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A Study on the Thermal Diffusivity of Potato Slabs in Various Conditions

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(Fig. 1~9; Table 1~3)

INTRODUCTION

In order to optimum design and operate the various equipments and processes such as cooling [1,2] and so on, it is necessary to determine the thermal diffusivity of food materials.

In a previous paper[3], we have studied the thermal diffusivity of spherical root vegetables such as potato and so on. For softish foods such as fish and meat materials, we can not cut then to the spherical shape specimens as shown in the previous paper.

For these foods, we must use a slab shape sample and adapt two gurded plates or nets on both the surface parts of samples. In this paper, we studied the thermal diffusivity of slab materials in various conditions by using a potato sample which has a uniform tissue and physical properties for the wide portions.

THEORETICAL CONDITIONS

1. Heat-conduction equation

The general heat conduction equation that results from an energy balance on an infinite slab material is given in various textbooks and so on[4,5] as follow:

$$\partial t/\partial \theta = \alpha\,(\partial^2 t/\partial x^2) \qquad (1)$$

$$\text{where,}\quad \alpha = k/(C_P \cdot \rho)$$

where, t(°C) is temperature, θ(min) is processing time, x(cm) is the distance from the surface in the thick direction, α (cm^2/min) is thermal diffusivity, k(cal/cm·min·°C) is thermal conductivity, C_p (cal/g·°C) is specific heat, and ρ (g/cm^3) is density.

Eq.(1) takes the following assumptions: 1. one dimensional heat flow, 2. no internal heat generation, and 3. indipendent of position and temperature. By making these appropriate assumptions, a solution of Eq. (1) can be obtained analytically or numerically. One of the most widely known analytical solutions has been developed by making the following initial and boundary conditions.

Initial condition:

$$\theta = 0, \quad 0 < x < 2L \qquad : t = t_0$$

Boundary conditions:

$$\theta \geqq 0, \quad x = 0 \text{ and } 2L \qquad : t = t_e$$

$$\theta \geqq 0, \quad x = L \qquad : \partial t / \partial \theta = 0$$

$$\theta = \infty, \quad 0 \leqq x \leqq 2L \qquad : t = t_e$$

where, L(cm) is half of the slab thickness, and t_0 and t_e (°C) are the initial and terminal temperatures. Abobe conditions are formed by considering the material initially at t_0, in contact with a stirred fluid at t_e, and assuming that the heat transfer resistance of the film at the surface of the samples is negligible.

Under these conditons, Eq.(1) and the analytical solution are presented as follows[5~7]:

$$\partial Y / \partial \theta = \alpha \, (\partial^2 Y / \partial x^2) \tag{2}$$

$$Y = (2/\pi) \sum_{n=1}^{\infty} \Big[\{(1-(-1)^n)/n\} \{\sin (n\pi x/(2L))\} \times \{\exp(-n^2 \pi^2 X/4)\} \Big] \tag{3}$$

where, $Y = (t - t_e)/(t_0 - t_e)$

$X = \alpha \theta / L^2$

Dimensionless temperature $Y(-)$ and time $X(-)$ are given by the above equations. The value of T changes $1 \rightarrow 0$ with respect to $t = t_0 \rightarrow t_e$.

The numerical calculation of Eq.(1) made as a finit difference form [4,5]. A body is divided into nodal points that are a distance Δx apart, and a time step is given as $\Delta \theta$. The dimensionless temperature in the body can be expressed with respect to position and time by means of the subscripts i = 1,2, - - - n and j = 1,2, - - - m, respectively. The differential term in Eq.(2) can be specified as:

$$\partial Y / \partial \theta = (Y_{i,j+1} - Y_{i,j}) / (\Delta \theta) \tag{4}$$

$$\partial^2 Y / \partial x^2 = (Y_{i-1,j} - 2Y_{i,j} + Y_{i+1,j}) / (\Delta x)^2 \tag{5}$$

