Lattice Study of Low-lying Nonet Scalar Mesons in Quenched Approximation

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Abstract

Using lattice QCD simulation in the quenched approximation, we study the κ meson, which is $^3P_0$ in the quark model, and compare experimental and other lattice data. The κ is the lowest scalar meson with strangeness and constitutes the scalar nonet. The obtained mass is much higher than the recent experimental value, and therefore the $\kappa(800)$ is difficult to consider as a simple two-body constituent-quark structure, and may have another unconventional structure.

1 Introduction

We are in the age of renaissance of hadron spectroscopy, initiated by the announcement of the pentaquark baryon [1], which is followed by the discovery of many other possible exotic hadrons with a mass larger than 2 GeV containing heavy quarks[2]. These experimental developments prompted the intensive theoretical studies of QCD dynamics with new as well as old ideas on the structure and dynamics of the exotic hadrons, such as chiral dynamics[3], multi-quark states with diquark correlations or molecular states and hybrids[2].

Such a controversy on the structure of hadrons is also the case for the scalar mesons below 1 GeV: the existence of the $I = 0$ and $J^{PC} = 0^{++}$ meson, i.e., the $\sigma(400-600)$, has been reconfirmed [4, 5] after around twenty years not only in $\pi\pi$ scattering but also in various decay processes from heavy-quark systems, *e.g.* , D $\to \pi\pi\pi$ and $\Upsilon(3S) \to \Upsilon\pi\pi$ [6, 7, 8, 9]. Moreover, the resonance of a scalar meson with $I = 1/2$ is also reported to exist in the K-π system with a

mass m_κ of about 800 MeV [9, 10, 11]. This meson is called the κ meson and may constitute the nonet scalar state together with the σ meson. See Fig.1.
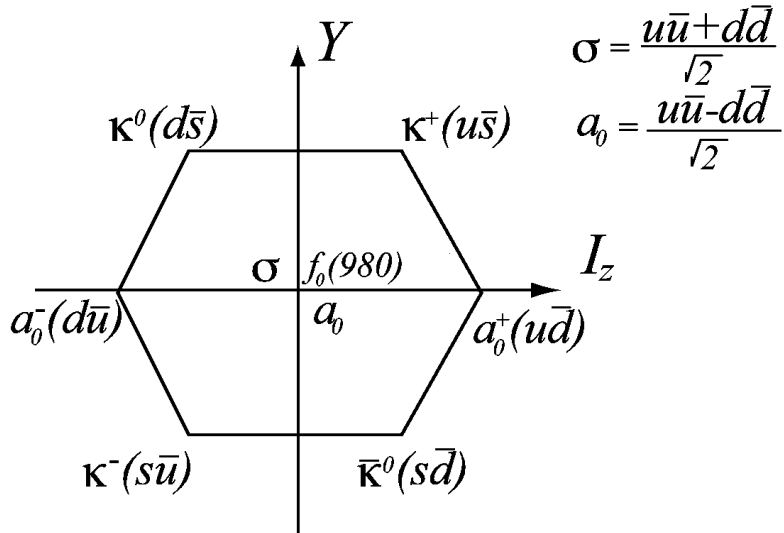

Figure 1: Scalar meson nonet. The σ and $f_0(980)$ mesons may be ideal mixing states of singlet state $\frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})$ and octet state $\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})$. There is , however, experimental evidence that the σ meson consist of only $u\bar{u}$ and $d\bar{d}$ components. Hence, we take the σ wave function given in the figure.

The problem is the nature of these low-lying scalar mesons [12]: they cannot be ordinary $q\bar{q}$ mesons as described in the non-relativistic constituent quark model since in such a quark model, the $J^{PC}=0^{++}$ meson is realized in the $^3P_0$ state, which implies that the mass of the σ meson must be as high as 1.2 $\sim$ 1.6 GeV. Thus, the low-lying scalar mesons below 1 GeV have been a source of various ideas of exotic structures, as mentioned above: they may be four-quark states such as $qq\bar{q}\bar{q}$ [13], or $\pi\pi$ or Kπ molecules as the recent high-lying exotic hadrons can be. These mesons may be *collective* $q\bar{q}$ states described as a superposition of many *atomic* $q\bar{q}$ states [14, 15]. A mixing with glueball states is also possible [16, 17, 18, 19].

