The "Bargaining Power" Theory of the State: A Synthesis of Economic Approaches to the Origin of the State

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Abstract

Based on the recent historical and archaeological studies of the early states in irrigation societies, I demonstrate the hypothesis that the states were brought into being by the selfish motives of the chieftains of irrigation societies for securing the better terms of trade with external societies providing new necessary goods vital to survival of those societies. The logic of the hypothesis is proved in the analytical framework of a two-stage bargaining game and that of a hierarchical cooperative game. The main results of the analysis are corroborated by some historical evidences.

Key Words: Constitutional Approach, States, Economic Associations, Game Model

JEL Classification: D74, C72, C71

1. Introduction

The state is a social organization with sovereignty over a territory. [1] The sovereignty is secured by "power" as *ultra ration*, realizable by use or threat of force. The administrative authority of a state becomes effective by the back up of the power whose instrumental elements such as military forces are used to defend any aggression from within and without. Why such a social organ came into existence at some stage of human histories has been explained either by the "contract theory" of the state or by the "coordination theory," from the neoclassical point of view. [2] Even though each of those traditional theories could abstract some aspects of the processes of forming a state, however, they should be criticized for their dismissing a missing link, i.e., "the fundamental historical fact that inter-society trades among preceding societies had been spontaneously grown prior to forming a state." If we take it into due

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[1] See Lowie (1927, pp.116-117), North (1981, pp.21) and Weber (1911, pp.8).

[2] The contract theory is comprised of the *constitutional contract models* of the state defined by Buchanan (1975). They are classified into a social contract type and a slave contract type. Though the so-called "predatory theory of the state" defined by North (1981), formulated as the "rational bandits" theory of Olson (2000, 1993), McGuire and Olson (1996) and Kurrild-Kligaard and Svendsen (2003), has not been subsumed under the contract theory defined in this paper, it can be considered as a special case of the slave contract models because the reservation utility of the governed in the predatory models is considered to be set at the utility level under death threat, i.e., less than zero level. In any way, political actions aimed at any constitutional contract are taken under a prisoner's dilemma game. On the other hand, the coordination theory is a modern version of the organic theory of a Hegelian type, formulated by Hardin (1995). It attributes the origin of the state to a positive-sum payoff obtainable from coordinating strategies under a coordination game setting. It sets the reservation utility at the zero level, meaning nothing obtained before the state is formed. However, neither the contract theory excluding the slave contract models nor the coordination one took into consideration collective action for organizing the members of a stateless society into a state. In this sense they fall into circulation logic. Thus, they had to appeal for normative motives or charisma. The predatory models are free from the problem of the collective action, but seem to presuppose the existence of a type of the state, because a military force is required for predating players to be able to attack for plundering. Or societies attacked by the rational bandits were a state such as China dynasty and feudal societies in Western Europe. Thus, the predatory models also fall into a circulation argument.

consideration, we can derive a synthesized hypothesis on the origins of the state subsuming the main elements of the traditional theories. In this paper I name it the "bargaining power" theory of the state, whose main logic is as follows: When the preceding societies without regular force often fell into disadvantageous positions in inter-societal trades with some external societies owing to a weak bargaining-power in conflicts, the chieftains were motivated to transform those preceding societies into a state with military force, provided it could secure a more advantageous position in the inter-society trade to procure necessity goods for their survival. A larger payoff obtainable from the trade was the private incentive of the chieftain for transformation from a stateless society into a state.[3]

As was emphasized by Ortega (1950), the state is a dynamic or evolutionary concept.[4] Actually, the historical processes of forming a state are classified into three stages as in the following.

In the first stage, preceding societies, i.e., stateless communities, spontaneously form an economic association or union through forming a stable network of intra-societal trades. Since there is no overwhelming violence in those communities and their locations are fixed, those communities are considered to be put in a repeated-game setting. More than one association may come into existence, each of which is more or less autarkic. The state of nature of a Locke ＝ Nozick type is applicable to such situations in this stage.

In the second stage, they encounter an opportunity to acquire new necessary goods vital for their survival from an external society. In the beginning those goods are acquired by trading with the external partner. However, the new inter-societal trade is put under a prisoner's game structure, since the external partner has not yet been combined with the existent association of a stable nature. The trade is subjected to a relative power in bargaining on each occasion. The state of nature of a Hobbes＝Buchanan type is applicable to such a conflicting situation. When some chieftains recognize that the bargaining power is dependent on whether or not they have a violent force as *ultra ration*, they are motivated to take leadership for transforming the existent social system so as to adapt to the new external circumstance. To form a state is the most effective adaptation. Those chieftains are driven to organize the preceding communities into as a large organ as possible in order to pursue the scale merits of a military force.[5] The coordination model of a Hegel＝Hardin type abstracts such processes of producing the scale merits. If in bargaining in the external trade the bargaining power becomes so strong as to enforce the external partner to be subjugated, it is conquered and a predatory model is applicable to this situation.

In order to be combined into a new organ, however, each constituent preceding community has to abandon its heterogeneous preferences for local public goods such as the chieftainship of a preceding community. The cost of forming a new social organ depends on the relative power between the leading group and the following group. If violence power is not so different among them, then the cost is not negligible and thus a bargaining process follows and the new social organ takes a *federal system* as a result. The social contract models of not only a Hobbes＝Buchanan type but also

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[3] As to the original version of the hypothesis, see Ueda (2007, 2008). As long as we inquire into the origin of the state, the "early state" may be a more appropriate terminology. However, I follow the traditional terminology. As to the concept and historical discussions of the early state, see Claessen and Stalnik (1978, 1981).

[4] The evolutionary approach of Carneiro (1970) dismissed the existence of inter-societal trades before conquest wars began. Service (1971) contributed to distinguishing the early stage from chiefdom but did not explain the private motives for transformation from the latter into the former. On the other hand, Smith (1956) rejects distinguishing s state from its preceding societies, and argued that all societal forms are in a continual process. However, society with a regular military force should be classified as one category distinguished from others.

[5] The size of a society approximates the strength of a military power. It is a combination of the "coordination power" (the personnel) with the "exchange power" (logistic capability). As to these concepts, see Hardin (1995, pp.35).

a Locke＝Nozick type fit well to this situation. By contrast, if the leading group or one chieftain has so overwhelming a power, then the cost is low and thus they appeal to the use or threat of power so as to enforce the other group to accept a *centralized system*. A slave contract model of a Hobbes＝Buchanan type including Olsonian type as its special case is applicable to this situation.

In the third stage, the governed people have to be willing to accept the authority of the government in order for the governed people to comply with laws without appeal to arms, even if the authority must be backed up by power. Thus, the utility level after forming a state must be larger than before.[6] This condition is common to all models, if the "rational bandits" model is considered as a special case in the sense that the reservation utility of the conquered people is set at a level below zero.

In this paper I prove the causal logic of the "bargaining power" theory of the state by analyzing two game models which abstract the above historical processes of forming a state in the irrigation societies of Japan. For this purpose, firstly I give a mathematical formulation to the irrigation societies by applying the hierarchical cooperative game developed by Demange (2004). Secondly, the historical process of the second stage is formulated by the analytical framework of a two-stage bargaining game developed by Querido (2007). For simplicity, however, the case to which a federal system is applicable is dismissed in this paper. From the analysis of those abstractive models, I derive the conclusion that the early state was driven into existence when the chieftains were driven to enhance their bargaining power. The conclusion is corroborated by comparing the main analytical results with relevant historical evidences.

In what follows, this paper is organized as follows: In the second section the base model of this paper is presented. A simplified irrigation society is formulated by a hierarchical cooperative game model. The stability of this society is proved. In the third section, the main logic of the "bargaining power" theory is demonstrated by analyzing a two-stage bargaining game, and is corroborated with historical evidences. In the fourth section, a federal system in irrigation societies is formulated and the rational bases argued by Alesina and Spolaore (2005) are confirmed. In the fifth section the main results and conclusions are summarized. In the Appendix 1, the base model of a simple irrigation society is illustrated. In the Appendix 2, the mathematical proofs for the second section are given. Some analytical results support the hypothesis of Wittfogel (1957).

