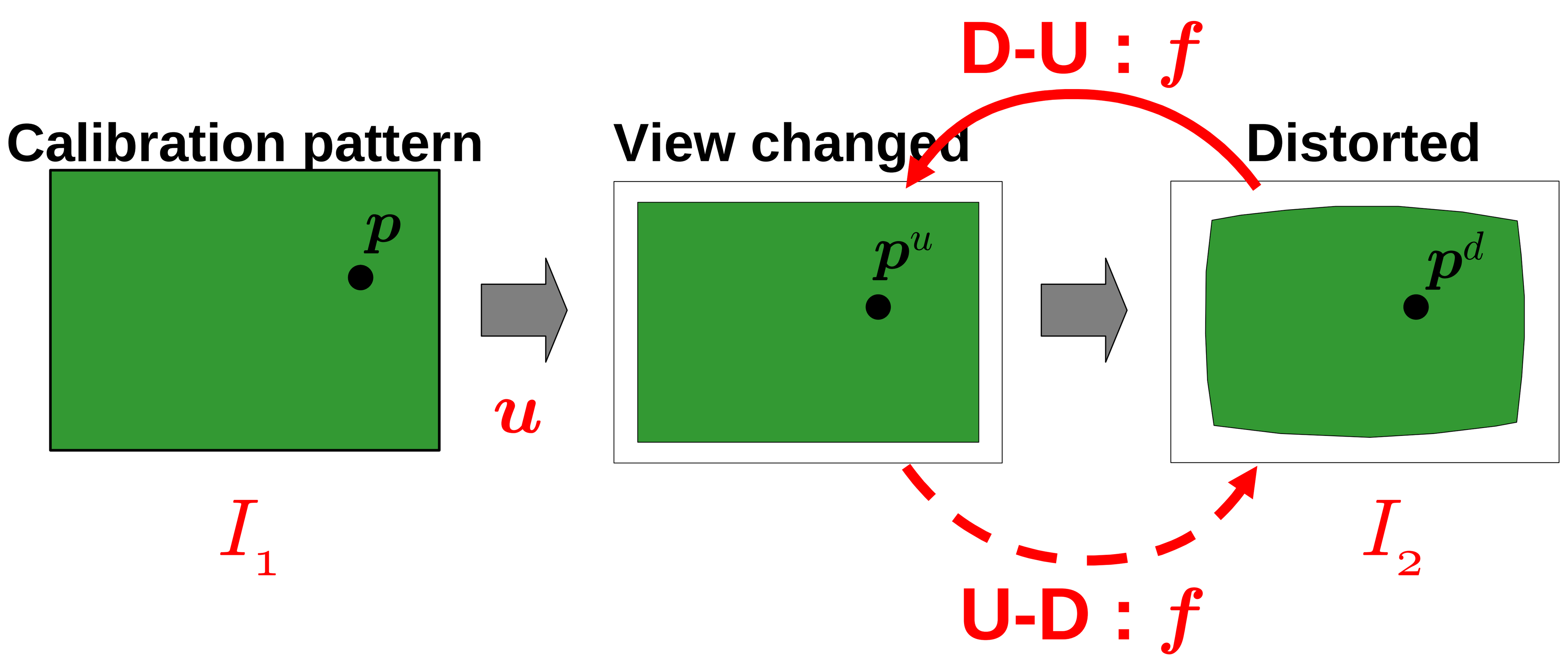
Unified Approach To Image Distortion

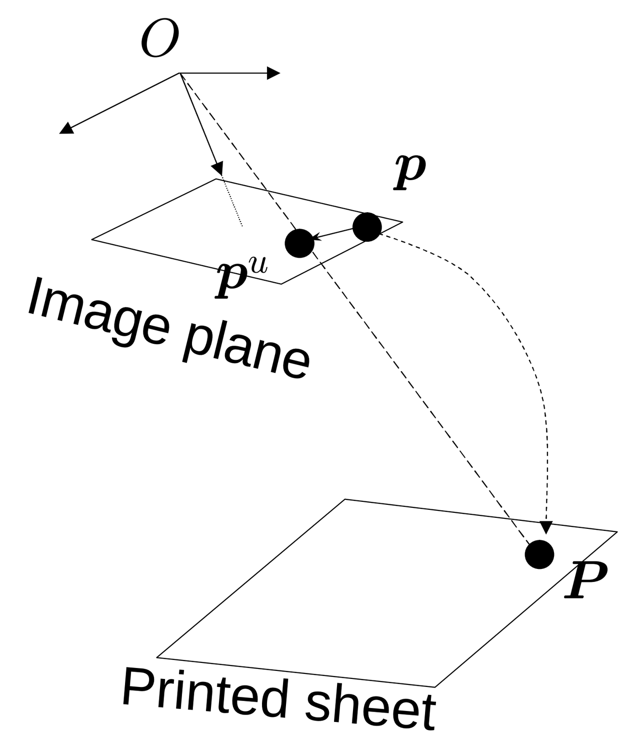
Toru Tamaki (*Niigata Univ.*)
Tsuyoshi Yamamura (*Aichi Pref. Univ.*)
Noboru Ohnishi (*Nagoya Univ. & RIKEN*)

ABSTRACT

We propose a new unified approach to deal with two formulations of the *image distortion* and a method for estimating the distortion parameters by using the both formulation. Although all of conventional researches are based on the same distortion model proposed in an early study in photogrammetry, two different formulations have been used and developed by different papers separately, and this has caused a confusion for developing calibration methods. The proposed method is based on *image registration* and consists of nonlinear optimization to estimate parameters including *view change* and *radial distortion*.



