \left(\frac{1}{\beta}\right)\mu(L)q_t + \left(\frac{1}{\beta}\right)[\delta(L) - v(L)]x_t + u_t \quad (91)$$

follows where $u_t = -\left(\frac{1}{\beta}\right)(v_t - w_t)$ is assumed to be expressed as a function of white noise errors:

$$u_t = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} = \phi(L)\varepsilon_t \quad (92)$$

Here ε_t is a white noise. By arbitrariness of $\phi(L)$, the correlogram of u_t is also arbitrary ($\phi_0 = 1$ is assumed for normalization).

Agents use all available information consisting of all of the past price and quantity values, and of all of the past and current values of x_t . Based upon this assumption of the full utilization of information, the expectation formation function is

$$p_t^e = \lambda(L)p_t + \psi(L)q_t + \theta(L)x_t \quad (93)$$

where $\lambda_0 = \psi_0 = 0$, because current values of p_t and q_t are unknown when agents make expectations.

From equations (91) – (93),

$$\begin{aligned} p_t = & \left[1 - \phi^{-1}(L) - \left(\frac{\gamma}{\beta}\right)\phi^{-1}(L)\lambda(L) \right] p_t + \phi^{-1}(L) \left[-\left(\frac{\gamma}{\beta}\right)\psi(L) - \left(\frac{1}{\beta}\right)\mu(L) \right] q_t \\ & + \phi^{-1}(L) \left[-\left(\frac{\gamma}{\beta}\right)\theta(L) + \left(\frac{1}{\beta}\right)\delta(L) - \left(\frac{1}{\beta}\right)v(L) \right] x_t + \varepsilon_t \end{aligned} \quad (94)$$

obtains. Now, the Muth rational expectation means

$$E(p_t) = p_t^e \quad (95)$$

from which

$$\lambda(L) = 1 - \phi^{-1}(L) - \left(\frac{\gamma}{\beta}\right)\phi^{-1}(L)\lambda(L) \quad (96)$$

$$\psi(L) = \phi^{-1}(L) \left[-\left(\frac{1}{\beta}\right)\psi(L) - \left(\frac{1}{\beta}\right)\mu(L) \right] \quad (97)$$

$$\theta(L) = \phi^{-1}(L) \left[-\left(\frac{\gamma}{\beta}\right)\theta(L) + \left(\frac{1}{\beta}\right)\delta(L) - \left(\frac{1}{\beta}\right)\nu(L) \right] \quad (98)$$

obtains (due to comparison between equations (93) and (94)).

Equations (96) – (98) show what condition $\lambda(L)$, $\psi(L)$, and $\theta(L)$ must satisfy if agents expectations are Muth rational.

Let $\lambda(R)(L)$, $\psi(R)(L)$, and $\theta(R)(L)$ be lag polynomials satisfying equations (96) – (97), then,

$$\lambda(R)(L) = [\phi(L) - 1] \left[\phi(L) + \left(\frac{\gamma}{\beta}\right) \right]^{-1} \quad (99)$$

$$\psi(R)(L) = -\left(\frac{1}{\beta}\right)\mu(L) \left[\phi(L) + \left(\frac{\gamma}{\beta}\right) \right]^{-1} \quad (100)$$

$$\theta(R)(L) = \left(\frac{1}{\beta}\right)[\delta(L) - \nu(L)] \left[\phi(L) + \left(\frac{\gamma}{\beta}\right) \right]^{-1} \quad (101)$$

hold. However, does the system of equations (96) – (98) converge to equations (99) – (101) ? Starting with $\lambda(0)(L)$, $\psi(0)(L)$, and $\theta(0)(L)$ in (93), the actual price evolution mechanism (94) makes rational agents change the structure of their expectation formation function (This further changes price evolution structure, and so on.) in the manner of equations (96) – (98). Therefore, the equations (96) – (98) give the difference equations representing the agents' learning process:

$$\lambda(n)(L) = 1 - \phi^{-1}(L) - \left(\frac{\gamma}{\beta}\right)\phi^{-1}(L)\lambda(n-1)(L) \quad (102)$$

$$\psi(n)(L) = \phi^{-1}(L) \left[-\left(\frac{\gamma}{\beta}\right)\psi(n-1)(L) - \left(\frac{1}{\beta}\right)\mu(L) \right] \quad (103)$$

$$\theta(n)(L) = \phi^{-1}(L) \left[-\left(\frac{\gamma}{\beta}\right) \theta(n-1)(L) + \left(\frac{1}{\beta}\right) \delta(L) - \left(\frac{1}{\beta}\right) \nu(L) \right], \quad (104)$$

solutions of which are

$$\lambda(n)(L) = [\lambda(0)(L) - \lambda(R)(L)] \left[-\left(\frac{\gamma}{\beta}\right) \phi^{-1}(L) \right]^n + \lambda(R)(L) \quad (105)$$

$$\psi(n)(L) = [\psi(0)(L) - \psi(R)(L)] \left[-\left(\frac{\gamma}{\beta}\right) \phi^{-1}(L) \right]^n + \psi(R)(L) \quad (106)$$

$$\theta(n)(L) = [\theta(0)(L) - \theta(R)(L)] \left[-\left(\frac{\gamma}{\beta}\right) \phi^{-1}(L) \right]^n + \theta(R)(L). \quad (107)$$