2. The Basic Model

In this section, we present the base model of this paper in order to demonstrate a hypothesis on the origins of the state. As a preceding society prior to forming a state, we assume a simple irrigation society where its "irrigation systems" stand for economic infrastructures with sub-national economies of scale and its "inter-society trades" to procure iron resources stand for the most essential goods with nationwide economies of scale. The members of the irrigation society are networked with an irrigation system, each located from an upstream site to a downstream one under the authority of an agricultural chief. The chief takes entrepreneurship not only for constructing and maintaining the irrigation system but also for an inter-society trade to procure iron resources which are necessary goods indispensable for agricultural production. In what follows, we show that the characteristics of such an irrigation society can be described by a cooperative game with hierarchies.

[6] The more strictly speaking, the social contract models are further classified into a Hobbes (1651)=Buchanan (1975) type and a Locke(1690)=Nozick (1974) type. It has been argued that a crucial difference between them lies in the concept of the "nature of state," and that whilst the former type recognizes it as a warlike situation, the latter as a peaceful anarchy. However, both types assume that every agent takes defensive actions against any aggression as the natural rights, and that there is someone who has an incentive to steal. Then, both models are bound to a warlike situation in the end, even if the beginning situations are different. For both cases, the payoff under a constitutional contract is larger than before.

2-1. Simple Irrigation System[7]

Suppose a river is flowing down from mountain areas in its riverhead region, and that a canal system for irrigation is set up along the river as follows: One reservoir or a set of trunk canal is constructed which can irrigate prospective *n* paddy fields, numbered *1, 2,..., n*, in order distanced away from the river. I note here that "*n*" is a generic but not fixed number. Each paddy field is cultivated by one farmer. The reservoir or the trunk canal intakes water for irrigation from one point of the river, called *sluice gate*. Paddy fields are developed and located one by one in line[8] However, in order for each prospective paddy field to intake irrigation water from the reservoir or the trunk canal, each farmer has to dig one branch canal so as to be directly or indirectly connected with it. It costs *K* to construct the reservoir or the trunk canal, and furthermore, it costs *C(i)* not only to construct the branch canal *i* locating in the *i*-th distance from the sluice gate numbered *zero*, but also to communicate and transport between *i* site and the zero site. In what follows, the number *i* is often treated as a natural number in order to stand for *i* player locating in *i*-th distance or *i*-th rank in a hierarchy. For example, if there exist *n+1* players in a society, it is denoted by a players' set $N = (0, 1, 2,..., n)$. The zero player stands for an agricultural and military chieftain of this society, who takes entrepreneurship not only for constructing and maintaining the irrigation system but also for procuring necessities indispensable for the whole irrigation society. Those necessities are represented by iron resources in what follows. If we refer to a generic coalition S, $S \subseteq N$, it is defined as $S \equiv (0, 1, 2, ..., s)$, for $s \equiv |S| \leqq |N| \equiv n$.

The paddy field locating in the *i*+1 site cannot be developed without permissions of players locating in the *i* to *0* site in turn. $C_i = C(i)$, for $i = 1, 2,...,n$, is assumed to be an increasing function of the distance from the sluice gate. Furthermore, the more distanced, the more rapidly it increases. These assumptions are formulated by (1).

$$C_i = C(i), \quad \text{s.t., } C(1)<C(2)< ,..., < C(n), \text{ and } C(i+1) - C(i) < C(i+2) - C(i+1), \text{ for } i \in [1, 2, ..., n]. \tag{1}$$

The above assumptions on C(・) are justified by the geographic condition that the further the distance from the sluice gate, the costlier to communicate with the zero player. The cost of communication is comprised, for example, of the cost to bring back seeds from the chief and to transfer a part of harvests as a tribute to him. Of course, iti is not so unrealistic to assume the conditions of (1) on the geotopographic basics.

The reservoir or the trunk canal is constructed by iron tools denoted by *M* in *masse* replacing stoneware. Therefore, given an irrigation technology, the cost of constructing and maintaining the reservoir or the trunk canal, *K*, is assumed as a decreasing function of *M*[9], as follows.

$$K = K(M),\ K' < 0, \text{ and } K'' > 0 \tag{2}$$

The above assumptions on *K* are justified, because, if the more of stone tools are replaced with iron tools to construct one set of reservoir, the less costly it can become.

At the *ex ante* stage of rice farming, the main part of an irrigation system, the reservoir or the trunk canal, must be constructed by the collective work of *s* prospective farmers coordinated by the zero player. Each farmer shares the cost, *K*, on an equal basis. Thus, the cost burden of *i* farmer amounts to K/s, $i = 1,2, ...,s$. It may appear to be a slave labor, but in a contractual term, it stands for *entrance fee* for irrigation canal system.[10] On the other hand, the cost of

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[7] As to the rough image of the irrigation system below, see *Appendix 1* to this paper.

[8] Though paddy fields are assumed to be located in line along one canal, the model can be extended to more complex irrigation systems in which each paddy field has its own hierarchical irrigation systems, or to those where the reservoir is subsumed in a more inclusive canal system as one branch. The proof in the next subsections can be applied to those more complex cases.

[9] Iron tools for construction and farming were usually lent to farmers by chieftains playing the role of an agricultural entrepreneur.

the i- th branch canal, $C(i)$, is assumed to be borne by i farmer.

Crops are harvested after each farmer is engaged in farming work denoted by e which is assumed as a constant for all farmers. This assumption is for simplicity but justified by the historical background that each farmer could not have so much option for leisure on those days. The harvest of each cropland is assumed as an increasing function of iron tools, and it is denoted by $f(M)$, $f' > 0$, and $f'' < 0$, as usual. This function is assumed to be the same for all croplands. Difference in the fertility is reflected in the increased marginal cost of the branch canal. A fixed percentage of $f(M)$, denoted by α, $0 \leqq \alpha \leqq 1$, is paid to the zero player as a *variable charge* for consuming irrigation water. It is a contractual expression for annual tributes.[11] In Appendix 1 of this paper, the case where $K(M)$ is the cost of a set of trunk canal is illustrated.

2-2. Coalitions with Hierarchies

Iron resources, M, have to be procured by the zero player. For this purpose, he engages in an external trade to acquire those iron resources. In this subsection, the circumstances of the external or inter-society trade are classified into two cases: the case (i) that it costs him a given P per unit of M under the preceding stateless society and the case (ii) that it costs him $\phi(s)$ per unit of M under a state comprised of $(s + 1)$ members including him. The size of the state, denoted by s, is a surrogate for the strength of a regular military force, the consolidation of bargaining processes and the scale effect of the production of a means of payment for the iron resources. The cost to organize and govern s members is denoted by G(s) that covers the cost of maintaining a regular military force and other administrative organizations.[12] It is assumed, as usual, that $\phi_1 = \partial\phi/\partial s < 0$, $G' = \partial G/\partial s > 0$, and $G'' = \partial^2 G/\partial s^2 > 0$.

Then, the total cost to the zero player of procuring iron resources M, denoted by Ψ, is assumed as follows:

$\Psi = \Psi(M;\ P) = P \cdot M$, for a stateless community with P given, and

$\Psi = \Psi(M;\ s) = \phi(s)M + G(s)$, for the state comprised of $s+1$ members.

If it is assumed that the terms of trade are the worse for the stateless society than for a state, it holds that $P > \phi(s)$.[13]

Suppose that the process of organizing an irrigation society is begun with the zero player's contractual offers to each prospective farmer, as follows: Just before he develops an irrigation system, he offers a set of *contracts* to prospective s farmers in order of the distance, i.e., in order of the cost of branch canal. That is, only after i farmer accepts the offer, the zero player can have access to $i+1$ player. The contracts are comprised of a fixed percentage charge for irrigation system, denoted by α, the production technology represented by $f(M)$, and the cost of the irrigation system for a set of prospective members, denoted by $K(M)$ and $C(i)$ for $i \in S$. Under the conditions of a given technology and those of geotopographic features, the contract offer is denoted by $\{\alpha, M, S\}$. In what follows, we use the following notations: $a(S) \equiv \{\alpha, M, S\}$ and $A(S) \equiv \{a(S) | \exists S \subseteq N\}$, where $A(S)$ is assumed to be finite and closed, for mathematical

[10] For example, under the centralized monarchy system in the 7th to 8th century, each farmer was liable to do "sixty day work" per year under the supervision of a local chief. This work is considered to be allocated to construction and maintenance of irrigation systems of the local community. On the other hand, each brunch canal is considered to have been maintained by family unit.