MODELING



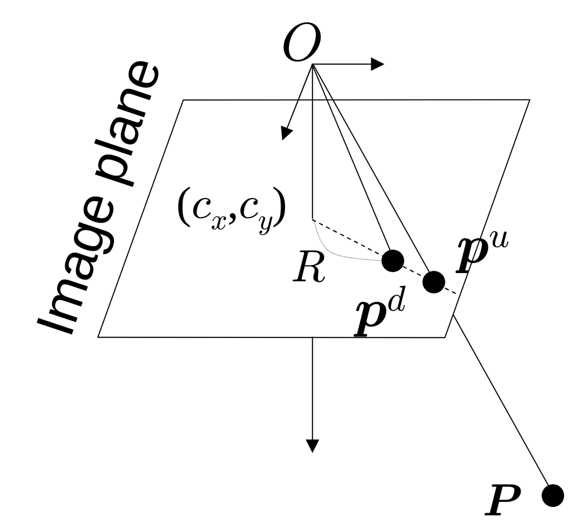
Two step observation of distortion

- A point $\boldsymbol{p}$ on a calibration pattern is projected to $\boldsymbol{p}^u$ on the image plane through a camera lens.

$$\boldsymbol{p}^u = \boldsymbol{u}(\boldsymbol{p};\boldsymbol{\theta}^u) = \frac{1}{\theta_1^u x + \theta_2^u y + 1}\begin{pmatrix}\theta_3^u x + \theta_4^u y + \theta_5^u \\ \theta_6^u x + \theta_7^u y + \theta_8^u\end{pmatrix}$$

- The projected point is moved on the image by the distortion.

$\boldsymbol{p}^u = (x^u, y^u)^T$: Undistorted (projected) point
$\boldsymbol{p}^d = (x^d, y^d)^T$: Distorted point



Distortion model

$$\boldsymbol{f}(\boldsymbol{p};\boldsymbol{\theta}^d) = \begin{pmatrix}\frac{x-c_x}{s_x}(1+k_1R^2+k_2R^4)+c_x \\ (y-c_y)(1+k_1R^2+k_2R^4)+c_y\end{pmatrix} \qquad R = \sqrt{\left(\frac{x-c_x}{s_x}\right)^2 + (y-c_y)^2}$$

