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A PERISHABLE INVENTORY MODEL WITH UNKNOWN TIME HORIZON

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Abstract—**Traditionally, the time (planning) horizon over which the inventory for a particular item will be controlled is often assumed to be known (finite or infinite) and the total inventory cost is usually obtained by summing up the cost over the entire time horizon. However, in some inventory situations the period over which the inventory will be controlled are difficult to predict with certainty, as the inventory problems may not live up to or live beyond the assumed planning horizon, thereby affecting the optimality of the model. This paper presents a deterministic perishable inventory model for items with linear trend in demand and constant deterioration when time horizon is unknown, unspecified or unbounded. The heuristic model obtains replenishment policy by determining the ordering schedule to minimize the total cost per unit time over the duration of each schedule. A numerical example and sensitivity analysis are given to illustrate the model.**

I. INTRODUCTION

Developing an inventory model requires a thorough understanding of the properties of the inventory system like demand and replenishment. Models in which demand is known in advance are referred to as deterministic models. Harris in 1915 developed the simplest inventory model, the Economics Order Quantity (EOQ) model, which was later popularized by Wilson [5]. Relaxation of some assumptions in the formulation of the EOQ model led to the development of other inventory models that effectively tackles several other inventory problems occurring in day-to-day life.

Ghare and Shrader [13] extended the classical EOQ formula to include exponential decay, wherein a constant fraction of on hand inventory is assumed to be lost due to deterioration. Covert and Philip [12] and Shah and Jaiswal [11] carried out an extension to the above model by considering deterioration of Weibull and general distributions respectively.

Dave and Patel [9] developed the first perishable inventory model with linear trend in demand. The model generates optimal replenishment schedules for items with linearly changing demand rate and constant rate of deterioration. Kim [4] developed another heuristic solution procedure to obtain replenishment schedules for items with linearly changing demand rate and constant rate of deterioration when the time horizon is unknown. Kim's heuristic model makes computation easier and allows for unequal replenishment period but does not allow for shortages.

In some real competitive markets, shortages do occur and lost sales are either fully or partially backordered. Abad [2] considered partial backordering and lost sales in solving lot-sizing problem for perishable goods under finite production and exponential decay whilst Wee et al [1] developed a model that incorporated backordering in a two-warehouse inventory situation. Both models assumed demand to be constant and time horizon to be known and finite.

For several practical situations however, demand varies and the time horizon may be unknown, unspecified or unbounded. Setting up time horizon beyond one life cycle of a system is highly risky in the present dynamic environment where rapid changes occur due to new developments, hence the need to relax the specific time horizon assumptions.

In this paper a model that relaxes the specific time horizon assumption is presented. It considers linear variation in demand, unequal replenishment, shortages and full backlogging in generating a replenishment policy for perishable goods.

II. MODEL ASSUMPTIONS AND NOTATIONS

The model is based on the following assumptions:

a. Demand is known and changes linearly with time.
b. Replenishment rate is infinite.
c. Shortages are allowed with full backlogging.
d. No repair or replacement of deteriorated items during the period under review
e. Inventory holding cost, replenishment cost, cost of deteriorated items, and shortage cost are known and constant.
f. A single item inventory is being considered.
g. Cycle time of system is *T*.
h. Lead-time is zero.
i. Item in inventory deteriorate at a constant rate, θ

The notations used are:

$D(t)$ = $a+bt$ is the demand rate at any time t, $a \geq 0, b \geq 0$

C_I = Inventory holding cost per unit per year.

C_R = Replenishment cost per order.

C_D = Cost of a deteriorated unit.

C_S = Shortage cost per unit.

$T.C$ = Total Inventory Cost per unit time over the first replenishment schedule.

T = Cycle time of system

θ = Fraction of On-hand Inventory that deteriorate.