Picking up $\lambda(n)(L)$, for example, the convergence to rational expectations ($\lambda(n)(L) \rightarrow \lambda(R)(L)$) is assured if $\lambda(0)(L) = \lambda(R)(L)$ (that means no need to learn,

or mere introduction of the REH), or $\lim \left[-\left(\frac{\gamma}{\beta}\right) \phi^{-1}(L) \right]^n = 0$. This limit condition of

sufficiency is hard to rephrase in economic sense. The necessary condition of convergence, however, is reasonably interpreted: By long division of

$$\phi^{-1}(L) = \frac{1}{1 + \phi_1 L + \phi_2 L^2 + \dots} = 1 - \phi_1 L + (\phi_1^2 - \phi_2) L^2 + \dots, \text{ and applying multinomial}$$

theorem to $[\phi^{-1}(L)]^n$, we get

$$[\phi^{-1}(L)]^n = 1 - n\phi_1 L + \left[\frac{n(n+1)}{2} \phi_1^2 - n\phi_2 \right] L^2 + \dots \quad (108)$$

from which the limit condition turns out to be satisfied only if $|\frac{\gamma}{\beta}| < 1$ holds. Thus, he concludes that when excluding the mere assumption of the REH, the learning process represented by equations (102) – (104) based upon a forecasting function of full information utilization does not guarantee convergence to rational expectations.

Cyert and Degroot [1974] applies the Bayesian learning process to the convergence problem.⁽²⁵⁾ Their motivation of developing learning process toward rational expectations is to make the hypothesis “a scientific truth rather than a religious belief”.

(25) In Cyert and Degroot [1970], they applied the Bayesian analysis to the context of duopoly in which learning was made on the rival's reaction function. The same analytical framework was directed to the agents' expectation formation process.

This is identical with saying that the Muth's second statement is not necessarily consistent with the first.

They introduce the three types of model. (1) the inconsistent model, (2) the consistent model, and (3) the control model. In the inconsistent model, agents do not know the true function that generates the market price. Their decision making is based on an incorrect model. Suppose the true market model is

$$C_t = d_1 - \beta p_t \quad (\text{Demand}) \quad (109)$$

$$Q_t = d_2 + \gamma E_{t-1}(p_t) + u_t \quad (\text{Supply}) \quad (110)$$

$$Q_t = C_t \quad (111)$$

where C_t , Q_t , P_t , $E_{t-1}(p_t)$, and u_t are consumption, output, the market price, the conditional expectation of the price, and a random error. d_1 , d_2 , β , and γ are fixed coefficients ($d_1 = d_2$ assumed for simplicity).

The equilibrium market price is from equation (111)

$$p_t = \frac{d_1 - d_2}{\beta} - \frac{\gamma}{\beta} E_{t-1}(p_t) - \frac{1}{\beta} u_t = -\frac{\gamma}{\beta} E_{t-1}(p_t) - \frac{1}{\beta} u_t. \quad (112)$$

But agents believe that the price is generated by

$$p_{t+1} = ap_t + v_{t+1}, \quad t = 0, 1, 2, \dots \quad (113)$$

where agents learn through observing the actual market price, and v 's are assumed to be normally, and identically distributed with the mean zero and the known precision r .⁽²⁶⁾ If agents form a posterior distribution for a after observing p_t , then this distribution turns into the prior distribution from which agents form the posterior of a by observing p_{t+1} , and so on. If the prior of a at the beginning of the period $t+1$ (or at the end of the period t) is normal with the mean m_t , and the precision h_t , then the distribution of a belongs to conjugate families in the normal distribution, and the posterior at the end of $t+1$ is normal with the mean m_{t+1} and the precision h_{t+1} ⁽²⁷⁾ where

$$m_{t+1} = \frac{h_t m_t + r p_t p_{t+1}}{h_t + r p_t^2} \quad (114)$$

$$h_{t+1} = h_t + r p_t^2 \quad (115)$$

This relation tells that at the end of the period t , the agents' price expectation $E_t(p_{t+1})$ is (by equation (113))

$$E_t(p_{t+1}) = E_t(a) p_t + E_t(v_{t+1}) = m_t p_t. \quad (116)$$

(26) r is the reciprocal of the variance in case of the normal distribution.

