

Basic relations between Social Accounts and Walras' Law with some economic identities

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1 Introduction

This paper aims to represent basic relations and some variations of economic identities which are described in the simplest monetary transaction, social aggregate transaction and international transaction. These are the fundamental of a social account rule and a general equilibrium economic system. It will be clear that all dimension in transactions have the same essential relation to a accounting method. It's just the *Walras' Law*. *Stock* and *Flow*, which are separated in National Income, are joined and treated together in this paper.¹ Because it must be easy to understand a reality of economic transaction. This paper is a trial which gives a macro economic theory and one of micro or accounting foundation.

Firstly, we describe the Accounting method in the simplest monetary economy and it is same as the *Double Entry Book-keeping* style. This style is the most important fact in this paper. This is maintained in describing social account and an international trading.

Secondly, it is represented that this account rule is enlargeable to transaction among countries. That is a normal social account or national account. SNA is well known as a standard social account and this is very similar to Double Entry Book-keeping style¹.

Thirdly, we'll treat a social account system including financial institutions. This treatment is a little complicated because of the existence of securities, but the Accounting method or Walras Law will be unchanged.

Finally, we adapt the account to international transaction and describe

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¹However, *Imputation*, such as imputed rent and imputed interest, is not treated in this paper for simplification.

the Walras Law's formula on the whole world model. It'll be shown how economic identities in the Two Countries model of U.S.A and Japan change maintaining the same Accounting method.

2 Budget Constraint and Double Entry Book-keeping

2.1 The simplest monetary economy

In this subsection we assume the simplest monetary economy. Now endowments which any economic agent i have initially are \bar{Z}_i and \bar{M}_i . \bar{Z}_i is goods and \bar{M}_i is money. And we assume that a kind of goods is only one and goods Z are production goods and consumption goods. It transacts through the period and has the resulted assets Z and M at the end of a term. Then formulative balance equation of goods will be as follows;

$$\bar{Z}_i + b_i = s_i + Z_i. \quad (1)$$

B_i is buying volume of agent i and s_i is selling volume. This formula just describes a quantitative relation of goods transaction.

On the other hand, the balance formula of money is as follows ²;

$$\bar{M}_i + ps_i = pb_i + M_i. \quad (2)$$

p is a price. This price is decided in the market now.

Then we can transform formula (2) by substituting formula (1), agent i 's balance formula in this time is as follows;

$$\bar{M}_i + p\bar{Z}_i = pZ_i + M_i. \quad (3)$$

This is a balance of income and expenditure formula of pure exchange economy of a single good and one period. From this formula (3), profit formula π_i of agent i is as follows ³;

$$\pi_i := (M_i - \bar{M}_i) = p(\bar{Z}_i - Z_i). \quad (4)$$

The above rule mentioned means that income and expenditure relation of the simplest transaction of agent i , but profit is defined as only change of assets in the term.

²This equation represents a similar meaning of income and expenditure. That means an account described a quantitative balance of money.

³Actually, profit means just changing of choice of assets here. It is written for a comparison with later formula.

2.2 Production and Transaction

In this subsection let us assume a more general model than above, which is the case when agent i will buy raw materials and will produce goods. Quantitative balance formula of raw materials is as follows;

$$\overline{X}_i + b_i = x_i + X_i \tag{5}$$

\overline{X}_i is endowments of raw materials at the beginning of a term, b_i is amounts of buying, x_i is input and X_i is inventories of raw at the end of a term.

Next quantitative formula of product is shown as follows;

$$\overline{Z}_i + y_i = s_i + Z_i \tag{6}$$

\overline{Z}_i are inventories at the beginning of a term, y_i are amounts of the products, s_i are amounts of sales and Z_i are final inventories.

Then a balance of income and expenditure of agent i is shown as follows;

$$\overline{M}_i + ps_i = qb_i + M_i \tag{7}$$

This is formally similar to formula (2). p is the selling price of a product and q is the buying price of raw materials. And we assume again that these prices are already decided in the markets. Considered formula (5) and (6), balance of income and expenditure formula follows above;

$$(\overline{M}_i + q\overline{X}_i + p\overline{Z}_i) + py_i = qx_i + (qX_i + pZ_i + M_i). \tag{8}$$

Equation (8) represents that a sum of initial assets and amounts of product on the left hand side is equal to a sum of input and final assets on the right hand side ⁴. Transformed formula (8) to an excess demand formula, it follows ⁵

$$(M_i - \overline{M}_i) + q[(X_i + x_i) - \overline{X}_i] + p[Z_i - (\overline{Z}_i + y_i)] \equiv 0. \tag{8'}$$

The first term on the left hand side is an excess demand of money, the second term is an excess demand of raw materials and the third term is an excess demand of product ⁶.

⁴As we will show later, this formula is equivalent to *Trial Balance Formula* of Accounting or *Double Entry Book-keeping*.

⁵Each term in this equation(8) can be interpreted as an excess demand of each good or asset.

⁶As we'll show in detail later, a social sum of all agents is *General Equilibrium System* and then derived formula (8') is the *Walras' Law*. The meaning of *General Equilibrium* we said here is a system like each market depends respectively.

Then profit π_i of agent i is as follows ⁷;

$$\pi_i := (pZ_i + qX_i + M_i) - (p\bar{Z}_i + q\bar{X}_i + \bar{M}_i) = py_i - qx_i. \quad (9)$$

The first formula gets profit from a difference between initial assets and final assets. The second formula gets it from an amounts of product minus input cost. Equation (9) shows that the sum of each formulas is equal.

2.3 Generalizing Production Factor

In this subsection we'll assume a production more concretely. Now assume that agent i buys not only raw materials but another factors ⁸ and employs labor forces N . Then quantitative balance is shown as follows;

$$\bar{X}_i + b_i = x_i + X_i \quad (5)$$

$$\bar{K}_i + I_i = d_i + K_i \quad (10)$$

$$\bar{N}_i = N_i \quad (11)$$

$$\bar{Z}_i + y_i = s_i + Z_i. \quad (6)$$

Equation (10) is a quantitative balance of production factor. \bar{K}_i are endowments of production equipments (or fixed capital, production factor), I_i are amounts of investments ⁹, d_i are amounts of depreciation (or *Consumption of Fixed Capital*, scraped equipments) and K_i are production equipments at the end of a term ¹⁰.

And formula (11) is a balance formula of labor forces who are constantly employed. The other formulas are the same as above.

Then balance of income and expenditure formula is as follows;

$$\bar{M}_i + ps_i = qb_i + rI_i + w\bar{N}_i + M_i. \quad (12)$$

But r is buying price of fixed capital and w is wage rate.

Transformed formula (12) from formula (5), (10), (11) and (6),

⁷As mentioned above, profit π_i is defined as an amount increased of total assets, but it finally results the differences of an amount production and raw materials. The similar definition is gotten by Hicks. See Hicks[2]Chap14,15.The profit is equivalent to value added, because there are no labor, land and depreciation.

⁸For example, these are equipments, lands, buildings and so on. Here, these are treated as investment in the lump.

⁹This is gross investment which involves depreciation cost.

¹⁰We assume that this agent uses employee just equal to planned volume of employee. We omit balance of labor as household here, but show it later.

$$\begin{aligned} & \overline{M}_i + p\overline{Z}_i + q\overline{X}_i + r\overline{K}_i + py_i \\ = & qx_i + rd_i + wN_i + pZ_i + qX_i + rK_i + M_i. \end{aligned} \quad (13)$$

Terms from the first to the fourth on the left hand side are corresponding to endowments of assets and the fifth term means amounts of product. Terms from the first to the third on the right hand side are amounts of input and terms from the fourth to the seventh represent final assets.

