International Cooperation Project
Towards the Endogenous Development of Mathematics Education

開発途上国における理数科教育協力の評価指標に関する実証的研究
ー農村部児童の基礎学力の充実を中心にー

Empirical Study on the Evaluation Method for International Cooperation in Mathematics Education in Developing Countries
- Focusing on Pupils' Learning Achievement -

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Final Report

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Preface

まえがき

基礎教育の完全普及を目指す「万人のための教育(以下、EFA：Education for All)」の理念は、1990年タイのジョムティエンで一堂に会した国際機関、先進国、開発途上国に共有されている教育開発目標であった。しかし、2000年にセネガル共和国の首都ダカールで開催された世界教育フォーラムでは、途上国の教育を取り巻く様々な問題が未解決のままであることが指摘された。行動目標型で評価を語る試みは、様々な形でかつ様々な国際協力機関でなされているが、ありうべき評価の枠組みを確立するためには継続的かつ基礎的な研究が必要不可欠である。国際理数科教育協力においても「授業」に注目する意義が語られる一方で、我々は途上国の授業の実相とその歴史的・社会文化的背景を真に把握しているとはいいがたい。

教育協力における評価の枠組みは、各国の教育経験に大きく影響される。換言すれば、評価指標は対処療法的である。現行の国際教育協力においても授業研究は重要な位置を占めているものの、教育開発の一手段的な位置づけであり、プロジェクト評価全体の枠組みはいまだ事前と事後の比較による行動主義的な評価という傾向が強い。行動主義的評価における明確さ、客観化、システム化、数量化、効率化を特徴とする方法論の重要性を軽視するものではないが、そうした評価枠組みだけでは国際教育協力にとって本質なこと（たとえば、組織の内発性、自立発展性、教師の自律性、職業意識、子どもの創造的な思考力、生きる力など）を明確にできない。従って、評価手法を開発する際に「測れないもの」の存在を意識し、手法の有効性と同時に限界を十分に踏まえて多面的な視座から開発にあたることが重要である。複雑で重層的な授業を対象にして、それを適切に評価分析するためには、事前-事後の行動主義的評価が一項目となるような、新たな評価枠組みの開発が急務といえよう。

本研究の目的は、行動主義的な「測定」と「予測」よりはむしろ「記述」と「説明」を重視し、複数の国を対象としてプロセス重視の評価項目と手法を確立することであった。途上国の特に算数における教授-学習環境や児童の基礎学力の実際を把握する際に、これらに影響を及ぼす教育の歴史性、社会文化性に注目したことは必然であったといえよう。

本研究では、異なる歴史的、社会文化的背景をもつ途上国の具体的な教育情報は不可欠であり、平成16年度と平成17年度に現地調査を実施した。また、次のように毎年ワークショップを日本で開催し、課題の具体化とその解決に向けた議論を共有し蓄積した。

平成16年度　数学教育の内発的発展に向けた第1回国際ワークショップ
　日時：平成17年3月15日より17日まで
　場所：広島大学大学院国際協力研究科
平成17年度　数学教育の内発的発展に向けた第2回国際ワークショップ
　日時：平成17年12月11日より13日まで
　場所：広島大学大学院国際協力研究科
平成18年度　国際教育協力シンポジウム2007「数学教育の自立的発展に向けた国際協力のあり方」
　日時：平成19年1月14日
　場所：国際協力機構(JICA)国際協力総合研修所

まず、この 3 年間をとおして、オーストラリア国モナシュ大学名誉教授の Alan J. Bishop 先生の助言を仰ぐことができたことに感謝したい。また、第３回研究集会は、３年間の研究成果を確認する意味で、筑波大学教育開発国際協力センター(CRICED)の支援を得て、文部科学省・国際協力機構(JICA)・日本数学教育学会・日本科学教育学会の後援の下、東京で開催することができた。教育開発に関心をお持ちの多くの参加者に向けて、米国イリノイ州立大学の Norma C. Presmeg 先生による「数学教育における文化の役割」と題する基調講演を企画できたことは、本科研の掉尾を飾るにはできすぎた展開であった。本報告書は第３回国際ワークショップで発表された論文をもとにまとめたものである。したがってワークショップの詳細については、本文をお読みいただきたい。

最後になったが本書をこのような形に編集できたのも、また３度にわたるワークショップを何とか実施できたのも、研究分担者・協力者の方々の尽力に負うことがほとんどであった。ここにその名や研究発表実績を記して謝意を表しておきたい。

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Levi Elipane	埼玉大学・教育学部・院生

【国外研究組織】

氏名	所属・部局・職名
研究協力者	
Alan J. Bishop	Professor Emeritus, Monash University, Australia
Wee Tiong Seah	Lecturer, Professor Emeritus, Monash University, Australia
A.H.M. Mohiuddin	Specialist, NAPE (The National Academy for Primary Education), Bangladesh
Dai Quin	Professor, Inner Mongolia University of Education, China
Jin Kangbiao	Graduate of IDEC, China

Ernest Davis Kofi	Lecturer, University of Cape Coast, Ghana
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Narh Francis	Lecturer, Mount Mary College of Education, Ghana
Milagros D. Ibe	Emeritus Professor, Dean, University of the Philippines-Diliman, the Philippines
Maitree Inprasitha	Associate Professor, Khon Kaen University, Thailand
Norma Presmeg	Professor, Illinois State University, USA
Bentry Nkhata	Lecturer, University of Zambia, Zambia

【研究発表】

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(2) 学会発表

内田豊海、中村聡、馬場卓也「ザンビア基礎学校における生徒の分数概念獲得の到達段階について」国際開発学会第 17 回全国大会、2006 年 11 月 24・25 日、東京大学.

【交付決定額（配分額）】

	直接経費	間接経費	合計
平成１６年度	3,500,000	0	3,500,000
平成１７年度	2,800,000	0	2,800,000
平成１８年度	3,000,000	0	3,000,000
総計	9,300,000	0	9,300,000

以上

平成１９年３月

研究代表者　岩崎 秀樹
（広島大学大学院教育学研究科教授）

Preface

The principle of "Education for all (EFA)" has been a common goal in educational development among United Nations agencies and developed countries as well as developing countries since the World Conference on Education for All at Jomtien, Thailand in 1990. In World Education Forum at Dakar, Senegal in 2000, however, it was pointed out that the various issues surrounding the education in developing countries remain unsolved. While the attempt of evaluation has been made by establishing specific goals for action in various organizations, a continuous and foundational research is essential to build the framework of evaluation as it ought to be. While the significance to focus on reality of "lessons" is often mentioned in international cooperation in education, it is doubtful that we are fully aware of the reality of lessons in developing countries and its historical and sociocultural background.

A framework of evaluation in international cooperation in education is greatly affected by the educational experience in each country. And also, evaluation indicators tend to be used for symptomatic treatments. While "Lesson study", which put emphasis on the process of teaching and learning, is regarded as an important measure in international cooperation in education by Japan, the framework of project evaluation is still based on a behavioristic way of ex ante and ex post analysis. We never understate the significance of clarity, objectivity, systemicity, quantification, and efficiency advocated in the behavioristic evaluation. We, however, would like to make aware of and place greater value on the essence in international cooperation in education, such as endogenousness of educational body, autonomy of teaching profession, and creativity of children. In developing an alternative framework of evaluation from a multidimensional perspective, it is important for us to have a consciousness of unmeasurable factors and think of both the effectiveness and the limitation of certain methodology. In order to appropriately evaluate intricate and multitiered lessons, it is urgent issue to develop an alternative framework of evaluation in which the behavioristic evaluation become part of the indicators.

This research project aims at developing the indicators and procedure with the emphasis on the process of teaching and learning for some participating countries, valuing "description" and "interpretation" rather than behavioristic "measurement" and "prediction". When we address the reality of children's achievement and teaching-learning environment in mathematics education, it is inevitable for us to focus on historical and sociocultural aspects of education. Since the living educational information in developing countries with a great variety of historical and sociocultural background is essential in this research project, we implemented field surveys in 2004 and 2005. In addition, we held the following international workshops to share and accumulate the result of the field surveys to clarify and discuss wide-ranging issues and the solutions.

i) The first international workshop towards the endogenous development of mathematics education,
   Date and venue: March 15-17, 2005, in IDEC, Hiroshima University.
ii) The second international workshop towards the endogenous development of mathematics education,
   Date and venue: December 11-13, 2005, in IDEC, Hiroshima University.
iii) The international symposium on international cooperation towards the endogenous development of mathematics education,
   Date and venue: January 14, 2007, in the Institute for International Cooperation, Tokyo.

Here, I would like to make a most cordial acknowledgment to those who contributed to this three-year project. Fist of all, I would like to express my sincere gratitude for the professional assistance by Professor Emeritus Alan J. Bishop, Monash University, Australia. There could not have been progress without his insightful messages.

The third international meeting mentioned above was held to ensure the result of our three-year activities as the international symposium in cooperation with the Center for Research on International Cooperation in Educational Development (CRICED), Tsukuba University, under the auspices of the Ministry of Education, Culture, Sports, Science and Technology; Japan International Cooperation Agency (JICA); Japan Society of Mathematics Education; and Japan Society for Science Education. It is indeed an honor for us to have the keynote speech, "The world role of culture in mathematics education" by Professor Norma C. Presmeg, Illinois State University.

This final report is mainly based on the reports presented in the international symposium in 2007. I would like readers to go through with interest the full report of this research project.

Lastly, I greatly appreciate efforts made by all the research members towards the progress of this research project, with regards to implementing of the field surveys, holding of the international workshops and the symposium, and compiling the final report.