In finite difference form, Eq.(2) becomes:

$$Y_{i,j+1} = Y_{i,j} + (Y_{i-1,j} - 2Y_{i,j} + Y_{i+1,j}) / M \tag{6}$$

where, $M = (\Delta x)^2 / (\alpha (\Delta \theta))$

By using Eq.(6), the dimensionless temperature $Y_{i,j+1}$ at node x = i(Δx) for a time, θ = (j+1) ($\Delta \theta$) can be solved from the values of $Y_{i-1,j}$ $Y_{i,j}$ and $Y_{i+1,j}$ for time θ = j ($\Delta \theta$). The numerical solution can be obtained for the above indicated initial and boundary conditions. The dimensionless number M (–) must be represented as $M = 2$.

2. Calculation of thermal diffusivity

It is possible to obtain the value of α by employing the dimensionless heating curves developed by Gurney and Lurie[4,6,8] or by employing the tables by Olson and Schultz[9]. When the heat transfer resistance of the film at the surface can be neglected, the central

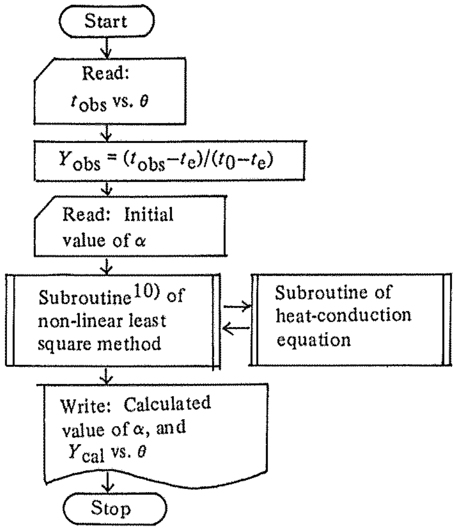

Fig. 1. Flow chart for the calculation of the thermal conductivity

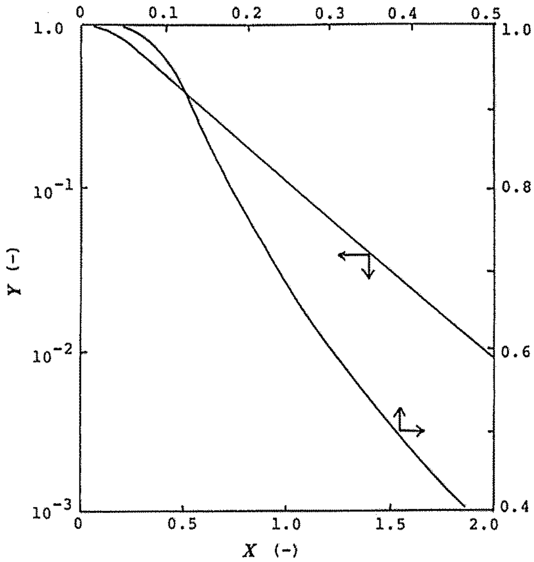

Fig. 2. Central temperature history in a slab material

temperatuer history in a infinite slab material is shown in Fig. 1. Fig. 1 gives a straight line between log (Y) and X for the region of a very smaller Y, but does not give a simple line for the ordinaly region of a larger Y. the value of α can be obtained by using Fig. 1. However, this method is complicated, and also lacks sufficient accuracy. Therefore, the following method that uses a digital computer is more efficient and accurate.

A computer program[5] using finite differences was used to calculate thermal diffusivity from the changing temperature measurements. The basic flow chart of the program is shown in Fig. 2. This program include a non-linear least square method[10] which is an interation procedure for reaching a more accurate value of α. The values of the following standard deviation σ (–) for the Y were minimized.

$$\sigma = \left[\sum_{i=1}^{N} (Y_{obs} - Y_{cal})_i^2 / N \right]^{0.5} \qquad (7)$$

where, Y_{obs} and Y_{cal} are the observed and calculated values of Y, and N is the total number of the experimental points. For the calculation of α, we used the digital electric computer "HITAC M-200H" in the Computation Center of Hiroshima University.