[11] The tribute from annual harvest, called *So*, was about 3 to 5 per cent of the harvest. Seeds were lent at about fifty percent of interests. Payment in cloth, called *Cyo* and *Yo*, is also subsumed in α f(M) for simplicity.

[12] According to the ancient centralized dynasty system called the *Rituryo* system, the regular force was comprised of about 200 thousands military services and the cost of maintaining it was financed by the dynasty government. They were exempt from both payment in cloth called *Cho* and 60 days work for construction called *Zoyo*. These exemptions are considered as a payment to the military servicemen. The cost of constructing roads and metropolis was also financed by taxes.

[13] This assumption is justified in the third section.

simplicity.

In order for a contract offer, *a(S)*, to be accepted by |S| farmers of *S*, it must be able to meet their participation constraints and must be feasible. The participation constraints are satisfied, if the payoff of each player is larger or at least equal to his reservation utility or opportunity cost. The feasibility condition of *a(S)* is satisfied, if the total net payoffs, i.e., the *coalitional value*, are nonnegative.

If each of these |S| farmers accepts a contract offer *a(S)* and it is feasible, the payoff of the zero player, Π_0, and that of i farmer, Π_i, $i \in S-\{0\}$, are defined by (3) and (4), respectively.

$$\Pi_0(a(S)) = |S|\,\alpha f(M) - \Psi(M) \tag{3}$$

$$\Pi_i(a(S)) = (1-\alpha) f(M) - K(M)/|S| - C(i) - e, \text{ for } i \in S-\{0\} \tag{4}$$

The above payoff functions are defined over the compact set *A(S)*, and they are assumed to be *continuous* over *A(S)*, when the variables are defined to be *real*. Furthermore, the reservation utilities are normalized to zero, and thus the participation constraints of those players are defined by $\Pi_0(a(S)) \geqq 0$ and $\Pi_i(a(S)) \geqq 0$ for $i \in S-\{0\}$. Under the condition that these payoffs are transferable, the feasibility condition of *a(S)* is defined by (5).

$$|S| \cdot f(M) - \Psi(M) - K(M) - [C(1) + C(2) + \ldots + C(|S|)] - |S|e \geqq 0 \tag{5}$$

In what follows, e is omitted without loss of generality.

According to the terminology of Demange (2004), $(A(S), \pi_0, \pi_1, \ldots, \pi_s$, for $\forall S \subseteq N)$ is called the "problem" for coalition *S*. The next subsection demonstrates that this irrigation society is of a stable nature.

2-3. Stability of Irrigation Society

In this subsection, it is proved that the irrigation society is of a stable nature in the sense that no one of the society has an incentive to deviate from it. For this purpose, firstly it is proved that the "hierarchical outcome" defined by Demange (2004) exists for more elaborate irrigation systems than the basic irrigation system assumed in the previous subsections, and secondly that the hierarchical outcome is stable. We begin with several definitions[14] in what follows.

Hierarchy: Hierarchy *R* is defined as $R^h(h) = 0$, for h = *1,2,...,n*. It means that *h* player has the *h*-th rank in the hierarchy at the top of which the player 0 is placed.

Team: Given a hierarchy *R*, a coalition *T* is defined as a *team*, if there is a member *i* of *T* who is in a position superior to any other member *j* of *T*, and furthermore, the interval [*j, i*] is included in *T*. T^i is called the *full team* of *i*, which is composed of the *i player* and all his subordinates. D^i is called the *direct team* of *i*, which is composed of the *i* player and all his direct subordinates.

Blocking Condition: Given a superadditive problem $(A(S), \pi_0, \ldots, \pi_s$ for $\forall S \subseteq N)$, a contract offer $a(N) \in A(N)$ is defined to be *blocked* by a coalitional team $S \subset N$, if and only if there exists $\exists b(S) \in A(S)$ such that $\pi_i(b(S)) > \pi_i(a(N))$, for $\forall i \in S$.

$\mathcal{T}$*-Stability*: Let $\mathcal{T}$ denote a set of teams of *N*. Then, a contract offer $a(N) \in A(N)$ is $\mathcal{T}$*-Stable*, if *a(N)* is feasible and not blocked by any team coalition of $\mathcal{T}$.

Guarantee Levels: Given a feasible problem $\{A(S), \pi_0, \ldots, \pi_s : \forall S \subseteq N\}$ where *A(S)* is feasible for $\forall S \subseteq N$, the guarantee levels, denoted by $(g_n, g_{n-1}, \ldots, g_1, g_0) \equiv g$, are defined by the mathematical algorism as follows: *At the step 0*, the guarantee level of the player with the maximum rank, denoted by g_n, is determined by his reservation utility. That is, $g_n = 0$. *At the step r* (r = 1, 2, ...,n−1), the guarantee level of the player with rank $n-r$, denoted by g_{n-r}, is determined by maximizing his payoff subject to the condition that the payoffs of the players with higher ranks are larger than or at least equal to their guarantee levels. That is,

$g_{n-r} = \max [\pi_{n-r}(a)$ over $a \in A(T^{n-r})$| s.t., $\pi_k(a) \geqq$

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[14] As to the details of these definitions, see Demange (2004).

g_k, for $\forall k \in T^{n-r} - \{n-r\}$]

At the step n, the guarantee level of the player with the top superior status, denoted by g_0, is determined by the following.

g_0 = max [$\pi_0(a)$ over $a \in A(T^0)$| s.t., $\pi_k(a) \geqq g_k$, for $\forall k \in T^0 - \{0\}$].

Hierarchical Outcome: The contract offer $\exists a(N) \in A(N)$ which brings about the guarantee levels $g = (g_n, g_{n-1}, ..., g_1, g_0)$ solved according to the above algorism is defined as the *hierarchical outcome*.

Based on the above definitions, we can prove both the existence of the hierarchical outcome for the irrigation system and its stability. It turns out that the irrigation system can be extended to more multilayered and complex types than the one defined by the base model in the first and second subsection. For example, each branch canal can have its own hierarchical irrigation systems by extending smaller branch canals from it and connecting them. For another example, more than two irrigation systems each of which is set up with a separate reservoir or a trunk canal can be organized into a more inclusive hierarchic system. The theorem below proves the existence and stability of any type of irrigation system under the superadditive condition.[15]

Theorem:

Given a hierarchy R, teams' set T, and a superadditive[16] problem $\{A(S), \pi_0, ..., \pi_s : \forall S \subseteq N\}$ where $A(S)$ is feasible for $\forall S \subseteq N$, then, (i) the guarantee levels g are finite, (ii) the hierarchical outcome is not blocked by any team coalition of T, and (iii) T is the maximum stable set of teams which satisfies (i) and (ii).

The Proof of (i): By the assumptions on π_j for $\forall j \in N$ defined over the compact sets of A(S) for $\forall S \subseteq N$, the superaddivity, and the feasible contract offers, the existence of a finite g is obvious. For example, the finite $g = (g_0, g_1, ..., g_s)$ for a team S is obtained by setting g_i at π_i for $i \in S$ defined by (3) and (4), if π_0 is maximized subject to π_i for $i \in S / \{0\}$ satisfying each participation constraint. By the way, if the top superior of each team coalition is fixed, for example, by some traditional authority, the guarantee levels become unique.[17]

The Proof of (ii): If a team coalition $S \in T$ could block the hierarchical outcome, $a(N) \in A(N)$, then, there is a contract offer $b(S) \in A(S)$ such that $\pi_j(b(S)) > \pi_j(a(N))$ for $\forall_j \in S$. But this is in contradiction with the definition of the guarantee levels of those players.