Hideki IWASAKI
Professor
Graduate School of Education
Hiroshima University,
March 2007

Chapter 1

Research Background and Methodology

Chapter 1 Research Background and Methodology

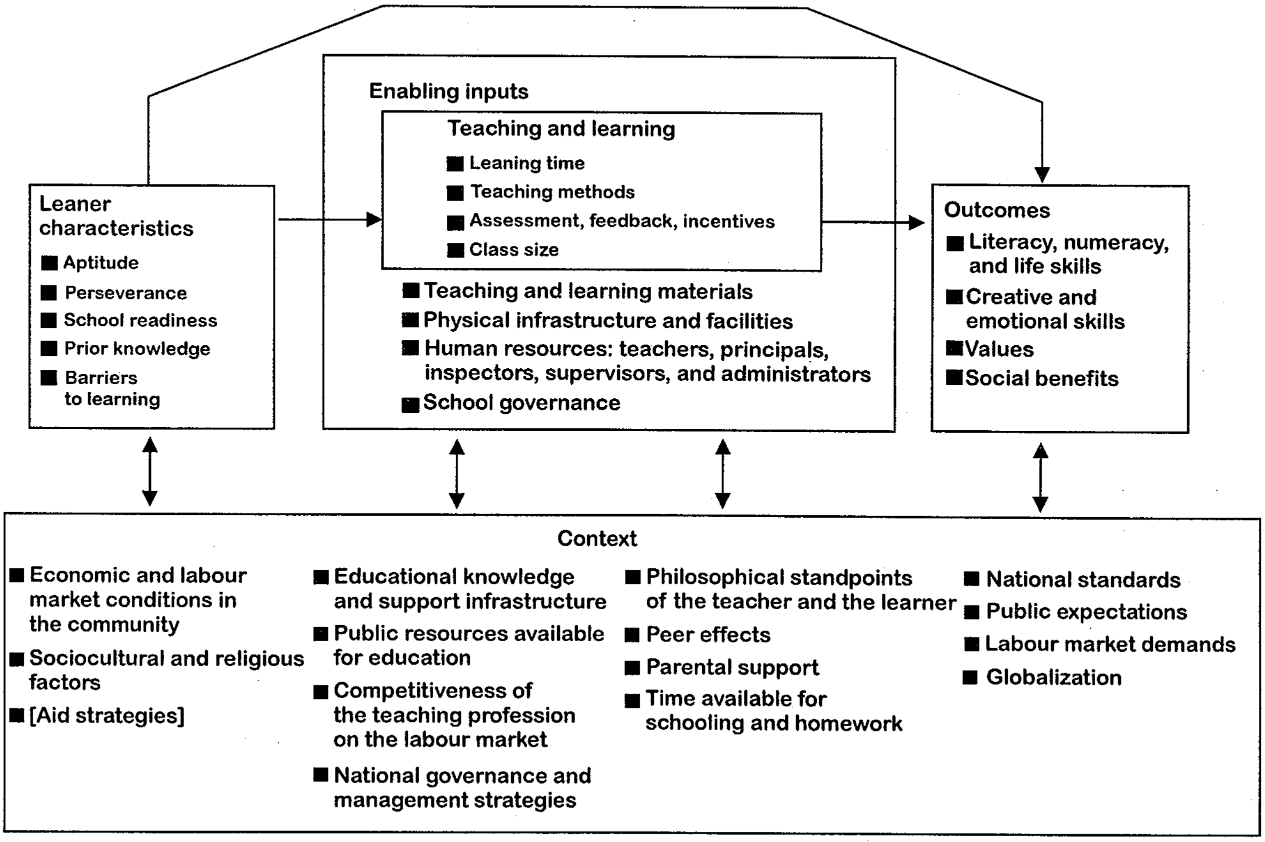
1.1 Research Background and Objective

1.1.1 Background

Since the World Conference on Education for All at Jomtien, Thailand in 1990, international organizations and individual countries have been seriously engaged in improving the quality of basic education in developing countries. In this global context, since 2004, our research group has been engaged in identifying the status of mathematics education from the viewpoints of children's learning and its several interrelated factors as well as teaching in the participating countries through collaborative and joint research.

In this research project, we especially attach importance to the context of teaching and learning in mathematics education. As Figure 1 shows below (UNESCO, 2005), teaching and learning process is in the center of whole education quality framework. And also, the involvement of several interrelated factors and interpretations make it difficult to define and realize quality education, which is a major task since 1990's conference. It is a very urgent and long-term issue to grasp the status of teaching and learning and its context.

Figure 1. A framework for understanding education quality



1.1.2 Objective

This research group has conducted an international comparative study with above-mentioned background for the period 2004–2006. The specific objectives of this project are as follows:

(1) To compare and analyze the children's achievement in mathematics education in the participating countries with both an internationally recognized test such as TIMSS and an alternative test. Simple comparisons, however, cannot be made to decide whether one is better than any other country.

(2) To exchange views and information on the issues found in (1). Through this process, several factors which were unconsciously embedded in teaching and learning would be sought for an alternative framework of the evaluation of children's achievement.

(3) Finally, to establish a professional and long-term relationship or network among the various participating countries in the field of curriculum development and teacher education especially in mathematics education through the process of knowledge sharing. In this research project, we build the foundation to find the way towards "the endogenous development" (Tsurumi and Kawata, 1989) in mathematics education.

1.2 Research Members

The list of the research project members both in Japan and overseas is provided below. Apart from the members on the list, there are three special advisors—renowned professors in this field—whose valuable contributions added another dimension to this report.

Table 1. Members in Japan (As of March, 2007)

NAME	Affiliation	Position	Role
Hideki Iwasaki	Faculty of Education, Hiroshima University	Professor	Overall manager, China
Atsumi Ueda	Faculty of Education, Hiroshima University	Professor	China
Takuya Baba	IDEC, Hiroshima University	Associate professor	Coordinator, Bangladesh
Masami Isoda	CREICED, Tsukuba University	Associate professor	Thailand
Hideki Maruyama	National Institute of Educational Policy Research	Researcher	Development of research tools, Ghana
Hisashi Kuwayama	Faculty of Education, Hiroshima University	Research Associate	Coordinator, Ghana
Hiroyuki Ninomiya	Faculty of Education, Saitama University	Associate professor	Philippines
Kazuma Ohara	Naruto University of Education	Associate professor	Thailand
Satoshi Nakamura	IDEC, Hiroshima University	Researcher	Zambia
Uddin MD Mohsin	IDEC, Hiroshima University	Graduate Student	Bangladesh
Toyomi Uchida	IDEC, Hiroshima University	Graduate Student	Zambia
Chikara Kinone	IDEC, Hiroshima University	Graduate Student	Japan
Levi Elipane	Saitama University	Graduate Student	Philippines

Table 2. Members Overseas (As of March, 2007)

NAME	Affiliation	Position	Role
A.H.M. Mohiuddin	NAPE (The National Academy for Primary Education)	Specialist	Bangladesh
Dai Quin	Inner Mongolia University of Education	Professor	China
Jin Kangbiao	IDEC, Hiroshima University	Graduate	China
Ernest Davis Kofi	University of Cape Coast	Lecturer	Ghana
Ghartey Ampiah	University of Cape Coast	Senior Lecturer	Ghana
Narh Francis	Mount Mary College of Education	Lecturer	Ghana
Milagros D. Ibe	University of the Philippines-Diliman	Emeritus Professor, Dean	Philippines
Maitree Inprasitha	Khon Kaen University	Associate Professor	Thailand
Bentry Nkhata	University of Zambia	Lecturer	Zambia

Table 3. Special Advisor (As of March, 2007)

NAME	Affiliation	Position	Country
Alan J. Bishop	Monash University	Professor Emeritus	Australia
Wee Tiong Seah	Monash University	Lecturer	Australia
Norma Presmeg	Illinois University	Professor	USA

1.3 Contents of Report

This report consists of four sections, namely, Research background and Methodology, Special Contribution, Country Report, and Discussion and Future Issues. In Special Contribution, research trends and issues have been intensively discussed.

With regard to the objectives, Country Report is the primary section of this report. It presents the data in the format that conforms to our prior agreement, which is as follows:

(Composition of Country Report)

- Basic Information
  - School Curriculum
  - Schooling Age
  - School Calendar and Examination
  - Promotion System
  - Medium of Instruction
  - Class Organization
  - Period of Survey
  - Target Schools

- Results from the Field Survey
    - Coverage of the Achievement Test in Curriculum
    - Results of the Achievement Test
    - Results of the Pupils' Questionnaire and Interview
- Discussion

The Country Report presents at first basic data such as schooling age and promotion system, which we think have given impact on pupils' learning. And result and discussion are presented afterwards. The result presents the data collected through the field survey. And the discussion part presents some points of interests.

The "Future Issues" constitutes also very crucial part of this research. It provides the comprehensive discussion over the whole research and projects regarding some important issues for future activities. This cycle "projection –research (field survey) - discussion" gives us foundation for our ultimate goal of the Endogenous Development of Mathematics Education.

Besides these, in order to strengthen the discussion, some past researches will be reviewed and summarized.

1.4 Research Methodology

1.4.1 Target Schools and Grade

Average schools from both urban and rural areas were chosen after consultation with local experts. Basically, we conducted the field survey in the same schools (for 2004 and 2005). In some countries, where language problem appeared to influence the result, an additional comparative survey was conducted (e.g., no explanation vs. explanation in local language).

Grade four was chosen as the original focus in the planning of this research. The reasons were as follows: 1) TIMSS tools were available for grade four in the first year survey. 2) Grade four was perceived as a transitional stage from the concrete operational period to the formal operational period. 3) Besides this, countries like Bangladesh had five years for primary education and the final year was influenced by other factors such as the scholarship examination. However, there were some exceptions in target grade due to the timing of each field survey in participating counties.

1.4.2 Research Tools in 2004

The research tools such as a questionnaire and tests were prepared by the Japanese team and revised through discussion among co-researchers.

As for Pupils' achievement test, the prototype was designed on the basis of items in TIMSS 1995 and 2003. Table 4 shows the structure of test items by content domains in the pupils' achievement test. The ratio in numbers of the items by content domains in our tests was nearly the same as in TIMSS, such as 40% for Number, 15% for Patterns, Relations and Functions, 21% for Measurement, 14% for Geometry, and 10% for Data. Some items were, however, excluded from TIMSS items, as this survey did not focus on complicated problem solving, but basic learning achievement. As a result, the ratio in numbers of the items by cognitive domains in TIMSS 2003 was not preserved in this project. The ratio of multiple choice question to free answer question were not maintained either, such as 67% for multiple choice question and 33% for free answer question, while 57% for the former and 43% for the latter in TIMSS 2003.

Table 4. No. of Items in Mathematics Test Items by Content Domains

	Items from TIMSS 2003 (Involving 37 'Trend' items (51% of full items))				Items from TIMSS 1995 except 'Trend' Items	Total
	Knowing Facts and Procedure	Using Concepts	Solving Routine Problems	Reasoning		
Number	8	8	6	1	6	29(40%)
Patterns, Relations, and Functions	0	2	2	1	6	11(15%)
Measurement	5	1	3	2	4	15(21%)
Geometry	4	0	3	1	2	10(14%)
Data	0	0	3	1	3	7(10%)
Total	17(24%)	11(15%)	17(24%)	6(8%)	21(29%)	72(100%)

Table 5 shows the main topics in the mathematics achievement test items. Especially, some free response questions were set for Fraction and Decimals to investigate how pupils understand concepts. Mathematical content of each item were basically preserved to examine the process and result of TIMSS critically from the perspective of endogenous development of mathematics, while the setting of some questions were modified to exclude culturally unfamiliar terms in some countries (not all the participating countries).

Table 5 Contents of Mathematics Test Items

Content Domains	Main Topic	No. of Items
Number	Whole Numbers	18
	Fractions and Decimals	10
	Ratio, Proportion and Percent	1
Patterns, Relations, and Functions	Patterns	4
	Equations and Formulas	4
	Relationships	3
Measurement	Attributes and Units	8
	Tools, Techniques and Formulas	7
Geometry	Lines and Angles	1
	Two- and Three-dimensional Shapes	2
	Congruence and Similarity	1
	Locations and Spatial Relationships	2
	Symmetry and Transformations	4
Data	Data Representation	4
	Data Interpretation	3
Total		72

Questionnaire for pupils were also based on TIMSS research tools. It was slightly modified to suit to the situation in developing countries. It focused on affective aspect of pupils' achievement and their learning environment including out-of-school surroundings.