(27) See Degroot [1970], p 167 for the relevant Bayes theorem.

Substituting from equation (116) into equation (112) while letting $v_t = -\left(\frac{1}{\beta}\right)u_t$,

$$P_{t+1} = -\frac{\gamma}{\beta}E_t(p_{t+1}) + v_{t+1} = -\frac{\gamma}{\beta}m_t p_t + v_{t+1} \quad (117)$$

obtains where v 's are iid (normal) with the mean 0 and the precision r .

They performed several Monte Carlo runs to see the time paths of m_t , h_t , p_t , etc. while assigning arbitrary numbers to $\frac{\gamma}{\beta}$, r , p_0 and m_0 where p_0 and m_0 are initial values of p 's and m 's (throughout the runs a is fixed at the same constant), and found that after ten (when $\frac{\gamma}{\beta} = 0.1$) to one hundred ($\frac{\gamma}{\beta} = 1$) iterations, m_t tends to get sufficiently close to zero, and h_t to grow rapidly (representing that agents get more confident about the accuracy of their prior distribution). Substituting $m_t = 0$ ⁽²⁸⁾ into equation (117) thus gives the same result as the one of Muth [1961]: The market price is a white noise under the REH.

In a consistent model, the agents' expectation function is given by the system (109) – (111). While no longer assuming $d_1 - d_2 = 0$, agents' expectation is written as

$$E_{t-1}(p_t) = E_{t-1} \left(\frac{d_1 - d_2}{\beta} \right) - E_{t-1} \left(\frac{\gamma}{\beta} \right) E_{t-1}(p_t) - E_{t-1} \left(\frac{1}{\beta} u_t \right), \quad (118)$$

thus

$$E_{t-1}(p_t) = \frac{E_{t-1} \left[\left(\frac{d_1 - d_2}{\beta} \right) \right] - E_{t-1} \left[\left(\frac{1}{\beta} \right) u_t \right]}{1 + E_{t-1} \left(\frac{\gamma}{\beta} \right)} \quad (119)$$

Now they assume that the values of β and γ are known, but $D = d_1 - d_2$ is unknown while u_t is iid (normal) with the mean 0 and the known precision τ . Then, from equation (119)

$$E_{t-1}(p_t) = \frac{E_{t-1}(D)}{\beta + \gamma} \quad (120)$$

follows. Suppose further that the posterior of D at the end of $t-1$ is normal with the mean m_{t-1} and the precision h_{t-1} , then

(28) In equation (116), $m_t = 0$ gives $E_t(p_{t+1}) = 0$. Therefore, prices and quantities should be interpreted as being measured from their equilibrium levels.

$$E_{t-1}(p_t) = \frac{m_{t-1}}{\beta + \gamma} \quad (121)$$

holds. To conform with the Bayes theorem, equation (112) must be rewritten as

$$\beta p_t + \gamma E_{t-1}(p_t) = D - u_t, \quad (122)$$

where given D the mean of the left side is D .

From the Bayes theorem, the posterior of D at the end of the period t is normal with the mean m_t and the precision h_t where

$$m_t = \frac{h_{t-1}m_{t-1} + \tau[\beta p_t + \gamma E_{t-1}(p_t)]}{h_{t-1} + \tau}, \quad (123)$$

$$h_t = h_{t-1} + \tau. \quad (124)$$

Substituting from equation (121) into equation (123) gives

$$m_t = \frac{\left[h_{t-1} + \left(\frac{\tau\gamma}{\beta + \gamma} \right) \right] m_{t-1} + \tau\beta p_t}{h_{t-1} + \tau} \quad (125)$$

Equations (112) and (125) combine to produce

$$p \lim m_t = D, \quad (126)$$

and, therefore, the relation

$$p \lim E_{t-1}(p_t) = \frac{D}{\beta + \gamma} \quad (127)$$

holds as the probability limit of the agents' price expectation.

The actual market price, on the other hand, turns out to converge to the same probability limit. Using equation (127) in equation (112) shows that the distribution of the actual market price becomes asymptotically normal with the mean $\frac{D}{\beta + \gamma}$ and the precision $\beta^2\tau$.

Therefore, it is concluded that as time goes on the agents' expectation which is based on the consistent model tends to be identical with the relevant model's prediction implying convergence to rational expectations.

The control model is introduced to give another definition of rational expectations.

