

Public Debt and the Corporate Financial Structure

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1. Introduction

The main objective of this paper is to trace out the effect of an increase in public debt on corporate capital structure (debt-equity ratio) by employing the asset markets general equilibrium model of Tobin (1969).

Empirically, the followings are observed in the US. After the World War II, public debt to GNP ratio has been decreasing, and corporate debt to GNP ratio has been increasing. The overall non-financial sector total debt to GNP ratio has remained about constant level (Friedman, 1985). Public debt and corporate debt are substitutes such that the increase in one may cause the decrease in another, i. e., crowding out effect (McDonald, 1983). Corporate debt-equity ratio has been increasing. The average corporate debt ratio (corporate debt divided by asset value) is about 17% between 1955 and 1967, 20% between 1968 and 1977 (Ciccolo and Bawn, 1985).

To explain these observations, I employed Tobin's general equilibrium model of asset markets in which demands and supplies of assets are equated through the adjustment of each assets' rate of return. If there is an excess supply of an asset in one market, then it is absorbed in general not only through the increase in the asset's rate of return but also through the decreases in all the other assets' rate of

return. This then will induce the portfolio reshuffling of both demand and supply of assets. From the view point of firms, this implies an adjustment of debt-equity ratio.

This paper is organized as follows. In section 2, the structure of model will be explained. To clarify the adjustment process of corporate debt-equity ratio, the short-run and the long-run situations are considered separately. The short-run implies the time periods not long enough for firms to adjust their debt-equity ratio initiated by changes in exogenous variables, and the long-run implies the time periods long enough for firms to complete the adjustment. In section 3, comparative static analysis will be performed for the short-run and the long-run, respectively. Section 4 deals with special case in which public debt is very small so that income effect can be neglected. Then, the results of comparative static analysis in section 3 will be reinterpreted in terms of the elasticity of demand for each assets.

2. Model

Consider a model economy described as follows. There are three sectors (government, firms and households) and three assets (government bonds, corporate bonds and corporate stock) in the economy. The

Table 1

Asset \ Sector	Government	Firms	Households	Net rate of return
Government bonds	$-F$		F	$\tilde{r}_F = (1 - c_F)r_F$
Corporate bonds		$-B$	B	$\tilde{r}_B = (1 - c_B)r_B$
Corporate stock		$-S$	S	$\tilde{r}_S = (1 - c_S)r_S$
Physical capital		$q \cdot K$		
(Tax)	(T)		$(-T)$	
Net wealth	0	0	$W - T^*$	

balance sheet of the economy is summarized in table 1. F , B and S are for the market value of government bonds, corporate bonds and corporate stock, respectively. The government bonds may be redeemed by future taxation on household sector. However, it is assumed that households regard the government bonds as net wealth.¹⁾ K is the stock of physical capital of firms, and q is the price of one unit of capital.

Asset markets are in equilibrium when the following equations are satisfied.²⁾

$$(1) \quad F = f_F(\tilde{r}_F, \tilde{r}_B, \tilde{r}_S) W$$

$$(2) \quad B = f_B(\tilde{r}_F, \tilde{r}_B, \tilde{r}_S) W$$

$$(3) \quad S = f_S(\tilde{r}_F, \tilde{r}_B, \tilde{r}_S) W$$

where

$$(4) \quad W = F + B + S$$

$$(5) \quad f_F + f_B + f_S = 1$$

f_F , f_B and f_S are the fractions of the households' total asset W invested in government bonds, corporate bonds and corporate stock, respectively. \tilde{r}_F , \tilde{r}_B and \tilde{r}_S are the after-tax rate of return on government bonds, corporate bonds and corporate stock, respectively, defined as

$$(6) \quad \tilde{r}_i = (1 - c_i) r_i, \quad i = F, B, S$$

where r_i is the gross rate of return on asset i , $i = F, B, S$, and c_i is the tax rate imposed on asset i , $i = F, B, S$. Each asset is assumed to be a substitute, i. e.,

1) McDonald (1983) used a parameter $\gamma \in [0, 1]$ such that $\gamma \times 100\%$ of F is recognized as net wealth by households. For the discussion about the situations in which the Ricardian equivalence does not hold, see Barro (1974).

2) Auerbach and Kind (1983) considered an portfolio equilibrium in a model with investors who have mean-variance argued preference facing differing tax treatments.

$$(7) \quad \left. \begin{array}{l} \frac{\partial f_i}{\partial r_j} > 0, \quad i=j \\ \frac{\partial f_i}{\partial r_j} < 0, \quad i \neq j \end{array} \right\} \text{ for } i, j = F, B, S.$$

With respect to firms,

$$(8) \quad qK = B + S$$

holds identically. The cost of capital is defined as

$$(9) \quad \rho = (1 - \theta) \frac{\tilde{r}_S}{1 - c_S} + (1 - \tau) \theta \frac{\tilde{r}_B}{1 - c_B}$$

or

$$(10) \quad \rho = (1 - \theta) r_S + (1 - \tau) \theta r_B$$

where

$$(11) \quad \theta = B / (B + S)$$

is the corporate debt ratio. τ is corporate tax rate so that (10) implies that the interest payment on corporate bond is tax deductible. q and ρ are related through the marginal product of capital R in such a way that

$$(12) \quad q = R / \rho.$$

Households are assumed to expect R to be constant. For a fixed level of K , the maximization of the value of firm is equivalent to the minimization of the cost of capital. Therefore, the firms' decision rule with respect to θ is described as

$$(13) \quad \theta = \begin{cases} = 1 & \text{if } r_S > (1 - \tau) r_B \\ \in [0, 1] & \text{if } r_S = (1 - \tau) r_B \\ = 0 & \text{if } r_S < (1 - \tau) r_B. \end{cases}$$

In order to trace out the adjustment of θ explicitly, two cases will be considered with respect to the treatment of θ . In the *short-run*, time periods are not long enough for firms to adjust θ in response to changes in exogenous variables. In the *long-run*, firms complete the adjustment of θ .

Since there are three asset markets which are related through the

budget constraint (4), we omit the government bonds market by Walras' law and take r_F as numeraire. Therefore, in the short-run, the system determines seven endogenous variables (r_B , r_S , B , S , ρ , q , W) given eight exogenous variables (c_F , c_B , c_S , F , K , R , τ , θ), and θ will be endogenized in the long-run.