$I(t)$ = Inventory level at any time t during the first replenishment cycle

III. MODEL FORMULATION

The behavior of the inventory can be described thus:

1. During the time between $t = 0$ and $t = t_1$, Consumption (due to demand and deterioration) bring inventory level to zero.
2. From $t = t_1$ up to time $t = T$, shortages occur.
3. Replenishment with full backlogging of shortages occurs at time T.

Considering the first replenishment period only, the differential equation for the system is given by:

$$\frac{dI(t)}{dt} + \theta I(t) + D(t) = 0,\ 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} + D(t) = 0,\ t_1 \le t \le T \tag{2}$$

The solutions to the differential equations (1) and (2) are respectively represented by

$$I(t) = e^{-\theta t}\left(\int_0^t e^{\theta t} D(t)dt\right)\ 0 \le t \le t_1 \tag{3}$$

$$I(t) = -\int_{t_1}^{t} D(t)dt, \qquad t_1 \le t \le T \tag{4}$$

Let

I_D: Number of Units that deteriorate during the first replenishment.

I_I: Number of Units in inventory during the first replenishment

I_S: Amount of shortage during the first replenishment

According to Goswami and Chaudhuri [7]

$$I_D = \int_0^{t_1} (a+bt)e^{\theta t} dt - \int_0^{t_1} (a+bt)dt \tag{5}$$

$$I_I = \int_0^{t_1} t(a+bt)e^{\theta t} dt \tag{6}$$

Using Eq. (4) above, I_S is given by

$$I_S = \int_{t_1}^{T} -(a+bt)dt \tag{7}$$

The total inventory cost per unit time (TC) is given by:

$$TC = \frac{1}{T}\left(C_R + C_D I_D + C_I I_I + C_S I_S\right) \tag{8}$$

Since θ is usually very small, expression for I_D and I_I can be simplified by neglecting second and higher – order terms in the expansion of exponential term, hence

$$I_D = \tfrac{1}{2}\left(a\theta t_1^2\right) + \tfrac{1}{3}\left(b\theta t_1^3\right) \tag{9}$$

$$I_I = \tfrac{1}{2}\left(a t_1^2\right) + \tfrac{1}{3}\left(a\theta t_1^3 + b t_1^3\right) + \tfrac{1}{4}\left(b\theta t_1^4\right) \tag{10}$$

$$I_S = -\left(a(T - t_1) + \frac{b}{2}(T^2 - t_1^2)\right) \tag{11}$$

Substituting (9), (10) and (11) into (8) gives

$$TC(t_1, T) = \frac{K}{T} \tag{12}$$

$$\begin{aligned} K = C_R &+ C_D\left[\frac{a\theta t_1^2}{2} + \frac{b\theta t_1^3}{3}\right] \\ &+ C_I\left[\frac{a t_1^2}{2} + \frac{a\theta t_1^3}{3} + \frac{b t_1^3}{3} + \frac{b\theta t_1^4}{4}\right] \\ &- C_S\left[a(T - t) + \frac{b}{2}(T^2 - t_1^2)\right] \end{aligned} \tag{13}$$

Let α be the fraction of the first replenishment interval for which there is no shortage. Then t_1 can be written as

$$t_1 = \alpha T \tag{14}$$

Hence Eq. (12) becomes

$$TC(T) = \frac{K}{T} \tag{15}$$

Using $t_1 = \alpha T$ in Eq. (13) gives

$$K = C_R + C_D\left[\frac{a\theta\alpha^2T^2}{2} + \frac{b\theta\alpha^3T^3}{3}\right] + C_I\left[\frac{a\alpha^2T^2}{2} + \frac{a\theta\alpha^3T^3}{3} + \frac{b\alpha^3T^3}{3} + \frac{b\theta\alpha^4T^4}{4}\right] - C_S\left[a(1-\alpha)T + \frac{b}{2}(1-\alpha^2)T^2\right] \quad (16)$$