The Proof of (iii): Allow any coalition to block the hierarchical outcome and take $\exists S_0 = (k, k+2, ..., k+m) \not\subseteq T$. Then, we can constitute the Condorcet triple by selecting $S_1 = (k, k+1)$ and $S_2 = (k+1, k+2, ..., k+m)$ for $\forall k \geqq 2$, such that $S_i \cap S_j \neq \phi$ for $i, j = 0, 1, 2, i \neq j$, and $S_0 \cap S_1 \cap S_2 = \phi$. Thus, T is the maximal stable set of teams. *Q.E.D.*

The above *Theorem* proves that under the superadditive condition an irrigation society with hierarchies be of a stable nature in the sense that no member of the irrigation society has an incentive for deviating from it. This leads us to the conjecture that it is not right to trace the origins of the state back to the fissiparous tendencies which many evolution archaeologists consider had caused preceding societies to break up in the end.[18]

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[15] The theorem below is not applicable to the more inclusive irrigation system in which more than two constituent systems are separately set up, each of which is equipped with its own reservoir or trunk canal. Such a more inclusive system does not meet the superadditive condition.

[16] The superadditive condition is defined as follows: for example, take $\{A(S), \pi^s, S: \forall S \subseteq N\}$ as the *problem*, where *A(S)* is feasible, the participation constraints are met, and $\pi^s \equiv (\pi S_0, \pi S_1, \cdots, \pi S_s)$ with *So* being the chief of S. Then, the *problem* is *superadditive*, if and only if for $\forall S_1$ and S_2, $S_1 \cap S_2 = \phi$, there is $\exists a \in A(S_1 \cup S_2)$ such that $\pi_k(a) \geqq \pi_k(a_1)$ for $\forall k \in S_1$, and $\forall a_1 \in A(S_1)$, and $\pi_k(a) \geqq \pi_k(a_2)$ for $\forall k \in S_2$, and $\forall a_2 \in A(S_2)$.

[17] The ancient monarch of Japan had the authority due to the monopoly of technologies on rice-farming such as those on the maintenance and improvement of seeds, as well as those on irrigation system.

[18] As to their arguments, see Classen and Skalnik (1978, 1981).

The transformation from a preceding society into an early state was brought about through a series of lengthy processes, but not by an abrupt change. It is hard to find out documental evidences to show definitely when the early state was established. It must be induced from archeological evidences supporting its formation, such as the existence of regular forces, of military leaders, of social stratification. Many archaeological evidences show that local conflicts for water use among irrigation societies locating along a river disappeared in the 1st to 2nd century in Japan. They gradually formed a sophisticated trade-network having the common center. Local chiefs are considered to have voluntarily joined in such a more inclusive hierarchy because they could acquire necessities by forming direct links with a hierarchical intra-society network. The irrigation societies on those days did not have to form a state because a regular force was not required for enforcing those intra-society trades. It is because those trades were in a repeated-game setting as well as the conditions of the core are satisfied.

However, when the intra-society trade networks did extend to foreign societies, a new "inter-society" trade with them was put out of the repeated-game setting. This new game setting inflicts a high transaction cost on the preceding society without enforcing power. Actually it was when an "international order of trade" organized by the Han dynasty was broken up that the irrigation societies of Japan were impelled to be armed with a regular force with the aim of securing iron resources vie Korean peninsular. According to many archaeological evidences such as enormous tombs and their burial accessories suggesting not only the role of agricultural entrepreneur but also that of a military leader, irrigation societies in Japan grew into an early state in the late 3 rd century.[19]

Owing to the superadditive condition, the scale economies of iron materials can be extended to a national level. However, such an extension enters into a new historical stage where it is faced with those external societies. A new external trading relation with them was put under a one-stage game setting, motivating the chieftains to resort to power as *ultra ration* in order to reduce the bargaining cost in external conflict. In the next section, under the same conditions of this subsection we demonstrate the hypothesis on the origins of the state in the analytical framework of a two-stage game characterizing the external trade.

3. State Formation in Inter-Society Trade Network

In this section the causal logic of the "bargaining power" hypothesis is proved by analyzing a two-stage game in which the irrigation societies are engaged in an inter-societal trade to procure iron resources from a neighbor society called "supplier" under the condition that there is no common enforcer. In the first subsection, the inter-societal trade without a regular military force is examined. In the second subsection, the effect of installing a regular military force on the terms of trade is examined, and the rational basis of state formation is confirmed.

3-1. Inter-Society Trade without Regular Force[20]

Suppose an inter-societal trade between the zero player of an irrigation society (called *buyer*) without regular military force and an external society (called *supplier*), as following: The former has to procure iron resources from the latter who, we assume for simplicity, can supply those iron resources at no cost. Each side of the inter-societal trade can expend only a fixed amount of resources for an emergency need in conflict, which is denoted by V_0 for the former and V for the latter. For example, V_0 is composed of some guards and shipping agents accompanying each inter-

[19] Many archaeologists support this hypothesis based on recent archaeological studies. See, for example, Tsude (2005, 1991, 1989), Murakami (2007) and Matuki (2007).

[20] The main idea of the two-stage game below is based on Querido (2007).

societal trade.

The inter-societal trade is assumed to be composed of two stages as following: At the first stage, the supplier calls on the zero player to pay *P* per one unit of iron resources. That price level may be an "unreasonable demand" and surpass the reservation utility level of the zero player.[21] On the contrary, if reasonable and thus if the demand price is accepted by the zero player, then his payoff, $\Pi_0(\cdot)$, and the payoff of the supplier, $\Pi(\cdot)$, are defined by (6) and (7), respectively.

$$\Pi(P) = PM \quad (6)$$

$$\Pi_0(P) = s\alpha f(M) - PM \quad (7)$$

The society size, *s*, is determined together with *M* in accordance with the assumption of the superadditivity, with *P* given (See Appendix 2). This and the next subsection derive the optimal level of *P* and $\phi(\cdot)$.

If the zero player rejects the above demand, the game proceeds to the second stage. The condition (8) is necessary to cause the zero player to reject the demand.

$$s\alpha f(M) - V_0 > s\alpha f(M) - PM \quad (8)$$

The above condition (8) means that *M* can be taken away, for example, by exercising guards' service at the cost of V_0.

If the zero player rejects *P* at the first stage, then, at the second stage both players fall into a conflict in which the zero player and the neighbor expend V_0 and V, respectively. Let's assume that the zero player wins the conflict with the probability of λ, a given parameter. If he wins the conflict, his payoff amounts to $\{s\alpha f(M) - V_0\}$. If he loses, he has to pay *PM* and thus his payoff is reduced to $\{s\alpha f(M) - PM - V_0\}$. Then, the expected payoff of the supplier and that of the zero player are defined by (9) and (10), respectively.

$$\Pi(\lambda) = (1-\lambda)(PM - V) + \lambda(-V) \quad (9)$$

$$\Pi_0(\lambda) = \lambda(s\alpha f(M) - V_0) + (1-\lambda)(s\alpha f(M) - PM - V_0) \quad (10)$$

Let's denote by *P** the maximizing *P* which can induce the zero player to accept at the first stage. It is determined by the following equation:

$$\Pi_0(P) = s\alpha f(M) - PM = \lambda(s\alpha f(M) - V_0) + (1-\lambda)(s\alpha f(M) - PM - V_0) = \Pi_0(\lambda)$$

By arranging the above equation, we obtain (11).

$$P^* = V_0 / (\lambda M) \quad (11)$$

Substitute (11) into $\Pi(P)$ and $\Pi(\lambda)$ and compare the results, and then the optimality of P* for the supplier is proved by the following inequality (12).

$$\Pi(P^*) - \Pi(\lambda) = V_0/\lambda - (1-\lambda)(V_0/\lambda - V) - \lambda(-V) = V_0/\lambda + V > 0 \quad (12)$$

It is obvious from the inequality (11) that the higher is λ, the lower is P*M. This apparently causal relation between λ and P*M may well impel the zero player to try raising λ anyhow. Then, if the chieftain believes that λ, a determinant of the default payoff, can be raised by installing a regular military force, we can derive the hypothesis as follows; It is because the traditional irrigation societies were often faced with a disadvantageous position in inter-society trades with an external society that regular military force was brought into being and thus that the preceding societies were transformed into a state.

