

Is Balanced Growth Path Subgame Perfect?

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1. Introduction

In many endogenous economic growth models, the possibility of an economy to grow is often discussed whether the growth rate g on the balanced growth path expressed in terms of the parameters in the model is larger than one. It is known that, if in a model, the technologies in consumption goods production sector and capital goods production sector combined together exhibit increasing returns to scale, then the model can generate an unbounded growth.¹⁾ For example, in Romer (1986), Lucas (the second model, 1988) and Yuen (1992), a consumption goods production technology, which exhibits increasing returns to scale because of the Marshallian externality, combined with a linear capital accumulation technology generates an unbounded economic growth. In Tamura (1991) and Glomm and Ravikumar (1992), a capital accumulation technology, which exhibits increasing returns to scale, combined with a linear consumption goods production technology generates an unbounded economic growth.

Consider an endogenous economic growth model in which the decision about improving the quality of existing capital (R & D activities) is the engine of growth. The question I would like to address in this

1) The increasing returns to scale is not necessary to have an unbounded growth. See Jones and Manuelli (1990) for the discussion.

paper is whether an economic agent has an incentive to spend resources for the R & D activities so that the economy will grow. If the agent lives forever, then the answer is yes since the future benefits from the improved capital will accrue to the same individual. On the other hand, if the agent lives a finite life, then the benefit from the R & D activities may accrue to future generations, after his/her death. The agent may be better off by spending resources for activities which yield immediate payoff rather than for the R & D activities.

What is missing here is the existence of capital market where people can buy and sell capital assets. With a capital market, the improved quality of capital through the R & D activities should be capitalized in the price of the assets. That is, even if an agent lives a finite life, and the benefits from the current R & D activities may accrue to future generations, the agent can earn capital gains in the form of an increase in the asset price by spending resources for the R & D activities. Therefore, an economic agent will have an incentive to spend for the R & D if the capital gain is large enough relative to the return on activities which yield an immediate payoff.²⁾

In this paper, it will be shown that in a decentralized overlapping generations model, depending on technologies which determine the rate of return on the R & D activities and other alternative activities, each

2) Jones and Manuelli (1992) and Boldrin (1992) showed that the asymptotic growth rate in an one sector overlapping generations model with a convex technology is zero since the income of young agents does not grow as fast as the value of the capital assets owned by old agents does so that the young agents can not purchase the capital assets from the old agents. Therefore, it is necessary to have an unbounded growth that the incomes of the young agents grows at least as fast as the value of the capital assets owned by the old agents which is possible in an overlapping generations model with two production sectors (consumption goods and capital goods), or in an one sector overlapping generations model with a non-convex technology. In this paper, the latter case is considered.

agent may not have an incentive to spend resources for the R & D activities so that an economy, even if it can potentially grow, may stay at a stationary (no growth) state.

The intuitive reason for this observation can be explained by a centipede game. The structure of the game is described in figure 1. There are two players, X and Y. Player X takes either one of two actions, a or b. Similarly, player Y takes either one of two actions, α or β . They alternate the moves in each stage. In the figure, the two numbers in each parenthesis are the payoff to player X, the first element, and the payoff to player Y, the second element, respectively. In the first stage if player X takes action b, then the game ends with the outcome in which the payoff to X is 2 and the payoff to Y is 2. If X takes action a instead, then the game continues to the second stage, player Y's turn. If Y takes action β , then the game ends with the outcome in which the payoff to X is 1 and the payoff to Y is 4. If Y takes action α instead, then the game continues to the third stage, player X's turn again. They repeat this process until one of them chooses either b or β so that the game ends, or until the last stage arrives. It can be shown that the unique subgame perfect equilibrium outcome of this game is that player X takes action b in the first stage and the game ends immediately with payoff (2, 2). If, instead, they keep taking a and α so that the game continues to the last stage, they

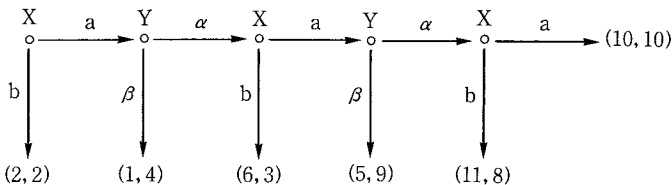


Figure 1 Centipede Game

Two numbers in each parenthesis are the payoff to player X, the first element, and the payoff to player Y, the second element, respectively

can attain the payoff (10, 10) at the end. This could have been the outcome if X and Y were the same person. However, since there is no way to transfer the payoff from the later stages to the first stage, player X has no incentive to take action a in the first stage.