The conditions for minimizing total cost (*TC*) is given by

$$\frac{d(TC)}{dT} = 0 \quad (17)$$

$$\frac{d^2(TC)}{dT^2} > 0 \quad (18)$$

From Eq. (16) it can be deduced that

$$\frac{d(TC)}{dT} = \frac{1}{T^2}\left(T\frac{dK}{dT} - K\right) \quad (19)$$

Employing condition (17) in Eq. (19) gives:

$$\frac{3}{4}C_I b\theta\alpha^4T^4 + \frac{2}{3}\alpha^3\left[C_D b\theta + C_I b + C_I a\theta\right]T^3 + \frac{1}{2}\left[\alpha^2 a(C_D\theta + C_I) - C_S b(1-\alpha^2)\right]T^2 - C_R = 0 \quad (20)$$

Eq. (20) is a bi-quadratic equation that can be solved iteratively to obtain the value of T (for b>0).

When shortages are not allowed, $\alpha = 1$ from Eq. (14).
Setting $\alpha = 1$ and $C_D = C_P$ in Eq. (20) gives:

$$\frac{3}{4}C_I b\theta.T^4 + \frac{2}{3}\left[C_P b\theta + C_I b + C_I a\theta\right]T^3 + \frac{1}{2}\left[C_P a\theta + C_I a\right]T^2 = C_R \quad (21)$$

Eq. (21) is the same as Eq. (10) in Kim's model where shortages are not allowed (See [4]).

Putting $C_R = S, C_I = H, C_D = P, \alpha = 1 \ and \ \theta = 0$ in Eq. (20) yields:

$$\frac{2}{3}HbT^3 + \frac{1}{2}HaT^2 = S \quad (22)$$

Eq. (22) is the same as Eq. (8) in Silver's model for non-deteriorating items (See [10]).

From Eq. (17), the expression on the left hand side can be written as:

$$\frac{d(TC)}{dT} = \frac{3}{4}C_I b\theta\alpha^4T^2 + \frac{2}{3}\alpha^3\left[C_D b\theta + C_I b + C_I a\theta\right]T + \frac{1}{2}\left[\alpha^2 a(C_D\theta + C_I) - C_S b(1-\alpha^2)\right] - \frac{C_R}{T^2} \quad (23)$$

Then Eq. (18) becomes

$$\frac{d^2(TC)}{dT^2} = \frac{3}{2}C_I b\theta\alpha^4T + \frac{2}{3}\alpha^3\left[C_D b\theta + C_I b + C_I a\theta\right] + \frac{2C_R}{T^3} \quad (24)$$

From Eq. (24) it is obvious that $\frac{d^2(TC)}{dT} > 0$ for all values of T.

Hence, all conditions for a minimum value of *TC* are satisfied by Eq. (20).

IV. SOLUTION PROCEDURE

In using the heuristic proposed for solving inventory problem with deterioration, shortages and time varying demand, rather than determining all the replenishments to minimize the total relevant cost up to the horizon, we determine the size of replenishment to minimize the total cost per unit time over the duration of the first replenishment only. The procedure is then repeated for other replenishments.

Once the parameter values at the beginning of each replenishment cycle are specified, the value of T_i, the i^{th} replenishment duration, is then determined to minimize total inventory cost over that duration. Solving the bi-quadratic Eq. (20) gives four values of T_i of which only one is relevant under any situation being considered (other values being either negative, complex or outside the desired range of values). With the value of T_i known, the minimized total cost for the i^{th} replenishment duration, TC_i, can be determined using Eq. (15) and Eq. (16).

This solution procedure is summarized in the algorithm outlined below:

Step 1: Input all parameter values (e.g. costs, rate of deterioration etc)

Step 2: Compute all possible values of T_i by solving the bi-quadratic Eq. (20) for the first replenishment duration.

Step 3: Select the appropriate value of T_i

Step 4: Compute *TC.*

The above steps are used for all replenishment schedules using appropriate parameter values. In order to obtain the value of T_i we need to solve the bi-quadratic Eq. (20) using any robust numerical equation solver e.g. "NSolve", in MATHEMATICA package or "Roots" in the MATLAB package.