Both mathematics achievement tests and questionnaires are attached to this report.

1.4.3 Research Tools in 2005

In 2005, the research tools such as test and interview items were designed to evaluate children's understanding about fraction. The reason was that fraction had been perceived as the first topic in abstract mathematics, so that it would be suitable for grade four pupils who were on a transitional stage from the concrete operational period to the formal operational period.

In principle, any additional explanation which affected how to get the correct answer was prohibited to give to pupils. However, since we implemented mathematics achievement test, not English test, the following three ways of implementation were allowed depending upon the conditions of participating countries.

(a) Neither of reading in mother tongue nor additional explanation of how to express the answer (Standard method)

(b) Reading of test item in mother tongue, but no additional explanation of how to express the answer (Alternative method I)

(c) Both Reading of test item in mother tongue and additional explanation of how to express the answer. (Alternative method II)

In case of (b) or (c), any additional sample had better be chosen for comparative survey with the case (a), if possible.

As for the procedure for interview, we selected ten pupils, top five pupils and bottom five pupils, according to the result of the latest end-of-term examination. We used the Newman procedure to investigate their understanding about the questions and the strategy for problem-solving.

Both mathematics achievement tests and interview items are attached to this report.

[Newman Procedure]

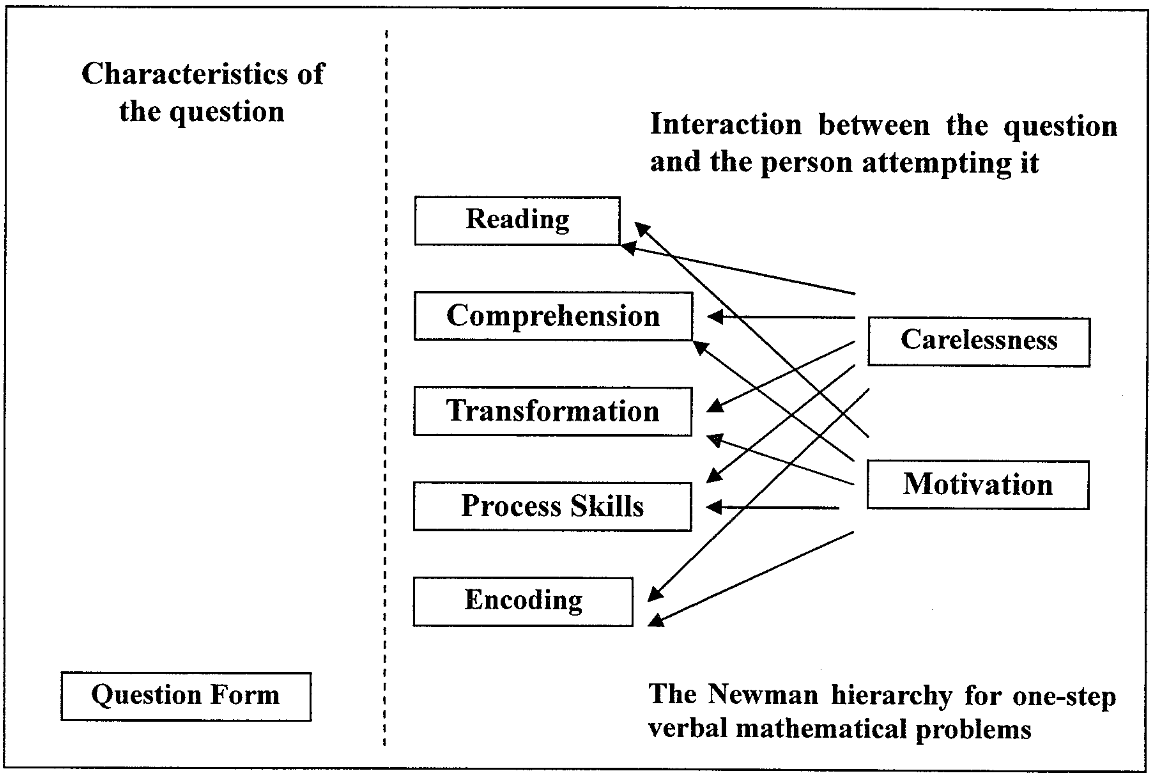
The Newman Procedure for analyzing errors on written mathematical tasks. Since 1977, when the Australian educator M. A. Newman published data based on a system she had developed for analyzing errors made on written tasks, a steady stream of research papers has been published in many countries in which data from many countries have been reported and analyzed along line suggested by Newman.

Analyses of data based on Newman Procedure have drawn special attention to (a) the influence of language factors on mathematics learning (b) the inappropriateness of many 'remedial' mathematics programs in schools in which there is an over-emphasis on the revision of standard algorithms.

According to Newman Procedure with a one-step written problem has to read the problem, then comprehend what he/she read, then carry out the transformation from the words to the selection of an appropriate mathematical 'model', then apply the necessary process skills, then encode the answer.(see Figure 2)

Errors due to the form of the question would appear to be essentially different from errors in the other categories shown in the above because the source of difficulty resides fundamentally in the question itself rather than in the interaction between the question and the person attempting it. This difference is indicated by the category labeled 'Question Form' being placed beside the five-step hierarchy. Two other categories 'Carelessness' and 'Motivation' have also been shown as separate from the hierarchy although, as indicated, these types of errors can occur at any stage of problem-solving process. A careless error, for example, could be a reading error, or a comprehension error, and so on; similarly, someone might decide not to try to get the correct answer for a problem even after he has read and comprehended it satisfactorily, and carried out an appropriate 'transformation'

Figure 2. The Newman Procedure



Newman recommended that the following 'questions' or requests be used in interviews that are carried out in order to classify students' errors on written mathematical tasks:

1. Please read the question to me. If you don't know a word leaves it out. (Reading)
2. Tell me what the question is asking you to do. (Comprehension)
3. Tell me how you are going to find the answer. (Transformation)
4. Show me what to do to get the answer. Tell me what you are doing as you work. (Process skills)
5. Now write down the answer to the question.

1.5 Plan of Activity

FIRST YEAR: To conduct a preliminary study in order to generally describe the status of mathematics education and its problems in each participating country.

(1) To contact the local expert

The following countries were selected: Ghana and Zambia in Africa; Bangladesh in South Asia; Myanmar, Thailand, and Philippines in Southeast Asia; and China in East Asia. In each country, we selected a curriculum developer or a teacher trainer. Apart from this, we presupposed that everybody could be contacted by email.

(2) To discuss the research methodology

We provided the first draft plan for methodology and then, for further refinement, discussed it with the local experts. Although, on the whole, we desired to describe the general features of mathematics education in each country, a comparative evaluation of the same was not our intention. We analyzed the points of similarity and difference. Further, in order to describe the average status of each country, we compared contrasting entities, e.g., urban vs. rural.

(3) To conduct the preliminary study

We requested each local expert to conduct the preliminary study. They were allocated a budget, which enabled them to conduct the preliminary study, along with video shooting, translation of the questionnaires, execution of the survey, and its analysis.

(4) The Japanese experts visited different countries for quality control and went on site to confirm the data collected.

(5) International workshop

An international workshop was held to discuss and share information and problems, and to plan for the following year.

(6) Compilation of the report of the first year study

The results of the first year preliminary study were compiled as a report.

Figure 3. Plan of Activity

	2004				2005												2006		
	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
Preliminary Study3) & 4)	—	—	—	—	—	→													
Workshop 5)							★									★			
Report 6)									✹										✹

SECOND YEAR: To conduct the main study and to analyze and discuss the findings.

(1) To discuss the research methodology

Based upon the discussion during the first international workshop, the Japanese experts developed a draft of research tools and discussed it with the local experts with regards to the main study.

(2) To conduct the main study

The local and Japanese experts conducted the main study. The Japanese data was also collected for the discussion of the first year.

(3) International workshop

The workshop discussed and shared information and the results of the research.

(4) Compilation of the report of the second year study

THIRD YEAR: To write a summary report of the three-year activity and to plan for the next three years.

(1) To conduct a complementary study

If necessary, a complementary study will be done.

(2) To conduct an international seminar in Tokyo, Japan

(3) Compilation of the final report

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Chapter 2

Country Report

Chapter 2 Country Report

2.1 Bangladesh

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2.1.1 School Curriculum

Since 1992, a curriculum known as the Essential Learning Continuum (ELC) comprising 53 competencies has been introduced at the primary education level in Bangladesh. The ELC is a listing of competencies that serves as a guide for determining what to teach and assess among students at the primary level. Competency has been defined as "the acquired knowledge, ability, and viewpoint when these could be applied in real life at the right time" (translated from the original mathematics curriculum by the author). These competencies are based on the psychological maturity, age, and physical capability of the children, relate to the changes taking place in our society, demands for the future, existing physical facilities in schools, and the preparation of teachers at the school level. Children are expected to acquire these competencies in the five-year term of primary education; these competencies are referred to as the "terminal competencies of primary education." It is claimed that this goal provides the all-round profile or vision of development for a child who completes the five-year cycle of primary education.

From among the 53 competencies, 5 are mathematical. These are as follows:

- To gain the basic ideas of number and to be able to use them.
- To know the 4 fundamental operations and to be able to use them.
- To apply the simple methods of computation/calculation to problem-solving in everyday life.
- To know and use the units of money, length, weight, measurement, and time.
- To know and understand geometrical shapes and figures.

2.1.2 Basic Information

(1) Schooling Age

The official age for beginning school is 6 years; however this varies from case to case. There is no birth registration system in Bangladesh. Certain parents, particularly in rural areas, are unaware with regard to the age of their children. Occasionally, they measure age according to the height of the children.

(2) School Calendar and Examination

The school academic year begins from January and ends in December. In general, there are 3 school terms in Government Primary School (GPS): The first term (January–April); the second term (May–August), and the third term (September–December). Table 1 shows the school calendar in 2005; shaded portions represent long holidays. These holidays depend on the lunar calendar and vary with each year.

Table 1. School calendar in 2005

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
▨									▨	▨	

There is no nationally established examination system from grades 1 to 4 at the primary level in Bangladesh. There is a term examination at the end of each term in individual schools. Moreover, there is a special national examination in grade 5 known as the primary scholarship examination. Every GPS is required to send students in the top 20 percent, who are completing grade 5, for this examination. Merely 31.7 percent of the examinees passed the scholarship examination in 2001 (Directorate of Primary Education, DPE, 2002). Table 2 shows the school calendar in 2005; shaded portions represent term examinations.