The equations system described above is reduced to the following three equations.

$$(14) \quad \theta = f_B \left(1 + \frac{F}{RK} ((1-\theta)r_S + (1-\tau)\theta r_B) \right)$$

$$(15) \quad 1 - \theta = f_S \left(1 + \frac{F}{RK} ((1-\theta)r_S + (1-\tau)\theta r_B) \right)$$

$$(16) \quad r_S = (1-\tau)r_B$$

(14) and (15) correspond to the market equilibrium conditions for corporate bonds market and corporate stock market, respectively. (16) captures the optimization behavior of firms. The existence of the interior solution with respect to θ is assumed.

In the short-run, (14) and (15) determine the equilibrium value of r_B and r_S given the exogenous variables and θ . In the long-run, (14), (15) and (16) determine the equilibrium value of r_B , r_S and θ .

For the comparative static analysis which will be performed in the later sections, stability conditions for the system are described by the correspondence principle (Samuelson, 1947). First, let us assume the following adjustment process.

$$(17) \quad \dot{r}_B = \alpha_B \left(\theta - f_B \left(1 + \frac{F}{RK} ((1-\theta)r_S + (1-\tau)\theta r_B) \right) \right)$$

$$(18) \quad \dot{r}_S = \alpha_S \left(1 - \theta - f_S \left(1 + \frac{F}{RK} ((1-\theta)r_S + (1-\tau)\theta r_B) \right) \right)$$

$$(19) \quad \dot{\theta} = \alpha_\theta (r_S - (1-\tau)r_B)$$

where

$$\alpha_B(0) = \alpha_S(0) = \alpha_\theta(0) = 0$$

$$\alpha_B' > 0, \quad \alpha_S' > 0 \quad \text{and} \quad \alpha_\theta' > 0.$$

(17) ((18)) implies that the excess supply in corporate bonds market (corporate stock market) induces an increase in r_B (r_S). (19) implies that θ will be increased when the cost of equity finance is larger than the cost of debt finance.

By taking the linear approximation of the system around equilibrium values, r_B^* , r_S^* and θ^* , we obtain the following stability conditions.

$$(20) \quad (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* > 0$$

$$(21) \quad (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\theta^*) > 0$$

$$(22) \quad \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \right\}$$

$$\times \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\theta^*) \right\}$$

$$- \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\theta^*) \right\}$$

$$\times \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\tau) \theta^* \right\} > 0$$

$$(23) \quad \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \right\}$$

$$+ (1-\tau) \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\theta^*) \right\}$$

$$+ \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\tau) \theta^* \right\}$$

$$+ (1-\tau) \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\theta^*) \right\} > 0$$

where $\rho^* \equiv (1-\theta^*)r_S^* + (1-\tau)\theta^*r_B^*$. f_i^* and $\partial f_i^*/\partial \bar{r}_j$, $i, j=B, S$ imply

that these functions are evaluated at equilibrium values. We assume that

$$(23)' \quad (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \\ + (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\tau) \theta^* > 0$$

and

$$(1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\theta^*) \\ + (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\theta^*) > 0$$

which is sufficient for (23) to hold. The interpretation is that the change in r_B (r_S) causes the larger change in f_B (f_S) than in f_S (f_B), i. e., the “own” effect is larger than the “cross” effect which is equivalent to assuming the dominant diagonality (c. f. Takayama, 1985, p. 380).

The graphical exposition of the long-run equilibrium values of r_B and r_S (and θ implicitly) is shown in figure 1-2, 1-2 and 1-3. BB is the locus of (r_B, r_S) which satisfies (14), and SS is the locus of (r_B, r_S) which satisfies (15). The ray from the origin with slope $1-\tau$ is (16). Notice that $r_S > (1-\tau)r_B$ holds above the ray so that $\dot{\theta} > 0$ and $r_S < (1-\tau)r_B$ holds below the ray so that $\dot{\theta} < 0$. Since the long-run equilibrium (r_B^*, r_S^*) is on the ray (the intersection of BB and SS is on the ray), $\dot{\theta} = 0$.

The slope of BB is

$$(24) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} = - \frac{(1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\tau) \theta^*}{(1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_B^* \frac{F}{RK} (1-\theta^*)}$$

and the slope of SS is

$$(25) \quad \left. \frac{dr_S}{dr_B} \right|_{SS} = - \frac{(1-c_B) \frac{\partial f_S^*}{\partial r_B} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\tau) \theta^*}{(1-c_S) \frac{\partial f_S^*}{\partial r_S} \left(1 + \frac{F}{RK} \rho^*\right) + f_S^* \frac{F}{RK} (1-\theta^*)}$$

The sign and the relative size of these slopes are indeterminate because of the countervailing substitution effects and income effects. For example, the first term of the denominator of (24) represents the substitution effect which is negative and the second term represents the income effects which is positive. We assume that all the assets are gross-substitutes, i. e., the substitution effects dominate the income effects so that the slope of BB and the slope of SS are both positive. Furthermore, we assume that the slope of BB is larger than that of SS so that the stability condition (22) is satisfied.³⁾

The implicit function representation of BB is

$$(26) \quad r_S = \Phi_B(r_B, \theta, F, K, R, \tau, c_F, c_B, c_S).$$

The + sign above r_B implies the positive slope of Φ_B in (r_S, r_B) plane, and the other signs (+ or -) above each exogenous variable imply the direction of the shift of Φ_B . For example, + sign above F imply that the increase in F causes an upward shift in Φ_B . Similarly, the implicit function representation of SS is

$$(27) \quad r_S = \Phi_S(r_B, \theta, F, K, R, \tau, c_F, c_B, c_S).$$

3) The numerator and the denominator of (24) are the partial derivatives of the total cost of debt finance, $f_B W\rho$, with respect to r_B and r_S , respectively. Since $f_B W\rho = f_B(RK + F\rho)$ by substitution, the increase in r_B (r_S) causes the positive (negative) change in f_B which is the substitution effect and the positive (positive) change in $W\rho$ which is the income effect. Similar interpretation can be applied to (25). If we assume that the slope of BB is larger than that of SS in absolute value, then the stability condition (22) is satisfied which is weaker than the gross-substitutability assumption.