V. NUMERICAL EXAMPLE

To illustrate the heuristic model developed above, we present a numerical example having the same parameters as one of the problems solved by Giri et al [3]:

$D(t) = 20 + 2t$ is the demand rate at any time *t,*

C_I = Inventory holding cost per unit per year = 5

C_R = Replenishment cost per order = 90

C_D = Cost of a deteriorated unit. = 0.5

C_S = Shortage cost per unit. = 1.5

θ = Fraction of On-hand Inventory that deteriorate = 0.01

A value of α = 0.8 was used in this example and using the procedure outlined above we obtained the value of the optimal time for the first replenishment to take place (T_1*) as well as the minimized total inventory cost for the first replenishment duration (TC_1*).

We equally generate the values of T_i* and TC_i* for the next five replenishments as shown below:

1st replenishment: T_1* = 1.5513, TC_1* = 119.105
2nd replenishment: T_2* =1.4621, TC_2* = 127.481
3rd replenishment: T_3* = 1.3900, TC_3* = 135.006
4th replenishment: T_4* = 1.3304, TC_4* = 141.874
5th replenishment: T_5* = 1.2796, TC_5* = 148.215
6th replenishment: T_6*= 1.2360, TC_6* = 154.662

VI. SENSITIVITY ANALYSIS

To study the effect of changes in parameters on the replenishment duration, T_i, and total inventory cost, *TC*, a sensitivity analysis was performed using the numerical example. The value of each parameter was varied between the range ±100% while others were kept constant and the corresponding values of T_i and *TC* were determined. Table 1 to Table 8 below shows the percentage change in the values of the parameters and the associated changes in the values of T_i and *TC*.

Table 1:
Sensitivity analysis with respect to θ

% Change in θ	% Change in T_i	% Change in *TC*
-90.00%	0.73%	-0.37%
-70.00%	0.57%	-0.29%
-50.00%	0.41%	-0.21%
-30.00%	0.24%	-0.12%
-10.00%	0.08%	-0.04%
0.00%	0.00%	0.00%
10.00%	-0.08%	0.04%
30.00%	-0.23%	0.12%
50.00%	-0.39%	0.20%
70.00%	-0.55%	0.29%
90.00%	-0.70%	0.37%

Table 2:
Sensitivity analysis with respect to *b*

% Change in *b*	% Change in T_i	% Change in *TC*
-100.00%	7.11%	-4.40%
-80.00%	5.47%	-3.47%
-60.00%	3.96%	-2.56%
-40.00%	2.55%	-1.69%
-20.00%	1.24%	-0.83%
0.00%	0.00%	0.00%
20.00%	-1.16%	0.81%
40.00%	-2.26%	1.60%
60.00%	-3.29%	2.37%
80.00%	-4.29%	3.13%

Table 3:
Sensitivity analysis with respect to *a*

% Change in a	% Change in T_i	% Change in *TC*
-100.00%	93.47%	-60.31%
-80.00%	59.60%	-44.93%
-60.00%	36.89%	-31.79%
-40.00%	20.91%	-20.17%
-20.00%	9.10%	-9.66%
0.00%	0.00%	0.00%
20.00%	-7.25%	9.00%
40.00%	-13.17%	17.47%
60.00%	-18.13%	25.48%
80.00%	-22.34%	33.13%

Table 4:
Sensitivity analysis with respect to α

% Change in α	% Change in T_i	% Change in *TC*
-87.50%	7649.04%	171.86%
-75.00%	482.45%	-43.34%
-62.50%	202.64%	-41.29%
-50.00%	112.09%	-34.17%
-37.50%	64.84%	-26.04%
-25.00%	35.31%	-17.52%
-12.50%	14.94%	-8.82%
0.00%	0.00%	0.00%
12.50%	-11.45%	8.89%
25.00%	-20.52%	17.82%