$\boldsymbol{\theta}^d = (k_1, k_2, c_x, c_y, s_x)^T$: Intrinsic camera parameters

- Radial distortion parameters k_1 and k_2
- Image center $(c_x, c_y)^T$
- Horizontal scale factor s_x

Two formulations of the distortion model

$\boldsymbol{p}^d = \boldsymbol{f}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)$ from-Undistorted-to-Distorted (**U-D**) formulation

$\boldsymbol{p}^u = \boldsymbol{f}(\boldsymbol{p}^d, \boldsymbol{\theta}^d)$ from-Distorted-to-Undistorted (**D-U**) formulation

$$\boldsymbol{p}^d = \boldsymbol{f}^{-1}(\boldsymbol{p}^u, \boldsymbol{\theta}^d) \equiv \boldsymbol{d}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)$$

An *implicit function* $\boldsymbol{d}()$ is defined.

ESTIMATION

Image registration seeks to minimize the residuals r_i of intensities of I_1 (calibration pattern) and I_2 (distorted image). The function to be totally minimized is the sum of the squares of the residuals over the image I_1.

- **Cost function**

$$\min_{\boldsymbol{\theta}} \sum_i r_i^2 \qquad r_i = I_1(\boldsymbol{p}_i) - I_2(\boldsymbol{p}_i^d)$$

Estimating the parameters $\boldsymbol{\theta} = (\boldsymbol{\theta}^u, \boldsymbol{\theta}^d)$, the cost function is minimized by the Gauss-Newton method. To calculate the decent direction of the cost function, the following Jacobian of r with respect to $\boldsymbol{\theta}$ is required.

- **Gradient**

$$\frac{\partial r}{\partial \boldsymbol{\theta}} = \left(\frac{\partial r}{\partial \boldsymbol{\theta}^u}, \ \frac{\partial r}{\partial \boldsymbol{\theta}^d} \right)$$

For each formulation, the Jacobian is derived as follows by *the implicit function theorem*.

- **Jacobian**

For U-D $\quad \boldsymbol{p}_i^d = \boldsymbol{f}(\boldsymbol{p}_i^u, \boldsymbol{\theta}^d) \qquad \boldsymbol{p}_i^u = \boldsymbol{u}(\boldsymbol{p}_i, \boldsymbol{\theta}^u)$

$$\frac{\partial r}{\partial \boldsymbol{\theta}} = \left(-\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^u} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u}, \ -\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} \right)$$

For D-U $\quad \boldsymbol{p}_i^d = \boldsymbol{d}(\boldsymbol{p}_i^u, \boldsymbol{\theta}^d) \qquad \boldsymbol{p}_i^u = \boldsymbol{u}(\boldsymbol{p}_i, \boldsymbol{\theta}^u)$

$$\frac{\partial r}{\partial \boldsymbol{\theta}} = \left(-\nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{\theta}^u}, \ \nabla I_2(\boldsymbol{p}^d) \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}^d} \right)$$