3-2. Inter-Society Trade under Threat by Regular Force

Suppose the zero player is armed with a regular military force. The formation of a state is

[21] The "unreasonable demand" took various forms. For example, a part of an agreed volume may be subtracted on delivery time, or may be stolen during transportation. In this section, all of those cases are represented by the terms "unreasonable demand for price," which is denoted by P.

characterized with an institutional shift from the preceding society without any military force into a social organ armed with a regular military force. Given a military technology, the cost of governing the state, denoted by G, is assumed to be an increasing function of the size of the society. That is, G = G(s) and under the "as usual" assumption, G'(s) >0 and G"(s) >0.

On the other hand, the probability of winning in conflict with the suppler is assumed as an increasing function of F(s) where F(s) with F' >0, F" <0 stands for the combination of the "coordination power" and the "exchange power" defined by Hardin (1996). Thus, the probability of winning is dependent ultimately on the size of the society which is under governance of the zero player. This assumption is justified because the society size approximates the effects of the personnel and logistic capacity which are crucial factors of the strength of regular military force, and furthermore because it approximates the consolidation of the society. Then, by applying the idea of the "conflict success function"[22], the probability of winning is defined by (13).

$$\lambda(s;\theta)=\frac{F(s)\theta_0}{F(s)\theta_o+\theta_1 V}=\frac{F(s)}{F(s)+\theta V} \qquad (13)$$

In the above (13), $\theta=\theta_1/\theta_0$ where θ_1 and θ_0 stand for the military technology of the supplier and that of the zero player, respectively. It is obvious that $\partial\lambda/\partial s>0$, $\frac{\partial^2\lambda}{\partial s^2}<0$, $\partial\lambda/\partial\theta<0$, and $\partial^2\lambda/\partial\theta^2>0$.

Here, we can redefine the probability of winning, λ, of the previous subsection *3−1* as $\lambda=\inf_{s\in s}\lambda(s:\theta)$, which is achieved when s takes a minimum threshold value.

Then, the payoff of the supplier, $\Pi(\cdot)$, and that of the zero player, $\Pi_0(\cdot)$, are defined by (14) and (15), respectively, if a price demanded by the supplier, P, is accepted by the zero player at the first stage.

$$\Pi(P)=PM \qquad (14)$$

$$\Pi_0(P)=s\alpha f(M)-PM-G(s) \qquad (15)$$

However, if the zero player rejects the demand, the game proceeds to the second stage. In order for him to choose "rejection," the following condition must be met: $s\alpha f(M)-V_0-G(s)>s\alpha f(M)-PM-G(s)$. This condition is the same as (8).[23] Then, the conflict arises at the second stage, and the expected payoff of the supplier, $\Pi(\lambda(s:\theta))$, and that of the zero player, $\Pi_0(\lambda(s:\theta))$, are defined by (14)' and (15)', respectively.

$$\Pi(\lambda(s:\theta))=(1-\lambda(s:\theta))(PM-V)+\lambda(s:\theta)(-V) \qquad (14)'$$

$$\begin{aligned}\Pi_0(\lambda(s:\theta))=&\ \lambda(s:\theta)(s\alpha f(M)-G(s)-V_0)\\ &+(1-\lambda(s:\theta))(s\alpha f(M)-G(s)-V_0-PM)\end{aligned} \qquad (15)'$$

If there exists some s satisfying $\lambda^*=Sup_{s\in s}\lambda(s)\fallingdotseq 1$, the zero player can acquire M at the cost of V_0 bringing about his payoff, $\Pi_0(\lambda^*)\fallingdotseq\{s\alpha f(M)-G(s)-V_0\}$. This corresponds to the situation where the predatory theory of the state prevails.

In order for the supplier to induce the zero player to accept the demand at the first stage, P must be set at P** which satisfies the equation, $\Pi_0(P)=\Pi_0(\lambda(s:\theta))$. From this equation we obtain (16).

$$P^{**}=P^{**}(s)=V_0/[\lambda(s:\theta)M] \qquad (16)$$

The above P**(s) is optimal for the supplier, since by substituting (16) into (14) and (14)', it is proved that $\Pi(P^{**})-\Pi(\lambda(s))=V_0(s)+V>0$. The payoff of the zero player at this optimal level of demand price is obtained by substituting (16) into (15) and (15),' leading to the equation (17) below.

$$\begin{aligned}\Pi_0(P^{**})&=s\alpha f(M)-P^{**}(s)M-G(s)\\ &=\Pi_0(\lambda(s:\theta))\end{aligned} \qquad (17)$$

It is be obvious from (16) that $\partial P^{**}/\partial s<0$.(The Proof is given in Appendix 2) That is, the zero player

[22] As to the original concept of the conflict success function, see Skaperdes (1992).

[23] It is the case that $V_0=V_0(s)$, $V_0'<0$. This is because the cost of guards can be reduced if a regular military force is standing. This more realistic assumption can strengthen the causal logic below.

conjectures that he can make the terms of trade more advantageous by strengthening the state power. i.e., by strengthening its regular military force. Therefore, if $P^{**}(s)M$ replaces $\phi(s)M$ defined in the 2nd section, the equation (17) is the same as (3) where $\Psi(M)=\phi(s)M+G(s)$ subject to the constraint that both M and s are determined by the zero player's maximization. They are denoted by M^{**} and s^{**}, respectively, and $dM/ds>0$ in the neighborhood of the optimal point.

By comparing the above results, we can demonstrate the private incentive of a selfish chieftain (the zero player) for being armed with a regular military force. Even though he did not have to resort to threat by force for the sake of keeping an irrigation society in order, he needs it for the aim of avoiding the trouble of often falling into the worse terms of trade. The private incentive of the zero player for being armed with a regular military force is assured by (18).

$$\Pi_0(P^{**})=s^{**}\alpha f(M^{**})-V_0/\lambda(s^{**}:\theta)-G(s^{**})>s^{*}\alpha f(M^{*})-V_0/\lambda=\Pi_0(P^{*}) \qquad (18)$$

Under the superadditive condition, it is derived that $M^{**}>M^{*}$ and $s^{**}>s^{*}$.[24] In the next subsection, the condition (18) is corroborated by some historical evidences of the irrigation society of Japan.

3-3. The Private Motives for State Formation

Rearranging the inequality (18), the conditions that the rational chieftain of the preceding communities is motivated to integrate as many of them as possible into a state are summarized by the inequality (19).

$$s^{**}\alpha f(M^{**})-s^{*}\alpha f(M^{*})>G(s^{**})-[V_0/\lambda-V_0/\lambda(s^{**}:\theta)] \qquad (19)$$

First of all, I note that whilst the left side of (19) means an increase in the benefits from transforming from the preceding society into a state, the right side means an increase in the cost of the transformation. Therefore, the inequality (19) demonstrates that if the net benefit to the zero player of transforming into the state is positive subject to the participation constraints of other members, he is motivated to do it. It is the mathematical formulation of the "bargaining power" hypothesis on the origins of the state.

Next, it is noted that the inequality (19) can hold provided that a combination of the following three possibilities is realized: (i) the left side is large enough, (ii) $G(s^{**})$ is small enough, (iii) $\lambda(s^{**})$ is large enough relative to the given λ. In what follows, we examine how those conditions were satisfied in the process of forming the early state in Japan.