All but one firm (viewed as an entrant, or as a firm with new management) are assumed to be in equilibrium in the competitive market. That firm takes the long-run equilibrium price for granted, and wants to minimize the loss function to achieve the long-run equilibrium output over a certain time horizon (finite or infinite).⁽²⁹⁾

(29) The first two models were assuming that agents try to form rational expectations period by period. Here a certain number of time periods is assumed over which optimization is made due to consideration of adjustment costs.

Starting from Q_0 (initial output), the firm's output follows

$$Q_j = Q_{j-1} + u_j + e_j, \quad j = 1, \dots, n \quad (128)$$

where u_j is the adjustment to move into period j from $j-1$, and e_j is a serially independent random factor with the mean 0 and the variance σ_j^2 . Denoting the long-run equilibrium output by Q^* , the quadratic loss function

$$\sum_{j=1}^n E \left[\alpha_j (Q_j - Q^*)^2 + \beta_j u_j^2 \right] \quad (129)$$

is assumed where α_j and β_j are nonnegative constants, and the first and second terms in the parenthesis represent the cost of being away from the target and the adjustment cost, respectively. Optimization is made over n periods.

By the backward induction method of the adaptive stochastic control theory,⁽³⁰⁾ the problem of minimizing function (129) is solved for the time path of the value of the control variable u_j , and the result is

$$u_j = a_j Q_{j-1} + b_j \quad (130)$$

where a_j and b_j depend on Q^* , α 's and β 's (but not on σ^2 's), so that

$$Q_j = (1 + a_j) Q_{j-1} + b_j + e_j \quad (131)$$

obtains.⁽³¹⁾

In accordance with this result, they propose the another definition of rational expectations. In the above example, agents knew the long-run equilibrium price implying that they have perfect foresight. However, in general agents must solve the control problem based on their price expectation, so that the relation

$$Q_j = f[E_{j-1}(p_j)] + e_j \quad (132)$$

(30) See Degroot [1970] pp 411 - 414.

(31) When the time horizon is infinite, the optimization problem, i.e.,

$$\text{minimize } \sum_{j=1}^{\infty} \rho^j E \left[\alpha (Q_j - Q^*)^2 + \beta u_j^2 \right]$$

where $\alpha_j = \alpha \rho^j$ and $\beta_j = \beta \rho^j$ are assumed is proved to have the solution

$$u_j = \gamma (Q^* - Q_{j-1}), \quad 0 < \gamma < 1,$$

and thus the equation

$$Q_j = (1 - \gamma) Q_{j-1} + \gamma Q^* + e_j$$

holds.

holds where e_j is a random factor with the mean 0 and the variance σ_j^2 . When

$E_{t-j}(p_j)$ is identical with the equilibrium price level, function (132) will give the

optimal quantity. $E_{t-j}(p_j)$ is thus the rational expectation if

$$f[E_{j-1}(p_j)] = (1 + a_j)Q_{j-1} + b_j \quad (133)$$

holds.

Taylor [35] uses an adaptive learning process, and analyzes the effects of a monetary policy during the transitional period to rational expectations.

His analysis is based on the continuous time version of an expectation augmented Phillips Curve:

$$f = \phi(u) + x, \quad \phi'(\cdot) < 0, \quad \phi(u^*) = 0, \quad u > 0 \quad (134)$$

where f , x , u and u^* are the actual and expected inflation rates, and the actual and natural rate of unemployment. f is assumed to be controlled by the authorities (through money supply), thus a particular monetary policy is defined by the time path of $f(t)$.

Let $I(t_0)$ be the information set known to the public at time t_0 , then under the REH

$$x(t|t_0) = E[f(t)|I(t_0)] \quad (135)$$

holds where E is the expectation operator. If the monetary policy is deterministic,

then $f(t) = x(t|t_0)$ holds for all t , and from equation (134) this means $u = u^*$.

But if the policy is random (due to some intrinsic uncertainties in executing the policy), then agents form their expectations based on the subjective (identical with the objective) probability distribution of $f(t)$, and the expectation error $f(t) - x(t|t_0)$ becomes a white noise ($\varepsilon(t)$) which is independent of $I(t_0)$.

Substituting this into equation (134) gives

$$\phi(u) = \varepsilon(t), \quad (136)$$

and therefore $E[\phi(u)] = 0$, implying the policy ineffectiveness no matter what random policy is used. He discusses, however, that the REH disregards the transition period during which agents accumulate learning from their erroneous predictions, and learning from new beliefs. To take account of the effects of the randomized policy during the transitional period, he introduces the inflation policy such that $\log p(t)$ (the log of the price level) follows a diffusion process with the constant mean μ and

the serially independent variance $\sigma^2(t)$ characterized by the stochastic differential equation

$$d[\log p(t)] = \mu dt + \sigma(t)dv, \quad t \geq 0 \quad (137)$$

where $v(t)$ is the Wiener process with the mean 0 and the unit variance (He puts $d[\log p(t)]$ as $f(t)dt$ to make the notation conformable with the previous ones). Then, from results in stochastic calculus,⁽³²⁾

$$E[f(t)dt] = \mu dt \quad (138)$$

$$\text{var}[f(t)dt] = \sigma^2(t)dt \quad (139)$$

implying the stochastic nature of $f(t)$ given above.

