Influences of the Conditional Second Moment of Some Macroeconomic Ratios*

ISAMU GINAMA

Abstract

This paper investgates the time series properties of "the great ratios of Economics" for the presence of a unit root and for the reliability measured in terms of the conditional variance. It is argued that these ratios must be stationary to conform with the Klein=Kosobud growth model. Cyclical implications of the growth paths of some macroeconomic aggregates are analyzed through simple regressions of the conditional variances of those ratios on the economic growth rates.

Key words: Great ratios, Growth path, Unit root, Structural breaks, Conditional variance, Conditional covariance, EGARCH

Using several ratios of macroeconomic variables, Klein and Kosobud (1961) represented "the Great Ratio Model" of economic growth. It was a remarkable example of simplified model construction, in that the key ratios were assumed to be stable or or to exhibit the systematic variation over time based on the empirical testing procedures. "The Great Ratio of Economics" tested and applied were the savings-income ratio (S/Y) (or the

* I would like to thank Professor Koichi Maekawa for helpful discussions. The usual caveat, however, should be acknowledged.

average propensity to consume as (1 – S/Y)), the capital-output ratio (K/Y), labor's share of income (wN/pY), income velocity of circulation (pY/M) (or its reciprocal, the Marshallian k), and the capital-labor ratio (K/N). This paper investigates the time series properties of these ratios for the presence of a unit root and for a sense of reliability the latter of which is to be measured in terms of the conditional variance. It will be argued that trend stationarity or stationary fluctuations around a constant of the key ratios conform with the Klein=Kosobud growth model, but nonstationarity of the ratios does not. The cyclical implications of the conditional variances of the key ratios will turn out to be useful in interpreting that model. The paper is organized as follows. Time series of the key ratios and the summary of the Klein=Kosobud model are represented in section one. The method and the results of unit root tests on the key ratios are explained in section two. Usefulness of the key ratios as a simplifying factor in constructing the growth model is discussed in section three. Concluding remarks are presented in the last section.

1. The Klein=Kosobud Model

Figure 1 represents time series of the natural logarithm[1] of the key ratios where Y stands for the real net national product, C for the real consumption, K for the real capital stock, w for the nominal wage, N for persons engaged (employment), p for the general price level, and M for the cash balances.[2]

Klein and Kosobud (1961) regressed the logarithm of those ratios on a semiannual linear time trend (t) to get the following results[3] where log

1) The common logarithm of the key ratios was used in Klein and Kosobud (1961).

2) Real variables are expressed in terms of 1929 dollars.

3) Parameter estimates here slightly differ from what Klein and Kosobud (1961) reported. Statistics but the t-value for the coefficient of a time trend were not originally presented.