Suppose we interpret that actions b and β are “to consume all the resources immediately and leave nothing for future”, and that actions a and α are “not to consume all the resources immediately and leave something for future (so that the payoff may grow gradually)”. To assume that player X and Y are the same person is interpreted that there is an individual who lives over the length of the entire game. Such a long-lived individual will keep taking actions a and α to leave something for future so that his/her payoff gets larger. On the other hand, to assume that X and Y are different individuals may be interpreted that there is a sequence of players each of whom lives a short life. Depending on the payoff structure, even if the total payoff is growing at each stage, none of the players has incentive to leave something for future unless there is a way to transfer resources from the players in the later stages to the players in the early stages. Therefore, no economic growth is observed in such a situation.

This paper is organized as follows. In section 2, the basic structure of the model is explained in which capital assets are introduced *a la* Lucas’ tree (Lucas, 1978) with endogenous determination of the assets’ quality through R & D activities in an overlapping generations model. Then, the model is solved for the balanced growth path in which all the endogenous variables grow at the same rate. In section 3, it will be shown that the balanced growth path may not be subgame perfect. I will propose a situation in which all the agents ignore the R & D efforts and the economy stays at the stationary (no growth) path. Then, I will seek the possibility whether the economy can

takeoff toward the balanced growth path by comparing the utility level of agents under the balanced growth path and that under the stationary path at the beginning of the economy. Conclusion and discussion follow in section 4.

2. Asset Pricing on the Balanced Growth Path

One way to incorporate capital asset pricing in a dynamic economic model can be described as follows.³⁾

Consider an economy inhabited by a large number of identical consumers who lives forever. The consumption planning of a typical consumer is to choose a consumption stream $c(t)$, $t=0, 1, 2, \dots$ subject to a budget constraint so as to maximize the life time utility

$$\sum_{t=0}^{\infty} \delta^t u(c(t))$$

where $\delta \in (0, 1)$ is the subjective discount rate and $u: R_+ \rightarrow R$ is a concave increasing felicity function. The optimal consumption schedule should satisfy the Euler equation

$$u'(c(t)) = \delta(1+R(t))u'(c(t+1))$$

where R is the rate of return in the economy.

The capital asset in this economy, which is known as the Lucas' tree (c. f. Lucas (1978)) is regarded as a tree which yields dividend $d(t)$ per tree in each period t , measured in terms of consumption goods. Assume that there are as many trees as the number of people in the economy so that each consumer has one tree in equilibrium. The rate of return on the tree consists of capital gain and dividend yield so that the following is true in the absence of arbitrage opportunity.

$$1+R(t) = \frac{p(t+1) + d(t)}{p(t)}$$

3) For the general discussion of capital asset pricing, see Sargent (1987).

where $p(t)$ is the price of the tree measured in terms of consumption goods. By the assumption that all the people are the same, $c(t) = d(t)$ for all t in an equilibrium. Therefore, from the Euler equation the asset pricing formula is obtained as

$$\begin{aligned} p(t) &= \sum_{j=0}^{\infty} \delta^j \left[\prod_{s=0}^{j-1} \left(\frac{u'(d(t+s-1))}{u'(d(t+s))} \right) \right] d(t+j) \\ &= \sum_{j=0}^{\infty} \delta^j \left[\frac{u'(d(t+j))}{u'(d(t))} \right] d(t+j) \end{aligned}$$

provided that the transversality condition is satisfied.