The initial values of α were given by 0.05 in accordance with the previous paper[3]. For the calculation of α, we give $M = 3.0$ and $n = 13$ in $\Delta x = L/n$. The larger value of n is better than the smaller one, but the calculating time increased. The values of n and M were determined by the comparing calculations with assumed various values. The results obtained are shown in Table 1.

Table 1. Results for the comparing of n and M on the supposed data

Supposed data[5] : potato slab of thickness 3.0 cm assumed $\alpha = 0.088$ cm²/min, $t_0 = 75.0$°C,
$t_e = 10.0$°C, $\theta = 1 \sim 20$ min ($N = 20$)

n	Initial value		Calculated value	
	α (cm²/min)	σ (–)	α (cm²/min)	σ (–)
5	0.05	0.196	0.0883	0.0058
9	0.05	0.193	0.0881	0.0015
13	0.05	0.193	0.0880	0.0008

θ (min)	Supposed data	Calculated results Y (–)		
	Y (–)	$n = 5$	$n = 9$	$n = 13$
0	1.000	1.000	1.000	1.000
1	0.998	1.000	1.000	1.000
2	0.977	0.991	0.981	0.979
3	0.922	0.936	0.925	0.924
4	0.852	0.862	0.855	0.853
5	0.780	0.787	0.782	0.781
	(excepted intermediate results)			
20	0.185	0.182	0.184	0.184

where, $M = 3.0$

M	Initial value		Calculated value	
	α (cm²/min)	σ (–)	α (cm²/min)	σ (–)
1.0	0.05	——	——	——
2.0	0.05	0.193	0.0881	0.0014
3.0	0.05	0.193	0.0880	0.0008
4.0	0.05	0.192	0.0880	0.0006

θ (min)	Calculated results Y (–)			
	$M = 1.0$	$M = 2.0$	$M = 3.0$	$M = 4.0$
0	——	1.000	1.0000	1.0000
1	——	1.000	1.000	0.999
2	——	0.980	0.979	0.978
3	——	0.925	0.924	0.923
4	——	0.855	0.853	0.853
5	——	0.782	0.781	0.781
	(excepted intermediate results)			
20	——	0.184	0.184	0.185

where, $n = 13$, —— : can not calculate

EXPERIMENTAL

1. Samples

As samples, we used potato slabs. These potatoes were bought in the market. Their specific descriptions as well as their producing districts were unknown. They were cut into slabs by a sharp cutter. The formation of parallel surface was made by a pair of parallel cutters which were fixed with a acrylic resin slab.

For the slab of samples, the comparing experiments were tested in order to know the guarded plate and net effects on both surface parts of the samples. Plates of two types were used, made of brass, different in thickness: the smaller one was 0.05 X 5.0 X 5.0 cm and the other was 0.1 X 5.0 X 5.0 cm. The net used was a 30 mesh stainless steel net which was 0.05 X 5.0 X 5.0 cm.

In order to repeat the previous experiments using spherical materials, the spherical one was made by a sharp cutter, too. The samples used in this study are shown in Table 2.

Table 2. Samples and experimental conditions

Sample: Potato, t_0 = 10°C, t_e = 50.0°C

Run	Dimension (cm)	Stirrer (rpm)	Thermocouple insulator	Sample guard
1	1.1 X 4.0 X 5.0	100	except	except
2	〃	300	〃	〃
3	1.1 X 3.5 X 5.0	300	except	except
4	〃	〃	used	〃
5	1.02 X 1.03 X 1.03	300	used	except
6	1.10 X 2.07 X 2.07	〃	〃	〃
7	1.06 X 3.11 X 3.11	〃	〃	〃
8	1.09 X 4.11 X 4.11	〃	〃	〃
9	1.10 X 5.24 X 5.24	〃	〃	〃
10	1.07 X 3.28 X 3.28	300	used	except
11	1.50 X 4.51 X 4.51	〃	〃	〃
12	1.84 X 5.90 X 5.90	〃	〃	〃
13	0.97 X 5.09 X 5.09	300	used	used(**)
14	〃	〃	〃	except
15	1.00 X 5.18 X 5.18	300	used	used (***)
16	〃	〃	〃	except
17	1.02 X 5.25 X 5.35	300	used	used(****)
18	〃	〃	〃	except
19	2.72(*)	300	used	except

where, * : sphere by previous method
** : brass plates (0.05 X 5.0 X 5.0 cm)
*** 〃 (0.10 X 5.0 X 5.0 cm)
**** : stainless steel nets (0.05 X 5.0 X 5.0 cm)