Table 5:
Sensitivity analysis with respect to C_S

% Change in C_S	*% Change in T_i*	*% Change in TC*
-83%	-0.55%	-4.79%
-67%	-0.44%	-3.83%
-50%	-0.33%	-2.87%
-33%	-0.22%	-1.92%
-17%	-0.11%	-0.96%
0%	0.00%	0.00%
17%	0.12%	0.96%
33%	0.23%	1.92%
50%	0.34%	2.88%
67%	0.45%	3.84%

Table 6:
Sensitivity analysis with respect to C_I

% Change in C_I	% Change in T_i	% Change in TC
-80%	110.77%	-49.27%
-60%	53.19%	-32.87%
-40%	26.95%	-20.19%
-20%	11.03%	-9.46%
0%	0.00%	0.00%
20%	-8.19%	8.56%
40%	-14.64%	16.38%
60%	-19.86%	23.78%
80%	-24.18%	30.68%
100%	-27.92%	37.20%

Table 7:
Sensitivity analysis with respect to C_D

% Change in C_D	% Change in T_i	% Change in TC
-80%	0.04%	-0.04%
-60%	0.03%	-0.03%
-40%	0.02%	-0.02%
-20%	0.01%	-0.01%
0%	0.00%	0.00%
20%	-0.01%	0.01%
40%	-0.01%	0.02%
60%	-0.03%	0.03%
80%	-0.03%	0.04%
100%	-0.05%	0.04%

Table 8:
Sensitivity analysis with respect to C_R

% Change in C_R	% Change in T_i	% Change in TC
-66.67%	-40.32%	-41.08%
-55.56%	-31.59%	-32.51%
-44.44%	-23.99%	-24.91%
-33.33%	-17.20%	-18.00%
-22.22%	-11.02%	-11.61%
-11.11%	-5.31%	-5.63%
0.00%	0.00%	0.00%
11.11%	5.00%	5.35%
22.22%	9.71%	10.46%
33.33%	14.18%	15.36%
44.44%	18.45%	20.07%
55.56%	22.53%	24.62%
66.67%	26.45%	29.02%

From the tables it can be observed that:

1. *TC* is directly proportional to all the parameters as it value increases or decreases with increase or decrease in their values.
2. The total inventory cost (*TC*) obtained by the model is very sensitive to changes in the values of α, a, C_I and C_R (±10% and above).
3. The level of sensitivity of the total inventory cost to C_D, C_S, b and θ is very low (±5% downward).
4. The model is most sensitive to the fraction of time for which there are no shortages (α). *TC* increases and tends towards infinity as α tends towards zero, showing that α is a very critical factor whose range must be carefully determined.
5. The total inventory cost (*TC*) is least sensitive to the cost of deteriorating items (C_D).

The following can be deduced from these observations:

a. Lots of care should be taken in estimating the values of α, a, C_I, and C_R.
b. More research efforts need to be focused on the accuracy in determination of the extent of shortages in a deteriorating inventory system than what is presently obtained.
c. In inventory system with linear trend in demand the initial demand before the commencement of inventory (represented by "*a*" in this study) is a crucial factor in determining the total inventory cost.

VII. CONCLUSIONS

The heuristic model presented above is an improvement of the model presented by Kim [4] as it covers shortages and full back ordering. It is also different from other models in that it assumes no specific time horizon in its formulation and also relaxes the equal replenishment, constant demand and no shortage assumptions in earlier models. Thus it is an improvement on the earlier works of Bahari-Kashani [8], Chung and Ting [6] and Kim [4].

The fact that the model does not depend on any time horizon implies that it can be easily adjusted to changes in value of parameters as they occur in real life. This is more practical than assuming that the values will be the same throughout a particular time horizon.

VIII. RECOMMENDATIONS

Future studies should incorporate a general time-varying demand pattern and time value of money into the model for more realistic results.

Relaxing the specific time-horizon restrictions in inventory models is recommended to cater for the rapid changes that occur nowadays due to new developments.

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