I am going to transplant the lucas' tree in an overlapping generations model which has the following structure. In each period t , there are a large number of identical agents who live two periods, period t and period $t+1$. Population is constant so that in each period t , there are the same number of agents in their first period (young) who were born in period t , and agents in their second period (old) who were born in period $t-1$. The size of a generation is normalized to be one. At the beginning of period t , a young agent has one unit of time. The young supplies $h(t)$ units of labor (out of one) for old agents at wage $w(t)$, measured in units of consumption goods, borrows $b(t)$ units of consumption goods at the interest rate $R(t)$, purchases $x(t)$ units of capital asset (tree) of quality $A(t)$ from old agents at price $p(t)$ per tree, consumes $c_1(t)$ units of consumption goods and spends $1-h(t)$ units of time (R & D activity) to improve the quality of the capital asset purchased from the old agents. Therefore, the budget constraint of the young in period t is expressed as

$$c_1(t) + p(t)x(t) = w(t)h(t) + b(t). \quad (1)$$

The quality of the capital is improved by the R & D activity, $1-h(t)$, as

$$A(t+1) = A(t)[\beta(1-h(t)) + 1], \quad \beta > 0.$$

In period $t+1$, when the young becomes old, he/she employs $h(t+1)$ units of labor of young agents at wage $w(t+1)$ to produce $y(t+1)$ units of consumption goods, repays $(1+R(t))b(t)$ units of consumption goods to lenders, sells $x(t)$ units of the capital asset (tree) of quality $A(t+1)$ to the young agents at price $p(t+1)$ per tree, and consumes $c_2(t+1)$ units of consumption goods. Therefore the budget constraint of the old in period $t+1$ is expressed as

$$c_2(t+1) + (1+R(t))b(t) + w(t+1)h(t+1) = p(t+1)x(t) + y(t+1). \quad (2)$$

By employing $h(t+1)$ units of labor of the young agents

$$y(t+1) = A(t+1)h(t+1)^\alpha, \quad 0 < \alpha < 1$$

units of consumption goods are produced.

A young agent in period t has a preference over $c_1(t)$ and $c_2(t+1)$ described by a well-behaved utility function $u(c_1(t), c_2(t+1))$. The young in period t chooses $c_1(t)$, $c_2(t+1)$, $h(t)$, $h(t+1)$, $b(t)$ and $x(t)$, given $p(t)$, $p(t+1)$, $w(t)$, $w(t+1)$, $R(t)$ and $A(t)$ to maximize the utility subject to the budget constraints (1) and (2).

In period zero, an old agent employs $h(0)$ units of labor of the young agents at wage $w(0)$ to produce $A(0)h(0)^\alpha$ units of consumption goods, sells $x(0)$ units of the capital asset to the young agents at price $p(0)$ and consumes $c_2(0)$ units of consumption goods. Assume that he/she has no credit obligation in period zero ($b(-1) = 0$). Therefore, the old chooses $h(0)$, given $p(0)$, $w(0)$ and $A(0)$, to maximize $c_2(0)$ subject to the budget constraint

$$c_2(0) + w(0)h(0) = p(0)x(0) + A(0)h(0)^\alpha.$$

It is assumed that there are as many trees as the number of people of one generation so that in the absence of arbitrage opportunity each young agent purchases one tree in equilibrium. Therefore, I put $x(t) = 1$ for $t=0, 1, \dots$ in the following analysis.

In this model, even if each individual has a finite life, it is likely that

he/she has an incentive to spend resources for the R & D activity to improve the quality of the capital asset. It is the later generations who can enjoy the improved quality (productivity) of the asset rather than the generation who makes the R & D effort. However, the higher quality can be capitalized in the selling price of the asset.⁴⁾

The budget constraints (1) and (2) are combined to yield a difference equation with respect to $p(t)$ which is solved as

$$p(t) = [w(t)h(t) - c_1(t)] + \sum_{i=1}^{\infty} \left[\prod_{j=0}^{i-1} \left(\frac{1}{1+R(t+j)} \right) [A(t+i)h(t+i)^{\alpha} - (c_1(t+i) + c_2(t+i))] \right]$$

provided that the transversality condition is satisfied (in equilibrium) which will be shown later.