In the previous work[20, 21, 22, 23], we have presented a lattice calculation for the σ meson, by full lattice QCD simulation on the $8^3 \times 16$ lattice using the plaquette action and Wilson fermions: We have shown that the disconnected diagram plays an essential role in order to make the σ meson mass light. The importance of the disconnected diagram suggests that the wave function of the σ meson may have a significant four-quark, a collective q-$\bar{q}$ or an even glueball component, although the smallness of the lattice requires caution in giving a definite conclusion. In contrast to the σ meson, the κ is a flavor non-singlet

state with which a glueball state cannot mix. In previous reports [23, 24], we reported also a preliminary analysis on the κ meson using the dynamical fermion for the $u(d)$ quark but using the valence approximation for the s quark, which shows that the $I = 1/2$ scalar meson has a mass as large as about 1.8 GeV and cannot be identified with the κ meson observed in experiments.

The lattice volume in the previous investigations was admittedly too small to yield a definite conclusion at all, and the lattice cutoff was not appropriately chosen to accommodate large masses : $m_\kappa a > 1$, where a is the lattice spacing. Hence, we present a simulation with weaker couplings on a larger lattice than any other previous simulations although in the quenched level. We perform quenched level simulations on the κ meson so as *to clarify the structure of the mysterious scalar meson rather than to reproduce the experimental value of the mass*; a precise quenched-level simulation should give a rather clear perspective on whether the system can fit with the simple constituent-quark model picture or not.

2   Simulation

We perform a quenched QCD calculation using the Wilson fermions, with the plaquette gauge action, on a relatively large lattice ($20^3 \times 24$).

The values of the hopping parameter for the u/d quark are $h_{u/d} = 0.1589, 0.1583$ and 0.1574, while $h_s = 0.1566$ and 0.1557 for the s quark. Using these hopping parameters except for $h_s = 0.1557$, CP-PACS collaboration performed a quenched QCD calculation of the light meson spectrum with a larger lattice ($32^3 \times 56$) [25], which we refer to for comparison. The gauge configurations are generated by the heat bath algorithm at $\beta = 5.9$. After 20000 thermalization iterations, we start to calculate the meson propagators. On every 2000 configurations, 80 configurations are used for the ensemble average.

We emply the point-like source and sink for the κ^+ meson

$$\hat{\kappa}(x) \equiv \sum_{c=1}^{3} \sum_{\alpha=1}^{4} \bar{s}_\alpha^c(x) u_\alpha^c(x)   , \tag{1}$$

where $u(x)$ and $s(x)$ are the Dirac operators for the u/d and s quarks, and the indices c and α denote the color and Dirac-spinor indices, respectively. The point source and sink in Eq.(1) lead a positive spectral function $\rho(m^2)$ in the correlation function $\langle \hat{\kappa}(t)\hat{\kappa}(0) \rangle = \int dm \rho(m^2) \exp(-mt)$. The result obtained here is thus an upper bound of κ mass, because our result should include excited states.

First, we check finite lattice volume effects by comparing our results for the π and ρ masses as well as the mass ratio m_π/m_ρ with those of the CP-PACS group. The results are summarized in Table 1. Our result for the ρ meson mass is only slightly (< 5 %) larger than the CP-PACS's result. The resulting larger value is reasonable because the smaller lattice size gives rise to a mixture of higher mass states. We rather emphasize that the deviation between our results

and the larger lattice result (CP-PACS) is so small in spite of the large difference in the lattice size.

Table 1: Summary of results for $\bar{q}q$ type mesons.

$h_{u/d}$	0.1589	0.1583	0.1574	0.1566	0.1557
m_π	0.2064(62)	0.2691(36)	0.3401(29)	0.3935(28)	0.4478(28)
m_ρ	0.442(13)	0.461(06)	0.496(05)	0.527(04)	0.563(03)
m_π/m_ρ	0.467(21)	0.584(10)	0.686(05)	0.746(03)	0.796(03)
m_{σ_v}	1.12(74)	0.84(23)	0.886(98)	0.857(52)	0.897(35)
CP-PACS					[25]
m_π	0.20827(33)	0.26411(28)	0.33114(26)	0.38255(25)	–
m_ρ	0.42391(132)	0.44514(96)	0.47862(71)	0.50900(60)	–
m_π/m_ρ	0.491(2)	0.593(1)	0.692(1)	0.752(1)	–