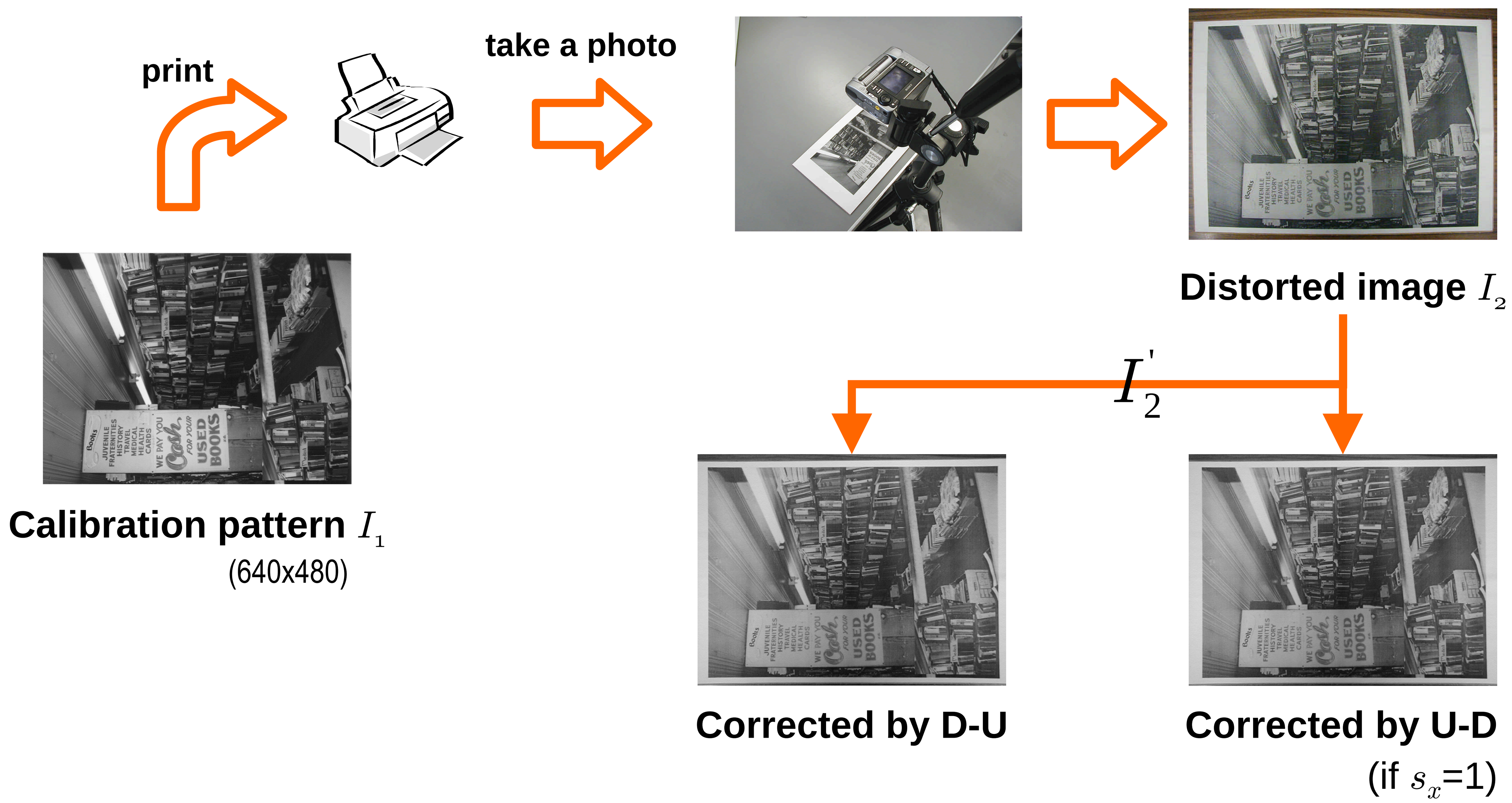
For every point $\boldsymbol{p}^u$ in the corrected image I'_2, the intensity is decided by that of the corresponding point in the distorted image I_2.

- **Correction**

$$I'_2(\boldsymbol{p}^u) = I_2(\boldsymbol{f}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)) \quad \text{for U-D model}$$

$$I'_2(\boldsymbol{p}^u) = I_2(\boldsymbol{d}(\boldsymbol{p}^u, \boldsymbol{\theta}^d)) \quad \text{for D-U model}$$