Transformed this to an excess demand formula, it follows;

$$\begin{aligned} & [(pZ_i + wN_i) - p(y_i + \overline{Z}_i)] + q[(x_i + X_i) - \overline{X}_i] \\ & + r[(d_i + K_i) - \overline{K}_i] + (M_i - \overline{M}_i) \equiv 0. \end{aligned} \quad (14)$$

Formula (14) is an accounting rule of agent i which is identically resulted¹¹. And from formula (12) profit π_i of agent i follows¹²;

$$\begin{aligned} \pi_i := & pZ_i + qX_i + rK_i + M_i - (p\overline{Z}_i + q\overline{X}_i + r\overline{K}_i + \overline{M}_i) \\ = & py_i - (qx_i + rd_i + wN_i). \end{aligned} \quad (15)$$

As mentioned before, it is shown that profit is acquired by deducting endowments of assets from final assets or input costs from amounts of product¹³.

2.4 Double Entry Book-keeping and General Equilibrium

Now it is already made clear above, that all balance of income and expenditure equations are corresponding with accounting of firms, which is *Double Entry Book-keeping*¹⁴. In order to show this clearly, we assume the previous model (5)(10)(11)(6).

We assume that there are liabilities of bonds and stock, and loans by buying securities¹⁵. Then a balance of income and expenditure formula of agent i is enlarged as follows¹⁶;

$$\overline{M}_i + B_{si} + ps_i = qb_i + rI_i + w\overline{N}_i + B_{di} + M_i \quad (16)$$

¹¹We assume that wage part is involved in a demand of product, but this assumption is not always absolute. Equation (14) just shows a increase or decrease of costs of raw materials and production factors. This equation means that an amounts of product involved wage part and other term identically remains zero.

¹²Generally, this is called *Closing Books* in Accounting.

¹³This profit π_i is just company's earnings.

¹⁴See Numata[7] which is a basic and famous literature of accounting.

¹⁵But we assume that there are not any rate of yield of bonds for simplification.

¹⁶It's easy to enlarge a model including inventories. But we omit it because the essence of this story would not change.

This formula is nothing but *Trial Balance Formula* on accounting of firms. Profits π_i of a firm from formula (16) are as follows;

$$\pi_i := (M_i + B_{di}) - (\overline{M}_i + B_{si}). \quad (17)$$

Then derived a relation like a *balance sheet* of firm from formula (17), it follows;

$$\overline{M}_i + B_{si} + \pi_i = B_{di} + M_i. \quad (18)$$

π_i shows *Net Profit*. The first and second terms on the left hand side are a *Liabilities* term (*Creditor*) and the right hand side is a *Assets* term (*Debtor*).

Moreover, the *Profit and Loss* formula as follows;

$$ps_i = qb_i + rI_i + w\overline{N}_i + \pi_i. \quad (19)$$

The left hand side shows amounts of sale, terms from the first to the third on the right hand side are cost of sales and the final term shows Net Profit ¹⁷.

Then it is clear that each formula means balance sheet table and, profit and loss table on accounting.

After all, balance of income and expenditure formula of each agent above and Trial Balance Formula of each firm (or production agent) are the same with formula (12) or (16).

A difference between balance formula of agents and accounting of firms is only a formal one where the former deducts a number of variables by substituting quantitative balance to income and expenditure and, on the other hand, the latter remains amounts of sales and buying terms. Of course, the formal difference is important when we interpret a economic meanings. For example, it is easy for balance formula of agents to interpret a relation of supply and demand, and budget constraint, while it is appropriate for accounting to interpret balance of income and expenditure, and a relation on Double Entry Book-keeping.

Now transformed formula (16) considering (5),(10),(11) and (6), it follows;

$$\begin{aligned} & \overline{M}_i + p\overline{Z}_i + q\overline{X}_i + r\overline{K}_i + B_{si} + py_i \\ & = qx_i + rd_i + wN_i + B_{di} + pZ_i + qX_i + rK_i + M_i. \end{aligned} \quad (20)$$

¹⁷In the Profit and Loss of financial statements, profit π are represented amounts of a difference between sales and input costs directly. This representation is formally different from the right hand side of above formula (15), which is usually represented in economic theory.

Terms from the first to the fourth on the left hand side represent endowments of assets, the fifth term are liabilities and the sixth term are amounts of product. Terms from the first to the third on the right side represent amounts of input, the fourth term means loan and terms from the fifth to seventh mean assets at the end of a term.

Transformed this equation to an excess demand, identity included bonds follows;

$$\begin{aligned} & [(pZ_i + wN_i) - p(y_i + \bar{Z}_i)] + q[(x_i + X_i) - \bar{X}_i] \\ & + r[(d_i + K_i) - \bar{K}_i] + (B_{di} - B_{si}) + (M_i - \bar{M}_i) \equiv 0. \end{aligned} \quad (21)$$

Term of $(B_{di} - B_{si})$ on the left hand side shows an excess demand of securities.

2.5 Household's balance

We have until here omitted a balance of income and expenditure of labor forces or household. Because we've been treating a simple economic model which only production agents exchange goods each other. However, as it became clear on generalizing production factor above, it happens that some agents employ labor forces paying wage in a production society over a level of scale. Because labor has usually no production methods, he doesn't trade raw materials and production factor. So he gets wage by offering labor force itself and lives by buying consumption goods.

Therefore, quantitative formulas of labor j are as follows;

$$\bar{c}_j = c_j. \quad (22)$$

$$\bar{N}_j = N_j. \quad (23)$$

We assume that household plans amounts of consumption \bar{c}_j and labor supply \bar{N}_j and does the same amounts¹⁸.

Then considering these formulas, a balance of income and expenditure of labor j is simply follows;

$$\bar{M}_j + w\bar{N}_j = p\bar{c}_j + M_j. \quad (24)$$

c_j is volume of consumption.

If we assume that household borrows some money and purchases some securities, this formula is generalized as follows;

¹⁸Of course, we can assume endowments and inventories of consumption goods, and leisure part and real labor part of a day. But as it's not essential, so we omit that assumption.

$$\overline{M}_j + B_{sj} + wN_j = pc_j + B_{dj} + M_j. \quad (25)$$

B_{sj} are amounts of liabilities and B_{dj} are amounts of purchasing securities.

And then considering profit of labor, that is known as *saving*. Represented it by π_j , it follows;

$$\pi_j := (M_j - \overline{M}_j) + (B_{dj} - B_{sj}) = wN_j - pc_j. \quad (26)$$

This saving part π_j is known well as investment part loaned in economic circulation.

This balance formula of labor will be needed when we consider a social account framework as it is shown below.

3 Social Accounts and Walras' Law

As we confirmed in the previous section, the description of transaction by production agent i and the accounting of firms are equivalent. In this section, we show that *the Framework of Social Accounts* is derived by summing up the accounting socially¹⁹.

Now, summed up socially above (5), (10), (11) and (6), it is as follows;

$$\overline{X} + b = x + X \quad (27)$$

$$\overline{K} + I = d + K \quad (28)$$

$$\overline{N}_d = N_d \quad (29)$$

$$\overline{Z} + y = s + Z. \quad (30)$$

For example, \overline{X} is $\sum_{i=1}^n \overline{X}_i$ and all variables without subscript mean summing up socially. And \overline{N}_d shows total demand of labor by all production agents²⁰.

And a summed balance of income and expenditure of production agent i is as follows;

$$\overline{M}_p + B_{sp} + ps = qb + \tau I + wN_d + B_{dp} + M_p. \quad (31)$$

Subscript p means summing on all production agents.