The optimization program of a young agent in period t is

$$\max u(c_1(t), c_2(t+1))$$

subject to

$$c_1(t) + \frac{c_2(t+1)}{1+R(t)} = w(t)h(t) + \frac{1}{1+R(t)} [A(t+1)h(t+1)^{\alpha} - w(t+1)h(t+1)] + \left[\frac{p(t+1)}{1+R(t)} - p(t) \right] x(t).$$

Notice that for a young agent in period t , the sale price of a tree depends not only on his/her action, $h(t)$, through $A(t+1)$ but also on the actions of all the future generations, $h(t+s)$, $s=1, 2, 3, \dots$. For simplicity, log-linear utility with a subjective discount rate $\delta \in (0, 1)$ is assumed, i. e.,

$$u(c_1(t), c_2(t+1)) = \ln c_1(t) + \delta \ln c_2(t+1).$$

Starting at period zero with a state variable $A(0)$, the competitive equilibrium is a sequence of prices $\{p(t), R(t), w(t)\}$, $t=0, 1, \dots$, and

4) The balanced growth path solution of the model with infinite-life agents is given in the appendix at the end of the paper.

a sequence of allocation $\{c_1(t), c_2(t), h(t)\}$, $t=0, 1, \dots$, such that the sequence of allocation is a solution to the utility maximization program of each agent given the sequence of prices, and the sequence of prices clears the goods market, the capital asset market, the credit market and the labor market. On the balanced growth path, $A(t)$, $p(t)$, $c_1(t)$ and $c_2(t)$ grow at the same rate g , and $w(t)$ grows at least as fast as $p(t)$ so that the young agents can purchase the capital asset from the old agents.

In order to solve for the balanced growth path, we have to impose the following conditions. For all $t=0, 1, \dots$,

$$\begin{aligned} b(t) &= 0 \quad (\text{the credit market clearing condition}) \text{ and} \\ p(t+1)/p(t) &= 1+R(t) \quad (\text{no arbitrage condition}). \end{aligned}$$

Then the balanced growth path solution is calculated as

$$\begin{aligned} h &= (\alpha/\beta) ((1+\beta)/(1+\alpha)) \\ g=1+R &= (1+\beta)/(1+\alpha) \tag{3} \\ c_1(t) &= g^t c_1(0), \quad t=0, 1, \dots \\ c_1(0) &= (1/(1+\delta)) A(0) h^\alpha \\ c_2(t+1) &= g^t c_2(1), \quad t=0, 1, \dots \\ c_2(1) &= \delta(1+R) c_1(0) \\ c_2(0) &= (\delta/(1+\delta)) A(0) h^\alpha \\ A(t) &= g^t A(0) \\ w(t) &= g^t w(0) \\ w(0) &= \alpha A(0) / h^{1-\alpha} \\ p(t) &= g^t p(0) \\ p(0) &= [w(0) h(0) - c_1(0)] \tag{4} \\ &+ \sum_{t=1}^{\infty} \left(\frac{1}{1+R} \right)_0^t [A(t) h(t)^\alpha - (c_1(t) + c_2(t))] \\ &= w(0) h(0) - c_1(0) \end{aligned}$$

$$= \left[\frac{\alpha(1+\delta)-1}{1+\delta} \right] A(0) h^\alpha.$$

The second equality in (4) follows from the fact that

$$A(t)h(t)^\alpha - (c_1(t) + c_2(t)) = 0, \quad t=0, 1, \dots$$

which is nothing but a period-wise resource constraint.

From (3), it is necessary and sufficient that $\beta > \alpha$ to have a positive growth on the balanced growth path. Notice that for all $t=0, 1, \dots$

$$w(t)h/p(t) = (\alpha(1+\delta)) / (\alpha(1+\delta) - 1) > 1.$$

Therefore, the young agent can purchase the capital asset from the old agent.