The specific gravity of the samples was determined by means of a picnometer. The average specific gravity of potatoes was 1.043 g/cm^3.

The weights of the completely dryed state of the samples were estimated as being the values of 7 houres drying at 105 °C in a dryer. The average initial water content of the potatoes was 7.56 g-H_2O/g-dry material, and the average initial moisture percentage of wet basis was 88.2%.

2. Apparatuses and experimental method

The apparatuses used in these experiments are shown in Fig. 3. The potatoes were

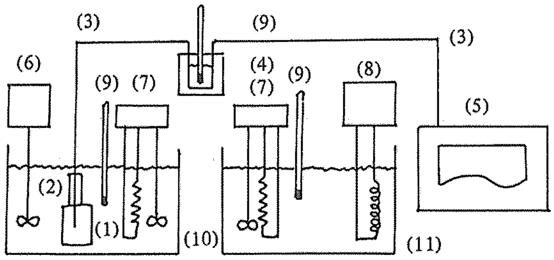

Fig. 3. Experimental apparatuses
(1) sample, (2) insulator, (3) thermocouple (4) ice box, (5) recorder, (6) stirrer (7) thermo-heater unit, (8) cooler unit (9) thermometer, (10) heat-side water bath (11) cool-side water bath

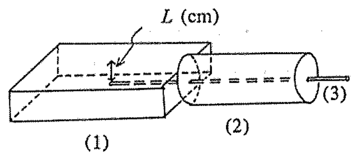

Fig. 3. (Cont.) Sample of slab

previously placed in a cool-side water bath which was controlled at 10°C, and then put onto a heat-side water bath controlled at 50°C. The water bath capacity was 45 × 30 × 15 cm and a thermo-heater unit which had a stirrer (Taiyo Kagaku Co., Type Minder Jr) was placed in the bath. A cooling unit (Yamato Kagaku Co., Type Neocool BD-11) was used for the cool-side water bath.

The center temperature changes of the samples were measured by means of a chromel-alumel thermocouple connected to a recorder (Yokogawa Denki Co., Type 3056-22).

The thermocouple was bought in the market. The each diameter of two fine wires enclosed in 1.0 mm diameter stainless steel sheath was 0.16 mm, respectively. The thickness of the sheath was 0.15 mm. The thermocouple probe was inserted into a hole made with a fine steel needle. The probe was extended beyond the longitudinal axis of the samples, and the top point was kept at the center of the samples.

The thermocouple probe was covered with a cylindrical insulator of polystyrol (1.5 × 1.5 × 8.5 cm). This was used in order to avoid the conduction of heat into the sample through the termocouple probe. The insulating effect was tested by comparing the experiments with and without insulator.

Stirring by using a stirrer (Tokyo Rikakikai Co., Type DC-3RT) was strong enough to reduce the heat transfer resistance of the film at the surface of the samples. The screw number of the stirrer was three, and the diameter at the revolution wing was 5.0 cm. The stirring effect was investigated by changing the number of revolutions.

The experimental conditions used in these studies are shown in Table 2. In order to comparing of the data, the same samples of potato were used for Run 1 and 2; Run 3 and 4; Run 10 and 11; Run 12 and 13; and Run 14 and 15, respectively.

RESULTS AND DISCUSSION

The relationships between the dimensionless temperature $Y(-)$ and the time θ (min) are shown in Figs. 4~6. The initial and calculated values of α are shown in Table 3 and Fig. 7. The calculated results made used of the obtained values of α in Table 3 are illustrated by the solid lines in Figs. 4~6 this in order to compare the observed values. All of the calculated results were satisfactory.