Next, a summed balance of income and expenditure of labor j is as follows;

¹⁹We referred Okishio[10][11] on this point

²⁰We omit there is no unemployment for simplification here.

$$\overline{M}_h + B_{sh} + wN_s = pc + B_{dh} + M_h. \tag{32}$$

Subscript h means summing on the whole labor. And N_s shows total supply of labor forces by the whole labor.

Therefore, We get social balance of income and expenditure by summing up formula (31) and (32).

$$\overline{M} + B_s + ps + wN_s = pc + qb + rI + wN_d + B_d + M \tag{33}$$

Here, $\overline{M} = \overline{M}_p + \overline{M}_h$, $B_s = B_{ps} + B_{hs}$, $B_d = B_{pd} + B_{hd}$, $M = M_p + M_h$. Equation (33) is a balance of transaction aggregated on the whole national economy ²¹.

Transformed formula (33) by formula (27) and (30), it follows;

$$\begin{aligned} &\overline{M} + B_s + p\overline{Z} + q\overline{X} + r\overline{K} + py + wN_s \\ &= pc + qx + rd + rK + wN_d + qX + pZ + B_d + M. \end{aligned} \tag{34}$$

Formula (34) is equivalent to Trial Balance Formula of a whole country and it is a balance of all domestic transactions by money. Transformed this formula, it follows;

$$\begin{aligned} &p[Z + c - (y + \overline{Z})] + q(x + X - \overline{X}) + r(d + K - \overline{K}) \\ &+ w(N_d - N_s) + (B_d - B_s) + (M - \overline{M}) \equiv 0. \end{aligned} \tag{35}$$

Equation (35) is equivalent to well-known *Walras' Law* of economic theory. Square brackets of the first term on the left hand side mean an excess demand on a consumption goods market, round brackets of the second term show an excess demand of an intermediate goods market. And the third, fourth, fifth and final terms on the left hand side show an excess of demand of fixed capital, labor force, securities and money market respectively. Sum of these six markets transaction must identically be equal to zero.

Therefore, this *Walras' Law* is always realized in any economy. If when prices are hold constant in all period, the prices are called *equilibrium prices*.

Formula (34) or (35) is usually an identity on economic theory, and it is

²¹We simplify an assumption integrated to one kind of good and raw materials, but it is easy to enlarge it to various kind of goods. Then formula follows;

$$\overline{M} + B_s + \sum_{i=1}^n \sum_{h=1}^o p_h s_{hi} + wN_s = \sum_{j=1}^m \sum_{h=1}^o p_h c_{hj} + \sum_{i=1}^n \sum_{k=1}^l q_k b_{ki} + wN_d + B_d + M. \tag{a}$$

Or used a vector formulation,

$$\overline{M} + B_s + \mathbf{p}'\mathbf{s} + wN_s = \mathbf{p}'\mathbf{c} + \mathbf{q}'\mathbf{b} + wN_d + B_d + M. \tag{b}$$

However, since increasing kinds of goods doesn't influence our conclusion, we consider only the simplest case here.

an fundamental framework on *the system of national account*, too.

Now, the return on a nation state level is usually called *Net Worth*²². Derived Net Worth Π from formula (34), as it follows;

$$\begin{aligned} \Pi &= (M + B_d + qX + rK + pZ) - (\overline{M} + B_s + q\overline{X} + r\overline{K} + p\overline{Z}) \\ &= py - qx - rd - [pc - w(N_s - N_d)]. \end{aligned} \quad (36)$$

We can interpret $[pc - w(N_s - N_d)]$ as sum of social consumption excluded unemployment. Therefore, we can get a return equivalent to Net Worth, as intermediate production and consumption are excluded from total production. Moreover, the left hand side of the formula shows that it is derived from increase or decrease of endowments and final assets, as well.

Then the national balance sheet of SNA is followed by formula(36);

$$M + B_d + q(X - \overline{X}) + r(K - \overline{K}) + p(Z - \overline{Z}) = B_s + \overline{M} + \Pi. \quad (37)$$

The first term of the left hand side is monetary asset(except stock), the second term is stock as assets, the third, fourth and fifth terms show Tangible Fixed Assets and Inventory. The first term of the right hand side are liabilities, the second is stock as Stockholder's Equity and the final is Net Worth²³.

Moreover, describing a relation with Input-Output table from formula (36), it follows;

$$py = qx + rd + [pc - w(N_s - N_d)] + \Pi. \quad (38)$$

The left hand side shows total product, the first term on the right hand side is raw materials and the second and third is Value Added (or an amount of *Final Demand*). Matched this to a fundamental model of Input-Output in money terms $\sum_{j=1}^n X_j = \sum_{j=1}^n \sum_{i=1}^n a_{ij} X_j + \sum_{j=1}^n V_j$ (or $\sum_{i=1}^n X_i = \sum_{i=1}^n \sum_{j=1}^n a_{ij} X_j + \sum_{i=1}^n F_i$);

²²New value yielded on domestic economy in a term is usually called *Value Added*. Since this Value Added usually includes consumption part of economic circulation, a part of net return carried forward to next term is Net Worth exactly and the concept is better than Value Added. Actually the national balance sheet in SNA is made such as it is. Therefore we define net worth as a return of the country. However, we'll tell later about Value Added including a consumption part with *GNP*, it is usually used as a kind of return in the country.

See Saitoh[12], Miyazawa[6], EPA[3] as to a basic framework of National Income Statistics. About a basic difference between Social Accounts and Double Entry Book-keeping, see Kurabayashi[4] and Kurabayashi and Sakuma[5].

²³Defined Net Worth on SNA exactly, it follows;

$$\begin{aligned} \text{NetWorth} &= \text{National Income} - \text{consumption} + \text{Reconciliation} \\ &= \text{Assets} - (\text{liabilities} + \text{Corporate Share}) \\ &= \text{Saving on Capital Finance Account} + \text{Net Capital Transfers from the Rest of the World} \\ &\quad + \text{Statistical Discrepancies} + \text{Net Worth on Reconciliation Account.} \quad (c) \end{aligned}$$

$$\sum_{j=1}^n X_j = py \tag{39}$$

$$\sum_{j=1}^n \sum_{i=1}^n a_{ij} X_j = qx \tag{40}$$

$$\sum_{j=1}^n V_j = rd + [pc - w(N_s - N_d)] + \Pi \tag{41}$$

Therefore, formula (38) is equivalent to Input-Output model directly.

Showing only Value Added part, we get a relation with the well-known National Income. It follows;

$$Y = W + R = C + S. \tag{42}$$

However, $Y = py - qx$, $W + R = rd + [pc - w(N_s - N_d)] + \Pi$, C is sum of consumption (that is pc) and S is sum of savings²⁴.

Moreover, from a view of economic circulation, formula (38) is as follows;

$$\begin{aligned} py &= qx + rd + p\bar{c} + \Pi \\ &= qx + rd + p\bar{c} + S \\ &= qx + rd + p\bar{c} + rI = qx + C + I' \\ &= qx + Y. \end{aligned} \tag{43}$$

But $p\bar{c} = [pc - w(N_s - N_d)]$, and I' is a sum of gross investment ($I' = rd + rI$). Then Y of the right hand side of formula (43) is Value Added part on National Income Statistics and it's just GNP. This formula is equivalent to the equivalence of three or four phases of National Income.

And from a definition of GNP, it follows;

$$Y = C + I' = n + rd + t. \tag{44}$$

t is indirect tax and n is National Income. Then this is an well-known formula describing that National Income is gotten by excluding depreciation cost and indirect tax from GNP.