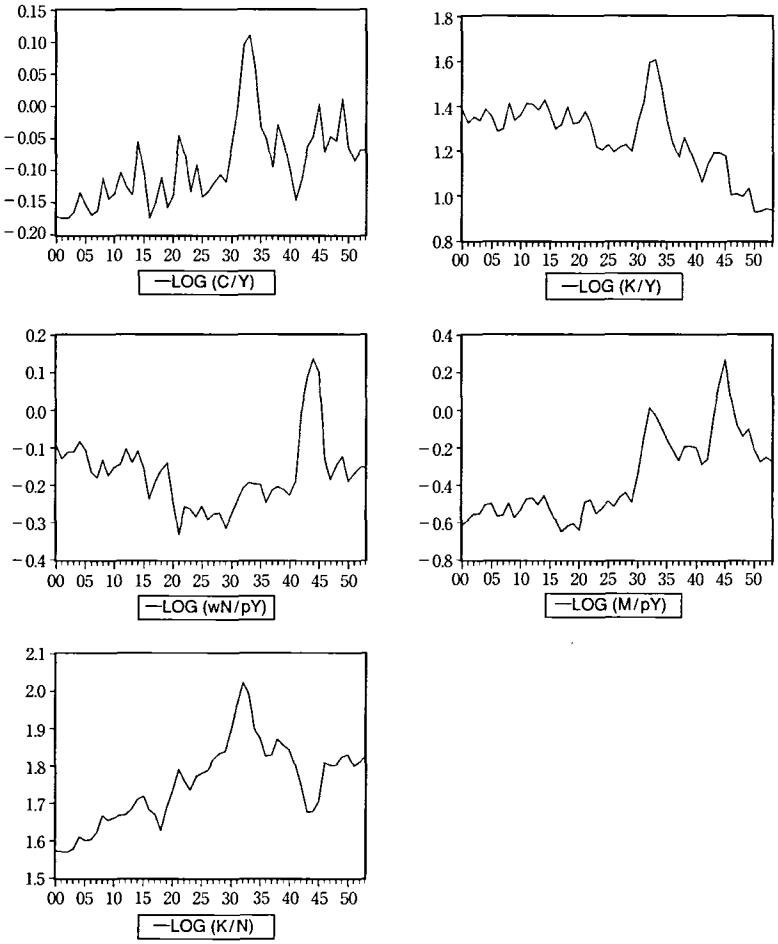

Figure 1

means the common logarithm, R^2 the coefficient of determination not adjusted for the degrees of freedom, DW the Durbin=Watson statistic, and the numbers in the parentheses t values.

$$\log C/Y = -0.0388 + 0.00054t \qquad R^2 = 0.3554$$
$$(-12.27) \quad (5.35) \qquad DW = 0.7143$$

or

$$C/Y = 1.0935\ (1.00124)^t$$

$$\log K/Y = \underset{(79.32)}{0.54557} - \underset{(-6.92)}{0.001527t} \qquad R^2 = 0.47966 \qquad DW = 0.3599$$

or

$$K/Y = 3.512\ (1.0035)^t$$

$$\log \frac{wN}{pY} = \underset{(-13.10)}{-0.073626} + \underset{(0.46)}{0.000083t} \qquad R^2 = 0.004 \qquad DW = 0.3746$$

or

$$\frac{wN}{pY} = 0.8441\ (1.00019)^t$$

$$\log M/pY = \underset{(-18.20)}{-0.1526} + \underset{(9.11)}{0.00245t} \qquad R^2 = 0.6150 \qquad DW = 0.3643$$

or

$$M/pY = 0.7037\ (1.00566)^t$$

$$\log K/N = \underset{(160.73)}{0.7623}^{4)} + \underset{(6.81)}{0.001036t} \qquad R^2 = 0.4717 \qquad DW = 0.2058$$

or

$$K/N = 5.7850\ (1.002388)^t$$

Using these estimates, the Klein=Kosobud model of great ratios can be written as

$$C/Y = 1.0935\ (1.00124)^t$$

$$K/Y = 3.512\ (1.0035)^{-t}$$

$$\frac{wN}{pY} = 0.8441^{5)}$$

4) The original estimate of this parameter was 3.76126 which is to have been an error. The corresponding estimate below this equation is supposed not to be 5571 but to be as indicated. The period-to-period growth rate of employment (N) is independent of this number, however. See op. cit., p187.

5) The coefficient of a time trend was set equal to zero based on the standard t test.

$$M/pY = 0.7037\ (1.00566)^t$$

$$K/N = 5.7850\ (1.002388)^t$$

$$Y - C = I + B$$

$$K - K_{-1} = I$$

where I stands for the net investment and B for a foreign balance. Ignoring a small foreign balance, the multiplier-accelerator system of this model leads to the period-to-period growth rates of the five endogenous variables, Y, p, K, N, and w. The derivation of the solutions for C and I are straightforward based on those solutions. A money supply, M, is defined to be exogenous throughout. The growth rates can be reresented for the five variables with explicit time subscripts as

$$Y_t/Y_{t-1} = \frac{3.512\ (1.0035)^{-t+1}}{1.0935\ (1.00124)^t + 3.512\ (1.0035)^{-t} - 1}$$

$$K_t/K_{t-1} = (1.0035)^{-1}\ Y_t/Y_{t-1}$$

$$N_t/N_{t-1} = (1.002388)^{-1}\ K_t/K_{t-1}$$

$$p_t/p_{t-1} = (M_t/M_{t-1})\ (1.00566)^{-1}\ Y_{t-1}/Y_t$$

$$w_t/w_{t-1} = (N_{t-1}/N_t)\ (p_t/p_{t-1})\ (Y_t/Y_{t-1})$$

Based on "the possibility of constancy" of labor's share, the numerical derivation of a Cobb=Douglas production function was represented in addition to the model's solution. The semilogarithmic regression equation of the capital-labor ratio can be written as $\log K - \log N = 0.7623 + 0.001036t$. Multiplying through both sides by 0.8441, the equation

$$0.8441 \log K - 0.8441 \log N = 0.64346 + 0.0008745t$$

obtains. Subtracting this equation from the regression equation of the capital-output ratio

$$\log K - \log Y = 0.54557 - 0.001527t$$

the equation

$$0.1559 \log K + 0.8441 \log N - \log Y = -0.09789 - 0.0024t$$

holds. A linearly homogeneous Cobb-Douglas production function which

this equation implies is $Y = 1.2528K^{0.1559} N^{0.8441} (1.00554)^t$.