In Fig. 2, we show m_π^2, m_ρ and m_{σ_v} in the lattice unit as a function of the inverse hopping parameter $1/h_{u/d}$ for the u/d quark. The chiral limit ($m_\pi^2 = 0$) is obtained at $h_{u/d} = 0.1598(1) \equiv h_{\mathrm{crit}}$ ($1/h_{\mathrm{crit}}$=6.2581). We find the lattice spacing $a = 0.1038(33)$ [fm] in the chiral limit from the value $m_\rho a = 0.406(13)$ at this point with the physical ρ meson mass being used for m_ρ. Note that these values are consistent with the CP-PACS's result, $h_{\mathrm{crit}} = 0.1598315(68)$ and $a = 0.1020(8)$ [fm], within the error bars.

In Table 1, the mass of the valence σ for each hopping parameter is shown; the valence σ, which is denoted as σ_v, is defined as the scalar meson described solely with the connected propagator. The mass ratio m_{σ_v}/m_ρ varies from 2.5 ($h_{u/d} = 0.1589$) to 1.6 ($h_{u/d} = 0.1557$), which is consistent with our previous results [23]. In other words, without the disconnected part of the propagator the "σ" mass becomes heavy.

The propagators of the K, K^* and κ mesons are calculated with the same configurations using the s-quark hopping parameter, $h_s = 0.1566$ and 0.1557. For $h_s = 0.1557$, the effective mass plots of the K^* and κ mesons are shown in Figs. 3 and 4. The masses of the K, K^* and κ mesons, which are extracted from the effective mass plots [26] , are summarized in Tables 2 and 3. Errors are estimated by jack-knife method. We find that the effective masses of the K and K^* mesons have only small errors and are taken to be reliable, while that of the κ meson suffers from large errors, especially at larger time regions. To avoid possible large errors coming from the data at large t, we fit the effective mass of the κ meson only in the time range $5 \leq t \leq 7, 8$ where the effective masses are almost constant with small errors. Since the effective mass of the K^* meson is reliable, we show the mass of the κ in terms of the ratio to m_{K^*}: Table 4 gives the mass ratios m_K/m_{K^*} and m_κ/m_{K^*} at the chiral limit together with m_ϕ/m_{K^*} for $h_s = 0.1566$ and 0.1577. For example, $m_\kappa/m_{K^*} = 0.89(29)/0.4649(69) = 1.92(61)$ at $h_s = 0.1566$ in Table 4. These calculated mass ratios are shown in
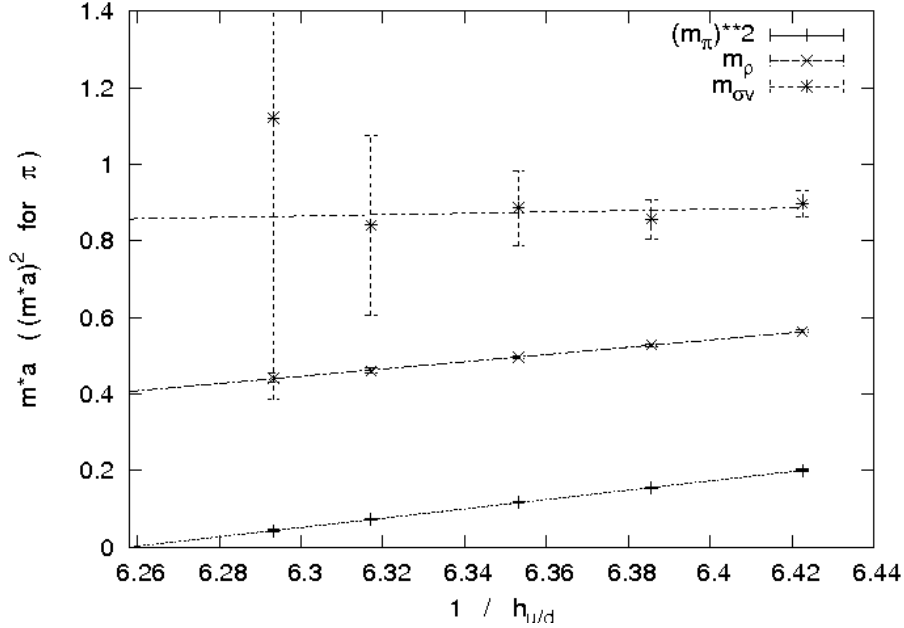
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Figure 2: m_π^2, m_ρ and m_{σ_v} in the lattice unit as a function of the inverse $h_{u/d}$. The chiral limit is obtained at $h_{\text{crit}} = 0.1598(1)$.