As mentioned above, it is clear that the system of national account as a social account framework is identified with Walras' Law in a general equilibrium system.

²⁴Here, Π includes companies' consumption except household exactly, but we omit this here.

4 Social Accounts with Financial Institutions and General Government

We treat a social account system including financial institutions and general government here. That means that there is not only liabilities by bonds and stock transaction, but also deposit as indirect financing and tax. These financial factors are all treated as assets here. However, we omit an existence of central bank and assume money supply is constant.

Firstly, quantitative relation of company i is enlarged from subsection 2.3.

$$\overline{X}_i + b_i = x_i + X_i \quad (5)$$

$$\overline{K}_i + I_i = d_i + K_i \quad (10)$$

$$\overline{N}_{di} = N_{di} \quad (11)$$

$$\overline{Z}_i + y_i = s_i + Z_i. \quad (6)$$

$$\overline{D}_i + \Delta D_i = D_i \quad (45)$$

$$\overline{B}_{di} + \Delta B_{di} = B_{di} \quad (46)$$

$$\overline{B}_{si} + \Delta B_{si} = B_{si} \quad (47)$$

$$\overline{S}_{di} + \Delta S_{di} = S_{di} \quad (48)$$

$$\overline{S}_{si} + \Delta S_{si} = S_{si} \quad (49)$$

D_i is deposit of company i , B_{di} is demand for bonds, B_{si} is supply or sale for bonds. S_{di} is demand for stock and S_{si} is supply or sale for stock. $\overline{}$ shows endowments.

Then, considering above formulas, balance formula of income and expenditure of company i is as follows;

$$\begin{aligned} \overline{M}_i + (1 + i_B)\overline{B}_i + (1 + v)\overline{S}_i + (1 + i)\overline{D}_i + p(\overline{Z}_i + y_i) + q\overline{X}_i + r\overline{K}_i + U_i \\ = pZ_i + q(x_i + X_i) + r(d_i + K_i) + wN_{di} + S_i + B_i + D_i + T_i + M_i. \end{aligned} \quad (50)$$

$B_i \equiv B_{di} - B_{si}$ is net demand for on bonds. $S_i \equiv S_{di} - S_{si}$ is net demand for stock. i_B is a rate of yield on bonds, assumed as a constant level. i is a interest rate of deposit, which is also constant. v is dividends per stock, which we assumed as to be at the same level. U_i represents subsidies from government. T_i is a corporation tax and equivalent to $t_p\pi_i$ ²⁵.

And profit π_i is also as follows;

²⁵ t_p is a corporatin tax rate and it is assumed constant for all companies here.

$$\begin{aligned} \pi_i &:= (pZ_i + qX_i + rK_i + B_i + S_i + D_i + M_i) \\ &\quad - (p\bar{Z}_i + q\bar{X}_i + r\bar{K}_i + \bar{B}_i + \bar{S}_i + \bar{D}_i + \bar{M}_i) \\ &= (py_i + i_B \bar{B}_i + v\bar{S}_i + i\bar{D}_i + U_i) - (qx_i + rd_i + wN_{di} + T_i). \end{aligned} \quad (51)$$

Second, quantitative relation of household j is also enlarged from subsection 2.5.

$$\bar{c}_j = c_j. \quad (22)$$

$$\bar{N}_{sj} = N_{sj}. \quad (23)$$

$$\bar{D}_j + \Delta D_j = D_j \quad (52)$$

$$\bar{B}_{dj} + \Delta B_{dj} = B_{dj} \quad (53)$$

$$\bar{B}_{sj} + \Delta B_{sj} = B_{sj} \quad (54)$$

$$\bar{S}_{dj} + \Delta S_{dj} = S_{dj} \quad (55)$$

$$\bar{S}_{sj} + \Delta S_{sj} = S_{sj} \quad (56)$$

Then a balance of income and expenditure of household j is as follows;

$$\begin{aligned} \bar{M}_j + (1 + i_B)\bar{B}_j + (1 + v)\bar{S}_j + (1 + i)\bar{D}_j + wN_{sj} + U_j \\ = pc_j + S_j + B_j + D_j + M_j + T_j. \end{aligned} \quad (57)$$

Notation meaning is the same as company's formula except subscript j . U_j represents welfare benefit from government. T_j is an income tax and equivalent with $t_j wN_{sj}$ ²⁶.

Saving part π_j is derived from this formula, it is as follows;

$$\begin{aligned} \pi_j &:= (S_j + B_j + D_j + M_j) - (\bar{M}_j + \bar{B}_j + \bar{S}_j + \bar{D}_j) \\ &= (wN_{sj} + i_B \bar{B}_j + v\bar{S}_j + i\bar{D}_j + U_j) - (pc_j + T_j). \end{aligned} \quad (58)$$

And third, quantitative relation of financial institutions b can be described as follows;

$$\bar{N}_{db} = N_{db}. \quad (59)$$

$$\bar{D}_b + \Delta D_b = D_b \quad (60)$$

$$\bar{B}_{db} + \Delta B_{db} = B_{db} \quad (61)$$

$$\bar{B}_{sb} + \Delta B_{sb} = B_{sb} \quad (62)$$

$$\bar{S}_{db} + \Delta S_{db} = S_{db} \quad (63)$$

$$\bar{S}_{sb} + \Delta S_{sb} = S_{sb} \quad (64)$$

²⁶We assume that t_j is constant for all household to simplify argument.

Then a balance of income and expenditure of financial institutions b is as follows;

$$\begin{aligned} \overline{M}_b + (1 + i_B)\overline{B}_b + (1 + v)\overline{S}_b + D_b + f_b + U_b \\ = wN_{db} + S_b + B_b + (1 + i)\overline{D}_b + T_b + M_b. \end{aligned} \quad (65)$$

f_b shows service fee of financial institutions. U_b represents subsidies from government. T_b is a corporation tax and equivalent to $t_p\pi_b$ ²⁷

Then, π_b is as follows;

$$\begin{aligned} \pi_b := (S_b + B_b + M_b - D_b) - [\overline{M}_b + \overline{B}_b + (1 + v)\overline{S}_b - (1 + i)\overline{D}_b] \\ = (f_b + i_B\overline{B}_j + v\overline{S}_j + i\overline{D}_j + U_b) - (wN_{db} + T_b). \end{aligned} \quad (66)$$

Since financial institutions are supplier of deposit, signs on it's term are negative.

In the fourth, quantitative relation of general government g can be described as follows;

$$\overline{K}_g + I_g = d_g + K_g \quad (67)$$

$$\overline{U}_g = U_g \quad (68)$$

$$\overline{N}_{dg} = N_g \quad (69)$$

$$\overline{D}_g + \Delta D_g = D_g \quad (70)$$

$$\overline{B}_{dg} + \Delta B_{dg} = B_{dg} \quad (71)$$

$$\overline{B}_{sg} + \Delta B_{sg} = B_{sg} \quad (72)$$

We'll omit stock transaction and consumption by general government.

U_g represents all subsidies from government and it is $U_g = \sum_{i=1}^n U_i + \sum_{j=1}^m U_j + \sum_{b=1}^k U_b$. Then a balance of income and expenditure of general government g is followed;

$$\begin{aligned} \overline{M}_b + (1 + i_B)\overline{B}_g + (1 + i)\overline{D}_g + T \\ = wN_g + B_g + D_g + G + M_g. \end{aligned} \quad (73)$$

T shows the revenue and $T = \sum_{i=1}^n T_i + \sum_{j=1}^m T_j + \sum_{b=1}^k T_b$. G is the government expenditure and $G = rI_g + U_g$.