2. A Unit Root of the Great Ratios

The constancy or stability of the key ratios on which the Klein = Kosobud growth model relies can be rephrased as stationarity or trend stationarity of the parametric ratios. In this section, the results of the unit root tests on the key ratios are represented[6]. The observation period of the annual aggregative data used in the Klein=Kosobud model is from the year 1900 through the year 1953[7]. When applying the unit root tests to time series which ranges over such a long period, it would perhaps be appropriate to control the possibility of structural changes of a data generating process (dgp). Perron (1989) pointed out the importance of taking structural breaks into account which a dgp may possess in conducting the unit root test. Examples of the structural breaks he considered explicitly were the Great Crash and the First Oil Crisis. The data set of the Klein=Kosobud model contains the year of the Great Crash, so that the Perron's methodology would be appropriate[8] in testing the key ratios for the presence of a unit root. He offered three models of the unit root tests for the cases where it can be assumed that there exists a structural break. The null and the alternative hypotheses of the three models are as follows.

Null hypotheses: H_0: $\alpha = 1$ in

6) Dickey and Fuller (1979) reports the test on the annual observations of the velocity of money over the period from 1869 through 1960 for the presence of a unit root. For this sample period, the velocity of money was identified as an I (1) process.

7) The data on the appropriate bond yield and population differ from other series with respect to the sample range.

8) The presence of the endogenous breaks (Nunes, Newbold and Kuan (1997)) are not considered in this paper.

Model (A) $y_t = \mu + dD\,(TB)_t + \alpha y_{t-1} + e_t$

Model (B) $y_t = \mu_1 + \alpha y_{t-1} + (\mu_2 - \mu_1)\,DU_t + e_t$

Model (C) $y_t = \mu_1 + \alpha y_{t-1} + dD\,(TB)_t + (\mu_2 - \mu_1)\,DU_t + e_t$,

where $D\,(TB)_t = 1$ if $t = T_B + 1$,

$= 0$ otherwise;

$DU_t = 1$ if $t > T_B$,

0 otherwise; and

$A\,(L)e_t = B\,(L)\,v_t$, $v_t \sim$ i. i. d. $(0, \sigma^2)$ with A (L) and B (L) pth and qth order polynomials in the lag operator L. T_B stands for the time of break.

Alternative hypotheses: H_1: $\alpha < 1$ in

Model (A) $y_t = \mu_1 + \beta t + (\mu_2 - \mu_1)\,DU_t + e_t$

Model (B) $y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)\,DT_t^* + e_t$

Model (C) $y_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1)\,DU_t + (\beta_2 - \beta_1)\,DT_t + e_t$

where $DT_t^* = t - T_B$ and $DT_t = t$ if $t > T_B$ and 0 otherwise. t denotes an annual time trend hereafter. In model (A), the null hypothesis states that the dgp of y_t has a unit root, and its intercept experiences a temporal shift $(D\,(TB)_t)$ at the time of break. Under the alternative hypothesis, the dgp of y_t is trend stationary (TS), and its intercept allows for a one-time shift (DU_t). In model (B), the null hypothesis embodies a unit root (DS) process of y_t with a one-time shift in the intercept. Under the alternative hypothesis, y_t is a TS process with a shift in the slope of a linear trend while structural break is effectively allowed for the intercept ("without any sudden change in the level"). Model (C) characterizes the null hypothesis that y_t is a unit root process with the sudden change in the level, and the alternative hypothesis that y_t is a TS process with both "a sudden change in the level" and a shift in the slope of a time trend. T_B is set equal to the year 1929.