EXPERIMENAL RESULTS



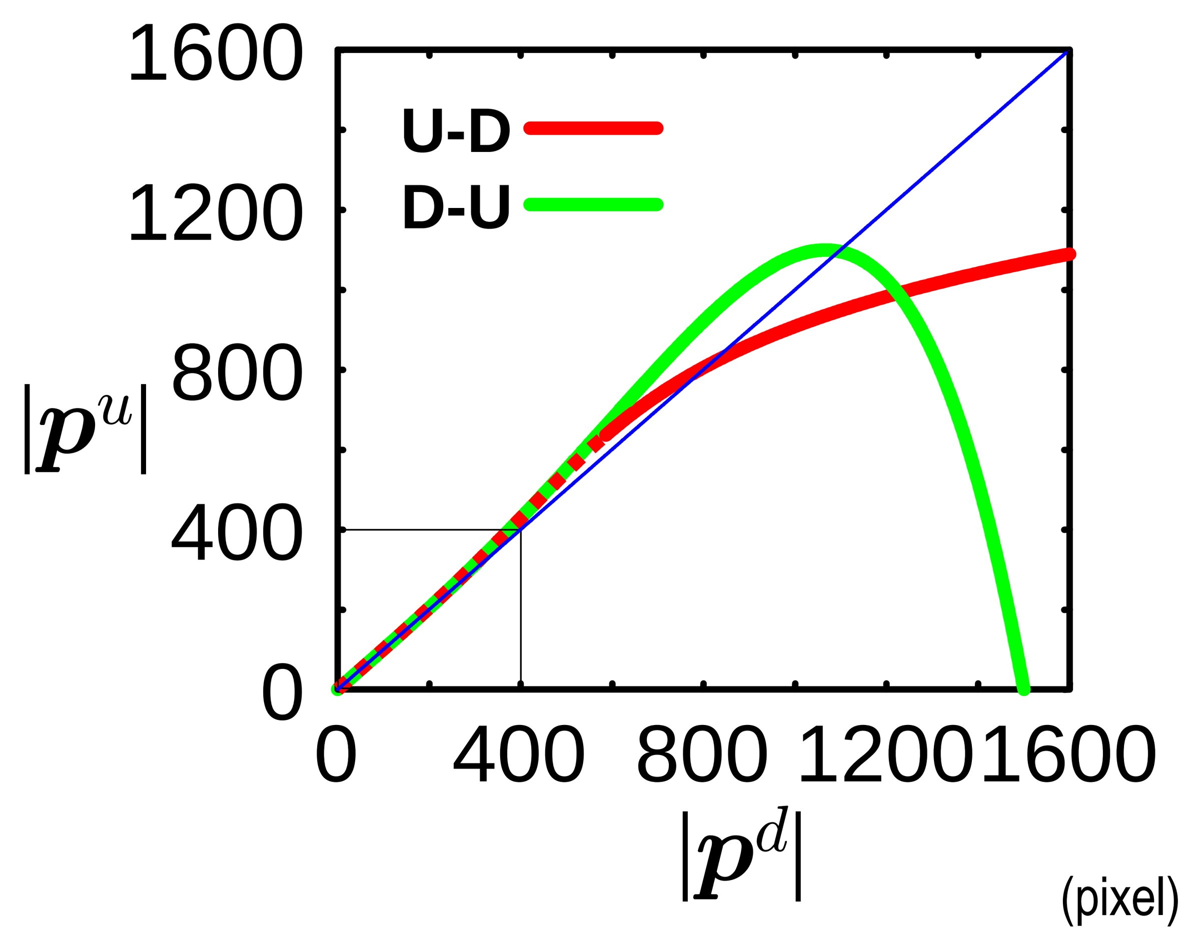
- **Estimated parameters of both formulations**

	k_1	k_2	c_x	c_y	s_x
U-D	-4.96e-7	7.49e-13	298.7	241.2	0.762
D-U	5.07e-7	-4.22e-13	297.7	241.2	0.978

s_x estimated by U-D is unreliable.

s_x is absorbed into $\boldsymbol{\theta}^u$ for U-D formulation ($\boldsymbol{\theta}^u$ stretches the image horizontally, and s_x makes it shrink)

- **Distortion curves of both formulations**



Distortions **by both U-D and D-U** ***have the same effect*** **where** $|\boldsymbol{p}^d|$***<400.***

$|\boldsymbol{p}^d|$ is the distance from the image center, and maximum distance for an image of 640x480 is less than 400.