Now it is assumed that the authorities know both μ and $\sigma^2(t)$, but agents know only $\sigma^2(t)$ and do not know μ (the lack of knowledge on μ makes distinction from the REH). Instead, agents have subjective distribution (normal) with the mean μ_0 and the variance σ_0^2 (this is true initial value of $\sigma^2(t)$), and are assumed to learn about the true mean μ according to the scheme

$$dx(t) = \frac{\omega}{\omega + z(t)} [f(t)dt - x(t)dt], \quad (140)$$

$$z(t) = \int_{i=0}^{i=t} y(s)ds, \quad (141)$$

where $y(t)$ is the precision equal to σ_0^{-2} , and ω which is equal to σ_0^{-2} is the precision of the agents' prior distribution.

This stochastic differential equation is integrated to give

$$x(t) = \left[\frac{\omega}{\omega + z(t)} \right] x_0 + \left[\frac{z(t)}{\omega + z(t)} \right] g(t), \quad (142)$$

$$g(t) = \frac{\int_{i=0}^{i=t} y(s)f(s)ds}{\int_{i=0}^{i=t} y(s)ds} \quad (143)$$

where $x(0) = \mu_0$ is the initial expectation. Equation (142) means that $x(t)$ is the weighted sum of $x(0)$ and the weighted sample mean $g(t)$. As t goes to infinity,

(32) See Kamien and Schwartz [1981].

$z(t)$ goes to infinity in equation (141) unless $y(t)$ converges to zero, and thus $x(t)$ converges to μ in probability (due to $g(t)$ going to μ). Convergence of $x(t)$ to μ implies that agents' learning process converges to rational expectations (characterized by $x(t) = \mu$).

Now he goes on to the policy effects during the transitional periods. The authorities try to choose $y(t)$ (μ is fixed) to maximize

$$E \int_{t=0}^{\infty} e^{-\rho t} W[x(t), u(t)] dt, \quad s.t. \quad 0 \leq y(t) \leq M \quad (144)$$

based upon the knowledge on the actual (chosen) distribution of $f(t)$. Here $W(\bullet)$ is the social utility function, ρ the continuous discount rate, and M some fixed positive number (the lower bound of the variance).

The solution to this optimization problem is characterized as

$$E[x(t)] = \frac{\omega}{\omega + z(t)} x(0) + \frac{z(t)}{\omega + z(t)} \mu \quad (145)$$

$$\text{var}[x(t)] = \frac{z(t)}{[\omega + z(t)]^2} \quad (146)$$

$$E[f(t)dt - x(t)dt] = \frac{\omega}{\omega + z(t)} [\mu - x(0)] dt \quad (147)$$

$$\text{var}[f(t)dt - x(t)dt] = y^{-1}(t)dt + \frac{z(t)dt}{[\omega + z(t)]^2} \quad (148)$$

From equation (147), it follows that the prediction error is not a white noise in contrast to the rational expectations case, but is influenced by μ and $z(t)$. Therefore, during the transitional period, the expected unemployment in equation (134) can be reduced by making μ larger, and/or by reducing $y(t)$'s (so that $z(t)$'s).

2-2 Characterization of Convergence Models.

In this section, we compare the models of Friedman, Decanio, and Cyert and Degroot, and try to sketch some aspects which seems to be in need for further developments.⁽³³⁾

The fundamental idea in introducing learning process into the REH is that the

(33) Taylor [1975] tells a little about convergence mechanism. His main concern can be said to have centered on the effects of monetary policy during a transitional period while assuming convergence to rational expectations. Convergence is mostly a matter of assumption in equations (140) and (141). When $z(t)$ goes to infinity (assuming $y(t)$ does not go to zero), price expectation $x(t)$ will cease to change, and this means the end of the error correction process as the state of the rational expectations is achieved.

mere fact of the efficient utilization of all available information⁽³⁴⁾ is not sufficient to rationalize the hypothesis of rational expectations. For the sake of reaching the state that agents' expectations are identical with predictions of the relevant economic model, or in order to recognize what the objective probability distribution of outcomes of agents' concern is, it is more realistic than a mere assumption to think that agents need to accumulate their experiences and knowledges on the actual generating processes of the relevant outcomes.