The models (A), (B), and (C) were applied to all of the key ratios, and the

Table 1 Model A

$$y_t = \hat{\mu} + \hat{\theta} DU_t + \hat{\beta} t + \hat{d} D\,(TB)_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$$

$T_B = 1929$	T	λ	κ	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\theta}$	$t_{\hat{\theta}}$	$\hat{\beta}$	$t_{\hat{\beta}}$	$\hat{d}$	$t_{\hat{d}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$S(\hat{e})$
C/Y	53	0.53	0	-0.05	-2.38	0.044	1.76	-0.00027	-0.359	0.022	0.515	0.57	-3.65	0.04
K/Y	52	0.52	1	0.438	3.21	0.062	1.56	-0.004	-2.62	0.103	1.54	0.719	-3.167	0.06
wN/pY	52	0.52	1	-0.036	-1.51	0.05	1.66	-0.0013	-1.25	-0.015	-0.26	0.72	-0.034	0.05
M/pY	52	0.52	1	-0.24	-4.02	0.177	3.49	0.000066	0.05	0.011	0.15	0.55	-4.75[c]	0.07
K/N	52	0.52	1	0.26	2.58	-0.0046	-0.23	0.0007	1.13	0.068	1.98	0.84	-0.012	0.03

Note: c stands for statistical significance at the 1 % level.

Table 2 Model B

$$y_t = \hat{\mu} + \hat{\beta}t + \hat{\gamma}DT_t^* + \hat{\alpha}y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \hat{e}_t$$

$T_B = 1929$	T	λ	κ	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\beta}$	$t_{\hat{\beta}}$	$\hat{\gamma}$	$t_{\hat{\gamma}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$S(\hat{e})$
C/Y	53	0.53	0	-0.072	-2.76	0.0018	1.998	-0.002	-1.30	0.608	-0.072	0.04
K/Y	52	0.52	1	0.457	3.49	-0.000047	-0.038	-0.006	-2.14	0.663	-0.048	0.06
wN/pY	52	0.52	1	-0.036	-1.614	-0.0024	-1.94	0.0058	2.29	0.592	-0.072	0.05
M/pY	52	0.52	1	-0.17	-2.74	0.003	1.88	-0.0006	-0.195	0.744	-0.0297	0.08
K/N	52	0.52	1	0.46	3.82	0.003	3.03	-0.0045	-2.72	0.702	-0.033	0.03

Table 3 Model C

$$y_t = \hat{\mu} + \hat{\theta} DU_t + \hat{\beta} t + \hat{\gamma} DT_t + \hat{d} D(TB)_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$$

$T_B = 1929$	T	λ	κ	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\theta}$	$t_{\hat{\theta}}$	$\hat{\beta}$	$t_{\hat{\beta}}$	$\hat{\gamma}$	$t_{\hat{\gamma}}$	$\hat{d}$	$t_{\hat{d}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$S(\hat{e})$
C/Y	53	0.53	0	-0.085	-3.25	0.177	2.63	0.0009	1.029	-0.0036	-2.12	-0.015	-0.34	0.45	-4.33[a]	0.04
K/Y	52	0.52	1	0.83	5.13	0.51	3.995	-0.0037	-2.59	-0.012	-3.65	-0.028	-0.41	0.417	-5.12[c]	0.06
wN/pY	52	0.52	1	-0.024	-1.05	-0.12	-1.40	-0.0035	-2.44	0.005	2.14	0.006	0.108	2.583	-3.968	0.05
M/pY	52	0.52	1	-0.24	-3.99	0.21	2.00	0.00039	0.24	-0.00099	-0.357	0.0039	0.048	0.554	-4.6379[b]	0.07
K/N	52	0.52	1	0.59	3.98	0.205	2.72	0.003	3.06	-0.0057	-2.86	0.03	0.91	0.621	-3.957	0.03

Note: a, b, and c stand for statistical significance at the 5 %, 2.5 % and 1 % level.

results are represented in Tables 1, 2, and 3, respectively.