Fig. 5. All the mass ratios are almost independent of h_s. Although the error bar for m_κ/m_{K^*} is large, the behavior as a function of h_s is reasonable.

We have searched for the physical value of the s quark hopping parameter h_s in the following two ways, both of which are found to give similar results: 1) By tracing a regression line for m_ϕ/m_{K^*} (Fig. 5), we have $h_s = 0.1563(3)$ (or $1/h_s = 6.396(13)$) for $m_\phi/m_{K*}=1019[\text{MeV}]/892.0[\text{MeV}]=1.143$ (input), taken from the PDG [4]. This hopping parameter gives the mass ratio $m_\kappa/m_{K^*} = 1.89(55)$. 2) We have also determined the hopping parameter so as to reproduce the mass ratio $m_K/m_{K^*} = 495.6[\text{MeV}]/892[\text{MeV}] = 0.5556$, with $m_K = 495.6$ [MeV] being the average value of the Kaon masses given in the PDG [4]. The resulting value is found to be $h_s = 0.1576(2)$ (or $1/h_s = 6.3452(80)$), which in turn gives the mass ratio $m_\kappa/m_{K^*} = 2.00(80)$. The mass ratios obtained using methods 1) and 2) are also presented in Table 4. Both methods give almost identical results for the masses of the κ, that are about twice that of the K^*.

3   Concluding remarks

The motivation of our lattice study is to reveal the nature of the scalar meson nonet, and the results should be important especially in clarifying how the κ
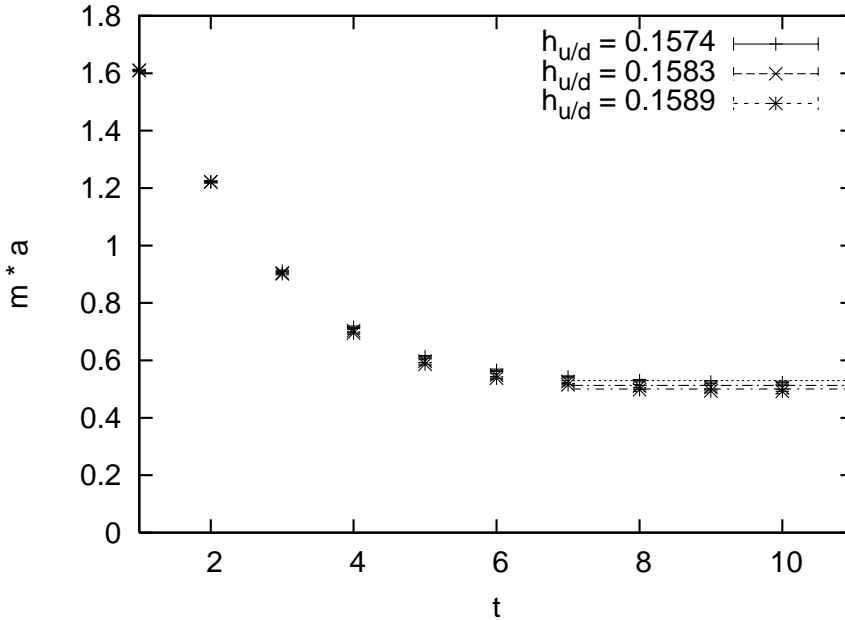
Figure 3: Effective mass plots of K^* for s quark hopping parameter $h_s = 0.1557$.

meson with a reported low mass ~ 800 MeV obtained from experiments can be compatible with the valence or constituent quark model: the κ is a P-wave $q\bar{q}$ bound state in the non-relativistic quark model, and the κ meson constitutes a nonet together with the σ meson and the a_0 mesons.