Depending on the relative slope of BB and SS to $1-\tau$, the following three cases are possible.

Case 1 (figure 1-1);

$$(28) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS} > 1-\tau$$

Case 2 (figure 1-2);

$$(29) \quad 1-\tau > \left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

Case 3 (figure 1-3);

$$(30) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} > 1-\tau > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

In the following sections, we will perform comparative static analysis for each of the three cases because the effects of changes in exogenous variables on r_B , r_S and θ may be different depending on the reaction of investors (households) which is represented by the slope of BB and SS .

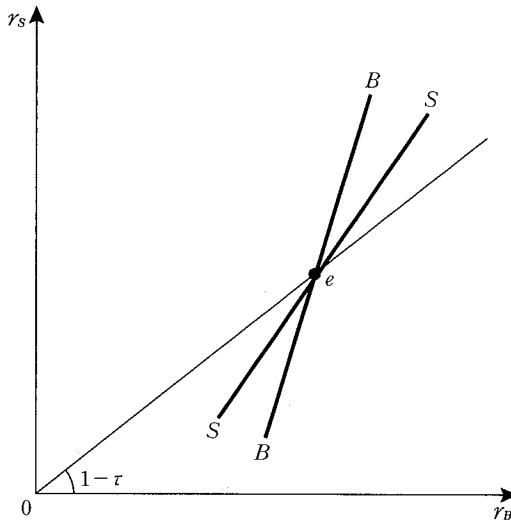


Figure 1-1

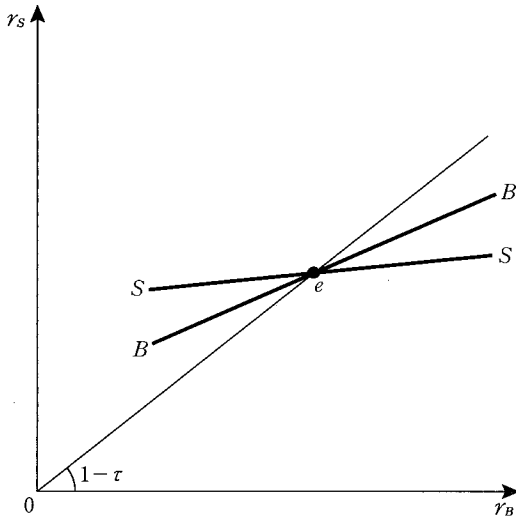


Figure 1-2

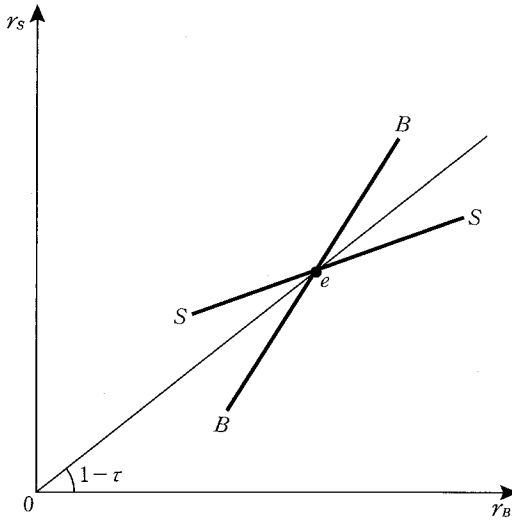


Figure 1-3

3. Comparative Statics

Consider an increase in government bonds. As we saw in the previous section, this will cause an upward shift in BB and a downward shift in SS . The short-run equilibrium by definition is attained at the intersection of BB and SS (r_B and r_S which solve (14) and (15) simultaneously). Depending on the relative slope of BB and SS to $1-\tau$, this short-run equilibrium may be attained either above or below the ray with slope $1-\tau$. Figure 2-1 shows the short-run equilibrium for case 1. e_0 is the initial long-run equilibrium, i. e., the intersection of B_0B_0 , S_0S_0 and $r_S=(1-\tau)r_B$. The increase in F causes B_0B_0 to shift up to B_1B_1 and S_0S_0 to shift down to S_1S_1 such that e_1 , the intersection of B_1B_1 and S_1S_1 , is the new short-run equilibrium. Since e_1 is below the ray with slope $1-\tau$, i. e., $r_S < (1-\tau)r_B$, Firms will increase θ in the long-run. It is sufficient for the system to be stable if r_B decreases and r_S increases as θ decreases so that the long-run equilibrium will be

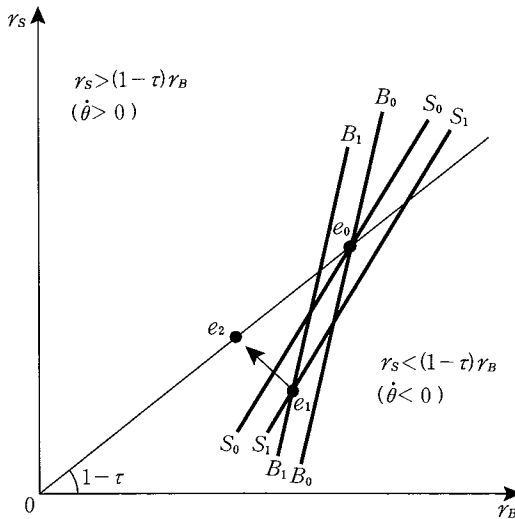


Figure 2-1

re-established, as suggested by the arrow in the figure (the direction of the adjustment process). This requirement is satisfied by imposing the stability condition (23)'. It is straightforward to show that

$$(31) \quad \frac{dr_B}{d\theta} = \frac{1}{H_0} \left[\left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\theta^*) \right\} \right. \\ \left. + \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1-\theta^*) \right\} \right] > 0$$

$$(32) \quad \frac{dr_S}{d\theta} = \frac{-1}{H_0} \left[\left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \right\} \right. \\ \left. + \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1-\tau) \theta^* \right\} \right] < 0$$

where

$$(33) \quad H_0 = \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \right\} \\ \times \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1-\theta^*) \right\} \\ - \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\theta^*) \right\} \\ \times \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1-\tau) \theta^* \right\} > 0 \quad (\text{by (22)}).$$

Therefore, the new long-run equilibrium may be re-established at the point like e_2 . The overall (long-run) consequence of the increase in F is the decrease in both r_B and r_S , and the decrease in θ , i. e., lower corporate debt ratio.