²⁷ t_p is the same corporation tax rate above.

Then, government surplus π_g is as follows;

$$\begin{aligned} \pi_g &:= (B_g + D_g + M_g) - (\overline{M}_g + \overline{B}_g + \overline{D}_g) \\ &= i_B \overline{B}_g + i \overline{D}_g + (T - G) - wN_g. \end{aligned} \quad (74)$$

Now, summed up above formulas (50), (57) and (65) and (73) aggregated in terms of each agent, balance of the whole society is as follows²⁸;

$$\overline{M} + (1+i)(\overline{D}_d - \overline{D}_s) + (1+i_B)\overline{B} + (1+v)\overline{S} + p(\overline{Z} + y) \quad (75)$$

$$\begin{aligned} &+ q\overline{X} + r\overline{K} + wN_s + f + U_g \\ &= p(c + Z) + q(x + X) + r(d + K) \end{aligned} \quad (76)$$

$$+ wN_d + S + B + (D_d - D_s) + G + M. \quad (77)$$

Here, $D_d \equiv \sum_{i=1}^n D_i + \sum_{j=1}^m D_j$ and $D_s \equiv \sum_{b=1}^k D_b$.

Therefore, Walras' Law can be described as follows in this case;

$$p[c + Z - (y + \overline{Z})] + q(x + X - \overline{X}) \quad (78)$$

$$\begin{aligned} &+ r(d + K - \overline{K}) + w(N_d - N_s) + (G - U_g) \\ &+ [S - (1+v)\overline{S}] + [B - (1+i_B)\overline{B}] \end{aligned} \quad (79)$$

$$+ [D - (1+i)\overline{D}] + (M - f - \overline{M}) \equiv 0. \quad (80)$$

Here, $D \equiv D_d - D_s$ and it shows net demand for deposit.

And derived Net Worth Π from this formula, it follows;

$$\Pi := (pZ + qX + rK + S + B + D + M) \quad (81)$$

$$- (p\overline{Z} + q\overline{X} + r\overline{K} + \overline{S} + \overline{B} + \overline{D} + \overline{M})$$

$$= (py + f + v\overline{S} + i_B \overline{B} + i \overline{D} + U_g) - qx \quad (82)$$

$$- rd - G - [pc - w(N_s - N_d)]. \quad (83)$$

As mentioned above, these formulas are more complex than formulas in section 3. However, their essential relation are unchanging.

5 International trade and General Equilibrium

Now, we have't treated an international transaction at all until above section. Then we have treated above only one currency in the world. However, foreign currencies and goods are usually bought in trade in the real world. How does above formulas in a general equilibrium system change?

²⁸Tax T is canceled out in both sides.

In this section, we consider an international trade between countries with different currencies and the identities in the world. We'll treat only the Two (small) Countries model Japan and America, since it is very simple.

Let us see how a balance of income and expenditure changes according to different currency. Here, we'll treat three cases which are (1) both countries pay by buyer's currency, (2) both pays by producer's currency, (3) both pays by the same currency (i.e. dollar).

5.1 Balance of Payment by buyer's currency

First, we'll treat a case in which both Japan and America pay by each country's currency when they buy goods. Here, we have next assumption to simplify a story.

1. Both countries buy no raw materials.
2. There is no employment.
3. There are no inventories.
4. We omit domestic transaction in each country.

The point is that there is no production activity in each country and both countries distribute endowments of goods to domestic consumption and export. We describe a balance of income and expenditure using above formula under this assumption. A quantitative relation of goods is as follows;

$$y_J = s_J + e_J \quad (84)$$

$$y_A = s_A + e_A. \quad (85)$$

A sum of domestic production y is equal to a sum of domestic sales s and export e , and there are no inventories. $e_J = m_A, e_A = m_J$. Subscription represents a name of the country, J is Japan and A is America.

Moreover, each country holds foreign currencies respectively on the trade, so it follows;

$$\overline{M_J^J} + \overline{M_J^A} \equiv \overline{M_J} \quad (86)$$

$$\overline{M_A^J} + \overline{M_A^A} \equiv \overline{M_A} \quad (87)$$

Here, M_J^A shows Japanese currency *yen* which America holds and M_A^J shows American currency *dollar* which Japan holds. $\overline{\quad}$ shows endowments.

Now, a balance of yen of Japan is as follows ²⁹;

$$\overline{M_J^J} + EF_s = Ep_A m_J + M_J^J. \quad (88)$$

²⁹We omit domestic transaction since it is cancelled out.

A balance of dollar is as follows;

$$\overline{M}_A^J + \frac{p_J e_J}{E} = F_s + M_A^J. \quad (89)$$

E is an exchange rate for yen, p_A is an import price from America, p_J is an export price from Japan and F_s shows supply for dollar by the exchange.

Substituting formula (89) for (88), it follows;

$$\overline{M}_J^J + E\overline{M}_A^J + p_J e_J = E p_A m_J + E M_A^J + M_J^J. \quad (90)$$

This formula (90) is a fundamental (yen-dominated) form of balance of payment of Japan with an international transaction. The first term on the left hand side means endowments of Japanese yen, the second term means endowments of American dollar and the third term is a sum of exports. The first term on the right hand side is a sum of (yen-dominated) import, the second term means an amount of dollar at the end of a term and the third term is amounts of yen at the end of a term.

Transformed it considering formula (86), it follows;

$$p_J e_J - E p_A m_J \quad (91)$$

$$= E(M_A^J - \overline{M}_A^J) - [(\overline{M}_J - M_J^J) - \overline{M}_J^J] \\ = [EM_A^J - (\overline{M}_J - M_J^J)] - (E\overline{M}_A^J - \overline{M}_A^J). \quad (92)$$

This formula (92) represents *Balance of Payment* of Japan. The left hand side shows *Trade Balance* of Japan and the first term on the final side is an increment of dollar held by Japan. The second term is an increment of yen held by America. These first and second terms show an increment of net external assets of Japan (or an external asset at the end of a term minus the assets at the beginning)³⁰.

Similarly, we can also describe *Balance of Payment* of America. First, a balance of yen of America is as follows;

$$\overline{M}_A^A + E p_A e_A = E F_d + M_A^A. \quad (93)$$

And balance of dollar is as follows;

$$\overline{M}_A^A + F_d = \frac{p_J m_A}{E} + M_A^A. \quad (94)$$

E is an exchange rate for yen, p_J is an import price from Japan, p_A is an export price from America and F_d shows demand for dollar by the exchange.

³⁰This simple example is a case in which there is no Services and Capital Balance, it shows Trade Balance and *Balance of Monetary Movements* as being balancing.

Substituting formula (94) for (93), it follows;

$$\overline{M}_J^A + E\overline{M}_A^A + Ep_Ae_A = p_Jm_A + EM_A^A + M_J^A. \quad (95)$$

Similarly above, formula (95) is a fundamental (yen-dominated) form of a balance of America with an international transaction. The first term on the left hand side means endowments of yen of America, the second term means endowments of dollar and the third term is a sum of (yen-dominated) export. The first term on the left hand side is a sum of import, the second term is an amount of dollar at the end of a term and the third term is an amount of yen at the end of a term.

Transformed this considering equation (87), it follows;

$$\begin{aligned} Ep_Ae_A - p_Jm_A &= (M_J^A - \overline{M}_J^A) - E[(\overline{M}_A - M_A^A) - \overline{M}_A^A] \\ &= [M_J^A - E(\overline{M}_A - M_A^A)] - (\overline{M}_J^A - \overline{M}_A^A). \end{aligned} \quad (96)$$

The left hand side shows Trade Balance of America, the first term on the final side shows an increment of yen held by America, the second term shows an increment of dollar held by Japan. These first and second terms show a net external asset of America, so formula (96) represents Balance of Payments by yen of America.