When Korean peninsula was put under the control of the Han dynasty in the 2nd century B.C., the irrigation communities of the Northern part of Kyushu region locating in the far west-south part of the Japanese Archipelagos began an inter-society trade with it. Then, they were integrated into one "union" which was later called *Nakoku*.[25] The aim of the union was to make better terms of trade in the inter-society trade by integrating bargaining processes and by cutting the cost of producing the means of payment. Though rice-planting had begun around1000 B.C. in Japan, iron tools began being utilized for agriculture in the later periods called the Yayoi. The conjecture that the union brought back iron resources vie the southern part of Korean peninsula is corroborated by the Gisyo Benshinden in the ancient Chinese document called the "history of the three countries" in English. Given the military balance on those days, however, it was hard to raise λ without bearing a high governance cost G. Furthermore, P^{*} was considered to be given by the Han dynasty. Though the coalition shows some characteristics of a state such as rulership and authority according to recent archaeological finding, it

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24 Those relations are proved in *Appendix 2.*

25 According to Kansyo Chirishi (the History of Han), fifteen commerce centers were built in the south part of the peninsula called Hinban Gun in 108 BC. Since 82BC, those centers were abolished and integrated into the northern center called Rakuro Gun, as Han dynasty was waning. Then, those irrigation communities were required to go abroad for inter-society trades by themselves.

is not clear whether it satisfied the crucial condition of the state, i.e., a regular military force.

When the Han actually broke down in the 2nd century AD, the Korean peninsula was put under anarchy in the sense that three preceding societies were in conflict for the hegemony of the peninsula. Under this new external circumstance, a new larger coalition was formed by the hegemony of the preceding society in the central region of the Japanese Archipelagos. The new coalition is called *Yamato*. The primary aim of this coalition was to keep the supply source of iron resources in stable order, or to carry out the diplomatic policy which can assure of procuring iron resources in advantageous terms. To maintain a regular military force was a necessary condition for that aim. The military balance in the peninsula had changed, and the new coalition could become predominant there, if it was armed. The armed Yamato coalition could pursue the diplomatic policy which assists one of those three hostile societies in the Korean peninsula, called *Kudara*, to cope with other two ones and to protect the supply route of iron resources. It was enough for the Yamato coalition to keep an advantageous inter-societal trade in order, because it was too costly to put the main part of the peninsula under a direct control. That is, it is enough to raise λ(s) relative to G(s). On the other hand, irrigation technologies could advance drastically by making use of more iron resources, and iron tools became major agricultural tools in these periods. Both factors are considered to have contributed to raising the left hand of (19).[26]

The above new external circumstances stimulated the chieftains of the preceding irrigation communities under geographically and historically favorable conditions to form a larger coalition with the aim of having more advantageous position in those inter-society trades for iron resources. For this purpose, they transformed the preceding communities into a state armed with a military force. Taking into consideration the historical and archaeological finding that such a new social organ was set up in the late third to early fourth century, it turns out to be right that the early state was formed in this period under the hegemony of the Yamato Coalition.

Since the 6th century, however, the irrigation societies of Japan could gradually self-supply iron resources and were losing interests in inter-societal trades for iron resources. It was for the sake of the defense against the military threat of the Tang dynasty that a centralized political system under the rule of the ancient dynasty was accepted in the 7th to 8th century. When the military threat waned away in the 8th century, the regular army comprised of 200-thousand soldiers was gradually disbanded and the centralized political system supporting it was being changed into a heterachy society in the late 8th century onward. In the end, the tribute which had continued to be brought by then ruling state of the Korean peninsula was rejected in the year 780. This meant the diplomatic declaration that Japan lost interests in diplomacy with the continent.[27]

4. The Rational Bases of Federalism

In the previous sections the superadditive characteristics or scale merits of defense services is assumed to be compatible with, if ever, heterogeneous irrigation systems. That is, it was assumed that an increase in the cost due to a longer distance is offset by the superadditive effects of a larger hierarchical coalition. However, an irrigation system is utilized by farmers with a various level of cost. Such a cost difference in the irrigation system was represented by *C(i)*, an increasing function of distance from water sources (sluice gate). In this section, this assumption is dismissed and we take up the case that an increase in *C(i)* located at some marginal site outdoes the scale economies of defense services, in the sense that the participation constraint can not be satisfied there. Such a farmer may prefer an alternative irrigation system connected to other water sources. Then, the economies

[26] As to those processes, see Hirose (1997), pp.135-138, and pp.151-152.

[27] See Shimomukai (1999).

of scale of defense services are in contradiction with heterogeneous preferences of farmers. This section shows the economic background of a federal system in irrigation society. It confirms the main hypotheses of Alesina and Spolaore (2005).

4-1. The Base Model of Heterogeneous Irrigation Societies

Suppose there are a set of preceding irrigation societies, consisting of S_1, S_2, ..., and S_L, each of which, despite being technically combinable with others, is managed independently and separated. Denote a contract offer to society S_i by $a_i(S_i) = (\alpha, M, \theta_i(0), S_i) \in A(S_i)$ for $\forall i \in \{1,2,...,L\}$ where θ_i is a parameter denoting the location of the sluice gate of society *i*, connected to its water sources. It stands for the distance from the shortest water sources. Each $\theta_i(0)$ of this autarky type of society is normalized to the zero level. Then, $\{a_i, S_i\}_{i \in \{1,2,...,L\}}$, is called the *coalition structure*. The utility functions of the members of S_i are defined below.

$$U_{i0}(a_i) = |S_i| \alpha f(M) - \psi_i(M;P) \tag{20}$$

$$\psi_i(M;P) = PM \tag{21}$$

$$U_{ij}(a_i) = (1-\alpha) f(M) - K_i(M)/|S_i| - C_i(j), \quad for\ j \in S_i,\ i \in \{1,2,...,L\} \tag{22}$$

If each of (a_i, S_i), i=1,2,..,L, is feasible and superadditive under the condition that any union $S_h \cup S_k$, $S_h \cap S_k = \phi$, is not superaddtive, then the coalition structure (a_i, S_i), $i \in \{1,2,\cdots,L\}$, is stable in the sense that any member of each coalition has no incentive to block it.[28] Under such circumstances, we can say that there exist *L* autonomous preceding societies.

If a state organ is brought into those preceding societies and furthermore, if through the positive effects of defense services on bargaining power in inter-societal trades with an "out-of-the coalition" society, the superadditivity can extend to *H* coalitions, i.e., to $S_1 \cup S_2 \cup \cdots \cup S_H \equiv S(H)$, for H< L, then it is more efficient to divide the prerogatives of each society in such a way that whilst supreme prerogatives represented by defenses are left to the central institution of S(H), the management of each irrigation system is transferred to local ones. In the next two subsections, it is shown that the "hierarchical hypothesis" and the "principle of subsidiarity" should be compatible in order for a federal system to be preferred.

4-2. Heterogeneity Cost under a Centralized Political System

Assume that the chief of S_1 is the hegemonic player of the coalition S(H), and that each of the original irrigation systems, S_h, $h = 1,2,\cdots, H$, are in turn connected to a chain type of hierarchy. This chain type of hierarchy stands for a centralized political system under which all public goods are provided under the direct control of the ruler of S_1. The "direct control" means that not only regular forces but also irrigation systems are determined by the preference of the hegemonic player. Under this centralized system each farmer has to directly communicate with the hegemonic player to gain seeds and to pay annual tributes, even if the water source of this new irrigation system is connected to the irrigation system located in the closest upper irrigation society.

Then, according to the mathematical algorism to derive the hierarchical outcome,[29] the chain type of hierarchical coalition is still stable due to the assumption of the superaddtivity of $(a_H, S(H))$, $a_H = \exists (\alpha, M, \theta_H, S(H)) \in A(S(H))$ where $\theta(H) = \{\theta_1, ..., \theta_H\}$ is the location of sluice gates under this chain coalition. They stand for factors influencing on the communication cost under the condition of *H* sluice gates being connected in a chain. The utility functions of this hierarchical coalition are defined below.

$$U_{10}(a) = |S(H)| \alpha f(M) - \psi(M;s), \quad for\ \exists a \in A(S(H)) \tag{23}$$

[28] See Demange (2004) for the proof.