The economic interpretation of this result can be described as follows for case 1. (28) can be rewritten as

$$(34) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} \frac{r_B}{r_S} > \left. \frac{dr_S}{dr_B} \right|_{SS} \frac{r_B}{r_S} > \frac{(1-\tau)r_B}{r_S}.$$

Especially, at the equilibrium value (r_B^*, r_S^*) ,

$$(35) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} \frac{r_B^*}{r_S^*} > \left. \frac{dr_S}{dr_B} \right|_{SS} \frac{r_B^*}{r_S^*} > \frac{(1-\tau)r_B^*}{r_S^*} = 1.$$

An increase in F causes an excess supply in the government bonds market which may be absorbed by decrease in r_B and r_S . However, (35) implies that 1% decrease in r_B should be accompanied with larger in magnitude decrease in r_S for the corporate bonds market and the corporate stock market to be in equilibrium. As a result of this requirement, $r_S < (1-\tau)r_B$ holds in the short-run equilibrium (initially, $r_S^* = (1-\tau)r_B^*$). For the firms, the cost of equity finance becomes lower than the cost of debt finance. Therefore, the debt ratio θ will be decreased in the long-run. (This observation will be restated more explicitly in terms of the elasticity of demand for assets in the next section in which the income effects are negligible.)

Figure 2-2 describes the effect of an increase in F for case 2. The increase in F , as before, causes an upward shift in BB and a downward shift in SS . As a result, contrary to case 2, the short-run equilibrium

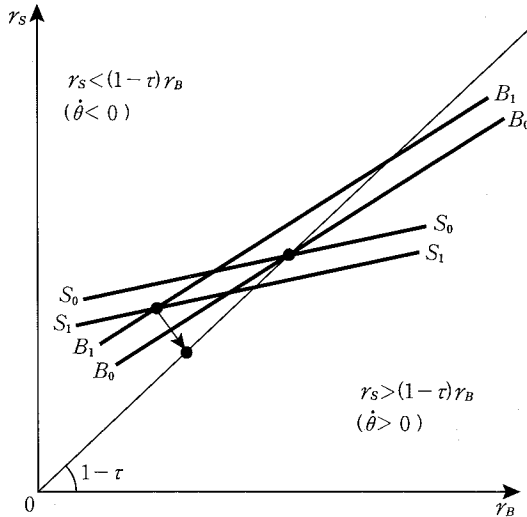


Figure 2-2

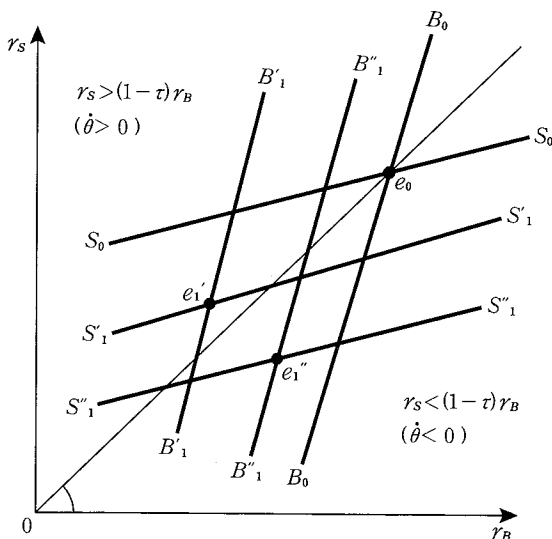


Figure 2-3

is attained above the ray with slope $1-\tau$. Since $r_s > (1-\tau)r_B$ holds in this short-run equilibrium, firms will increase θ in the long-run. Therefore, the overall consequence of the increase in F in case 2 is the decrease in both r_B and r_s , and the increase in θ , i. e., higher corporate debt ratio in the long-run equilibrium.

In case 3, the long-run effect of an increase in F on θ is ambiguous. Depending on the relative size of the shifts in BB and SS , as described in figure 2-3, the short-run equilibrium may be attained either above (e_1') or below (e_2') the ray with slope $1-\tau$. Consequently, the adjustment of θ may be either positive or negative. The mathematical exposition of above argument is shown in appendix A.

4. Pure Substitution Effects

As we saw in the previous section, the results of the comparative static analysis are ambiguous in some cases. One of the main reason

of this ambiguity is the countervailing substitution effects and income effects. In this section, we deal with the situation in which the government bonds outstandings are very small such that the income effects are negligible. This will be made possible by evaluating the equilibrium at $F=0$. Such analysis may be inappropriate for economies with a large amount of government bonds outstandings, however, it will clarify theoretical implications of the model.

The stability conditions are, in addition to the substitutability condition (7),

$$(36) \quad \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \right\} \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \right\} - \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \right\} \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \right\} > 0$$

$$(37) \quad (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} + (1-\tau) (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} + (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \\ + (1-\tau) (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} > 0.$$

We assume

$$(38) \quad \frac{\partial f_B^*}{\partial \bar{r}_B} + \frac{\partial f_S^*}{\partial \bar{r}_B} > 0$$

and

$$(39) \quad \frac{\partial f_B^*}{\partial \bar{r}_S} + \frac{\partial f_S^*}{\partial \bar{r}_S} > 0$$

which is sufficient for (37) to hold.

The slope of BB schedule is

$$(40) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} = - \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \right\} / \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \right\} > 0$$

and the slope of SS schedule is

$$(41) \quad \left. \frac{dr_S}{dr_B} \right|_{SS} = - \left\{ (1-c_B) \frac{\partial f_S^*}{\partial \bar{r}_B} \right\} / \left\{ (1-c_S) \frac{\partial f_S^*}{\partial \bar{r}_S} \right\} > 0.$$

The stability condition (37) is equivalent to

$$(42) \quad \left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS}.$$

Define the elasticity of demand for assets as

$$(43) \quad \eta(f_i, r_j) \equiv \frac{\partial f_i}{\partial r_j} \frac{r_j}{f_i} \\ = (1-c_j) \frac{\partial f_i}{\partial r_j} \frac{r_j}{f_i}, \quad i, j=F, B, S.$$

Then the three cases with respect to the relative slope of BB and SS to $1-\tau$ are rewritten in terms of elasticities as follows.