Now, how is the Walras' Law in a general equilibrium system changed in this simple model? Summing up both sides above (90) and (95), it follows;

$$\begin{aligned} p_J(m_A - e_J) + Ep_A(m_J - e_A) + [(M_J^J + M_J^A) \\ - (\overline{M}_J^J + \overline{M}_J^A)] + E[(M_A^J + M_A^A) - (\overline{M}_A^J + \overline{M}_A^A)] \equiv 0. \end{aligned} \quad (97)$$

Therefore,

$$\begin{aligned} p_J(m_A - e_J) + Ep_A(m_J - e_A) + (M_J^J + M_J^A - \overline{M}_J) \\ + E(M_A^J + M_A^A - \overline{M}_A) \equiv 0. \end{aligned} \quad (98)$$

The first term on the left hand side shows Trade Balance of goods market in Japan (net export, the sign is negative), the second shows Trade Balance in America, the third shows an excess demand of Japanese currency yen, the fourth term shows an excess demand of dollar in a currency market. A sum of these must be identically equal to zero.

Moreover, a transaction on foreign exchange market is also generated in the international transaction. For example, an excess demand on foreign exchange market in Japan is as follows by formula (89) and (93);

$$\begin{aligned} F_s - F_d &= (\overline{M}_A^J + \frac{p_Je_J}{E} - M_A^J) - (\frac{\overline{M}_J^A}{E} + p_Ae_A - \frac{M_J^A}{E}) \\ &= (\frac{p_Je_J}{E} - p_Ae_A) + [(\frac{M_J^A}{E} - \frac{\overline{M}_J^A}{E}) - (M_A^J - \overline{M}_A^J)]. \end{aligned} \quad (99)$$

The first term round brackets on the final side show (dollar-dominated) Trade Balance of Japan. The former round brackets in the second term square brackets show an increment of yen held by America and the latter round brackets show an increment of dollar held by Japan, so the whole square brackets show an increment of net external assets of America. Then formula (99) shows that the trading profit on foreign exchange market is a sum of Trade Balance of Japan and an increment of net external assets of America.

5.2 Balance of Payment by producer's currency

Now, how is the above fundamental relation changed by altering method of payment? To make it clear, let us change method of payment as each country pays by producer's currency respectively. This assumption is opposite to above one.

First, since a quantitative relation of goods and currency is unchanged, formula (84), (85), (86) and (87) remain the same.

A balance of yen of Japan is changed as follows;

$$\overline{M}_J^J + p_J e_J = E F_d + M_J^J. \quad (100)$$

$p_J y_J$ is a sum of Japanese production, and $p_J s_J$ is a sum of domestic transaction and $E F_d$ is (yen-dominated) a sum of buying dollar (selling yen).

A balance of dollar of Japan is also changed as follows;

$$\overline{M}_A^J + F_d = p_A m_J + M_A^J. \quad (101)$$

$p_A m_J$ shows a sum of (dollar-dominated) import from America.

Substituting (101) for (100), but it follows;

$$\overline{M}_J^J + E \overline{M}_A^J + p_J e_J = E p_A m_J + E M_A^J + M_J^J.$$

This formula is the same as (90). Then Trade Balance formula is also as follows;

$$\begin{aligned} p_J e_J - E p_A m_J &= E(M_A^J - \overline{M}_A^J) - [(\overline{M}_J - M_J^J) - \overline{M}_J^A] \\ &= [E M_A^J - (\overline{M}_J - M_J^J)] - (E \overline{M}_A^J - \overline{M}_J^A). \end{aligned} \quad (102)$$

And this formula is the same as (92).

On the other side, a balance of yen of America is as follows;

$$\overline{M}_J^A + E F_s = p_J m_A + M_J^A. \quad (103)$$

A balance of dollar is as follows;

$$\overline{M}_A^A + p_A e_A = F_s + M_A^A. \quad (104)$$

$E F$ shows buying yen (or selling dollar).

Considering (85), it follows from both formulas ;

$$\overline{M}_J^A + E\overline{M}_A^A + Ep_Ae_A = p_Jm_A + EM_A^A + M_J^A.$$

This formula is the same as (95).

Moreover, Walras' Law with international transaction is also as follows;

$$p_J(m_A - e_J) + Ep_A(m_J - e_A) + (M_J^J + M_J^A - \overline{M}_J) + E(M_A^J + M_A^A - \overline{M}_A) \equiv 0. \quad (105)$$

This is the same as (98) here, too.

Therefore, an international trading relation is unchanged by altering method of payment.

5.3 Balance of Payment by dollar

If both trading countries always does international payments by *key currency*, how does the above change? Now, we assume that key currency in the world is dollar and both Japan and America always pay by dollar.

Quantitative formulas of goods and money remain the above (84),(85),(86) and (87) here.

Then a balance of yen of Japan is as follows;

$$\overline{M}_J^J = EF_d + M_J^J. \quad (106)$$

EF_d shows a sum of buying dollar yen-dominated (or selling yen)³¹.

A balance of dollar is as follows;

$$\overline{M}_A^J + F_d + \frac{p_Je_J}{E} = p_Am_J + M_A^J. \quad (107)$$

$\frac{p_Je_J}{E}$ shows a sum of export to America (dollar-dominated).

Total balance of payment of Japan from both formulas is as follows;

$$\overline{M}_J^J + E\overline{M}_A^J + p_Je_J = Ep_Am_J + EM_A^J + M_J^J.$$

This formula becomes the same as (90) again.

Total balance of America is as follows;

$$\overline{M}_J^A = EF_d + M_J^A. \quad (108)$$

A balance of dollar is as follows;

$$\overline{M}_A^A + p_Ae_A + F_d = \frac{p_Jm_A}{E} + M_A^A. \quad (109)$$

³¹Similarly to the above subsection 5.1, we assume that domestic transaction is cancelled out.

$p_A y_A$ shows a sum of production of America, $p_A s_A$ shows a sum of domestic purchases and $\frac{p_J m_A}{E}$ shows a sum of import from Japan (dollar-dominated). Of course, this assumption means that both countries always need dollar.

Substituting (109) for (108), it follows;

$$\overline{M}_J^A + E\overline{M}_A^A + Ep_A e_A = p_J m_A + EM_A^A + M_J^A.$$

This formula becomes the same as (95), too.

Therefore, when we assume a key currency as means of settlement of international transaction, balance formulas of both countries become the same as the above result. Then Walras' Law is unchanged from the above result (98), too.

$$p_J(m_A - e_J) + Ep_A(m_J - e_A) + (M_J^J + M_J^A - \overline{M}_J) + E(M_A^J + M_A^A - \overline{M}_A) \equiv 0. \quad (110)$$

After all, it is clear that a balance formula with international trading does not depend on a method of payment. What is changed by a kind of currency is only a monetary balance formulas of each currency in each country. However, the difference is canceled out in Balance of Payment of each country.

5.4 Generalizing International trade System

Finally, we consider the more general model according to an assumption which two countries has transactions of raw materials, employments and securities³². Here, we assume that the means of settlement of international transaction is buyer's currency.