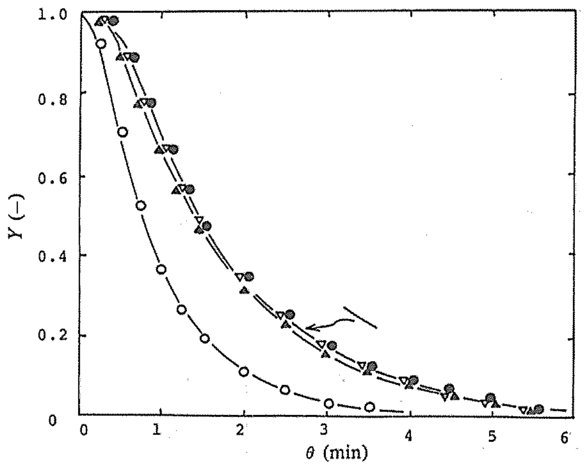

Fig. 4. Relations between dimensionless temperature and process time

Observed results: Run 5 6 7 9
○ ▲ ▽ ●
Calculated results: ————

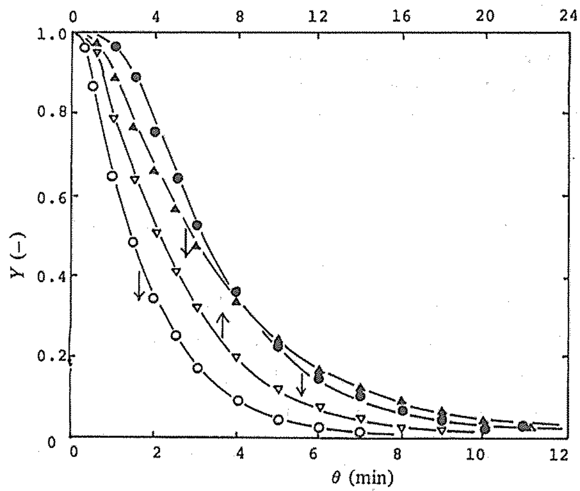

Fig. 5. Relations between dimensionless temperature and process time

Observed results: Run 10 11 12 19
○ ▲ ▽ ●
Calculated results: ————

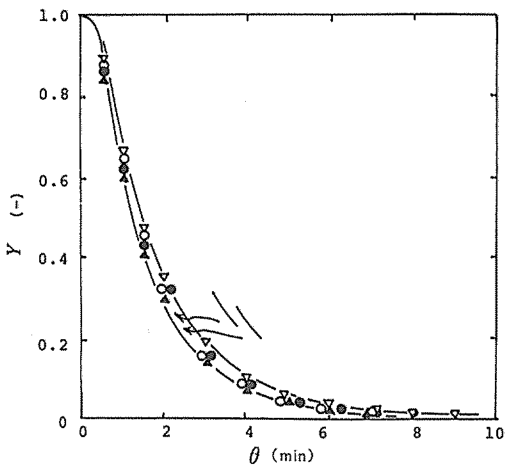

Fig. 6. Relations between dimensionless temperature and process time

Observed results:	Run	13	14	17	18
		○	▲	▽	●
Calculated results:	———				

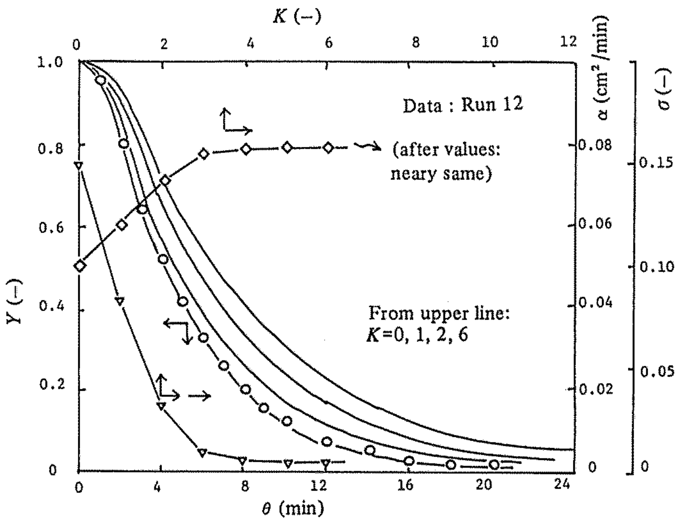

Fig. 7. Relations between thermal diffusivity, standard deviation and iteration number, and between dimensionless temperature and processing time where, K (–) : number of iteration