For case 1,

$$\left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS} > 1-\tau$$

is equivalent to

$$(44) \quad -\frac{\eta(f_B, r_B)}{\eta(f_B, r_S)} > -\frac{\eta(f_S, r_B)}{\eta(f_B, r_S)} > \frac{(1-\tau)r_B}{r_S}.$$

For case 2,

$$1-\tau > \left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

is equivalent to

$$(45) \quad \frac{(1-\tau)r_B}{r_S} > -\frac{\eta(f_B, r_B)}{\eta(f_B, r_S)} > -\frac{\eta(f_S, r_B)}{\eta(f_B, r_S)}.$$

For case 3,

$$\left. \frac{dr_S}{dr_B} \right|_{BB} > 1-\tau > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

is equivalent to

$$(46) \quad -\frac{\eta(f_B, r_B)}{\eta(f_B, r_S)} > -\frac{(1-\tau)r_B}{r_S} > -\frac{\eta(f_S, r_B)}{\eta(f_B, r_S)}.$$

Notice that in equilibrium

$$\frac{(1-\tau)r_B^*}{r_S^*}=1.$$

The Jacobian of the system (see (A4) in appendix A) is

$$(47) \quad H = - \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \tilde{r}_B} + (1-\tau)(1-c_S) \frac{\partial f_B^*}{\partial \tilde{r}_S} \right. \\ \left. + (1-c_B) \frac{\partial f_S^*}{\partial \tilde{r}_B} + (1-\tau)(1-c_S) \frac{\partial f_S^*}{\partial \tilde{r}_S} \right\}$$

which is negative by the stability condition (37).

The effect of an increase in F on r_B and r_S are shown as follows.

$$(48) \quad \frac{dr_B}{dF} = \frac{\rho^*}{HRK} (f_B^* + f_S^*) < 0$$

$$(49) \quad \frac{dr_S}{dF} = \frac{(1-\tau)\rho^*}{HRK} (f_B^* + f_S^*) < 0$$

The effect of the increase in F on θ is expressed in terms of the elasticities as

$$(50) \quad \frac{d\theta}{dF} = \frac{f_B^* f_S^* \rho^*}{HRK \rho^*} [(\eta(f_B^*, r_B^*) + \eta(f_B^*, r_S^*)) \\ - (\eta(f_S^*, r_B^*) + \eta(f_S^*, r_S^*))].$$

Therefore, by (44), (45) and (46)

$$(51) \quad \frac{d\theta}{dF} \begin{cases} < 0 \text{ for case 1} \\ > 0 \text{ for case 2} \\ (?) \text{ for case 3.} \end{cases}$$

The economic interpretation is straightforward. For case 1, (44) is equivalent to

$$(52) \quad \eta(f_B^*, r_B^*) > -\eta(f_B^*, r_S^*)$$

and

$$(53) \quad -\eta(f_S^*, r_B^*) > -\eta(f_S^*, r_S^*)$$

Initially, $(1-\tau)r_B^* = r_S^*$. The excess supply of government bonds will be absorbed by the decrease in r_B and r_S . However, (52) and (53) imp-

ly that the decrease in r_S should be larger in magnitude than the decrease in r_B for the corporate bonds market and the corporate stock market to be in equilibrium. Therefore, $(1-\tau)r_B > r_S$ holds in the short-run equilibrium. Since the cost of equity finance is lower than the cost of debt finance, the firms will decrease θ in the long-run.

The effect of the changes in other exogenous variables are summarized in appendix B.

5. Conclusion

Thus far, based on the simple general equilibrium model, we have obtained qualitative predictions about the effect of an increase in public debt on the cost of debt finance, the cost of equity finance and the corporate debt ratio. It turned out that in order to obtain unambiguous predictions, we need quantitative information such as the elasticity of demand for assets.

In addition to this, there are several points to be elaborated. First, money was not included in the model mainly because of simplification. Comparative static analysis of the four assets economy model further complicates the outcomes. However, if one can perform empirical research with appropriate data and information, money can be included in the model in a straightforward manner.

Second, the model was static. However, we can interpret the model in the following way. The households' decision making process is sequential. (i) the households decide the splitting of their disposable income between consumption and savings. (ii) having decided the amount of savings, the households then solve the portfolio selection problem.⁴⁾ Our analysis corresponds to the second stage of the pro-

4) It is known that such sequential decision making process is valid under

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blem. The households' preference about future consumption may be implicitly reflected in the demand function for assets.

One drawback of such static model analysis is to veil the "dynamic budget constraint" so that it becomes impossible to trace out theoretically the neo-Ricardian hypothesis of the effect of public debt (Barro, 1974). A dynamic re-formulation of the model will be necessary for this matter.

Finally, the firms' decision making problem is also treated as static problem. However, it is known that firms' decision criteria about their size (capital stock) is summarized in q . In a dynamic treatment of the firms' decision making problem, the time path of capital stock (through investment) will be determined endogenously.

Appendix A: Comparative Statics

Define

$$(A 1) \quad M_B(r_B, r_S, \theta; F, K, R, \tau, c_F, c_B, c_S) \\ = \theta - f_B((1-c_F)r_F, (1-c_B)r_B, (1-c_S)r_S) \\ \times \left(1 + \frac{F}{RK}((1-\theta)r_S + (1-\tau)\theta r_B)\right)$$

$$(A 2) \quad M_S(r_B, r_S, \theta; F, K, R, \tau, c_F, c_B, c_S) \\ = 1 - \theta - f_S((1-c_F)r_F, (1-c_B)r_B, (1-c_S)r_S) \\ \times \left(1 + \frac{F}{RK}((1-\theta)r_S + (1-\tau)\theta r_B)\right)$$

The total differentiation of (16), (A1) and (A2) evaluated at the long-run equilibrium values gives in matrix form the following expression.

some specific functional form specifications. Therefore, we cannot expect that our interpretation is valid in general situations (c. f. Ingersoll, 1987, ch. 11).