Considered the same social aggregation told in subsection 3, a quantitative formula in Japan follows the next one. A balance of production agencies and labor forces in Japan is as follows;

$$\overline{X}^J + b_J^J = x^J + e_{xJ} + X^J \quad (111)$$

$$\overline{K}^J + I_J^J = d^J + e_{KJ} + K^J \quad (112)$$

$$\overline{Z}_J + y_J = s_J + e_J + Z_J \quad (113)$$

$$N_{dJ}^J + N_{dA}^J = N_d^J \quad (114)$$

$$N_{sJ}^J + N_{sA}^J = N_s^J. \quad (115)$$

b_A shows a sum of purchases of raw materials from America, e_{xJ} is a sum of export of raw materials from Japan. Employment is described in supply side and demand side separately, both are not always equivalent. Each

³²Here, we omit stock and deposit in each country for simplification.

superscription J and A shows Japanese and American.

It is similar in America, too.

$$\overline{X^A} + b_A^A = x^A + e_{xA} + X^A \quad (116)$$

$$\overline{K^A} + I_A^A = d^A + e_{KA} + K^A \quad (117)$$

$$\overline{Z_A} + y_A = s_A + e_A + Z_A \quad (118)$$

$$N_{dJ}^A + N_{dA}^A = N_d^A \quad (119)$$

$$N_{sJ}^A + N_{sA}^A = N_s^A. \quad (120)$$

Holding currency and securities is as follows;

$$\overline{M_J^J} + \overline{M_A^J} + B_{sJ}^J = M_J^J + M_A^J + B_{dJ}^J + B_{dA}^J \quad (121)$$

$$\overline{M_A^J} + \overline{M_A^A} + B_{sA}^A = M_A^J + M_A^A + B_{dA}^A + B_{dA}^J. \quad (122)$$

B_{sN}^N is a sum of debts in country N and B_{dN}^N shows purchases of securities (or sum of loans) in country M .

Now, a quantitative relation of yen in Japan is as follows;

$$\begin{aligned} & \overline{M_J^J} + B_{sJ}^J + w_J N_{sJ}^J + E F_s + p_J s_J \\ & = p_J c_J + q_J b_J^J + r_A I_A^J + r_J I_J^J + w_J N_d^J \\ & + B_{dJ}^J + E p_A m_J + E q_A b_A^J + E B_{dA}^J + M_J^J. \end{aligned} \quad (123)$$

A quantitative relation of dollar is also as follows;

$$\overline{M_A^J} + \frac{p_J e_J}{E} + \frac{q_J e_{zJ}}{E} + \frac{r_J e_{KJ}}{E} + \frac{B_{dJ}^A}{E} = F_s + M_A^J. \quad (124)$$

Therefore, total balance of payment of Japan is followed;

$$(\overline{M_J^J} + E \overline{M_A^J}) + (B_{sJ}^J + B_{dJ}^A) + [p_J (s_J + e_J) + q_J e_{zJ}] \quad (125)$$

$$\begin{aligned} & + w_J N_{sJ}^J + r_J e_{KJ} \\ & = (p_J c_J + E p_A m_J) + (q_J b_J^J + E q_A b_A^J) + (r_A I_A^J + r_J I_J^J) \\ & + w_J N_d^J + (E B_{dA}^J + B_{dJ}^J) + (E M_A^J + M_J^J). \end{aligned} \quad (126)$$

Transformed it considering (115), it follows;

$$\begin{aligned} & [(M_J^J - \overline{M_J^J}) + E (M_A^J - \overline{M_A^J})] + [(B_{dJ}^J - B_{sJ}^J) + E B_{dA}^J - B_{dJ}^A] \\ & + [p_J (c_J - s_J) - (p_J e_J - E p_A m_J)] + [q_J b_J^J - (q_J e_{zJ} - E q_A b_A^J)] \\ & + [r_J I_J^J - (r_J e_{KJ} - r_A I_A^J)] + w_J (N_d^J - N_s^J + N_{sA}^J) \equiv 0. \end{aligned} \quad (127)$$

This (127) is an equilibrium balance of payment of Japan ³³ This is just Japanese Balance of Payment. Square brackets of the first term on the left hand side are Monetary Account and the second square brackets are equivalent to Capital Balance. The third square brackets show Current Balance in goods market and the fourth show Current Balance in intermediate goods market. The fifth square brackets show Current Balance in fixed capital market. An excess demand of labor market of the final term on the left hand side shows that labor supply by American in Japan is excluded.

If we assume a sum of goods and intermediate goods markets with an equilibrium state in the labor market, it shows current balance. Merging it with Capital Balance is Overall Balance and adding Monetary Balance to it derives well-known Balance of Payment.

Now, Balance of Payment of America is similarly as follows;

$$\begin{aligned} & [(M_J^A - \overline{M_J^A}) + E(M_A^A - \overline{M_A^A})] + [E(B_{dA}^A - B_{sA}^A) + B_{dJ}^A - EB_{dA}^J] \\ & + [Ep_A(c_A - s_A) - (Ep_A e_A - p_J m_A)] + [Eq_A b_A^A - (Eq_A e_{xA} - q_J b_J^A)] \\ & + [Er_A I_A^A - (Er_A e_{KA} - r_J I_J^A)] + Ew_A(N_d^A - N_s^A + N_{sJ}^A) \equiv 0. \end{aligned} \quad (128)$$

Then, we can derive Walras' Law of this world model from formula (127) and (128) ³⁴.

$$\begin{aligned} & (M_J^J + M_J^A - \overline{M_J}) + E(M_A^A + M_A^J - \overline{M_A^J}) + (B_{dJ}^J - B_{sJ}^J) + E(B_{dA}^A - B_{sA}^A) \\ & + p_J(c_J - s_J) + Ep_A(c_A - s_A) + q_J b_J^J + Eq_A b_A^A + r_J I_J^J + Er_A I_A^A \\ & + w_J(N_d^J - N_s^J + N_{sA}^J) + Ew_A(N_d^A - N_s^A + N_{sJ}^A) \equiv 0. \end{aligned} \quad (129)$$

Then, transformed it considering formulas (111), (112), (113), (116), (117) and (118), it follows;

$$\begin{aligned} & (M_J^J + M_J^A - \overline{M_J}) + E(M_A^A + M_A^J - \overline{M_A^J}) + (B_{dJ}^J - B_{sJ}^J) + E(B_{dA}^A - B_{sA}^A) \\ & + p_J[(c_J + e_J + Z_J) - (\overline{Z_J} + y_J)] + Ep_A[(c_A + e_A + Z_A) - (\overline{Z_A} + y_A)] \\ & + q_J(x^J + e_{xJ} + X^J - \overline{X^J}) + Eq_A(x^A + e_{xA} + X^A - \overline{X^A}) \end{aligned}$$

³³We can derive the whole net worth in Japan in the same way as above using quantitative formulas, but we omit it here.