The magnitude of κ, the number of lagged differences, was determined based on the usual t and F tests. Labor share $(\frac{wN}{pY})$ and the capital labor ratio (K/N) are identified as being nonstationary throughout the tables. The Marshallian k (M/pY) is stationary at the 1 % and 5 % level according to the estimation results in Table 1 and 3, although it is to be a unit root process under the framework of model (B). Visual inspection of the panel of (M/pY) in Figure 1 suggests that this variable is accompanied with a sudden change in the level at the time of break if it is regarded to be a TS process. Based on such a consideration and the values of $t_{\hat{\theta}}$, $t_{\hat{\alpha}}$, $t_{\hat{\beta}}$, and $t_{\hat{\gamma}}$ which should be compared (asymptotically) with a critical value taken from the standard normal distribution (Perron (1989), p1384), this ratio is identified as a stationary process with the one time shift in the intercept and no trend. A big spike at the time of break observed for the average propensity to consume (C/Y) and the capital output ratio (K/Y) does not seem to give rise to a permanent shift in the level of these variables (Figures 1). This would imply that these variables are stationary processes of some kind. The model C on Table 3 tells that the consumption ratio and the capital output ratio are stationary at the 5 % and 1 % level. Based on the value of $t_{\hat{d}}$, it can be argued that a temporary jump to be explained by the term $D\ (TB)_t$ is absent from these variable. The magnitude of $t_{\hat{\theta}}$, $t_{\hat{\beta}}$, and $t_{\hat{\gamma}}$ tells that the intercept and the slope of a time trend of these variables had structural breaks in the year 1929 prior to which a linear trend is not significant for the consumption ratio.

3. Constancy, Stability, and Reliability of the Key Ratios

Statistical tests of stationarity on the key ratios in the previous section show that three out of five ratios are stationary. They are the consumption ratio (TS), the capital output ratio (TS), and the Marshallian k (stationary

around a constant). It is, therefore, reasonable to use them as parameters or as a simplifying factor in constructing the aggregative macroeconomic model such as in Klein and Kosobud (1961). The other two, i.e., labor share and the capital labor ratio, however, are nonstationary, and it is inappropriate to treat them in a similar fashion to the cases of other "great ratios".

Focusing on the stationary key ratios, an investigation was made to estimate the measure of reliability of their conditional expectations in terms of the conditional variances. In so doing, the capital output ratio and the Marshallian k were regressed on the dummy variables and linear time trends in conformity to the results in Table 3. These equations were estimated without the logarithmic transformation of those ratios. It will turn out to be the case that the data should not be transformed in calculating time series of the conditional variances of those variables.

Table 4

	K/Y	M/pY
Const.	4.0553	0.5872
	(37.1559)	(32.9359)
DU_t	2.9596	0.2956
	(7.6704)	(11.0515)
t	-0.0188	
	(-3.0570)	
DT_t	-0.0681	
	(-6.4421)	
D/W	0.8302	0.6653
$\bar{R}^2$	0.7240	0.6956
obs.	54	54

Const. = constant term, t = time trend, D/W = Durbin-Watson Statistic, numbers in the parentheses = t-values, $\bar{R}^2$ = coefficient of determination adjusted for the degrees of freedom, obs. = number of observation,

$DU_t = 1$ if $t > T_B$ $DT_t = t$ if $t > T_B$
$= 0$ otherwise $= 0$ otherwise

Table 4 represents the results of estimation.

The residuals of these equations were used as the detrended series of those ratios. The autoregressive models of the detrended series then were estimated by the exponential GARCH (EGARCH) method (Nelson (1991)). The specification of the variance equation of the EGARCH (2, 1) model can be represented as

$$\log(\sigma_t^2) = \lambda_0 + \delta \log(\sigma_{t-1}^2) + \gamma_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \theta_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) + \gamma_2 \left| \frac{\varepsilon_{t-2}}{\sigma_{t-2}} \right| + \theta_2 \left(\frac{\varepsilon_{t-2}}{\sigma_{t-2}} \right),$$

where σ_t^2 is the conditional variance of the error term of the corresponding mean equation in the period t, ε_{t-i} the error term of the mean equation in the period t-i, log the natural logarithm, λ_0, δ, γs, and θs parameters. This specification assures that the conditional variance is to be nonnegative. The results are represented in Table 5.

The stationarity condition ($0 < \delta < 1$) is satisfied for DET (K/Y) equation, but the leverage effects ($\theta_i < 0$, $i = 1, 2$) (Nelson (1991)) which are often found for financial data are absent from the results in this table except θ_2 of DET (K/Y) equation. The experimental sampling distributions of the estimators of the variance equation, however, are not likely to be available for the sample size of the current paper (Deb (1996)). The time series of the conditional variances of these variables were calculated, and were used as the dependent variables in regression on the growth rate of real NNP. Due to the limitation of data availability in looking for variables which control business fluctuations for the relevant observation period, the economic growth rate is assumed to work as a proxy that reflects business cycles. Table 6 represents the results of such regressions in which the dependence of the conditional variance of the detrended Marshallian k on the growth rate of real NNP was estimated by the maximum likelihood method assuming the first order autocorrelation, and the cyclical implication of the conditional variance of the capital output ratio was estimated by OLS. The