$$(A 3) \quad \begin{bmatrix} \frac{\partial M_B^*}{\partial r_B} & \frac{\partial M_B^*}{\partial r_S} & \frac{\partial M_B^*}{\partial \theta} \\ \frac{\partial M_B^*}{\partial r_B} & \frac{\partial M_B^*}{\partial r_S} & \frac{\partial M_B^*}{\partial \theta} \\ -(1-\tau) & 1 & 0 \end{bmatrix} \begin{bmatrix} dr_B \\ dr_S \\ d\theta \end{bmatrix} = \begin{bmatrix} -\frac{\partial M_B^*}{\partial F} \\ -\frac{\partial M_S^*}{\partial F} \\ 0 \end{bmatrix} dF$$

$$+ \begin{bmatrix} -\frac{\partial M_B^*}{\partial K} \\ -\frac{\partial M_S^*}{\partial K} \\ 0 \end{bmatrix} dK + \begin{bmatrix} -\frac{\partial M_B^*}{\partial R} \\ -\frac{\partial M_S^*}{\partial R} \\ 0 \end{bmatrix} dR + \begin{bmatrix} -\frac{\partial M_B^*}{\partial \tau} \\ -\frac{\partial M_S^*}{\partial \tau} \\ 0 \end{bmatrix} d\tau$$

$$+ \begin{bmatrix} -\frac{\partial M_B^*}{\partial c_F} \\ -\frac{\partial M_S^*}{\partial c_F} \\ 0 \end{bmatrix} dc_F + \begin{bmatrix} -\frac{\partial M_B^*}{\partial c_B} \\ -\frac{\partial M_S^*}{\partial c_B} \\ 0 \end{bmatrix} dc_B + \begin{bmatrix} -\frac{\partial M_B^*}{\partial c_S} \\ -\frac{\partial M_S^*}{\partial c_S} \\ 0 \end{bmatrix} dc_S$$

where

$$\frac{\partial M_B^*}{\partial r_B} = - \left\{ (1-c_B) \frac{\partial f_B^*}{\partial \bar{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\tau) \theta^* \right\} < 0$$

$$\frac{\partial M_B^*}{\partial r_S} = - \left\{ (1-c_S) \frac{\partial f_B^*}{\partial \bar{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_B^* \frac{F}{RK} (1-\theta^*) \right\} > 0$$

$$\frac{\partial M_B^*}{\partial \theta} = 1 - f_B^* \frac{F}{RK} (-r_S^* + (1-\tau)r_B^*) = 1 > 0$$

$$\frac{\partial M_B^*}{\partial F} = -f_B^* \frac{1}{RK} \rho^* < 0$$

$$\frac{\partial M_B^*}{\partial K} = f_B^* \frac{F}{(RK)^2} \rho^* R > 0$$

$$\frac{\partial M_B^*}{\partial R} = f_B^* \frac{F}{(RK)^2} \rho^* K > 0$$

$$\frac{\partial M_B^*}{\partial \tau} = f_B^* \frac{F}{RK} \theta^* r_B^* > 0$$

$$\frac{\partial M_B^*}{\partial c_F} = \frac{\partial f_B^*}{\partial \tilde{r}_F} \left(1 + \frac{F}{RK} \rho^* \right) < 0$$

$$\frac{\partial M_B^*}{\partial c_B} = r_B^* \frac{\partial f_B^*}{\partial \tilde{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) > 0$$

$$\frac{\partial M_B^*}{\partial c_S} = r_S^* \frac{\partial f_B^*}{\partial \tilde{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) < 0$$

$$\frac{\partial M_S^*}{\partial r_B} = - \left\{ (1 - c_B) \frac{\partial f_S^*}{\partial \tilde{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1 - \tau) \theta^* \right\} > 0$$

$$\frac{\partial M_S^*}{\partial r_S} = - \left\{ (1 - c_S) \frac{\partial f_S^*}{\partial \tilde{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) + f_S^* \frac{F}{RK} (1 - \theta^*) \right\} < 0$$

$$\frac{\partial M_S^*}{\partial \theta} = -1 - f_S^* \frac{F}{RK} (-r_S^* + (1 - \tau) r_B^*) = -1 < 0$$

$$\frac{\partial M_S^*}{\partial c_S} = r_S^* \frac{\partial f_S^*}{\partial \tilde{r}_S} \left(1 + \frac{F}{RK} \rho^* \right) > 0$$

$$\frac{\partial M_S^*}{\partial c_B} = r_B^* \frac{\partial f_S^*}{\partial \tilde{r}_B} \left(1 + \frac{F}{RK} \rho^* \right) < 0$$

$$\frac{\partial M_S^*}{\partial c_F} = \frac{\partial f_S^*}{\partial \tilde{r}_F} \left(1 + \frac{F}{RK} \rho^* \right) < 0$$

$$\frac{\partial M_S^*}{\partial \tau} = f_S^* \frac{F}{RK} \theta^* r_B^* > 0$$

$$\frac{\partial M_S^*}{\partial R} = f_S^* \frac{F}{(RK)^2} \rho^* K > 0$$

$$\frac{\partial M_S^*}{\partial K} = f_S^* \frac{F}{(RK)^2} \rho^* R > 0$$

$$\frac{\partial M_S^*}{\partial F} = -f_S^* \frac{1}{RK} \rho^* < 0$$

The determinant of the coefficient matrix of (A3) is

$$(A 4) \quad H_1 \equiv \frac{\partial M_B^*}{\partial r_B} + (1-\tau) \frac{\partial M_B^*}{\partial r_S} + \frac{\partial M_S^*}{\partial r_B} + (1-\tau) \frac{\partial M_S^*}{\partial r_S}$$

which is negative by the stability condition (23).