³⁴Since we assume Two Countries model here, $e_J = m_A, e_A = m_J, e_{xJ} = b_J^A, e_{xA} = b_A^J, e_{KJ} = I_J^A, e_{KA} = I_A^J$. If we consider the more general one which does not guarantee these relations, formula (129) is as follows;

$$\begin{aligned} & (M_J^J + M_J^A - \overline{M_J}) + E(M_A^A + M_A^J - \overline{M_A^J}) + (B_{dJ}^J - B_{sJ}^J) + E(B_{dA}^A - B_{sA}^A) \\ & + p_J[c_J - (s_J + e_J - m_A)] + Ep_A[c_A - (s_A + e_A - m_J)] \\ & + q_J(b_J^J - e_{xJ} + b_J^A) + Eq_A(b_A^A - e_{xA} + b_A^J) \\ & + r_J(I_J^J - e_{KJ} + I_J^A) + Er_A(I_A^A - e_{KA} + I_A^J) \\ & + w_J(N_d^J - N_s^J + N_{sA}^J) + Ew_A(N_d^A - N_s^A + N_{sJ}^A) \equiv 0. \end{aligned} \quad (d)$$

$$\begin{aligned}
& +r_J(d^J + e_{KJ} + K^J - \overline{K^J}) + r_A(d^A + e_{KA} + K^A - \overline{K^A}) \\
& +w_J(N_d^J - N_s^J + N_{sA}^J) + Ew_A(N_d^A - N_s^A + N_{sJ}^A) \equiv 0. \quad (130)
\end{aligned}$$

Each term shows an excess demand respectively. The first brackets show an excess demand of money market in Japan, the second brackets show one of money market in America, the third brackets show a securities market in Japan, the fourth brackets show one in America, the fifth and sixth show goods markets in Japan and America, the seventh and eighth show intermediate goods markets in Japan and America, the ninth and tenth show fixed capital markets in Japan and America and the 11th and final show labor markets in Japan and America. An excess demand in each term is identically equal to zero. Further, intermediate goods, fixed capital and labor markets in both countries are excluding foreign raw materials, investments to foreign countries and demand of foreign labor forces respectively. This is Walras' Law in this world economic model ³⁵.

6 Concluding Remarks

As mentioned above, it is clear that various transactions in economy from the simplest transaction to an international one can be described by the same framework. The framework is changed in a form on each economic level or phase, and it is called *budget constraint* on micro economy or individual transaction, *Double Entry Book-keeping* on each company, *System of National Account* on the whole countries and *Balance of Payment* on international transaction. However, these are only various transformations of the same *general equilibrium system* and the condition under that is *Walras' Law*.

When we sum up of transaction in all of countries or international trade, it must be noted that labor forces aren't identical with production agency. Labour doesn't have an account like companies generally and he only acts as a consumer. Therefore, that gives a kind of variation of account to a

³⁵If we don't assume a balance of import and export between both countries, as it is told by equation (d) in the above footnote 34, (130) is changed as follows;

$$\begin{aligned}
& (M_J^J + M_J^A - \overline{M_J}) + E(M_A^A + M_A^J - \overline{M_A}) + (B_{dJ}^J - B_{sJ}^J) + E(B_{dA}^A - B_{sA}^A) \\
& +p_J[(c_J + Z_J + m_A) - (\overline{Z_J} + y_J)] + Ep_A[(c_A + Z_A + m_J) - (\overline{Z_A} + y_A)] \\
& +q_J(x^J + X^J + b_J^A - \overline{X^J}) + Eq_A(x^A + X^A + b_A^J - \overline{X^A}) \\
& +r_J(d^J + K^J + I_J^J - \overline{K^J}) + r_A(d^A + K^A + I_A^A - \overline{K^A}) \\
& +w_J(N_d^J - N_s^J + N_{sA}^J) + Ew_A(N_d^A - N_s^A + N_{sJ}^A) \equiv 0. \quad (e)
\end{aligned}$$

In this case, import terms are just substituted for export terms in formula (130) respectively. However, the economic interpretation of this formula is the same as the main text of this paper

general economic system. It causes that it's hard to consider individual account and social account are the same essentially.

And it is clear that an exchange rate and means of settlement by kinds of money are not essential in international transaction, too. As we told, it's easy to enlarge a general equilibrium system to the international model, and Walras' Law also remains the same here.

Thus, we see that various economic transactions and account rules can be treated as economic theory within one framework of a general equilibrium system. Many economic issues and topics are only one part of that. This fact is elementary but important.

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Appendix

A A Fundamental Framework of Open Macro Model

We'll describe a basic frame of an *open macro model* here. Accounting rules in this main subject does not need to receive influence fundamentally. For simplification, we exclude stocks and deposit, interest and dividends again here ³⁶.

At first, if we add government sector as an economy agency, budget constraint of government will follow;

$$\overline{M_{JG}} + \overline{M_{AG}} + B_{sG} + T = G + B_{dG} + M_{JG} + M_{AG}. \quad (131)$$

$\overline{M_{JG}}$ is a balance carried forward of yen, $\overline{M_{AG}}$ is a balance carried forward of dollar, B_{sG} is national debt publication, T is tax revenue, G is government expenditure, B_{dG} is bond purchase by government, M_{JG} is a balance of yen carried forward to the next term and M_{AG} is a balance of dollar carried forward to the next term here. When we consider this expression and think about total income and expenditure expression of macro economy, we get next equation equal to the former equation (127).

$$\begin{aligned} & [(M_J - \overline{M_J}) + E(M_A - \overline{M_A})] + [(B_{dJ} - B_{sJ}) + EB_{dA} - B_{dJ}^A] \\ & \quad + [p_J(c_J - s_J) - (p_J e_J - Ep_A m_J)] + (G - T) \\ & \quad \quad \quad + [q_J b_J - (q_J e_{xJ} - Eq_A b_A)] \\ & + [r_J I_J - (r_J e_{KJ} - r_A I_A)] + w_J(N_d - N_s + N_{sA}) \equiv 0. \quad (132) \end{aligned}$$

From this,

$$\begin{aligned} & [(M_J - \overline{M_J}) + E(M_A - \overline{M_A})] + (B_{dJ} - B_{sJ}) \\ & + [(r_J I_J + r_A I_A) - (p_J s_J - p_J c_J - q_J b_J - (G - T))] \\ & \quad \quad \quad + w_J(N_d - N_s + N_{sA}) \\ & = (B_{dJ}^A - EB_{dA}) + [(p_J e_J + q_J e_{xJ} + r_J e_{KJ}) - (Ep_A m_J + bEq_A b_A)]. \quad (133) \end{aligned}$$

The first term square brackets on the left side are an excess demand of money, the second term round brackets shows an excess demand of bond, the third term square brackets means an excess demand of goods or investment

³⁶The basis of income and expenditure type including these factors was shown at section4, but see Okishio[10] about a develop version including government sector and derivation of the equation.

/ savings balance and the fourth term is an excess demand of labor.

Because the first term round brackets on the right side show a capital balance and the second term square brackets show a current balance, both become a usual Balance of Payments.

Therefore if we simplify this and represent it, it follows;

$$\tilde{M} + \tilde{B} + \tilde{X} + \tilde{N} = P \quad (134)$$

\tilde{N} shows an excess demand here. P is a balance of payments. Now if we assume that there is an existence of disequilibrium in labor market or unemployment, this equation is as follows;

$$\tilde{M} + \tilde{B} + \tilde{X} = P. \quad (135)$$

Now an open macro model usually is expressed by the following basic relation.

$$\tilde{M}^J + \tilde{B}^J + \tilde{X}^J = P \quad (136)$$

$$\tilde{M}^A + \tilde{B}^A + \tilde{X}^A = -P \quad (137)$$

Five endogenous variables such as Japanese good price p_J , American good price p_A , Japanese interest rate i_J , American interest rate i_A and exchange rate E are decided by this system.

Walras law is as follows;

$$\tilde{M}^J + \tilde{B}^J + \tilde{X}^J + \tilde{M}^A + \tilde{B}^A + \tilde{X}^A = 0. \quad (138)$$

Therefore, an equilibrium condition can be written like next ³⁷.

$$\tilde{B}^J = 0, \tilde{X}^J = 0, \tilde{B}^A = 0, \tilde{X}^A = 0, P = 0.$$

³⁷Of course, it is possible to express this equilibrium condition like next.

$$\tilde{B}^J = 0, \tilde{X}^J = 0, \tilde{B}^A = 0, \tilde{X}^A = 0, \tilde{M}^A = 0.$$