In Table 3, the comparing results of Run 1 and 2 do not differ from each other. Therefore, the stirring effect from 100 to 300 rpm may be neglected.

The comparing results of Run 3 and 4 differ not from each other, So, the covering effect for the thermocouple probe can be neglected, too. If the thermocouple is of a larger diameter, internal temperature can not be accurately obtained[11], yet this effect did not appear in our study. The reason is perhaps that the thickness of the steel sheath part was very small and the wires in the sheath were very fine, and that we did not use copper wire on purpose because it shows a higher thermal conductivity. In the

Table 3. Calculated results of thermal diffusivity

Run	Initial value		Calculated value	
	α (cm²/min)	σ (–)	α (cm²/min)	σ (–)
1	0.05	0.119	0.0736	0.0114
2	〃	0.126	0.0728	0.0099
3	0.05	0.163	0.0819	0.0126
4	〃	0.160	0.0811	0.0154
5	0.05	0.315	0.130	0.0083
6	〃	0.172	0.0834	0.0125
7	〃	0.130	0.0739	0.0059
8	〃	0.140	0.0762	0.0070
9	〃	0.155	0.0797	0.0055
10	0.05	0.147	0.0770	0.0129
11	〃	0.138	0.0750	0.0049
12	〃	0.149	0.0792	0.0054
13	0.05	0.095	0.0669	0.0028
14	〃	0.123	0.0726	0.0069
15	0.05	0.075	0.0631	0.0135
16	〃	0.117	0.0725	0.0067
17	0.05	0.097	0.0679	0.0040
18	〃	0.130	0.0746	0.0069
19	0.05	0.200	0.0815	0.0082

experiments after Run 5, we used safer conditions, taking a stirring of 300 rpm and covering of thermocouple probe.

In Fig. 4, the results of Run 5 differ greatly from the others. This is due to the fact that the heating occured from three directions[12,13]. From these results as shown in Fig. 4 and Table 3, we may infer that the infinite slab model used in this paper can be only used when the ratios of width and length to thickness are larger than three.

The values of α in Run 5 ~ 10 in Table 3 are very scattered. The reason is that it is difficult to obtain the exact results on such thin samples, because it is difficult to insert the top point of the thermocouple into the center of the samples. For the infinit slab model, we may infer that the thickness must have larger values than about 2.0 cm.

The values of α which were obtained as the best results by Run 12 and 19 are smaller than the value of 0.097 cm²/min which was determined in the previous paper[3]. The reason is that the previous temperature resions were higher than those of the present studies.

The temperature effects of k and C_p in potatoes have been studied by Yamada[14]. However, the effects of α can not be obtained as unknown the effects of ρ. The effects of α in sweetpotato have been obtained by Wadswarth and Spadaro[15]. The values changed from 0.06 to 0.11 cm²/min at from 30 to 70°C.

The comparing results of Run 13 and 14, Run 15 and 16, and Run 17 and 18 in Table 3 differ from each other, respectively. From these results, we may infer that an other thermocouple has to be used in the intersurface position of the sample and the plate or net. For the samples of potato, we did not use this thermocouple, because it derives a space in the intersurface. For the softish foods such as fish and meat materials and so on, the space is perhaps not appearet.

The values in water calculated from the table values of k, C_p and ρ are from 0.086 to 0.100 cm² /min at from 20 to 90°C. The values of high water content materials may be approximate these values. The values of α in many foods seem to be 0.05 to 0.10 cm² /min. The relationships of the central dimensionless temperature and the processing time in slab materials of α = 0.05 and 1.0 cm² /min for the various half thicknesses are shown in Fig. 8.