There have not been many lattice studies of κ meson. Recently, estimations of the κ meson have been reported by two groups. Prelovsek et $al.$ [27] have presented a rough estimate of the mass of the κ as 1.6 GeV, which is obtained using the average quark mass of the u and s quarks from the dynamical simulations with the degenerate $N_f = 2$ quarks on a $16^3 \times 32$ lattice. Mathur et $al.$ have studied $u\bar{s}$ meson with the overlap fermion in the quenched approximation and obtained a mass of the $u\bar{s}$ scalar meson to be 1.41 ± 0.12 GeV [28]. The UKQCD Collaboration has studied to some extent the κ meson using the dynamical N_f=2 sea quarks and a valence strange quark on a $16^3 \times 32$ lattice [29]; they estimated the κ mass as about 1.1 GeV, which is much smaller than those in [23, 24, 27] but still far from the experimental value $\sim$800 MeV.

In this paper, we have presented the lattice simulation results in the quenched approximation for the κ meson; the results on the π, ρ, K and K^* mesons are also shown for comparison.

We have first checked that the masses of the π, ρ, K and K^* mesons obtained in our simulation are in good agreement with those on a larger lattice ($32^3 \times 56$) [25]; our results are only within five percent larger than the latter. Our
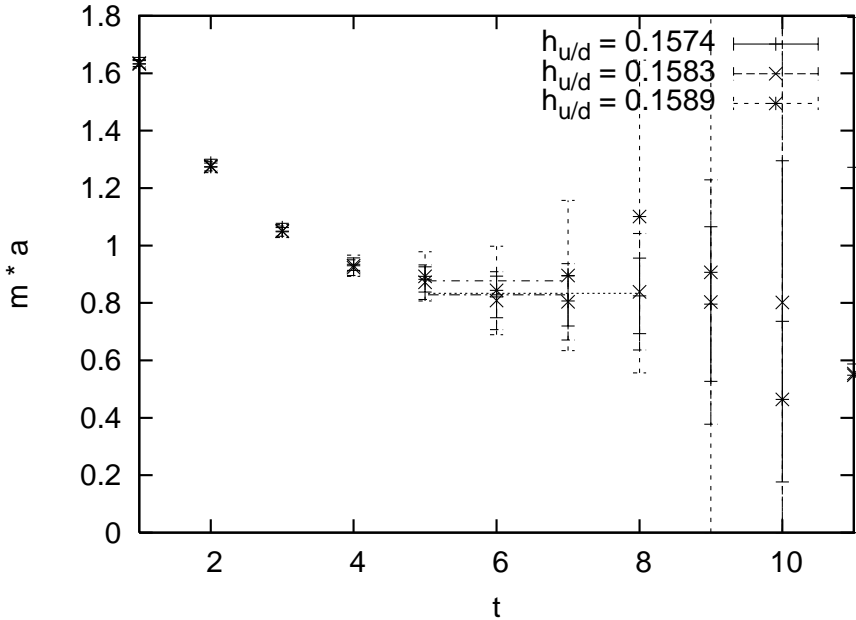
Figure 4: Effective mass plots of the κ meson for the s quark hopping parameter $h_s = 0.1557$.

estimated value of the mass of the κ is ~ 1.7 GeV, which is larger than twice the experimental mass ~ 800 MeV. This result was expected on the basis of our experience in calculating the σ meson. The relatively heavy mass of the κ may be at least partly attributed to the absence of the disconnected diagram in the κ propagator; the κ propagator is composed of only a connected diagram. While the disconnected diagram was essential for realizing the low-mass σ [23], it does not exist for the κ; therefore, the mass of the κ is not made lighter by the disconnected diagram. Indeed, the mass of the valence σ_v described solely with the connected propagator is far larger than the experimental value ~ 500-600 MeV, as seen in Table 1.

Our lattice study and the quark model analysis[30] suggest that the simple two-body constituent-quark picture of the κ meson does not agree well with the experimentally observed κ. Note that the quench simulation is a clean theoretical experiment in which a virtual intermediate like $qq\bar{q}\bar{q}$ is highly suppressed [13]. Therefore, if its existence with the reported low mass is experimentally established, the dynamical quarks may play an essential role for making the κ mass so lighter or the κ may contain an unconventional state such as a $qq\bar{q}\bar{q}$[31] or $K\pi$ molecular state[32], which are missing in the calculation here.

In order to establish this possible scenario, the systematic errors should be much reduced in future simulations. Our statistics here is reasonably high (80

Table 2: Summary of results for the K, K^* and κ mesons at $h_s = 0.1566$.