The effect of the increase in F on r_B and r_S are

$$(A 5) \quad \frac{dr_B}{dF} = \frac{-1}{H_1} \left(\frac{\partial M_B^*}{\partial F} + \frac{\partial M_S^*}{\partial F} \right) < 0$$

$$(A 6) \quad \frac{dr_S}{dF} = \frac{-(1-\tau)}{H_1} \left(\frac{\partial M_B^*}{\partial F} + \frac{\partial M_S^*}{\partial F} \right) < 0.$$

The effect of the increase in F on θ is analyzed as follows.

$$(A 7) \quad \frac{d\theta}{dF} = \frac{1}{H_1} \left[\frac{\partial M_S^*}{\partial F} \left(\frac{\partial M_B^*}{\partial r_B} + (1-\tau) \frac{\partial M_B^*}{\partial r_S} \right) + \frac{\partial M_B^*}{\partial F} \left(\frac{\partial M_S^*}{\partial r_B} + (1-\tau) \frac{\partial M_S^*}{\partial r_S} \right) \right]$$

Case 1;

$$\left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS} > 1-\tau$$

is equivalent to

$$(A 8) \quad -\frac{\partial M_B^* / \partial r_B}{\partial M_B^* / \partial r_S} > -\frac{\partial M_S^* / \partial r_B}{\partial M_S^* / \partial r_S} > 1-\tau.$$

That is

$$(A 9) \quad \frac{\partial M_B^*}{\partial r_B} + (1-\tau) \frac{\partial M_B^*}{\partial r_S} < 0$$

and

$$(A 10) \quad \frac{\partial M_S^*}{\partial r_B} + (1-\tau) \frac{\partial M_S^*}{\partial r_S} > 0.$$

Therefore

$$(A 11) \quad \frac{d\theta}{dF} > 0.$$

Case 2;

$$1-\tau > \left. \frac{dr_S}{dr_B} \right|_{BB} > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

is equivalent to

$$(A12) \quad 1-\tau > -\frac{\partial M_B^*/\partial r_B}{\partial M_B^*/\partial r_S} > -\frac{\partial M_S^*/\partial r_B}{\partial M_S^*/\partial r_S}.$$

That is

$$(A13) \quad \frac{\partial M_B^*}{\partial r_B} + (1-\tau)\frac{\partial M_B^*}{\partial r_S} > 0$$

and

$$(A14) \quad \frac{\partial M_S^*}{\partial r_B} + (1-\tau)\frac{\partial M_S^*}{\partial r_S} < 0.$$

Therefore

$$(A15) \quad \frac{d\theta}{dF} < 0.$$

Case 3;

$$\left. \frac{dr_S}{dr_B} \right|_{BB} > 1-\tau > \left. \frac{dr_S}{dr_B} \right|_{SS}$$

is equivalent to

$$(A16) \quad -\frac{\partial M_B^*/\partial r_B}{\partial M_B^*/\partial r_S} > 1-\tau > -\frac{\partial M_S^*/\partial r_B}{\partial M_S^*/\partial r_S}.$$

That is

$$(A17) \quad \frac{\partial M_B^*}{\partial r_B} + (1-\tau)\frac{\partial M_B^*}{\partial r_S} < 0$$

and

$$(A18) \quad \frac{\partial M_S^*}{\partial r_B} + (1-\tau)\frac{\partial M_S^*}{\partial r_S} < 0.$$

In this case, the sign of $d\theta/dF$ depends on the relative size of the shifts in $BB(\partial M_B^*/\partial F)$ and $SS(\partial M_S^*/\partial F)$.

The effect of the changes in other exogenous variables are summariz-

ed as follows.

$$K: \frac{dr_B}{dK} > 0, \quad \frac{dr_S}{dK} > 0, \quad \frac{d\theta}{dK} \begin{cases} > 0 \text{ for case 1} \\ < 0 \text{ for case 2} \\ (?) \text{ for case 3}^* \end{cases}$$

The sign depends on the relative shifts in $BB(\partial M_B^/\partial K)$ and $SS(\partial M_S^*/\partial K)$.

$$R: \frac{dr_B}{dR} > 0, \quad \frac{dr_S}{dR} > 0, \quad \frac{d\theta}{dR} \begin{cases} > 0 \text{ for case 1} \\ < 0 \text{ for case 2} \\ (?) \text{ for case 3}^* \end{cases}$$

The sign depends on the relative shifts in $BB(\partial M_B^/\partial R)$ and $SS(\partial M_S^*/\partial R)$.

$$\tau: \frac{dr_B}{d\tau} > 0, \quad \frac{dr_S}{d\tau} (?)^*, \quad \frac{d\theta}{d\tau} (?)^{**}$$

The sign is negative if $(\partial f_B^/\partial r_B) + (\partial f_S^*/\partial r_B) > 0$, which will be the case in section 4.

**The sign is positive if the shifts of $BB(\partial M_B^*/\partial \tau)$ and $SS(\partial M_S^*/\partial \tau)$ are small (see figure 3). The effect of τ is unambiguous if the income effects are small as will be discussed in section 4.

$$c_F: \frac{dr_B}{dc_F} < 0, \quad \frac{dr_S}{dc_F} < 0, \quad \frac{d\theta}{dc_F} \begin{cases} < 0 \text{ for case 1} \\ > 0 \text{ for case 2} \\ (?) \text{ for case 3}^* \end{cases}$$

The sign depends on the relative size of the shifts in $BB(\partial M_B^/\partial c_F)$ and $SS(\partial M_S^*/\partial c_F)$ (see (A7)).

$$c_B: \frac{dr_B}{dc_B} (?)^*, \quad \frac{dr_S}{dc_B} (?)^{**}, \quad \frac{d\theta}{dc_B} \begin{cases} (?)^{\$} \text{ for case 1} \\ (?)^{\$\$} \text{ for case 2} \\ < 0 \text{ for case 3} \end{cases}$$

*, **The signs are positive if $(\partial f_B^*/\partial r_B) + (\partial f_S^*/\partial r_B) > 0$, which will be the case in section 4.