This basic idea made convergence theorists introduce the learning mechanism by which agents may or may not achieve rational expectations.

Friedman interpreted the meaning of efficient information utilization in the sense of the minimum variance prediction. His learning mechanism was thus a series of least-squares prediction (based on a consistent model) by agents. Based on the common sense on rationality of human behavior in general (If we do our best in our circumstances, "our decision is perfectly rational), the least-squares prediction has a strong appeal. However, exactly due to this set up, this learning mechanism incurs the two specific constraints on the realization of the REH. (1) The possibility of misspecification, and (2) various limits to the availability of information. These constraints in the agents' learning processes made him skeptical about the REH even in the long-run. From this results, he rather supports an adaptive adjustment mechanism with a time varying coefficient as representing the least-squares learning process (therefore, the efficient utilization of information).

Decanio specifies the learning process by a set of difference equations. Unlike Friedman, he thinks that the assumption that economic agents know the true model is unrealistic, and uses an inconsistent model according to which agents form their expectations. A conspicuous feature of his model is that he explicitly took into account the structural change in the true model due to the modification of agents' expectations during the learning process (corresponding lag polynomials in equations (93) and (94) interact, and the generating processes of both the actual and expected prices develop under continuous structural change). This interaction is known as the endogeneity problem, and has been developed in the form of the stability of the REE⁽³⁵⁾ (rational expectations equilibrium: markets clearing with agents expectations being rational). The learning process with the interaction was analyzed through solving difference equations, and turned out that the process does not necessarily converge to rational

(34) It contains the statement that agents will not make systematic errors.

(35) See Blume, Bray, and Easley [1982]. They classify the learning models based on whether a model assumes the consistent or inconsistent expectation generating function, and whether a model contains the endogenous interaction between agents' subjective function and the relevant economic model (objective functions).

expectations depending on the time invariant structural parameters (of demand and supply equations). While starting with a different approach from Friedman, he is cautious of the unconditional adoption of the REH alike.

Cyert and Degroot worked with the Bayesian learning process, and in either cases (both inconsistent and consistent expectation formation functions), their results show that the REH, as a result of the agents' learning, is highly plausible.⁽³⁶⁾ Their model is evaluated for its distinctive way of learning and the Bayesian theoretic foundation of rational expectations. However, the specific assumption on the distribution of the error term (thus on the prior distribution of the parameter concerned: α in equation (113), and D in equation (118)) seems to suggest that Muth's original assertion that the agent's knowledge on the objective distribution of the relevant outcomes should be accepted on the a priori basis, and if v 's distribution in equation (113) can be assumed, and can somehow be rationalized empirically to be iid (normal) with the mean 0 and the precision r , then establishing the objective distribution of the market price in equation (117) does not seem to require further informational burden. Conversely, if the assumption on the distribution of stochastic errors are not precise, then the convergence in Bayesian learning will also be an approximation to the actual learning process (if it exists), so that the convergence property itself is open to skepticism.

Our purpose in investigating the convergence models, however, is not to find theoretical inferences to deny the plausibility of the REH, but to try to abstract any theoretically meaningful aspects of the economic agent's behavior. From such a point of view, it can even be said that those learning models discussed in this and the previous sections show the possibility of convergence in the long-run.

(36) In the inconsistent model (113), however, if the agents reaction to the previous observation is very sensitive, then it can be shown that the subjective expectation function is implicitly consistent with the economic model. Let $|a| > 1$, then by the forward expansion of (113) we have

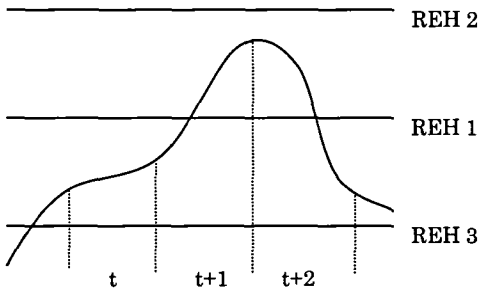
$$P_{t+1} = -\sum_{i=1}^{\infty} (1/a)^i v_{t+1+i},$$

so that

$$E_t(P_{t+1}) = -\sum_{i=0}^{\infty} (1/a)^i E_t(v_{t+1+i}) = 0.$$

Substituting from this into the middle of (117) gives $P_{t+1} = v_{t+1}$ implying the REH. This result is interpreted in the way that if the agents reaction is highly sensitive in the first order Markov process (113), then they will act as if they know the market equilibrium in forming price expectation, and if every agents happen to know the equilibrium, then the equilibrium will be actually achieved, and the equilibrium errors would be stochastic. If $|a| < 1$, however, then this is not the case.