Table 2. School examinations

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
			▨				▨				▨

(3) Promotion System

The promotion system in grades I and II is relatively relaxed. To a certain extent, it is an automatic promotion system. In grades III, IV, and V, it is compulsory for each student to give an annual exam in their individual schools and pass the exam. The pass percentage for each subject is 33.

(4) Medium of Instruction

The medium of instruction at the primary school level is Bangla. Bangla is the mother tongue for a majority of the people (e.g., Muslim and Hindu) and the national language (official language) in Bangladesh.

(5) Class Organization

At the primary level, teachers teach all subjects in different grades; there is no subject-specific teacher. Each lesson has a duration of 35 minutes.

A majority of the primary schools function with a two-shift schooling system. The first shift is from 9:30 am till 12:00 noon and is for grades I and II; the second shift is from 12:30 pm till 4:15 pm and is for grades III, IV, and V.

Schools that function with just one shift begin at 9:30 am and continue till 1:15 pm for grades I and II and till 4:15 pm for grades III, IV, and V.

In two-shift schools, the weekly class load for a teacher is 46.6 lessons in rural schools and 43.5 lessons in urban schools. In one-shift schools, the weekly class load of a teacher is 39.5 lessons in rural schools and 38.5 lessons in urban schools (The National Academy for Primary Education, NAPE, 2002).

Table 3. School hours

	First shift						Second shift			
Hour	8	9	10	11	12	13	14	15	16	17
Grade	▨	▨	Grade I and II			▨	Grade III, IV and V			▨

2.1.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 4. Schedule of data collection

Date	Activity
25th November, 2004	Arrival in Dhaka, Bangladesh
26th November, 2004	Mymensing (NAPE)
27th &28th November, 2004	NAPE (Meeting for the research, questionnaire revision, correction etc.)
29th & 30th November, 2004	Data collection in Urban school, Mymensing
1st & 2nd December, 2004	Data collection in Rural school, Mymensing
3rd December, 2004	Return to Dhaka
4th December, 2004	Stay in Dhaka
5th December, 2004	Departure from Dhaka, Bangladesh

(2) Target Schools and Samples

In Bangladesh, government primary schools are categorized into 4 levels—A, B, C, and D. This categorization is based on certain factors such as School Management Committee (SMC), Parent-Teachers Association (PTA), results of the students, results of scholarship examination, playground, plantation, scout group, etc. A majority of the schools belong to category B.

From among 1,249 GPS, 1 average GPS (B category school) was selected from an urban area and 1 average GPS (B category school) was selected from a rural area in Mymensingh district (Primary Education Statistics in Bangladesh, 2001). It must be noted that Mymensingh is one of the 64 districts in Bangladesh.

Table 5. Location of schools

	School location
Urban Primary School	Center of Mymensingh city
Rural Primary School	About 30 kilometers away from the main city

The selected urban school is situated in the center of Mymensingh city and the rural one is situated approximately 30 kilometers away from the main city. The former has a total of 6 teachers and the latter 5. These teachers have to teach all the subjects in different grades; there is no subject-specific teacher at the primary level.

There are a total of 6 rooms in both schools; 5 rooms are used as classrooms and the remaining as office-cum teachers' common room. The rooms used as classrooms are well decorated with colorful pictures and figures from textbooks of social studies, science, mathematics, and geometry. Verses from the holy Quran and a selection of morals are also written on the walls inside the classrooms.

Table 6. Distribution of sample

	Grade	Boys	Girls	Total
Urban Primary School	4	14	32	46
Rural Primary School	4	15	14	29
Total		29	46	75

Table 7. Distribution of students' age

Age (Years)	No. of Pupils		
	Urban	Rural	Total
8	4	11	15
9	13	3	16
10	17	8	25
11	3	1	4
12	4	4	8
13	1	1	2
14	4	1	5
Average	10.2	9.7	10

(3) Results of Student Questionnaire

(a) About how many books are there in your home?

Table 8. Distribution of students' books (%)

	0-10 books	11-25 books	26-100 books	100- books	No response	Crossed out
Combined	72.0	14.7	1.3	5.3	2.7	4.0
Urban school	67.4	21.7	2.2	4.3	2.2	2.2
Rural school	79.3	3.4	0.0	6.9	3.4	6.9

(b) Do you have any of these items at your home?

Table 9. Distribution of students' home items (Yes, %)

	Calculator	TV	Radio	Desk	Quiet place	Dictionary	Books (Excluding text book)	Computer
Combined	30.7	60.0	44.0	60.0	40.0	18.7	49.3	5.3
Urban school	34.8	71.7	43.5	60.9	41.3	26.1	52.2	2.2
Rural school	24.1	41.4	44.8	58.6	37.9	6.8	44.8	10.3

(c) How much do you agree with these statements about learning mathematics?

Legend: 1:agree a lot, 2: agree a little, 3: disagree a little, 4: disagree a lot

Table 10. students' attitude towards mathematics

Pupils' Perceptions	Average (%)		
	Combined	Urban	Rural
1. I usually do well in mathematics	1.3	1.1	1.7
2. I would like to do more mathematics in school	1.5	1.3	1.7
3. Mathematics is harder for me than for many of my classmates	2.2	2.5	1.6
4. I enjoy learning mathematics	1.5	1.4	1.5
5. I am just not good at mathematics	2.2	2.3	2
6. I learn things quickly in mathematics	1.6	1.4	1.8
7. I think learning mathematics will help me in my daily life	1.4	1.3	1.7
8. I need mathematics to learn other school subjects	1.5	1.4	1.5
9. I need to do well in mathematics to get into the university of my choice	1.3	1.2	1.5
10. I need to do well in mathematics to get the job I want	1.4	1.3	1.4

(d) How often do you do these things in your mathematics lessons?

Legend: 1: every or almost every lesson, 2: about half of the lessons, 3: some lessons, 4: never

Table 11. students' activities in their mathematics lessons

Students' activities	Average (%)		
	Combined	Urban	Rural
1. I practice adding, subtracting, multiplying, and dividing without using a calculator	1.6	1.4	1.7
2. I work on fractions and decimals	1.4	1.3	1.6
3. I measure things in the classroom and around the school	2.9	2.8	3.1
4. I make tables, charts, or graphs	2.0	2.2	1.6
5. I learn about shapes such as circles, triangles, and rectangles	1.7	1.6	1.7
6. I relate what I am learning in mathematics to my daily lives	1.8	1.8	1.7
7. I work with other students in small groups	1.7	1.8	1.4
8. I explain my answers	1.9	1.8	2
9. I listen to the teacher talk	1.2	1.1	1.2
10. I work problems on my own	1.9	1.7	2.1
11. I review my homework	1.6	1.5	1.6
12. I have a quiz or test	1.7	1.6	1.9
13. I use a calculator	2.9	2.8	3.2

(e) How much do you agree with these statements about your school?

Legend: 1:agree a lot, 2: agree a little, 3: disagree a little, 4: disagree a lot

Table 12. students' attitude towards school and teacher

Pupils' perceptions	Average (%)		
	Combined	Urban	Rural
1. I like being in school	1.1	1.1	1.2
2. I think that students in my school try to do their best	1.4	1.3	1.4
3. I think that teachers in my school care about the students	1.4	1.5	1.5
4.I think that teachers in my school want students to do their best	1.3	1.2	1.4

(f) On a normal school day, how much time do you spend before or after school doing each of these things?

Table 13. students' activities before or after school (%)

	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or more hours
1. I watch TV	46.7	26.7	9.3	1.3	5.3
2. I play or talk with friends	32.0	46.7	9.3	1.3	4.0
3. I do jobs at home	29.3	28.0	13.3	10.7	9.3
4. I play sports	34.7	36.0	17.3	1.3	1.3
5. I read a book for enjoyment	41.3	21.3	17.3	5.3	6.7
6. I use computer	34.7	8.0	5.3	14.7	16.0
7. I do homework	24.0	30.7	13.3	5.3	12.0

(g) Including yourself, how many people live in your home?

Table 14. Distribution of students' family member

Family member	Pupils' Distribution (%)		
	Combined	Urban	Rural
2-3 persons	6.7	1.7	1.7
4-6 persons	50.7	1.9	1.8
7-9 persons	9.3	2.5	2.1
10-12 persons	6.7	2	1.6
More than 12 persons	20.0	2.2	1.7

(4) Results of Achievement Test

(a) The distribution of right answers

Table 15. Distribution of students' right answer (%)

	Average	Boys	Girls	Highest	Lowest
Combined	33.9	35.1	33.1	52.1	15.1
Urban school	36.8	38.6	36.0	52.1	21.9
Rural school	29.1	31.7	26.4	42.5	15.1

Table 16. Distribution of students' right answer according to age

Age (years)	Average right answer (%)		
	Combined	Urban school	Rural school
8	21.8	24.8	20.7
9	28.1	30.5	18.0
10	23.9	25.1	21.5
11	17.7	23.7	18.0
12	28.4	30.0	26.8
13	25.0	30.0	20.0
14	22.4	23.5	18.0

(b) Analysis of the results with regard to content domains

Table 17. Average rates of correct answer by content domain

	Multiple choice		Short answer		Total	
Content domain	No. of questions	Rate of correct answers (%)	No. of questions	Rate of correct answers (%)	No. of Questions	Rate of correct answers (%)
Number	19	42.9	10	26.9	29	37.4
Measurement	12	40.4	3	20.4	15	36.4
Algebra	4	29.7	1	0.0	5	23.7
Data	4	55.0	4	31.3	8	43.1
Patterns, relations, and functions	5	33.3	1	0.0	6	27.7
Geometry	4	22.3	6	19.3	10	20.5
Total	48	37.2	25	16.3	73	31.4

Table 18. Average rates of correct answer of content domain by urban and rural school (%)

Content domains	Urban school			Rural school		
	Multiple choice	Short answer	Total	Multiple choice	Short answer	Total
Number	45.4	31.3	40.5	39.1	20.0	32.4
Measurement	41.7	17.4	36.8	39.1	25.3	36.3
Algebra	30.4	0.0	24.3	28.4	0.0	22.7
Data	57.1	42.9	50	51.7	12.9	32.3
Patterns, relations, and Functions	41.7	0.0	34.7	20.0	0.0	16.6
Geometry	24.5	21.7	22.8	18.9	15.5	16.9

2.1.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 19. Schedule of data collection

Date	Activity
9th November, 2005	Arrival in Dhaka, Bangladesh
10th November, 2005	Mymensing (NAPE)
11th &12th November, 2005	NAPE (Meeting for the research, questionnaire revision, correction etc.)
13th & 14th November, 2005	Data collection in Urban school, Mymensing
15th & 16th November, 2005	Data collection in Rural school, Mymensing
17th November, 2005	Departure from Dhaka, Bangladesh.