§, §§These signs depend on the relative size of the shifts in BB

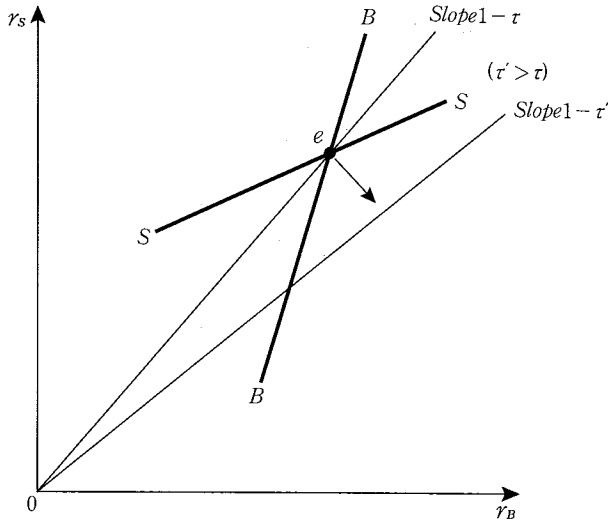


Figure 3

$(\partial M_B^* / \partial c_B)$ and $SS(\partial M_S^* / \partial c_B)$.

$$c_S: \frac{dr_B}{dc_S} (?)^*, \quad \frac{dr_S}{dc_S} (?)^{**}, \quad \frac{\partial \theta}{dc_S} \begin{cases} (?)^{\$} \text{ for case 1} \\ (?)^{\$\$} \text{ for case 2} \\ > 0 \text{ for case 3} \end{cases}$$

*, **The signs are positive if $(\partial f_B^* / \partial r_S) + (\partial f_S^* / \partial r_S) > 0$, which will be the case in section 4.

§, §§These signs depend on the relative size of the shifts in BB $(\partial M_B^* / \partial c_S)$ and $SS(\partial M_S^* / \partial c_S)$.

With respect to the effect of an increase in c_B and c_S on θ ,

$$\text{sgn}(d\theta/dc_B) = -\text{sgn}(d\theta/dc_S)$$

holds.

Appendix B: Comparative Statics (Pure Substitution Effects)

$$K: \frac{dr_B}{dK} = 0, \quad \frac{dr_S}{dK} = 0, \quad \frac{d\theta}{dK} = 0.$$

$$R: \frac{dr_B}{dR} = 0, \quad \frac{dr_S}{dR} = 0, \quad \frac{d\theta}{dR} = 0.$$

which can be shown as

$$\tau: \frac{dr_B}{d\tau} = \frac{-(1-c_S)r_B^*}{H} \left(\frac{\partial f_B^*}{\partial \bar{r}_S} + \frac{\partial f_S^*}{\partial \bar{r}_S} \right) > 0$$

$$\frac{dr_S}{d\tau} = \frac{(1-c_B)r_B^*}{H} \left(\frac{\partial f_B^*}{\partial \bar{r}_B} + \frac{\partial f_S^*}{\partial \bar{r}_B} \right) < 0$$

$$\frac{d\theta}{d\tau} = \frac{-(1-c_B)(1-c_S)r_B^*}{H} \left(\frac{\partial f_B^*}{\partial \bar{r}_B} \frac{\partial f_S^*}{\partial \bar{r}_S} - \frac{\partial f_B^*}{\partial \bar{r}_S} \frac{\partial f_S^*}{\partial \bar{r}_B} \right) > 0$$

$$c_F: \frac{dr_B}{dc_F} = \frac{-1}{H} \left(\frac{\partial f_S^*}{\partial \bar{r}_F} + \frac{\partial f_B^*}{\partial \bar{r}_F} \right) < 0$$

$$\frac{dr_S}{dc_F} = \frac{-(1-\tau)}{H} \left(\frac{\partial f_S^*}{\partial \bar{r}_F} + \frac{\partial f_B^*}{\partial \bar{r}_F} \right) < 0$$

$$\frac{d\theta}{dc_F} = \frac{-f_B^* f_S^*}{(1-c_F)r_B^* H} [\eta(f_S^*, r_F) (\eta(f_B^*, r_B^*) + \eta(f_B^*, r_S^*)) \\ - \eta(f_B^*, r_F) (\eta(f_S^*, r_B^*) + \eta(f_S^*, r_S^*))]$$

$$\frac{d\theta}{dc_F} \begin{cases} < 0 \text{ for case 1} \\ > 0 \text{ for case 2} \\ (?) \text{ for case 3} \end{cases}$$

$$c_B: \frac{dr_B}{dc_B} = \frac{-r_B^*}{H} \left(\frac{\partial f_S^*}{\partial \bar{r}_B} + \frac{\partial f_B^*}{\partial \bar{r}_B} \right) > 0$$

$$\frac{dr_S}{dc_B} = \frac{-(1-\tau)r_B^*}{H} \left(\frac{\partial f_S^*}{\partial \bar{r}_B} + \frac{\partial f_B^*}{\partial \bar{r}_B} \right) > 0$$

$$\frac{d\theta}{dc_B} = \frac{-f_B^* f_S^*}{(1-c_B)r_B^* H} [\eta(f_S^*, r_B) (\eta(f_B^*, r_B^*) + \eta(f_B^*, r_S^*)) \\ - \eta(f_B^*, r_B) (\eta(f_S^*, r_B^*) + \eta(f_S^*, r_S^*))]$$

which can be shown as

$$\frac{d\theta}{dc_B} \begin{cases} (?) \text{ for case 1*} \\ (?) \text{ for case 2**} \\ < 0 \text{ for case 3.} \end{cases}$$

*, **The signs are indeterminate but opposite.

$$c_S \cdot \frac{dr_B}{dc_S} = \frac{-r_S^*}{H} \left(\frac{\partial f_S^*}{\partial \tilde{r}_S} + \frac{\partial f_B^*}{\partial \tilde{r}_S} \right) > 0$$

$$\frac{dr_S}{dc_S} = \frac{-(1-\tau)r_S^*}{H} \left(\frac{\partial f_S^*}{\partial \tilde{r}_S} + \frac{\partial f_B^*}{\partial \tilde{r}_S} \right) > 0$$

$$\begin{aligned} \frac{d\theta}{dc_S} = & \frac{-f_B^* f_S^*}{(1-c_S)r_B^* H} [\eta(f_S^*, r_S)(\eta(f_B^*, r_B^*) + \eta(f_B^*, r_S^*)) \\ & - \eta(f_B^*, r_S)(\eta(f_S^*, r_B^*) + \eta(f_S^*, r_S^*))] \end{aligned}$$

which can be shown as

$$\frac{d\theta}{dc_S} \begin{cases} (?) \text{ for case 1*} \\ (?) \text{ for case 2**} \\ > 0 \text{ for case 3.} \end{cases}$$

*, **The signs are indeterminate but opposite.

With respect to the effect of an increase in c_B and c_S on θ ,

$$\text{sgn}(d\theta/dc_B) = -\text{sgn}(d\theta/dc_S)$$

holds.

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