[29] See the second section of this paper.

$$\psi(M;s) = \phi(|S(H)|)M - G(|S(H)|) \qquad (24)$$

$$\begin{aligned} U_{ij}(a) &= (1-\alpha)f(M) - K_i(M)/|S_i| \\ &\quad - C(i,j:\theta(H)),\ for\ j \in S_i, \\ &\quad i \in \{1,2,...,H\}\ s.t., U_{ij}(a) \geq \\ &\quad U_{ij}(a_i(S_i)),\quad for\ \forall j \in S_i, \\ &\quad \forall i \in \{1,2,....,H\} \end{aligned} \qquad (25)$$

C(i, j: θ(H)) on the right side of (25) is larger than *$C_i(j)$* of (22) for any $j \in S_i$, for i = 2, 3,...,H. This is because under the chain hierarchy those irrigation systems except S_1 are connected to water sources farther distanced from their original ones. Therefore, the above chain hierarchy is considered to be more efficient, if the management of the centralized irrigation system is entrusted to each original coalition S_i. If possible, therefore, *C(i, j: θ(H))* on the right of (24) should be replaced with a new cost function, more approximate to $C_i(j)$ less than C(i,j: θ (H)).

4-3. The Rational Foundations of Federalism

According to the classification given by Riker (1964), federalism is classified into the "centralized federalism" and the "peripheral one." The Federation of USA after 1787 is an example for the first one and the Confederation before 1787 is for the second one, respectively. In either case, a constitution is defined to be "federal," if three conditions are satisfied, as follows: Firstly, two levels of government rule the same land and people, and secondly each level has at least one area of action in which it is autonomous. Finally, there is some guarantee of the autonomy of each government in its own sphere. (Riker, 1964, pp.11)

According to the summary by Riker, the necessary conditions for the creation of a federal system are as follows:

(1) The politicians who offer the bargain on federation desire to expand their territorial control, usually either meet an external military and diplomatic threat or to prepare for military or diplomatic aggression. But though they desire to expand, they are not able to do so by conquest, because of either military incapacity or ideological distaste. Hence, if they are to satisfy the desire to expand, they must offer concessions to the rulers of constituent units.

(2) The politicians who accept the federal bargain, giving up some independence for the sake of union, are willing to do so because of some external military and diplomatic threat. (Riker, 1964, pp.12)

Those conditions are corroborated by *The Federalist*(1787) In terms of the concepts of this paper, the origins of federalism should be sought in the trade-off between the scale economies of defense and the heterogeneous cost of irrigation systems subject to the participation constraints of constituent members. Here, suppose that without any dominant power no chieftain can guarantee the participation conditions for the centralized system to be accepted at the final stage. Furthermore, the transaction cost jumps up so high that it is harder to meet the participation conditions. In what follows, we show that without any hegemonic power, the irrigation societies have to trade off the scale economy against the heterogeneity cost by a contractual means, that is, by a federal system.

Suppose that each irrigation system is left to the original local society and is connected to its own water sources, where factors influencing on the communication and transportation cost are represented by θ_i' for i society. Here, θ_i' for $i = 2,3,...,H$, is not set at zero, because each constituent member has to directly be connected to a central institution represented by the chieftain of S_1, even under the federal system. This is because though the management of each irrigation system is left to each irrigation society, the regular military force has to be still put under the direct control of the central institution. Each constituent member has to do a military service under the centralized control. Furthermore, he has to pay a part of his harvests to the central institution but not to the chieftain of the local irrigation society to which he belongs, and he has to obtain or borrow seeds from the central ruler. In any case, those constituent members are put under the circumstance in which they have to communicate with the central ruler located in a site further away from the original chief. Thus, the new marginal cost function, C(i,j: θ_i') for j∈S_i, i∈{1,2,...,H}, is assumed to be between C(i,j: θ (H)) and C_i(j), that is, C_i(j)≦C(i,j: θ i')< C(i,j: θ (H)), for j∈S_i, i∈{1,2,....,H}.

The above federal system, under which defense services are under the direct control of the central institution but the irrigation systems are left to local control, is formulated by (26) to (28) below.

$$U_{10}(b) = |S(H)|\alpha f(M) - \psi(M;s), \quad for\ \exists b \in A(S(H)) \tag{26}$$

$$\psi(M;s) = \phi(|S(H)|)M + G(|S(H)|) \tag{27}$$

$$U_{ij}(b) = (1-\alpha)f(M) - K_i(M)/|S_i| - C(i,j:\theta_i'), \quad for\ j \in S_i,\ i \in \{1,2,...,H\}\ s.t., U_{ij}(b) \geq U_{ij}(a_i(S_i)), for\ \forall j \in S_i, \forall i \in \{1,2,....,H\} \tag{28}$$

Since C(i,j: θ i')< C(i,j: θ (H)), for ∀j∈S_i, ∀i∈{1,2,...,H}, the participation conditions, U_{ij}(b)≧U_{ij} (a_i(S_i)) for ∀j∈S_i, ∀i∈{1,2,...,H}, can be satisfied at the final stage, even if the value of ϕ (|S(H)|) and/ or that of G(|S(H)|) are not so low due to no hegemonic power. Then, the federal system is justified on a rational basis.[30] This is an irrigation version of the rational federalism formulated by Alesinae Spolaore (2005).

5. The Main Conclusions

Based on the recent historical studies of ancient irrigation societies, I formulate the dynamic process of transforming the preceding stateless society into an early state, and proved the causal logic of the "bargaining power" theory of the state. Furthermore, I showed that this theory can subsume the main characteristics of the traditional theories of the state. The main analytical results are as follows: Firstly, the private incentives stimulated the chieftains of the preceding societies to transform stateless societies into an early state. Concretely speaking, when they believed that their bargaining power in a new inter-societal trade with some external society can be strengthened enough by being armed with a regular force, they transformed the existent social system so as to adapt to the new external circumstance. As a result, the state came into being. Secondly, a federal system is justified, if the trade-off between economies of scale and heterogeneity of preferences can be coordinated more efficiently under a federal system.

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[30] This result is proved by analyzing a two-stage game. An outline of the proof is as follows: At the first stage the zero player of S_1 decides the size of S_H which is accompanied with the decision of *M*. At the second stage, given S_H and *M* determined by the zero player, each irrigation society decides the best site of its sluice gate on its own.

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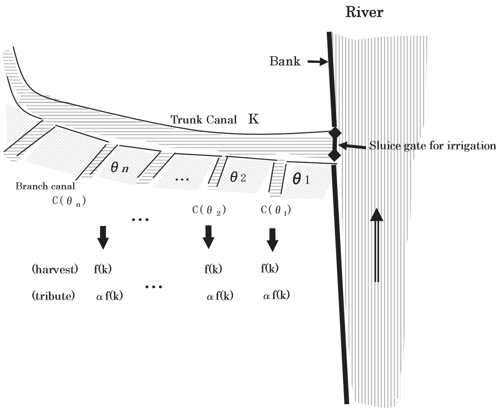
Appendix 1 The Base Model of Irrigation System



Assumptions on the Irrigation System

The marginal cost of the irrigation system: $C(\theta_1) < C(\theta_2) < \cdots < C(\theta_n)$

The cost of trunk canal: K = K (M), K' < 0 and K'' >0

Harvest per Cropland: f(M), f'>0 and f''<0

M:= provision of ironware, standing for irrigation technologies

Appendix 2 The Size of an Irrigation Society with Regular Military Forth

The zero player armed with a regular military forth is an agricultural and military entrepreneur engaged in inter-society trades. Then, the optimal size of the irrigation society for him is determined so as to satisfy (A1).

$$
\begin{aligned}
&Max\ \Pi_0(\alpha, M, s) = s\alpha f(M) - \phi(s)M - G(s) \\
&s.t.,\ \Pi_s(M, s) = (1-\alpha)f(M) - K(M)/s \\
&\qquad - C(s) \ge 0, M \ge 0, 1 > \alpha > 0, and\ s > 0. \qquad \text{(A1)}
\end{aligned}
$$

The Lagrangian of (A1), F(α ,M,s, μ), is defined by (A2).

$$
\begin{aligned}
F(\alpha, M, s, \mu) &= \Pi_0(\alpha, M, s) + \mu\Pi_s(\alpha, M, s) \\
&= \alpha s f(M) - \phi(s)M - G(s) \\
&\quad + \mu[(1-\alpha)f(M) - K(M)/s - C(s)] \qquad \text{(A2)}
\end{aligned}
$$