Table 5

	$DET(K/Y)$	$DET(M/pY)$
Constm.	0.0096	-0.0044
	(0.5826)	(-0.5645)
$DET(K/Y)_{t-1}$	0.8947	
	(16.8391)	
$DET(K/Y)_{t-2}$	-0.5184	
	(-5.9703)	
$DET(M/pY)_{t-1}$		0.4623
		(3.5176)
Constv.	-0.8102	-7.1623
	(-2.2245)	(-7.8975)
$\log(\sigma^2_{t-1})$	0.3679	
	(2.8696)	
$\left\lvert\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right\rvert$	-0.2715	0.8243
	(-0.4812)	(1.7045)
$\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$	0.4322	0.2317
	(2.3083)	(0.9474)
$\left\lvert\frac{\varepsilon_{t-2}}{\sigma_{t-2}}\right\rvert$	-1.3129	1.0483
	(-1.7500)	(1.9511)
$\frac{\varepsilon_{t-2}}{\sigma_{t-2}}$	-0.4538	0.3460
	(-2.9782)	(1.4919)
$\bar{R}^2$	0.3549	0.3148
obs.	52	53

$DET(K/Y)_i$ = detrended capital output ratio in the period i, $DET(M/pY)_i$ = detrended Marshallian k in the period i, Constm. = constant term of the mean equation, Constv = constant term of the variance equation, numbers in the parentheses = z statistics, $\bar{R}^2$ = coefficient of determination adjusted for the degrees of freedom, obs. = number of observations

autocorrelation LM tests (abbreviated) on these equations revealed the acceptance of the null hypotheses of no autocorrelations up to the time lag of four periods. The economic implication of those regression equations is as follows. The reliability of the expected Marshallian k which is considered to increase as its conditional variance declines gets augmented

with the time lag of three poriods as the growth rate increases. On the other hand, the reliability of the expected capital output ratio is likely to increase with the time lag of one period and to diminish with the time lag of two periods as the growth rate goes up. Among three stationary key ratios, the ARCH effects of the consumption income ratio were not successfully estimated in the framework of both the GARCH (Engle (1982) and Bollerslev (1986)) and the EGARCH (Nelson (1991)) methods (abbreviated).

These findings would contribute to understanding better an intertemporal property of the growth paths which are characterized based on the key ratios[9]. Let $C/Y=\alpha_t$, $K/Y=\beta_t$, and $M/pY=k_t$ then assuming that these ratios are random variables at the beginning of each time period, the period to period growth paths for Y_t, K_t, and p_t, in the Klein=Kosobud model presented above can be rewritten as follows[10].

$$Y_t/Y_{t-1} = \frac{\beta_{t-1}}{\beta_t + \alpha_t - 1},$$

$$K_t/K_{t-1} = \frac{\beta_t}{\beta_t + \alpha_t - 1},$$

$$p_t/p_{t-1} = \left(\frac{M_t}{M_{t-1}}\right)\left(\frac{k_{t-1}}{\beta_{t-1}}\right)\left(\frac{\beta_t + \alpha_t - 1}{k_t}\right),$$

Based on the second order approximation of the growth path of real NNP at the beginning of each time period, the relationship

$$\beta_{t-1} E_t \left(\frac{1}{\beta_t + \alpha_t - 1}\right) =$$

9) Assar and Kymn (1989) analyzed the effects of the declining saving rate on "the Great Ratios of Economic" and such macroeconomic aggregates as the total output, the capital stock, the wage level, the interest rate, and the labor productivity.