(2) Target Schools and Samples

Table 20. Location of schools

	School location
Urban Primary School	Center of Mymensingh city
Rural Primary School	About 30 kilometers away from the main city

The selected urban school is situated in the center of Mymensingh city and the rural one is situated approximately 30 kilometers away from the main city. There are a total of 6 teachers in the former and 5 teachers in the latter. These teachers have to teach all subjects in different grades; there is no subject-specific teacher in these schools.

There are a total of 6 rooms in both schools, where 5 rooms are used as classrooms and the remaining as office-cum teachers' common room. The rooms used as classrooms are well decorated with colorful pictures and figures from textbooks of social studies, science, mathematics, and geography. Verses from the holy Quran and a selection of morals are also written on the walls inside the classrooms.

Table 21. Distribution of sample

	Grade	Boys	Girls	Age	Total
Urban Primary School	5	8	16	10-13	24
Rural Primary School	5	11	12	10-14	23
Total		19	28	10-14	47

(3) Results of Achievement Test

(a) Overview of the result

Table 22. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Urban School	51.7%	55.3%	49.9%	68%	34%
Rural School	33.7%	33.8%	33.6%	53%	11%
All	42.9%	42.8%	42.9%	68%	11%

(b) Question-wise achievement of pupils in percentage

Table 23. Question-wise achievements of pupils in percentage

	School in urban area			School in rural area		
	Combined	Boys	Girls	Combined	Boys	Girls
Q1 (1)	91.6%	100%	87.5%	30.4%	54.5%	8.3%
(2)	89.5%	93.7%	87.5%	60.8%	63.6%	58.3%
Q2	54.1%	37.5%	62.5%	0%	0%	0%
Q3 (1)	96.5%	100%	94.7%	40.5%	36.3%	44.4%
Q3 (2)	79.1%	75%	81.2%	26.8%	6.1%	45.8%
Q4 (1)	92.7%	100%	89%	69.5%	43.1%	93.7%
Q4 (2)	0%	0%	0%	18.4%	29.5%	8.3%
Q4 (3)	33.3%	37.5%	31.2%	16.3%	25%	8.3%
Q5 (1)	35.4%	50%	28.1%	13%	9.1%	16.6%
Q5 (2)	2.1%	0%	3.1%	21.7%	45.4%	0%
Q5 (3)	6.2%	6.2%	6.2%	19.5%	9.1%	29.1%
Q6 (1)	94.7%	96.8%	93.7%	38%	37.5%	38.5%
Q6 (2)	51.5%	98.4%	28.1%	28.2%	19.3%	36.4%
Q6 (3)	0%	0%	0%	13%	9.1%	16.6%
Q7	27.1%	18.7%	31.2%	15.2%	31.8%	0%
Q8	58.3%	50%	62.5%	69.5%	81.8%	58.3%
Q9	60.4%	62.5%	59.3%	58.6%	54.5%	62.5%
Q10	12.5%	0%	18.7%	8.6%	18.1%	0%

(4) Results of Interview using Newman Procedure

We selected 10 students for an interview on the basis of their mathematics exam result. Five of them acquired a good grade in mathematics and the other 5 did not. The results of the students' interview are presented in the following tables.

Table 24. The level of pupils' errors of problem solving in urban and rural schools

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5 (3)	-	-	-	10	10	20	10	9	19	0	1	1
Q6 (1)	-	-	-	6	6	12	2	8	10	8	2	10
Q8	-	-	-	10	10	20	6	8	14	4	2	6

Table 25. The level of pupils' errors of problem solving according to their performance

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5 (3)	-	-	-	10	10	20	9	10	19	1	0	1
Q6 (1)	-	-	-	3	9	12	4	6	10	6	4	10
Q8	-	-	-	10	10	20	9	5	14	1	5	6

Table 26. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	Urban school	Rural School	High performance	Low performance
Difficult words and Expressions	"Correct", "appropriate"	"Correct", "appropriate"	"Correct", "appropriate"	"Correct", "appropriate"
Any remarks regarding problem solving process	1. 1/4 is greater than 1/3, since 4 is greater than 3. 2. A: 1/4, B: 1/3: A=B	1. 1/4 is greater than 1/3, since 4 is greater than 3. 2. A: 1/4, B: 1/3: B=A	1. 1/4 is greater than 1/3, since 4 is greater than 3. 2. A: 1/4, B: 1/3: A=B	1. 1/4 is greater than 1/3, since 4 is greater than 3. 2. A: 1/4, B: 1/3: B=A
Specific mistakes	1. If a denominator is greater than other denominators then that fraction is bigger than others. 2. Don't know how to find out, which is greater than other.	1. If denominator is big then that fraction's value is big 2. Don't know how to find out the solution.	1. If a denominator is greater than other denominators then that fraction is bigger than other fractions. 2. Don't know how to find out, which is greater than other.	1. If denominator is big then that fraction's value is big 2. Don't know how to find out the solution.

Table 27. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	Urban school	Rural School	High performance	Low performance
Difficult words and Expressions		"Process"	"Process"	
Any remarks regarding problem solving process	1. Calculation is accurate but could not explain the process.	1. Added the numerators and denominators. He/she has no idea regarding fraction. 2. Wrote the process but could not find out the solution.	1. Added the numerators and denominators. He/she has no idea regarding fraction.	1. Calculation is accurate but could not explain the process. 2. Wrote the process but could not find out the solution.
Specific mistakes	1. Only wrote the answer that was 5/3 instead of showing any process.	1. Could not find out the least common multiple. 2. Could not write the fraction correctly.	1. Could not find out the least common multiple.	1. Only wrote the answer that was 5/3 instead of showing any process. 2. Could not write the fraction correctly.

Table 28. Pupils' mistakes in problem solving in Q (8)

Q (8)	Urban school	Rural School	High performance	Low performance
Difficult words and Expressions	"Relationship", "in terms of"	"Bracket"	"Relationship", "in terms of", "Bracket"	"Bracket"
Any remarks regarding problem solving process	1. Only wrote the answer.	1. Only wrote the answer that was 5/3. 2. Only wrote the answer.		1. Only wrote the answer that was 5/3. 2. Only wrote the answer.
Specific mistakes	1. Pupils have no idea how to find out the relationship.	1. Pupils have no idea how to find out the relationship.	1. Pupils have no idea how to find out the relationship.	1. Pupils have no idea how to find out the relationship.

2.1.5 Discussion

2.1.5.1 Discussion from the First Year Field Survey

In general, the age of students in grade 4 ranges from 8 to 10 years. In the case of Bangladesh (Table 7), it varies from 8 to 14 years because of repetition of grade and tendency to begin school at a later age, etc. Therefore, approximately 25% students are in the age group of 11 to 14 years.

According to Table 8, over 65% students possess approximately 0–10 books at home excluding magazines, newspapers, and their schoolbooks in both urban and rural schools. In comparison, (Table 9) students in the urban school possess the following items, with the exception of a computer, in a greater number than students in the rural school: Calculator, TV, radio, study desk, a quiet place to study, dictionary, and supplementary books. In the survey, students in the rural school have reported possessing a greater number of computers than those in the urban school; however, it is assumed that the students in the rural school misunderstood the concerned item in the questionnaire.

Table 10 indicates that generally the students have a positive attitude toward mathematics. According to the findings, the average percentage of students who enjoy learning mathematics is 1.5, think learning mathematics will help them in their daily life is 1.4, usually do well in mathematics is 1.3, and need to do well in mathematics to get into the university of their choice is 1.3.

Table 11 indicates that students frequently do a large number of mathematical exercises such as 4 fundamental rules, fractions, draw geometrical shapes, etc. in their mathematics lessons and relate what they are learning in mathematics to their daily lives.

From Table 12 it is apparent that students like their school and teachers. They believe that teachers help students to improve their performance and the students themselves also attempt to do their best in this regard.

Table 13 indicates that students spend some time before or after school to watch TV, play or talk with friends, do a job at home, do homework for class, etc. It is impressive that certain students have reported reading books for enjoyment.

Although the test achievement results of the students is not at the expected level, the performance of students in the urban school (correct answer 36.8%) is better than that of students in the rural school (correct answer 29.1%). The performance difference between the two groups is 7.7%. Table 15 indicates that the performance of both boys and girls in the urban school is also better than that of the rural school. The performance difference between the two groups is 6.9% and 9.6%, respectively.

According to the analysis of content domain, the performance of students in the multiple-choice test is better than in the short-answer test. In this respect, teachers stated that students are unfamiliar with answering written problems. In general, the performance of students is comparatively good in data, number, and measurement content and poor in geometry and algebra content. It appears that teachers lay a greater emphasis on data, number, and measurement content, consequently causing students to perform better in these areas.

The performance of students in the urban school in content domain is better than that of students in the rural school, except in measurement content in the short-answer test where the latter have performed better. The underlying assumption of this exception appears to be that certain items in the measurement content are closely related to the rural context, thereby enabling the students in the rural school to have a better understanding than students in the urban school.

The most difficult question for students in the urban school in the multiple-choice test was Q1-3 and the rate of correct answers is 4.3%; in the short-answer test were Q2-2, Q4-2, Q4-7c, Q6-2, Q6-8, Q6-9, and Q6-12 and the rate of correct answers is 0%. On the other hand, the most difficult question for students in the rural school in the multiple-choice test was Q3-12 and the rate of correct answers is 0%; in the short-answer test the most difficult were Q2-8, Q2-9, Q4-7c, Q6-2, Q6-8, Q6-9, Q6-10, and Q6-12 and the rate of correct answers is 0%. According to the teachers, some of the abovementioned questions were out of syllabus and some were too difficult for grade-4 students in Bangladesh.

Further, the questionnaire reveals that although the general attitude of students toward mathematics is positive, it is not reflected in the result of their achievement test.

Teachers argued that students are not accustomed to this type of questionnaire format and sentence problems. They also stated that certain items in the test were not covered in the syllabus, such as Q1-9, Q3-2, Q3-12, and Q6-9.

Although additional explanation was provided during the examination, it was not sufficient for the students' understanding of the questionnaire and test.

2.1.5.2 Discussion from the Second Year Field Survey

(a) Analysis of the Results of the Students' Achievement Test

Table 22 presents the overall result of the students' score in the achievement test; this result was not at the expected level. The average score is 42.9%. The performance of students in the urban school (average score 51.7%) is better than that of students in the rural school (average score 33.7%). The performance difference between the two groups is 18%. It is evident that both boys and girls from the urban school performed better than those in the rural school. The performance difference between the urban and rural school is 21.5 for boys and 16.3 for girls.

The question-wise analysis of the achievement of students indicates that, in general, the performance of students is comparatively better in Q4 (1), Q1 (2), Q3 (1), and Q6 (1). It appears that to a certain extent students are familiar with these types of questions, which are similar to those in the classroom test. In general, the most difficult problems were Q6 (3), Q4 (2), Q10, Q5 (2), Q5 (3), and Q5 (1).