Now consider the following imaginary situation. Each generation $t=0, 1, \dots$ ignores the effect of the R & D effort on $A(t+1)$. Since the optimal level of labor in such a situation is $h(t)=1$ for all t , this will lead to the following stationary path (no growth) solution.

$$h=1$$

$$g=1+R(t)=1, \quad t=0, 1, \dots$$

$$c_1(t) = (1/(1+\delta))A(0), \quad t=0, 1, \dots$$

$$c_2(t+1) = \delta c_1(t), \quad t=0, 1, \dots$$

$$c_2(0) = (\delta/(1+\delta))A(0)$$

$$A(t) = A(0), \quad t=0, 1, \dots$$

$$w(t) = \alpha A(0), \quad t=0, 1, \dots$$

$$p(t) = p(0), \quad t=0, 1, \dots$$

$$p(0) = [w(0) - c_1(0)] + \sum_{t=1}^{\infty} [A(t) - (c_1(t) + c_2(t))]$$

$$= w(0) - c_1(0)$$

$$= ((\alpha(1+\delta) - 1) / (1+\delta)) A(0)$$

Notice that $w(t)h/p(t) = w(0)/p(0) > 1$ so that the young agent can purchase the capital asset from the old agent.

Denote the utility of generation t on the balanced growth path as U_g

(t) which is calculated as

$$U_g(t) = \ln \left[g^t \frac{A(0)h^\alpha}{1+\delta} \right] + \delta \ln \left[g^{t+1} \frac{\delta A(0)h^\alpha}{1+\delta} \right]. \quad (5)$$

Similarly, denote the utility of generation t on the stationary path as U_s (t) which is calculated as

$$U_s(t) = U_s = \ln \left[\frac{A(0)}{1+\delta} \right] + \delta \ln \left[\frac{\delta A(0)}{1+\delta} \right]. \quad (6)$$

From (5) and (6), it can be shown that

$$U_g(t) = U_s + \Psi(\alpha, \beta, \delta, t)$$

where

$$\Psi(\alpha, \beta, \delta, t) = [\delta + (\alpha + t)(1 + \delta)] \ln \left[\frac{1 + \beta}{1 + \alpha} \right] + \alpha(1 + \delta) \ln \left[\frac{\alpha}{\beta} \right].$$

In figure 2, $U_g(t)$ and U_s are plotted for two different sets of parameter values. Since Ψ increases monotonically (linearly) in t , U_g gets larger than U_s as t increases. However, even if $\beta > \alpha$ is the condition for $g > 1$, unless β is much larger than α , generation zero is worse off on the balanced growth path relative to the stationary path. Such observation leads one to suspect the incentive to spend resources for the R & D activity by generation zero at the beginning of the economy. In the next section, I will seek the possibility of an economy taking off toward the balanced growth path.

3. Sustainability of the Balanced Growth Path

The question addressed in this section is whether the balanced growth path can be sustained in a decentralized overlapping generations economy.

Consider the following two cases.

Case 1: Generations $t \geq 1$ follow the balanced growth path solution.

Case 2: Generations $t \geq 1$ follow the stationary path solution.

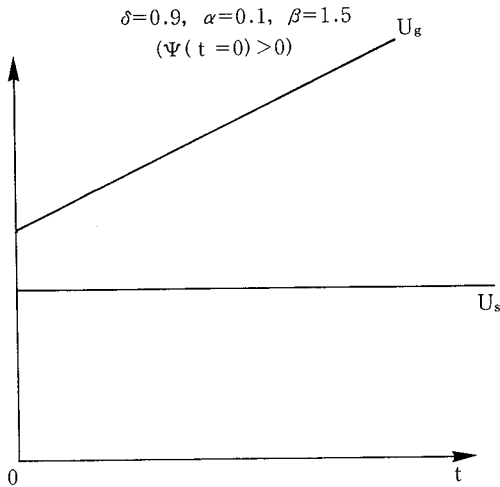
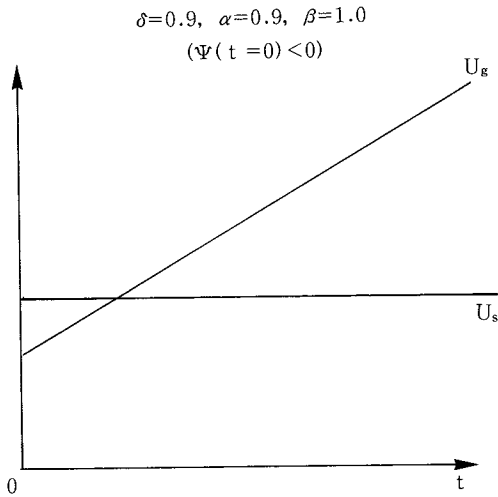