Now, is the long-run convergence of the agents' expectations sufficient for the REH to be realistic and therefore be a meaningful assumption in economic model building? If "the long-run" means a significantly long duration of nonrational expectations, the REH will be almost always be an invalid assumption. This does not deny the importance of the theoretical analysis in (or toward) the long-run steady state context. However, if that is the case, then the policy ineffectiveness results of the REH, for example, will almost always be invalid. From this consideration, we insist that in



order for the REH to be a meaningful assumption, quick convergence must be required in addition to the property of eventual convergence. Then, regardless of the interpretative aspects of the convergence models discussed in this chapter, we can point out that they neglect taking account of the structural

change of the model over time as a result of agents' optimal decision making process in the context of policy changes. This point should be clearly discriminated from the endogeneity problem mentioned in Decanio [1979]. The Lucas critique (and equivalently the condition (c) of Sargent and Wallace [1976]) indicated the changes in parameter values and functional forms of the economic model due to the change in the agents' optimal behavior as a result of the change in the policy rule. Decanio's model can be used to make the importance of this influence clear in the context of convergence to rational expectations. In the solution of the difference equations (105) – (107), the steady state equilibrium part $\lambda(R)(L)$, $\psi(R)(L)$, and $\theta(R)(L)$ with which agents' expectations are rational are defined by equations (99) – (101) where the structural lag polynomials and parameters ($\delta(L)$, $v(L)$, $\mu(L)$, and $\frac{\gamma}{\beta}$) enter the definition of these

equilibrium lag polynomials. The Lucas critique implication is that the equilibrium λ , ψ , and θ will shift around over time due to the changes in those structural coefficients. The situation⁽³⁷⁾ could be shown graphically. In the above figure, the three hypothetical states of the REH⁽³⁸⁾ corresponding to the three different states of the

(37) The solutions to the difference equations (102) – (104) will not take the form of equations (105) – (107) when $\delta(L)$, $v(L)$, $\mu(L)$, and $\frac{\gamma}{\beta}$ are not constant across the time. We use the equations (105) – (107) only to get

some intuition about the Lucas criticism implications in the framework of the model with time invariant structure (referring to equations (88) and (89)).

(38) The unique long-run state of the REH is assumed in the case of endogenous interaction.

combination of $\lambda(R)(L)$, $\psi(R)(L)$, and $\theta(R)(L)$ (supposing that these states arise according to the numerical order) are depicted. In each time period, the convergence process is disturbed, although it is the result of the agents' optimizing behavior. This shows that the convergence problem has another source of complication in addition to the endogeneity problem.

In coping with this problem, the four models discussed in this chapter do not seem to give much insight. In Friedman's model, if the true parameter vector continues to vary over time, the convergence will simply never be achieved (his misspecification arguments, and the inconsistency problem of available information due to the structural change will be further augmented). Decanio solved his difference equations of the learning process under the assumption of the time invariant structure. If the factor of changing structural coefficients is introduced, his system can not be solved in the manner that he used (In the interval between the policy changes, his approach will work. However, then the long-run convergence will lose its significance in general).

The Bayesian learning process of Cyert and Degroot[1974] analyzes how agents can learn about the distribution of fixed parameter. It is not clear what we can do with the Bayes theorem under the present context.

Taylor's convergence argument seems to be a matter of assumption. He paid only a little attention to the mechanism by which agents can learn about how to achieve rational expectations. The role put on $z(t)$ which goes to infinity as time passes has the definite implications under any conditions.

From these considerations, we propose to adopt another framework to analyze the implication of the REH. Davidson, Hendry, Srba, and Yeo [1978](DHSY) used the error correction model to reconcile the shortrun fluctuations of the average propensity to consume(APC) with its long-run stability. Suppose an economic theory tells that the following relationship holds between the two variables X_t and W_t ,

$$X_t = KW_t, \quad (149)$$

where K is constant on any fixed growth path of W_t , but varies with its growth rate. Taking the logarithms of the both sides of equation (149) gives

$$x_t = k + w_t, \quad (150)$$

where $x_t = \log X_t$, etc. The first difference version of equation (150) is

$$\Delta x_t = \Delta w_t. \quad (151)$$

They assume a general stochastic disequilibrium relationship between x_t and w_t due to the difficulty of specifying the dynamic adjustment of x_t to w_t ,

$$\alpha(L)x_t = k^* + \beta(L)w_t + v_t, \quad (152)$$

where $\alpha(L)$ and $\beta(L)$ are lag polynomials of high enough order to make v_t a white noise.