The relationships of the central dimensionless temperature and the processing time in slab materials of the half thickness L = 1.0 and 1.5 cm for α = 0.05, 0.10, 0.15 and 0.20 cm² /min are shown in Fig. 9. If we can obtain the central temperature history of foods

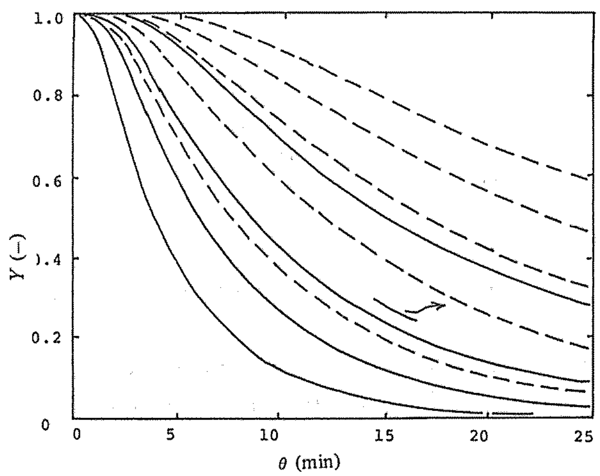

Fig. 8. Temperature history in slab materials for various conditions

α=0.05 cm²/min: - - 0.10 cm²/min:—
From upper line:
L=2.0, 1.75, 1.5, 1.25 and 1.0 cm

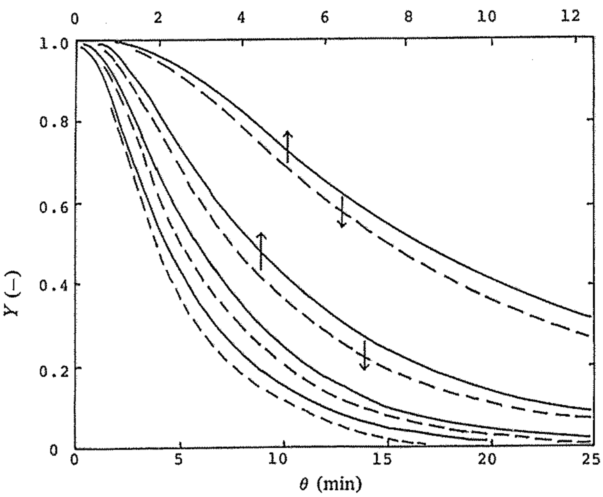

Fig. 9. Temperature history in slab materials for various conditions

L=1.0 cm: - - - 1.5 cm: ———
From upper line:
α=0.05, 0.1, 0.15 and 0.20 cm²/min

in a thickness of 2.0 or 3.0 cm, the values of α can be obtained approximatively by using the relationships in Fig. 9. If values with a high accuracy are needed, the above mentioned digital computering method used in this study is very usefull.

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SUMMARY

In order to design various food heating or cooling processes such as cooking apparatuses and so on, it is necessary to measure the thermal diffusivity of the food materials.

In a former paper[3], we studied the thermal diffusivity for the spherical root vegetables. In this paper, we studied the same for the slab potatoes. The values of thermal diffusivity of potatoes are considered to be in average 0.08 cm^2/min at a temperature range from 10 to 50°C. The results of the slab shape experiments are usuful for the softish foods such as fish and meat materials and so on which can not be cut in to spherical shape.

The central temperature history in slab material is shown. This figure can be used in order to obtain the approximate thermal diffusivity from the experimental data.

In this paper, we investigated a method which used the digital computer program in order to determine the thermal diffusivity. This method is more acculate and usuful than the other previous methods which used figures or tables.