Figure 2

In case 1, the asset price in period 1 is

$$p(1) = \left(\frac{\alpha(1+\delta)-1}{1+\delta} \right) \left(\frac{\alpha}{\beta} \frac{1+\beta}{1+\alpha} \right)^\alpha A(1).$$

If an agent in period zero also follows the balanced growth path solution, then the solution to the system is the same as the balanced growth path in section 2.

On the other hand, if the agent in period zero ignores the effect of the R & D effort $h(0)$ on $A(1)$, then the system is solved as follows.

$$\begin{aligned} h(0) &= 1 \\ c_1(0) &= (1/(1+\delta))A(0) \\ c_2(1) &= \left(\frac{\delta}{1+\delta}\right)A(0) \left(\frac{\alpha}{\beta} \frac{1+\beta}{1+\alpha}\right)^\alpha \\ w(0) &= \alpha A(0) \\ w(1) &= \alpha A(0) \left[\frac{\alpha}{\beta} \frac{1+\beta}{1+\alpha}\right]^{1-\alpha} \\ \dot{p}(0) &= ((\alpha(1+\delta) - 1)/(1+\delta))A(0) \\ \frac{\dot{p}(1)}{\dot{p}(0)} &= 1 + R(0) = \left(\frac{\alpha}{\beta} \frac{1+\beta}{1+\alpha}\right)^\alpha < 1 \\ A(1)/A(0) &= 1 \end{aligned}$$

Notice that the interest rate is negative, and the asset value shrinks between period zero and period one.

Denote the utility of generation zero as U_1 if he/she follows the balanced growth path solution and U_2 if he/she chooses $h(0)=1$. Then, it can be shown that the following relationship between U_1 and U_2 holds.

$$U_1 = U_2 + \Phi(\alpha, \beta, \delta) \tag{7}$$

where

$$\Phi(\alpha, \beta, \delta) = \alpha \ln\left(\frac{\alpha}{\beta}\right) + (\alpha + \delta) \ln\left(\frac{1+\beta}{1+\alpha}\right). \tag{8}$$

Therefore, if $\Phi > 0$, then $U_1 > U_2$ so that the balanced growth path can be sustained. On the other hand, if $\Phi < 0$, then $U_1 < U_2$ so that generation zero can be better off by ignoring the R & D effort ($h(0)$)

=1).

In case 2, the asset price in period 1 is

$$p(1) = ((\alpha(1+\delta) - 1) / (1+\delta))A(1).$$

If an agent in period zero follows the balanced growth path solution, then the system is solved as follows.

$$h(0) = \frac{\alpha}{\beta} \frac{1+\beta}{1+\alpha}$$

$$c_1(0) = (1/(1+\delta))A(0)h(0)^\alpha$$

$$c_2(1) = \left(\frac{1+\beta}{1+\alpha}\right) \left(\frac{\delta}{1+\delta}\right)A(0)$$

$$w(0) = \alpha A(0) / h(0)^{1-\alpha}$$

$$w(1) = \left(\frac{1+\beta}{1+\alpha}\right) \alpha A(0)$$

$$p(0) = ((\alpha(1+\delta) - 1) / (1+\delta))A(0)h(0)^\alpha$$

$$\frac{p(1)}{p(0)} = 1 + R(0) = \left(\frac{\beta}{\alpha}\right)^\alpha \left(\frac{1+\beta}{1+\alpha}\right)^{1-\alpha}$$

$$\frac{A(1)}{A(0)} = \frac{1+\beta}{1+\alpha}$$

On the other hand, if generation zero also ignores the effect of the R & D effort $h(0)$ on $A(1)$, then the solution to the system is the same as the stationary path solution in section 2.