$h_{u/d}$	$h_{\rm crit}^{1)}$	0.1589	0.1583	0.1574
m_K	0.2829(23)	0.3138(33)	0.3368(30)	0.3677(29)
m_{K^*}	0.4649(69)	0.4821(57)	0.4941(49)	0.5117(42)
m_κ	0.89(29)	0.88(23)	0.81(12)	0.814(81)
	CP-PACS			[25]
m_K	–	0.30769(28)	0.32833(26)	–
m_{K^*}	–	0.46724(84)	0.47749(74)	–

[1] $h_{\rm crit} = 0.1598(1)$.

Table 3: Summary of results for the K, K^* and κ mesons at $h_s = 0.1557$.

$h_{u/d}$	$h_{\rm crit}^{1)}$	0.1589	0.1583	0.1574
m_K	0.3188(25)	0.3474(31)	0.3684(29)	0.3971(28)
m_{K^*}	0.4835(61)	0.5006(52)	0.5126(44)	0.5299(37)
m_κ	0.89(21)	0.88(16)	0.828(96)	0.833(72)

[1] $h_{\rm crit} = 0.1598(1)$.

configurations separated by 2000 sweeps), and the standard meson masses have small error bars; see Fig.3. On the contrary, as seen in Fig.4, the effective mass of κ suffers from large errors for large t, which may be due to a small overlap of the physical states. This is not surprising because κ is a P-wave meson, and expected to be extended. Choosing more adequate extrapolation operators and with much higher statistics, we can study the dynamics of hadrons by comparing results in the quenched lattice QCD, full lattice QCD and various effective theories/models that include the constituent quark models with and without the tetra-quark structure, chiral effective theories.

Table 4: Summary of results for the mass ratios m_K/m_{K^*} and m_κ/m_{K^*} together with m_ϕ/m_{K^*} at chiral limit for u/d quarks.

h_s	0.1566	0.1557	0.1563(3)	0.1576(2)
$1/h_s$	6.3857	6.4226	6.396(13)	6.3452(80)
m_ϕ/m_{K^*}	1.135(10)	1.164(10)	$1.143^{1)}$	–
m_K/m_{K^*}	0.6086(79)	0.6593(63)	0.623(11)	$0.5556^{1)}$
m_κ/m_{K^*}	1.92(61)	1.84(43)	1.89(55)	2.00(80)

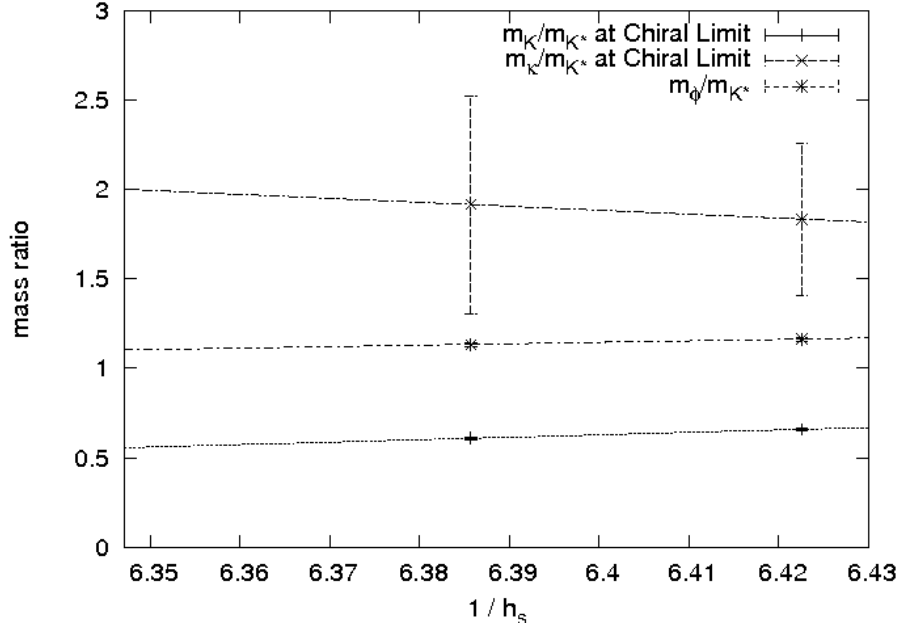[1] inputs for calculation of physical value of h_s. See the text.

Figure 5: The ratios m_K/m_{K^*} and m_κ/m_{K^*} at chiral limit, and m_ϕ/m_{K^*} for s quark hopping parameters $h_s = 0.1566$ and 0.1557.

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