Then, the necessary conditions of the Kuhn-Tucker theorem are derived, summarized by (A3-0) to (A3-3)' below:

$$sf(M) - \mu f(M) = 0 \qquad \text{(A3-0)}$$

$$\alpha s f'(M) - \phi(s) + \mu[(1-\alpha)f'(M) - K'(M)/s] \le 0 \qquad \text{(A3-1)}$$

$$M[\alpha s f'(M) - \phi(s) + \mu\{(1-\alpha)f'(M) - K'(M)/s\}] = 0 \qquad \text{(A3-1)'}$$

$$\alpha f(M) - \phi_1(s)M - G'(s) + \mu[K(M)/s^2 - C'(s)] = 0 \qquad \text{(A3-2)}$$

$$(1-\alpha)f(M) - K(M)/s - C(s) \ge 0 \qquad \text{(A3-3)}$$

$$\mu[(1-\alpha)f(M) - K(M)/s - C(s)] = 0 \qquad \text{(A3-3)'}$$

From (A3-3) it is derived that M>0 for s>0 and 0< α <1, resulting with μ >0 from (A3-0). Then, from (A3-1)', (A3-2), (A3-3) and (A3-3)', we obtain (A4-1), (A4-2) and (A4-3) as follows.

$$\alpha s f'(M) - \phi(s) + \mu\{(1-\alpha)f'(M) - K'(M)/s\} = 0 \qquad \text{(A4-1)}$$

$$\alpha f(M) - \phi_1 M - G'(s) + \mu\{K(M)/s^2 - C'(s)\} = 0 \qquad \text{(A4-2)}$$

$$(1-\alpha)f(M) - K(M)/s - C(s) = 0 \qquad \text{(A4-3)}$$

Next, since (1- α)f'(M)-K'(M)/s > 0, the equality (A5-1) is derived from (A4-1).

$$\alpha s f'(M) - \phi(s) < 0 \qquad \text{(A5-1)}$$

On the other hand, from (A4-1) and (A4-2) we obtain the equation (A5-2).

$$\frac{\alpha s f'(M) - \phi(s)}{\alpha f(M) - \phi_1 M - G'(s)} = \frac{(1-\alpha)f'(M) - K'(M)/s}{K(M)/s^2 - C'(s)} \qquad \text{(A5-2)}$$

The numerator of the left side of (A5-2) is negative from (A5-1), and that of the right side is positive due to the technological assumptions, f'(M) >0 and K'(M) <0. Thus, from the comparison of the denominators of (A5-2) we obtain the identical relation (A5-3) as follows.

$$\alpha f(M)-\phi_1 M-G'(s) \gtrless 0 \Leftrightarrow K(M)/s^2-C'(s) \lessgtr 0 \tag{A5-3}$$

The above identity relation (A5-3) tells us that in order for (A1) to have its positive extreme solutions, either of the following two cases must arise: (i)the case where an increase in the payoff of the zero player obtainable from one more increase in the size of irrigation society is positive and at the same time an increase in the payoff to the marginal farmer obtainable from an increase in the size of society is negative, or (ii) the case where the former is negative and the latter is positive. Each one of those cases is achievable, if, *ceteris paribus,* either the combination of a low G'(s) with a high C'(s), or that of a high G'(s) with a low C'(s) has to hold. The former case (latter case) can hold, if the governance cost does not so drastically increase (does drastically jump up) and the cost of constructing a branch canal drastically jumps up (does not increase so drastically) at the margin.

Under the geographical condition that each river is separated from its neighboring rivers by high mountains, an irrigation society developed along a river is often based on traditional networks of a clan community. In such a society G'(s) may be considered to be small. This case applies to the irrigation society of Japan. Then, the condition (A5-3) requires that C'(s) be sufficiently large. This society tends to become a heterarchy.

On the contrary, under the geographical condition that C'(s) is negligible, the right inequality of (A5-3) is always positive, and thus G'(s) must be so large as to satisfy the negativity of the left inequality. This case may be applicable to the irrigation society of a hierarchy type such as the Nile. The "hydraulic society" model of Wittfogel (1957) is applicable to such a geographical condition.

From (A5-3), we can derive *dM /ds* in the neighborhood of the optimal point, shown by (A5-4).

$$\frac{dM}{ds}=\frac{K(M)/s^2-C'(s)}{K'(M)/s-(1-\alpha)f'(M)} \tag{A5-4}$$

Since the denominator of the right hand of (A5-4) is negative, the sign of the derivative is dependent on the sign of the numerator. If the numerator is negative (positive), then *dM /ds* >0 (< 0). Therefore, in a heterarchy (hierarchy) type of irrigation society, *dM /ds* >0 (< 0).

Furthermore, by taking into consideration that K' <0, K">0,C'>0, C">0, f'>0, and f"<0, it is proved that the second derivative of (A5-4) is positive for *dM/ds* >0, as shown by (A5-5).

$$\begin{aligned}\frac{d^2M}{ds^2}=&\frac{1}{[K'/s-(1-\alpha)f']^2}[\{K'/s-(1-\alpha)f'\}\\ &\{(K's^2\frac{dM}{ds}-2Ks)/s^4-C''(s)\}\\ &-\{(K/s^2-C'(s)\}\{(K''s\frac{dM}{ds}-K')/s^2\\ &-(1-\alpha)f''\frac{dM}{ds}\}]>0\end{aligned} \tag{A5-5}$$

The analytical results are summarized by the following *Result.*

Result: For the optimal solution (A1), either of the following two cases must be satisfied: (i) the case where an increase in the payoff to the ruler of one more increase in the size of irrigation society is still *positive,* and at the same time an increase in the payoff to the marginal farmer of that increase in the size of society is *negative,* or (ii) the case where the former is *negative* and the latter is *positive*. The first case (the second case) is applicable to a heterarchy (hierarchy) type of irrigation society. It is shown that *dM /ds* > 0 for the first case and *dM /ds* < 0 for the second case. Furthermore, if *dM/ds* >0, it holds that $d^2M/ds^2>0$.

Finally, we have to derive *dM/ds* for the stateless society for comparison. The optimal offer by the chieftain is determined so as to satisfy (B1) below.

$$\begin{aligned}&Max\ \pi_0(\alpha,M,s)=s\alpha f(M)-PM\\ &s.t.\ \pi_s(\alpha,M,s)=(1-\alpha)f(M)-K(M)/s-C(s)\\ &\qquad 0<\alpha<1,\ 0<s,\ 0\le M.\end{aligned} \tag{B1}$$

The Lagrangian of (B1), $F^0(\alpha, M, s, \mu)$, is defined by (B2).

$$F^0(\alpha, M, s, \mu) = s\alpha f(M) - PM$$
$$+ \mu[(1-\alpha) f(M) - K(M)/s - C(s)] \quad \text{(B-2)}$$

From $\partial F^0/\partial \alpha = 0$, it is derived that μ = s >0. Thus, it holds that the equality *(B3)* with *M>0*.

$$(1-\alpha) f(M) - K(M)/s - C(s) = 0 \quad \text{(B3)}$$

From M>0, we obtain (B4)

$$\partial F^0/\partial M = s\alpha f'(M) - P + \mu [(1-\alpha) f'(M) - K'(M)/s] = 0. \quad \text{(B4)}$$

Furthermore, from $\partial F^0/\partial s = 0$, we obtain *(B5)*.

$$\alpha f(M) + \mu [K(M)/s^2 - C'(s)] = 0 \quad \text{(B5)}$$

From (B5), we obtain the inequality (B6).

$$K(M)/s^2 < C'(s) \quad \text{(B6)}$$

Then, by differentiation of (B3), by rearranging the result and by taking (B6) into consideration, we obtain (B7) in the end.

$$\frac{dM}{ds} = \frac{C'(s) - K(M)/s^2}{(1-\alpha) f'(M) - K'(M)/s} > 0 \quad \text{(B7)}$$