10) The time paths of N_t and w_t are ommitted bacause they are formed using nonstationary ratios.

$$\beta_{t-1}\left(\frac{1}{\bar{\beta}+\bar{\alpha}-1}+\frac{1}{2}\left[\frac{2}{(\bar{\beta}+\bar{\alpha}-1)^3}Var(\beta_t \mid \varphi_{t-1})+\frac{2}{(\bar{\beta}+\bar{\alpha}-1)^3}Var(\alpha_t \mid \varphi_{t-1})\right.\right.$$

$$\left.\left.+\frac{4}{(\bar{\beta}+\bar{\alpha}-1)^3}Cov(\beta_t, \alpha_t \mid \varphi_{t-1})\right]\right)$$

holds, where $\bar{\beta}$ and $\bar{\alpha}$ denote the conditional means of β_t and α_t, and $Var(\beta_t \mid \varphi_{t-1})$, $Var(\alpha_t \mid \varphi_{t-1})$ and $Cov(\beta_t, \alpha_t \mid \varphi_{t-1})$ the conditional variances of β_t and α_t, and the conditional covariance between β_t and α_t defined on the basis of the information set as of the beginning of the period t. Due to the constraints $0 < \bar{\alpha} < 1$ and $1 < \bar{\beta}$, the coefficients $\frac{2}{(\bar{\beta}+\bar{\alpha}-1)^3}$, $\frac{2}{(\bar{\beta}+\bar{\alpha}-1)^3}$, and $\frac{4}{(\bar{\beta}+\bar{\alpha}-1)^3}$ are positive. Let $Var(\alpha_t \mid \varphi_{t-1})$ for which the ARCH effects were not found be constant, and $Cov(\beta_t, \alpha_t \mid \varphi_{t-1}) = 0.0098$[11] the right hand side of which is the estimate of the unconditional covariance between β_t and α_t,

Table 6

	$Var(K/Y \mid \varphi_{t-1})$	$Var(M/pY \mid \varphi_{t-1})$
Const.	0.0413	0.0085
	(8.3775)	(2.0694)
g_{t-1}	-0.1511	
	(-2.6619)	
g_{t-2}	0.1315	
	(2.3323)	
g_{t-3}		-0.0389
		(-1.0569)
AR (1)		0.2813
		(1.9666)
D/W	1.5030	1.9431
$\bar{R}^2$	0.1564	0.0613
obs.	51	49

Const. = constant term, g_i = growth rate of real NNP in the period i, AR (1) = first order serial correlation coefficient, D/W = Durbin-Watson Statistic, numbers in the parentheses = t-values, $\bar{R}^2$ = coefficient of determination adjusted for the degrees of freedom, obs. = number of observation,

11) The estimation of the time series of $Cov(\beta_t, \alpha_t \mid \varphi_{t-1})$ using a multivariate GARCH method is beyond the scope of the current paper.

then it can be said that the second term in the right hand side of the above formula varies with $Var\ (\beta_t \mid \varphi_{t-1})$ which is nagatively related to the NNP growth rate in the previous period, and is positively related to the NNP growth rate of two periods ago (Table 6).

This result reveals the possibility that the change in the NNP's growth path affects itself through the change in the conditional variance of the capital output ratio. The similar inference to this can be made for the growth paths of the capital stock and price level. Taking the conditional expectation of the second order expansion of the growth path of the capital stock, the relationship

$$E_t\left(\frac{\beta_t}{\beta_t+\alpha_t-1}\right)=\frac{\bar{\beta}}{\bar{\beta}+\bar{\alpha}-1}+\frac{1}{2}\left[\frac{2\,(1-\bar{\alpha})}{(\bar{\beta}+\bar{\alpha}-1)^3}\,Var\,(\beta_t \mid \varphi_{t-1})\right.$$

$$\left.+\frac{2\bar{\beta}}{(\bar{\beta}+\bar{\alpha}-1)^3}Var(\alpha_t \mid \varphi_{t-1})+\frac{2\,(\bar{\beta}-\bar{\alpha}+1)}{(\bar{\beta}+\bar{\alpha}-1)^3}\,Cov\,(\beta_t,\alpha_t \mid \varphi_{t-1})\right]$$

holds. Assuming that $Var\ (\alpha_t \mid \varphi_{t-1})$ and $Cov\ (\beta_t, \alpha_t \mid \varphi_{t-1})$ are constants as in the above case, a channel of influences from the changes in the NNP's growth path to the growth path of the capital stock through the changes in the conditional variance of β_t can be found. As for the growth path of the price level, taking the conditional expectation of its second order expansion,

$$E_t\left(\frac{\beta_t+\alpha_t-1}{k_t}\right)=\frac{\bar{\beta}+\bar{\alpha}-1}{\bar{k}}+\frac{1}{2}\left[\frac{2\,(\bar{\beta}+\bar{\alpha}-1)}{\bar{k}^2}\,Var\,(k_t \mid \varphi_{t-1})\right.$$