NOTATIONS

C_p : specific heat (cal/g·°C)
K : number of iteration (–)
k : thermal conductivity (cal/cm·min·°C)
L : half thickness (cm)
M : dimensionless number (–)
m : number of divided time (–)
N : number of experimental points (–)
n : number of divided half thickness (–)
t : temperature (°C)
X : dimensionless time (–)
x : distance from surface (cm)
Y : dimensionless temperature (–)
α : thermal diffusivity (cm^2/min)
θ : processing time (min)
ρ : density (g/cm^3)
σ : standard deviation (–)

Subscripts;

0 and e	:	initial and terminal states
i and j	:	thickness position and processing time
obs and cal	:	observed and calculated values

REFERENCES

1) KUBOTA, K., TAKASAKI, K., FUJIMOTO, M., SUZUKI, K. and HOSAKA, H.: *Nippon Shokuhin Kogyo Gakkaishi*, 27, 157(1980).
2) KUBOTA, K., FUJIMOTO, M., SUZUKI, K. and HOSAKA, H. : *ibid*, 27, 381(1980).
3) KUBOTA, K., FUJIMOTO, M., SUZUKI, K., TAKASAKI, K. and HOSAKA, H. : *ibid*, 28, 68(1981).
4) Kagaku Kogaku Kyokai Ed.: Kagaku Kogaku Benran, 4th Ed., p.255, Maruzen, Tokyo(1978).
5) KUBOTA, K. : *Shokuhin Kogyo*, 22(14), 59(1979).
6) KUBOTA, K. and HOSAKA, H. : *ibid*, 24(16), 65(1981).
7) KAWAMURA, Y., NAKAMARU, H. and IMANISHI, M. : Kakou Sugaku Nyumon, p.134, Kagaku Kogyosha, Tokyo(1962).
8) GURNEY, H.P. and LURIE, J. : *Ind. Eng. Chem.*, 15, 1170(1923).
9) OLSON, F.C.W. and SCHULTSZ, O.T. : *ibid*, 34 ,874(1942).
10) KUBOTA, K. and MORITA, N. : *Computation Center News of Nagoya University (in Japanease)*, 4(4), 318 (1973).
11) HOSTETLER, R. I. and DUTSON, T.R. : *J. Food Sci.*, 42, 845(1977).
12) NEWMAN, A.B. : *Ind. Eng. Chem.*, 28, 545(1936).
13) DICKERSON, R.W. and READ, Jr., R.B. : *Food Tech.*, 22, 1533(1968).
14) YAMADA, T. : *Nougei Kagakukaishi*, 44, 578 (1970).
15) WADSWARTH, J. I. and SPADARO, J. J. : *Food Tech.*, 23,219(1969).

各種条件下における平板状ジャガイモの熱拡散率の算出に関する研究

久保田清・高瀬祐美子・鈴木寛一・江坂宗春

各種食品を加熱，冷却操作するクッキング装置などを設計するためには，食品の熱拡散率を求めておくことが必要となる。前報[3]において，球状食品の熱拡散率の測定に関する研究を根菜類食品を例として行ってきた。

本報では，平板状食品の熱拡散率の測定に関する研究をジャガイモを例として行った。ジャガイモの熱拡散率の値は，温度10～50℃において0.08 cm^2/minとなった。

伝熱方程式を差分法で解いて，非線形最小二乗法を用いて熱拡散率を算出する本報で用いた方法[5]は，従来からよく利用されている図表使用などによる方法よりも精度がよい値が得られることなどから優れているといえる。伝熱方程式を，前報で示したように解析法で解いて用いる場合には，初期，境界条件が固定される欠点があったが，本報で示したように差分法で解いていくと，任意の条件下に対応できるものとなる。試料表面に保護用の板などをつけると，試料表面温度が変化していき，非等温境界条件下での実験になると考えられるが，このような場合に対しても利用できるものとなる。参考のためとして，熱拡散率の概算値を得るのに便利な中心温度変化曲線を前報と同様の形式に従って示した。

本報で得られた各種条件下での測定結果ならびに熱拡散率の算出法に関する知見は，球状成型が困難な魚肉類のような軟質状食品を平板状成型をして，熱拡散率を求める場合に有用になると考えられる。