The most difficult questions for students in the urban school were Q4 (2), Q6 (3), and Q5 (2) and the average score is 0%, 0%, and 2.1%, respectively. On the other hand, the most difficult question for students in the rural school were Q2, Q10, Q 6 (3), and Q 5 (1) and the average score is 0%, 8.6%, 13%, and 13%, respectively. According to the teachers, the patterns of the abovementioned questions are not the same as those of the textbook questions or classroom tests. Teachers also reported that students are not accustomed to this type of questionnaire.

The performance of students in the rural school is comparatively better in Q4 (2) and Q6 (3) than that of those in the urban school. However, the reasons behind this are unclear. It is possible that students in rural areas help their parents in measurement in their agricultural activities; thus, they are familiar with these items.

On the other hand, the performance of students in the urban school is better in Q2 than that of those in the rural school. The reasons behind this, too, are unclear. It is possible that students in the rural school lack conceptual understanding of this topic, which is reflected in their inability to relate division and fractions.

(b) Analysis of Results from the Interview

Interview items for students are divided into the following four categories: (a) reading, (b) understanding of the concept, (c) process, and (d) specific errors (if any).

Table 24 shows the errors of students in problem-solving in urban and rural schools. According to the findings, all students were able to understand Q5 (3), Q6 (1), and Q8; however, none of them could understand the concept of Q5 (3) and Q8 and only a few were able to understand the concept of Q6 (1).

The process skill:

A majority of the students were unable to show the correct process of problem-solving. In Q6

(1) and Q8, half of the students were unable to show the correct process. Further, students in the urban school were able to solve more problems than those in the rural school.

Table 25 shows the level of errors in problem-solving according to the performance of the students. Students in all levels were able to understand Q5 (3), Q6 (1), and Q8; however, none of them were able to understand the concept of Q5 (3) and Q8, and few were able to understand the concept of Q6 (1).

In terms of the process skill, high performers are slightly better than low performers, with the exception of Q8 where the latter have performed better. High-performance students were able to solve a greater number of problems as compared with low-performance students, except in Q8.

In Q5 (3), the students of both schools found it difficult to understand the words "correct" and "appropriate". They committed some errors in the process skill. For example, according to them, 1/4 is greater than 1/3, since 4 is greater than 3 and again some stated that if a denominator is greater than other denominators then that fraction is greater than other fractions, etc.

In Q6 (1), the students of the rural school found it difficult to understand the word "process." They committed some mistakes in the process skill. For example, they added the numerators and denominators in order to find the solution.

In Q8, students of both schools found it difficult to understand the words "relationship," "in terms of," and "bracket." They committed some mistakes in the process skill. For example, they only wrote the answer, which was 5/3, instead of showing any process.

To conclude, it appears that lack of sufficient orientation to various types of problems, the so-called textbook-dependent lesson plan, the tendency of copying problems from the textbook for tests, and lack of adequate stimulation for innovative thinking are the main obstructions for students to achieve a satisfactory level of understanding in mathematics.

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2.2 China

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2.2.1 School Curriculum

- Mathematics education in China has witnessed drastic changes in the last decade. First, the policy of "One curriculum many books" was introduced. Absorbing foreign experiences, particularly that of Japan, the government designs the curriculum and private corporations are entrusted to write books for the same. As a result, there presently are nine textbook companies, although during the 1990s there was only one book that used to be published by the People Education Press.

- In July 2001, the new curriculum "Math Curriculum Standard (数学課程標準)" replaced the previous curriculum "Math Education Curriculum (数学教学大綱)." It will be tested until 2010, and thereafter, it will be implemented in the whole country.

- Among the 1st year samples, the urban schools used books based on "Math Curriculum Standard," and the rural schools used those based on "Math Education Curriculum." With regard to the 2nd year samples, both the urban and the rural schools used the latter.

2.2.2 Basic Information

(1) Schooling Age

Since the promulgation of the "Compulsory Education Law of the People's Republic of China" in 1986, primary education extends over five or six years; junior secondary school, three or four years, with a combination of 6 + 3 or 5 + 4. In both cases, at present, nine years of schooling in primary and junior secondary schools is prevalent in the current education system in China, and almost all of the districts opt for the 6 + 3 education system. Further, the entry-level age for primary schools is six or seven years; junior secondary schools, twelve or thirteen years.

(2) School Calendar and Examination

In China, the academic year includes two semesters: one from the beginning of September to the Chinese New Year (a day between the end of January and February; it varies every year); the other commences after the Chinese New Year and extends till July. Moreover, since 1995, schools have adopted a 5-day week.

Table 1. School calendar in (year)

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
▨		Spring Semester						Fall Semester			

In China, schools adopt the two-semester system: the fall semester (Sep–Jan) and the spring semester (Feb–Jul); they have introduced a 5-day week since 1995. The system of compulsory education is set at six years of elementary school and three years of junior high school. Although they are intended for children aged between six and twelve years, some elementary schools are allowed to accept 7-year-old-children as Grade 1 pupils.

(3) Medium of Instruction

In China, there are fifty-five ethnic minorities besides the Hans, which are in a majority. The Chinese government has adopted an education policy in which each of them is supposed to use both Mandarin Chinese (the language of the Hans) and their own language in school.

The 1st year sample schools are located around Beijing, and the 2nd year sample schools, are around Hohhot (the capital of Inner Mongolia). The inhabitants of both these areas use the Han language (Chinese) in both urban and rural areas.

(4) Class Organization

Daily classes commence at 8 am and end at 5 pm with a two hour lunch break in between. The duration of each lesson is forty-five minutes.

Table 2. School hours

Hour	8	9	10	11	12	13	14	15	16	17
Grade					▨	▨				▨

2.2.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 3. Schedule of data collection

Date	Activity
1st Nov.	CA154 arrival at Beijing
2nd Nov.	Meeting on research Prof. IWASAKI, Mr. LI JIAN ZHONG and Mr. JIN ①Reconfirming research schedule ②Rewarding
3rd Nov	Collecting materials on mathematics to find out the present situation of Chinese mathematics education; "Teaching Plan for Primary Education (2004)," and "Curriculum of Mathematics for Primary education "etc.
4th Nov	Visit Primary School A (at Beijing) and research Grade 4 pupils 10:00 Test part I , 11:00 Test part II , 11:50 Recording classes(Fraction)
5th Nov.	Visit Primary School B (at the province of Hebei) and research Grade 4 pupils. 9:00 Test Part I , 10:00 Test Part II 11:00 Discussion, 12:00 Lunch 14:00－15:00 Visit Primary School attached to Baoding normal school
6th Nov	Discussion at China National Institute for Educational Research

(2) Target Schools and Samples

Table 4. Location of schools

	School location
Urban Primary School	Xuanwu District, Beijing
Rural Primary School	Qingyuan County

Urban Primary School: Founded in 1957, this School has 14 classrooms in two of its buildings, which have special classrooms for music, art, and computers; all classrooms have modern teaching facilities. Further, the school has a 1,200 square meter playground and has also been designated as a "key export-oriented school" by the Xuanwu District.

Rural Primary School: The population of the village is 4,862; the number of pupils is 362; the enrollment rate of pupils is 100%. There are 35 teachers in total, of which 6 are mathematics teachers.

Table 5. Distribution of sample

	Grade	Male	Female	Total
Urban Primary School	4	39	40	79
Rural Primary School	4	33	38	71
Total		72	78	150

Table 6. Distribution of students' age

Age (years)	No. of Pupils		
	Urban	Rural	Total
8	2		2
9	47	1	48
10	29	18	47
11	1	46	47
12		4	4
13		2	2
14			
15			
Average	9.4	10.8	10.1

(3) Results of Student Questionnaire

Table 7. Distribution of students' books (%)

	0-10 books	11-25 books	26-100 books	100-books
Combined	22.82	34.90	31.54	10.73
Urban school	10.26	34.62	39.74	15.38
Rural school	36.62	35.21	22.53	5.63

Table 8. Distribution of students' home items (Yes, %)

	Calculator	TV	Radio	Desk	Quiet place	Dictionary	Books (Excluding text book)	Computer
Combined	64.7	80.1	54.7	68.7	63.3	74	68	40.7
Urban school	82.3	94.9	81.0	84.8	87.3	92.4	87.3	64.5
Rural school	45.1	64.8	25.4	50.7	36.6	53.5	46.5	14.1

Table 9. Students' attitude towards mathematics

Pupils' Perceptions	Average (%)		
	Combined	Urban	Rural
1. I usually do well in mathematics	1.7	1.5	1.8
2. I would like to do more mathematics in school	1.6	1.4	1.7
3. Mathematics is harder for me than for many of my classmates	3.1	3.3	2.8
4. I enjoy learning mathematics	1.4	1.3	1.5
5. I am just not good at mathematics	3.2	3.3	3.0
6. I learn things quickly in mathematics	1.8	1.6	1.9
7. I think learning mathematics will help me in my daily life	2.8	3.0	2.6
8. I need mathematics to learn other school subjects	2.6	2.7	2.5
9. I need to do well in mathematics to get into the university of my choice	3	3.1	2.9
10. I need to do well in mathematics to get the job I want	2.4	2.5	2.3

Table 10. Students' activities in their mathematics lessons

Students' activities	Average		
	Combined	Urban	Rural
1. I practice adding, subtracting, multiplying, and dividing without using a calculator	2.8	3.0	2.6
2. I work on fractions and decimals	2.5	2.7	2.5
3. I measure things in the classroom and around the school	2.9	3.1	2.9
4. I make tables, charts, or graphs	2.9	3.0	2.9
5. I learn about shapes such as circles, triangles, and rectangles	2.4	2.5	2.3
6. I relate what I am learning in mathematics to my daily lives	1.9	1.8	2.1
7. I work with other students in small groups	2.1	2.1	2.2
8. I explain my answers	2.3	2.2	2.3
9. I listen to the teacher talk	1.2	1.2	1.3
10. I work problems on my own	1.8	1.6	2
11. I review my homework	2.1	1.8	2.3
12. I have a quiz or test	2.4	2.2	2.6
13. I use a calculator	3.3	3.9	2.6

Table 11. Students' attitude towards school and teacher

Pupils' perceptions	Average		
	Combined	Urban	Rural
1. I like being in school	1.7	1.5	1.9
2. I think that students in my school try to do their best	1.6	1.5	1.6
3. I think that teachers in my school care about the students	1.2	1.1	1.2
4.I think that teachers in my school want students to do their best	1.2	1.1	1.3

Table 12. Students' activities before or after school (%)

	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or more hours
1. I watch TV	24.8	46.9	22.1	4.8	1.4
2. I play or talk with friends	21.8	35.2.	23.9	11.3	7.8
3. I do jobs at home	11.4	45	25	14.3	4.3
4. I play sports	12.1	36.4	37.9	7.1	6.4
5. I read a book for enjoyment	8.0	31.4	32.1	21.1	7.3
6. I use computer	46.4	37.1	12.9	0.7	2.9
7. I do homework	1.38	35.9	33.8	20.7	8.3