$$\left.-\frac{2}{\bar{k}^2}Cov\,(\beta_t, k_t \mid \varphi_{t-1})-\frac{2}{\bar{k}^2}\,Cov\,(\alpha_t, k_t, \mid \varphi_{t-1})\right]$$

obtains. Substituting the estimates of the unconditiional covariances between β_t and k_t (-0.0227) and between α_t and k_t (0.0082), it is found that the left hand side of this formula varies with $Var\ (k_t \mid \varphi_{t-1})$. Based on the restriction that $\frac{2\,(\bar{\beta}+\bar{\alpha}-1)}{\bar{k}^2} > 0$, this implies that when the reliability of the expected Marshallian k declines, inflation rate (p_t/p_{t-1}) is expected to get

accelerated. The negative dependence of the conditional variance of the Marshallian k on the NNP's growth rate represented in Table 6 implies that when the growth rate increases, the expected inflation rate declines with the time lag of three periods.

On the other hand, the derivation of a linearly homogeneous Cobb=Douglas production function with technical progress is not pursued here due to nonstationarity of the capital labor ratio. If such a production function was formulated in terms of the stationary key ratios, then the reliability of the technical relationship embodied in the production function could have been numerically represented by means of the conditional variances of those ratios. The concept of reliability in this sense could be significant in the occasions in which the conditional variances of β_t, κ_t, and α_t[12] show the particularly large magnitude of variations.

Concluding Remarks

Klein and Kosobud (1961) showed that if a set of ratios made from particular macroeconomic aggregates are thought of as being stable or forecastable over time, then those ratios can be treated as parameters, and be conveniently used in constructing the economic growth model they represented. By applying the test of a unit root to the data set they analyzed, this paper tried to look into the time series property of "the Great Ratios". If the ratios do not exhibit the presence of a unit root, then they would be worth being called important and key economic ratios. The three (the consumption income ratio, the capital output ratio, and the Marshallian k) out of five ratios were empirically interpreted to be stationary around the trend lines. The rest (labor share and the capital labor ratio) was interpreted to be nonstationary. The trend lines would not

12) As has already been noted, the conditional variance of α_t was not successfully estimated for the data set used in this paper.

provide useful information on the whereabout of the nonstationary variables, and it would be difficult to make use of those nonstationary ratios as the corner stone in constructing the Klein=Kosobud type of macroeconomic model. For the three key ratios considered to be trend stationary, their conditional variances were calculated. The economic implications of the conditional variances were investigated in the context of business cycles. The influences of the conditional variances of some of the key ratios on the growth paths of the macroeconomic aggregates were briefly analyzed. The behavioral implications of such influences are not explicitly considered in this paper. The analysis using a contemporary data set should be made immediately. Extension of the ARCH method from the univariate to the multivariate models is to be made especially to take into account the conditional covariances among the relevant ratios.

References

Assar, N. H. and Kymn, K. O. (1989), "The Impact of the Declining Saving Rate on the Great ratios of Economics", Rivista Internazionale di Scienze Economiche e Commerciali, vol. 36, N. 1, 29-38.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroscedasticity", Journal of Econometrics, 31, 307-27.

Deb, P. (1996), "Finite Sample Properties of Maximum Likelihood and Quasi-Maximum Likelihood Estimators of EGARCH Models", Econometric Reviews, 15 (1), 51-68.

Dickey, D. and Fuller, W. (1979), "Distribution of the Esimates for Autoregressive Time series with a Unit Root", Journal of the American Statistical Association, 74, 427-31.

Engle, R. F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", Econometrica, 50, 987-1007.

Enders W. (1995), Chap. 4, Applied Econometric Time Series, Wiley.

Klein, L. R. and Kosobud, R. F. (1961), "Some Econometrics of Growth: Great Ratios of Economics", Quarterly Journal of Economics, 2, 173-198.

Nelson, D. (1991), "Conditional Heteroscedasticity in Asset Returns: A New Approach", Econometrica, 59, 347-70.

Nunes, L., Newbold, P. and Kuan, C. H. (1997), "Testing for Unit Roots with Breaks: Evidence on the Great Crash and the Unit Root Hypothesis Reconsidered", Oxford Bulletin of Economics and Statistics, 59, 4, 435-448.

Perron, P. (1989), "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis", Econometrica, 57, 1361-1401.