For exposition purposes, they simplify equation (152) as

$$x_t = k^* + \beta_1 w_t + \beta_2 w_{t-1} + \alpha_1 x_{t-1} + v_t, \quad (153)$$

where equations (150) and (151) are the special cases of this relation in the sense that equation (150) and (151) are obtained if $\beta_1 = 1$, $\beta_2 = \alpha_1 = 0$, and if $\beta_1 = -\beta_2 = 1$, $k^* = 0$, $\alpha_1 = 1$, respectively. But as a technique to recover equation (150) from equation (153), a more general parameter restrictions are sufficient:

$$\beta_1 = -\beta_2 + \gamma, \quad (154)$$

$$\alpha_1 = 1 - \gamma \quad (155)$$

generating

$$\Delta x_t = k^* + \beta_1 \Delta w_t + \gamma(w_{t-1} - x_{t-1}) + v_t \quad (156)$$

where by equations (154) and (155) the long-run propensity (LRP) is unity (If x_t and w_t are cointegrated, then the Granger Representation Theorem assures that the relationship such as equation (156) exists (Engle and Granger [1987])). Comparing with the short-run model

$$\Delta x_t = k^* + \beta_1 \Delta w_t, \quad (157)$$

the term $\gamma(w_{t-1} - x_{t-1})$ in equation (156) is called an error correction term that carries the meaning of the adjustment through "the vital initial disequilibrium effect".

Let $\Delta x_t = g = \Delta w_t$ (equation (151)), $v_t = 0$, and $\gamma \neq 0$, then

$$g = k^* + \beta_1 g + \gamma(w_{t-1} - x_{t-1}), \quad (158)$$

and thus

$$X_t = KW_t \quad \text{with } K = \exp\left\{\frac{k^* - g(1 - \beta_1)}{\gamma}\right\} \quad (159)$$

are obtained. Therefore, having constant growth path in equation (156) is equivalent with equations (149) and (150).

To show the way to apply this method to the case of the fluctuating rational expectations target, suppose that the variable w_t (the log of money supply, for instance) is such that the monetary authorities control it to some random error u_t , and let x_t be the relevant variable for agents in forming expectations (the log of the price level, for instance). k is a fixed parameter in this generating scheme. Then, assuming, for example, a simple autoregressive form $w_t = w_{t-1} + u_t$, and

$u_t = \rho u_{t-1} + e_t$, where $|\rho| < 1$ and e_t is a white noise, the relationship

$$g = \Delta_1 x_1 = \Delta_1 w_t = u_t \quad (160)$$

follows through equation (150). Substituting from equation (160) into equation (159), it turns out that K is subject to the influence of the randomized policy rule. When agents form their expectations based on equation (149), and if equation (149) is assumed to be a true model, the rational expectations price level itself is influenced by the change in the way that the policy is conducted (some empirical devices would be necessary to estimate the exogenous shock variable u_t to make this model (Equations(156) and (159)) estimable. An example of such an innovation is the residual series of the data generating process for x_t and w_t).

This simple exercise can be summarized as follows. (1) Introducing γ in equation (156) implies that we imposed the parameter restriction in order to make a long run propensity unity. (2) Agents form their expectations based on the steady state relationship (149), thus their expectation is rational. (3) However, as a result of the agents' adjustment induced by the change in the policy rule, the parameter K in equation (159) varies. (4) Agents expectations continue to be rational while they continue to use equation (159) (or equation (149)) to form their expectations. (5) We can check if the differences between the actual and the expected price levels consist of white noise series. (6) The basic idea here is to see the possibility of justifying the REH under the Lucas criticism implication(i.e., g varies in equation (159) as u_t in equation (160) changes) when agents make their decision based on the equilibrium value of the relevant variable while the actual movement of the variable is determined by the market processes with variable adjustment speed, and with or without disequilibrium rigidities.

We tried a simple application of the error correction model to the investigation of the implication of the REH based on DHSY[1978]. The merits of this approach are that it enables us to take into account the effects of the structural change in the context of equilibrium analysis (in the sense that agents aim at achieving equilibrium), and it provides one of testable forms of the REH.

The error correction mechanism has been expanded to the vector error correction model(VECM) in relationship with the cointegration analysis of Engle and Granger[1987]. It is conceivable that the VECM technology can be applied to the analysis of convergence toward the REH in the context of the variables x_t and w_t in this section being the vectors of the variables which are modelled. The Granger Representation Theorem(Engle and Granger [1987]) indicates that a long run

equilibrium relationship can be written in the form of the corresponding error correction specification. If rational expectations are formed based on the long run equilibrium relationships in the way that were described by various convergence models exhibited in this paper, then the representation of convergence mechanism in terms of the error correction model seems to be important.

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