Table 13. Distribution of students' family member

Family member	Pupils' Distribution (%)		
	Combined	Urban	Rural
2-3 persons	39.7	61.0	16.0
4-6 persons	46.6	35.1	59.4
7-9 persons	10.3	2.6	18.8
10-12 persons	3.4	1.3	5.8
More than 12 persons	0	0	0

(4) Results of Achievement Test

Table 14. Distribution of students' right answer (%)

	Average	Male	Female	Highest	Lowest
Combined	72.4	70.5	70.6	98.7	13.1
Urban school	78.0	75.7	81.3	98.7	17.7
Rural school	66.1	67.1	65.3	98.6	8.5

Table 15. Average rates of correct answers by content domain

	Multiple-choice(%)	Short-answer(%)
Number	77.6	73.1
Measurement	70.7	56.0
Algebra	78.2	49.3
Data	81.7	57.0
Patterns, relations and functions	82.3	60.0
Geometry	67.3	67.6

Table 16. Average rates of correct answers of content domain by urban and rural school (%)

Content domains	Urban school			Rural school		
	Multiple -choice	Short -answer	Total	Multiple -choice	Short -answer	Total
Number	80.8	79.2	80	74.1	66.3	70.2
Measurement	75.8	60.8	62.8	64.9	50.7	57.8
Algebra	83.9	51.9	67.9	71.8	46.5	59.2
Data	88.0	69.3	78.7	74.7	43.3	59
Patterns, relations and functions	87.1	57.0	72.1	76.9	63.4	70.2
Geometry	82.6	73.0	77.8	50.4	61.5	56.0

2.2.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 17. Schedule of data collection

Date	Activity
18th November	CA154, (Hiroshima) 14:25–(Beijing) 17:25 CA1116, (Beijing) 20:20–21:25 (Mongolia) ※Reconfirmation of the schedule and activities.
19th November	A.M. : Lecture by Prof. Iwasaki at the Normal school. P.M. : Round-table talks with the mathematics teachers.
20th November	Research on the 5th graders at the urban primary school. Round-table talks with the teachers at the school. Video-shooting.
21st November	Research on the 5th graders at the rural primary school. Round-table talks with the teachers at the school. Video-shooting.
22nd November	Round-table talks with the graduate students at Inner Mongolia Normal University. Lecture at vocational normal school.
23rd November	Meeting for the research contents and marking.
24th November	Train travel : 21:12–7:30 (next morning).
25th November	Arrival in Beijing.
26th November	Back to Japan. CA153 (Beijing) 8:30–(Hiroshima) 1:30

(2) Target Schools and Samples

Table 18. Location of schools

	School location
Urban Primary School	Huhehaote city in Inner Mongolia
Rural Primary School	The suburbs of Inner Mongolia

Huhehaote city and the rural area that are situated in Inner Mongolia were selected on the basis of our 1st year field survey in which Beijing and the rural area were selected. It is extremely important to comprehend the basic achievements as well as the teachers' evaluation on mathematical education in China, where there exist more regional differences than other developing countries.

The selected urban and rural primary schools are situated in Inner Mongolia. The target students included Hans as well as other minority groups; the Han language is used as the medium of instruction.

The condition of the facilities as well as the teachers' academic backgrounds in both schools were mostly similar.

Table 19. Distribution of sample

	Grade	Boys	Girls	Age	Total
Urban Primary School	5	28	31	11.68	59
Rural Primary School	5	28	20	11.98	48
Total		56	51	11.83	107

(3) Results of Achievement Test

Table 20. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Urban School	64.9	65	64.4	98.3	6.8
Rural School	59.7	65	54	91.2	4.2
All	62.3	65	59.2		

Table 21. Question-wise achievements of pupils in percentage

	School in urban area			School in rural area		
	Combined	Boys	Girls	Combined	Boys	Girls
Q1 (1)	98.3	100	96.8	89.6	92.9	85
(2)	96.6	96.4	96.8	87.5	92.9	85
Q2	44.1	39.3	48.4	33.3	35.7	30
Q3 (1)	86.4	85.7	87.1	87.5	85.7	90
Q3 (2)	84.7	85.7	83.9	87.5	92.3	80
Q4 (1)	98.3	96.4	100	89.6	96.4	80
Q4 (2)	6.8	7.1	6.5	4.2	7.1	0
Q4 (3)	13.6	21.4	6.5	4.2	7.1	0
Q5 (1)	54.2	57.1	51.6	62.5	64.3	60
Q5 (2)	28.8	35.7	22.6	10.4	14.3	5
Q5 (3)	64.4	64.3	64.5	39.58	46.4	30
Q6 (1)	100	100	100	89.6	92.9	85
Q6 (2)	98.3	100	96.8	91.2	96.4	85
Q6 (3)	47.5	42.3	51.6	54.2	92.9	50
Q7	61.0	64.3	48.4	50	60.7	35
Q8	38.98	35.7	41.9	37.5	32.1	45
Q9	71.1	67.9	74.2	83.3	89.3	75
Q10	74.6	67.9	80.7	72.9	71.4	75

(4) Results of Interview using Newman Procedure

Table 22. The pupils' errors of problem solving level in urban and rural schools

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5 (3)	0	0	0	-	-	-	2	4	6	8	6	14
Q6 (1)	0	0	0	-	-	-	0	1	1	10	9	19
Q8	0	0	0	-	-	-	8	10	18	2	0	2

Table 23. The pupils' errors of problem solving level according to their performance

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5 (3)	0	0	0	-	-	-	1	5	6	9	5	14
Q6 (1)	0	0	0	-	-	-	1	0	1	9	10	19
Q8	0	0	0	-	-	-	8	10	18	2	0	2

Table 24. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	No	No	No	No
Any remarks regarding the problem solving process.	Did not learn how to solve the problem in a diagram. He thinks about 1/4 and 1/3.	1/4 is smaller than 1/3, because 1/4 has 4 parts, but 1/3 has only 3 parts.	1/4 is smaller than 1/3, because 1/4 has 4 parts, but 1/3 has only 3 parts.	The denominator of A is greater than that of B, and the numerator of A is same as that of B. So A is greater than B. However, she cannot get the correct answer by drawing a figure.
Specific mistakes	-	-	-	-

Table 25. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	No	No	No	No
Any remarks regarding problem solving process	-	Typically, did not use a diagram to solve this kind of problem, so was unable to do it, even though teacher had taught this method before.	Typically, did not use a diagram to solve this kind of problem, so was unable to do it, even though teacher had taught this method before.	-
Specific mistakes	-	-	-	-

Table 26. Pupils' mistakes in problem solving in Q (8)

Q (8)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	No	No	No	No
Any remarks regarding problem solving process	Meat B is 3/5 of meat A.	How much is the total weight of meats A and B? Meat A is 3 kg and meat B is 5 kg, so the answer is 3/5.	Meat B is 3/5 of meat A.	How much is the total weight of meats A and B? Meat A is 3 kg and meat B is 5 kg, so the answer is 3/5.
Specific mistakes	Can not draw a diagram to solve this problem.	She does not understand this problem.	Can not draw a diagram to solve this problem.	She does not understand this problem.

2.2.5 Discussion

(1)

- Students' experience in writing exams

The speed of answering test 1 was slower than that of test 2.

- The explanation was inadequate. For example, when being asked about the number of family members in a multiple-choice question, many pupils selected the number, which corresponded with the number of their actual family members. When queried about the kind of facilities available in their homes, some students chose to skip the question.

- No. of books or possession of items at home

According to Table 7, the highest ratio of possession of items at home in the urban area was in the range of 26 to 100 books (39.74%), but on the other hand, that of the rural area was in the range of 0 to 10 books (36.62%). The highest collective ratio including both areas combined was in the range of 11 to 25 books.

Based on Table 8, it is evident that people in the urban area have more items than those in the rural area. In general, people in the rural and urban areas have a TV (Q.V-b; 80.67%), a dictionary (Q.V-f; 74%) and a desk (Q.V-d; 68.67%).

- Students' attitude toward mathematics

Table 27. Students' attitude toward mathematics

	1	2	3	4
I like mathematics	102(70.8%)	31((21.5%)	6(4.2%)	5(3.5%)
I need to study mathematics to acquire a job of my choice	103(72.5%)	24(16.9%)	10(7.0%)	5(3.5%)
I need to study mathematics to obtain admission into a university of my choice	82(57.7%)	22(15.5%)	17(11.8%)	21(14.9%)

The percentage of pupils that liked mathematics was 70.8%; 72.5% of them thought that they needed mathematics in their daily lives; 58% of them thought that mathematics was important for obtaining admission into a university. These results indicate that the Chinese pupils' consciousness of mathematics is considerably high. Further, this consciousness is present equally in both urban as well as rural areas, suggesting the general positive attitude toward mathematics.

The most difficult question for the pupils was Q.6-9, and the percentage of correct answers was 31%. Teachers explained that it was because the pupils had not yet learned how to draw parallel lines. Fractions, proportions, and decimal numbers were also considered to be difficult questions by the Chinese pupils. Further analysis will be necessary with respect to the questions on fractions (Q.2-2 & Q.3-5) in which the Chinese pupils did not score well. The percentages of the correct answers for these questions are as follows.

Table 28. Percentages of the correct answers in Q 2-2 and Q3-5

	Q.2-2	Q.3-5
Urban	35(44.3%)	41(51.9%)
Rural	6(8.4%)	24(33.8%)
Average	41(27.3%)	65(43.3%)

Regarding the utilization of learning material sets, it was found that all the pupils in the urban area possessed a bag of learning materials and used them frequently in their class. On the other hand, it appeared that pupils in the rural area used the same sets sparingly. This fact appeared to affect the pupils' achievement in some domains. For instance, the pupils' achievements in locations and special relationships displayed the widest gap: 68%; between the pupils from the urban and the rural areas (urban area: 89%; rural area: 21%; total: 57%).

(2)

- Sample pupils G5 had not learned fractions to a great extent; they had only learned the "Basic Concepts of Fractions" at G4.
- The urban school had 1,405 pupils including 280 from among the minorities. There were 4 classes in G5 and 252 pupils. On the other hand, the rural school had 877 pupils, and 735 of them were from other districts.
- The percentage of the correct answer for Q2 was less than 50% and it meant that the concept of fractions was unclear to the pupils. The performance for Q7 too was also poor.
- Q10 required that asked the pupils frame a question. Most of the pupils framed it based on an aspect of algebra; few pupils attempted to frame the question by applying the concepts of quantity in the fractions.

Here, we will analyze the results of the teacher's responses to the questionnaire that required them to describe the appropriate method of teaching fractions. From Table 5, it can be observed that both the urban and rural mathematics teachers wished to teach Q14, "Which is longer 1/4 m or 1/3 m?" in terms of fractional concepts. This implies that the introduction of fractional concepts in China is consistent with the mathematics teachers' responses on Q14.