Denote the utility of generation zero as V_1 if he/she follows the balanced growth path solution and V_2 if he/she chooses $h(0)=1$. Then, it can be shown that the following relationship between V_1 and V_2 holds.

$$v_1 = v_2 + \Phi(\alpha, \beta, \delta) \tag{9}$$

where Φ is the same as (8).

From (7) and (9), if $\Phi > 0$ then $U_1 > U_2$ and $V_1 > V_2$. Therefore, all the generations $t=0, 1, \dots$ follow the balanced growth path, i. e., the

is intuitively clear since the immediate benefit from the asset $A(t)h(t)$ gets more attractive than the return from the R & D activity as the discount factor gets smaller.

4. Conclusion

As stated in the introduction, in the endogenous economic growth literature, the possibility of an economy to grow is often discussed in such a way that whether growth rate g expressed in terms of parameters in the model is larger than one. In our model, the growth rate on the balanced growth path is expressed in terms of two parameters which describe the productivity of the consumption goods production technology, α , and the productivity of the capital goods production technology (capital goods improvement technology), β . We saw that $g > 0$ if and only if $\beta > \alpha$. However, as we saw in section 3, $\beta > \alpha$ is not sufficient for the balanced growth path to be supported as a subgame perfect equilibrium. For an economy to grow, β should be relatively larger than α , not necessary in absolute size. In other words, the improvement in the quality of assets through the R & D effort should be relatively more efficient than the improvement in the immediate output through the alternative activities. Otherwise, in each period, an agent with finite life does not have an incentive to spend resources for the R & D activity which is the engine of growth in the economy since spending all the resources on the activities which yield the immediate benefit is more attractive to the agent.

One of the most important question about economic growth theory is how to explain the significant decrease in the growth rate of major industrialized countries after the oil shock of 1970s' (see Aaron (1989)). It may be the change in the characteristics of technologies which caused such a sudden drop in the growth rate. Even if the productivity

levels remained stable, a small change in the relative productivity between R & D investments and alternative activities, which do not improve the quality of capital but yield immediate benefits, may be able to cause a significant decrease in the growth rate through the change in the incentive of economic agents. Business managers are often criticized about seeking immediate benefits and not having long-run views which result in lower economic growth. However, these business managers may be acting rationally since there is no mechanism to reward them enough to have an incentive to R & D efforts. If an economy wants a higher economic growth, it may be necessary to introduce an investment subsidy scheme which will change the relative attractiveness between R & D activities and alternative activities which yield immediate benefits.⁵⁾

Appendix: The consumption planning of an agent with an infinite life is analyzed as follows. The notations used in the following model are the same as those used in the main text. An agent who lives forever with capital asset solves the following optimization program.

$$\max \sum_{t=0}^{\infty} \delta^t \ln c(t)$$

$$\{h(t)\}_{t=0}^{\infty}$$

subject to

$$c(t) + (1+R(t-1))b(t-1) = A(t)h(t)^\alpha + b(t), \quad t=0, 1, \dots$$

$$A(t+1) = A(t)[\beta(1-h(t))+1], \quad t=0, 1, \dots$$

given $A(0)$.

On the balanced growth path, it can be shown that

5) For the discussion of such policies, see Drazen (1978), Kotlikoff and Summers (1981) and Jones and Manuelli (1992).

$$h = \frac{\alpha(1-\delta)(1+\beta)}{\beta[\alpha+\delta(1-\alpha)]}$$

$$g = \frac{A(t+1)}{A(t)} = \frac{\delta(1+\beta)}{\alpha+\delta(1-\alpha)}$$

From this, unbounded growth, $g > 1$, is possible if $\delta(\alpha + \beta) > \alpha$. From noarbitrage condition,

$$\frac{A(t+1)}{A(t)} = \delta(1+R).$$

Therefore, an unbounded growth also implies that $1+R > 1/\delta$. The maximized utility is calculated as

$$\sum_{t=0}^{\infty} \delta^t \ln c(t) = \frac{\delta}{(1-\delta)^2} \ln g + \frac{\delta}{1-\delta} \ln(A(0)h^\alpha).$$

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