The lowest percentages of correct answers for the urban and rural areas were 6.8% and 4.2%, respectively. In response to Q4-2, fifty-two out of fifty-nine students from the urban area considered 1/2 m to be half of 2 meters. They had not yet learned mixed fractions at the time of the survey.

The highest percentages of correct answers were for Q1(1), Q1(2), Q6(1), and Q6(2). For the urban area: 98.3%, 96.6%, 100%, and 98.3%; for the rural area: 89.6%, 87.5%, 89.6% and 91.2%. It appears that the textbook generally containing a great amount of arithmetic and problem solving might have been the cause for these higher percentages with respect to the abovementioned questions.

Although the students faced problems in obtaining the answer of Q10, 2/3, most of them used algebraic methods; few students faced problems in terms of fractional concepts like quantity.

References

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2.3 Ghana

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2.3.1 School Curriculum

In Ghana, the school mathematics curriculum (syllabus) is centralized. According to the syllabus, the rationale behind teaching mathematics at the primary school level is to ensure that mathematics at this level emphasizes knowledge and skills that will help pupils to develop the foundation for numeracy. Pupils are expected to be able to read and use numbers competently, reason logically, solve problems efficiently, and communicate mathematical ideas effectively to other people. Their mathematical knowledge, skills, and competence at this stage should enable them to make more meaning of their world and should also develop their interest in mathematics as an essential tool for science, research, and development (MOE, 2001).

The general aims of teaching mathematics at the primary school level, as stated in the 2001 mathematics syllabus for primary schools, include the following:

- develop basic ideas of quantity and shapes
- develop the skills of selecting and applying criteria for classification and generalization
- communicate effectively using mathematical terms, symbols and explanations through logical reasoning.
- use mathematics in daily life by recognizing and applying appropriate mathematical problem-solving strategies.
- understand the processes of measurement and acquire skills in using appropriate measuring instruments.
- develop the ability and willingness to perform investigations using various mathematical ideas and operations.
- work co-operatively with other pupils to carry out activities and projects in mathematics.
- see the study of mathematics as a means for developing human values and personal qualities such as diligence, perseverance confidence and tolerance, and as preparation for professions and careers in science, technology, commerce and a variety of work areas.
- develop interest in studying mathematics to a higher level

The general objectives of teaching mathematics at the primary school level include the following: At end of the instructional period, each pupil will be able to

- socialize and cooperate with other pupils effectively
- adjust and handle number words
- perform number operations
- make use of appropriate strategies of calculation
- recognize and use patterns, relationships and sequence, and make generalizations
- recognize and use functions, formulae, equations and inequalities
- use graphical representations of equation and inequalities

- identify/recognize the arbitrary/ standard units of measures
- use the arbitrary/ appropriate unit to estimate and measure various quantities
- collect, process and interpret data

The primary school syllabus is designed to place significant emphasis on the development and use of basic mathematics knowledge and skills. The following are the major areas of content covered in all the six classes of primary schools:

- Number
- Shape and Space
- Measurement
- Collecting and handling Data
- Problem Solving
- Investigation with numbers

"Number" covers the reading and writing of numerals and the four operations of addition, subtraction, multiplication, and division. "Investigation with numbers" leads pupils to discover patterns and relationships and use the four operations meaningfully. "Shape and space" includes that which used to be known as "Geometry"; it is informally introduced at the primary level by using models and real objects in order to make the content more meaningful and interesting. "Measurement" is intended to involve pupils in practical activities that will enable them to appropriately understand and use the various units. "Collecting and handling data" requires pupils to be involved in the collection of data from various sources and to learn to organize, represent, and interpret the gathered information. "Problem solving" is not considered individually in the syllabus. Problem solving activities occur in nearly all the topics covered in the syllabus as an essential aspect of emphasizing the practical application of mathematics. It is hoped that textbook developers will include appropriate problems that will require mathematical thinking rather than the mere recall and use of standard algorithms.

2.3.2 Basic Information

(1) Schooling Age

Ghana presently practices the 6-3-3 system of preuniversity education, namely, six years of primary education, three years of junior secondary education, and three years of senior secondary education (Educational reforms, 1987). Pupils are supposed to begin school at the age of six, even though their admission to schools depends on their parents' financial situation.

(2) School Calendar and Examination

The school year begins in September and ends in June/July; the school year is divided into three terms, namely, the first, second, and third terms. The first term lasts from September to December; the second term, from January to March; and the third term, from April to June/July.

Table 1. School Calendar

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
▨ 2nd Term		▨	3rd Term			▨		1st Term			▨

(3) Promotion System

Promotions are conducted automatically from primary schools to junior secondary schools (JSS).

At the end of the third year in JSS, students take a national examination (BECE: Basic Education Certificate Examination), and the progression to senior secondary schools (SSS), technical institutes, or national vocational training institutes depends on the result of the BECE.

(4) Medium of Instruction

The Ghanaian language is usually the medium of instruction at the lower primary level (grades one to three); however, from the upper primary to the university levels, the medium of instruction is English (Language policy, 1973)

The controversial issue regarding the medium of instruction at the basic stages from grades one to three has the attendant problem of poor achievements since the larger bulk of pupils are in rural areas wherein English is far from being spoken or written on an everyday basis. At the lower primary level wherein the mother tongue is accepted as the medium of instruction, the lack of relevant materials and difficulties related to the ability of some teachers to communicate in children's native language—the children come from different ethnic areas—have been combined in order to render teaching at that level ineffective.

2.3.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 2 shows the schedule of our field survey. Since we decided to orally translate the test items from English to the local language, it took us considerable time to administer the instrument. The implementation of two tests on the same day seemed to be very tedious for the pupils. Further, we had to use the mathematics periods provided in the class time table in order to avoid a situation where teachers would be unable to teach certain subjects because of the achievement tests. We, therefore, decided to use two days for the mathematics achievement test. We administered questionnaires and conducted two achievement tests in a rural school between February 16 and February 18 and in an urban school between February 21 and February 23. The time duration for each test was also adjusted to provide sufficient time for the oral translation.

Table 2. Schedule of the First Year Field Survey

13th/Feb/2005	Arrival at Ghana
14th	Moving to Cape Coast and Meeting
15th	Necessary Arrangements for the field survey
16th	Questionnaire in a rural school (60min)
17th	Test I (75 min for P4, and 70 min for P5)
18th	Test II (70min for P4 and P5)
19th	Data Input
20th	— (Data Input)
21st	Questionnaire in an urban school (80 min for P4, 45 min for P5)
22nd	Test I (70 min for P4 and P5)
23rd	Test II (65 min for P4, and 70 min for P5)
24th	Departure for Japan

(2) Target Schools and Samples

Sample Procedure: Schools in Ghana are classified into three categories: schools A, B, and C. These categorizations were performed based on the schools' performances in the performance monitoring test (PMT) and the results of the BECE. Since this study focused on an "average school," two basic schools categorized as school B, one each from urban and rural areas, were randomly selected from a list of schools B that was obtained from the Ghana Education Service (GES) district office for the purpose of the study. The sample comprised two average schools, one each from urban and rural areas, in the Cape Coast municipal area in the central region of Ghana. Both schools were noted to be average in terms of fulfillment of equipment.

The urban school was located in the town of Cape Coast and had a staff strength of nineteen teachers, including five KG teachers, six primary school teachers, six JSS teachers, and two head teachers—one each for KG and primary/JSS—who were detached. The school comprised 300 pupils. On the other hand, the rural school was located approximately 25 km north of Cape Coast and had a staff strength of fifteen teachers, including two KG teachers, six primary school teachers, six JSS teachers, and a head teacher who was detached. This school consisted of 180 pupils.

During the planning of our field survey in October 2004, the local team in Ghana recommended that the test and questionnaires be administered to grade five pupils from the primary school, since the pupils who were currently in grade four had been promoted to this grade barely a one month ago. For this reason, the grade four pupils had not covered most of the topics included in the test to be administered. In reality, the field survey was implemented for both grade four and five pupils. In both the schools, six classes were held at the primary school level with class teaching and three classes were held at the JSS level with subject teaching. Both the schools held one class each under grades four and five so that the pupils could be treated as samples. Tables 3 and 4 present the basic information regarding sample pupils.

Table 3. Distribution of Sample

	Grade	Male	Female	Total
Urban Primary School	4	17	21	38
	5	13	15	28
Rural Primary School	4	16	8	24
	5	16	10	26
Total	-	62	54	116

Table 4. Distribution of Pupils' age

Age(years old)		Youngest	Oldest	Average
Rural Primary School	P4	9	14	11.2
	P5	10	16	11.8
Urban Primary School	P4	9	14	10.9
	P5	10	14	11.8

(3) Results of Student Questionnaire

Table 5. Distribution of Pupils' books (%)

		0-10 books	11-25 books	26-100 books	More than 100 books	n.a	Total
Rural	P4	83.3	12.5	0	0	4.2	100
	P5	38.5	46.2	11.5	3.8	0	100
	Total	60.0	30.0	6.0	2.0	2.0	100
Urban	P4	13.2	42.1	34.2	10.5	0	100
	P5	25	46.4	14.3	14.3	0	100
	Total	18.2	43.9	25.8	12.1	0	100
Combined		36.2	37.9	17.2	7.8	0.9	100

Table 6. Distribution of Pupils' Home Items (Yes, %)

	Calculator	TV	Radio	Desk	Dictionary	Computer
Rural	24.0	76.0	92.0	84.0	44.0	12.0
Urban	59.1	84.8	95.5	59.1	51.5	16.7
Combined	44.0	81.0	94.0	69.8	48.3	14.7

(Excludes "no-response" and "impossible to interpret")

Table 7. Pupils' Attitude Towards Mathematics

Pupils' Perceptions	Average				
	Combined	Rural		Urban	
		P4	P5	P4	P5
1. I usually do well in mathematics	1.7	2.0	1.6	1.3	2.0
2. I would like to do more mathematics in school	1.2	1.4	1.1	1.3	1.0
3. Mathematics is harder for me than for many of my classmates	2.2	2.2	2.4	2.1	2.1
4. I enjoy learning mathematics	1.3	1.5	1.3	1.2	1.4
5. I am just not good at mathematics	2.2	2.2	1.8	2.1	2.5
6. I learn things quickly in mathematics	1.8	1.7	2.0	1.6	2.0
7. I think learning mathematics will help me in my daily life	1.1	1.2	1.0	1.1	1.3
8. I need mathematics to learn other school subjects	1.4	1.4	1.6	1.4	1.4
9. I need to do well in mathematics to get into the university of my choice	1.1	1.2	1.2	1.1	1.0
10. I need to do well in mathematics to get the job I want	1.1	1.2	1.0	1.2	1.0

Excludes "no response" and "impossible to interpret"

Legend: 1 (greatly agree), 2 (slightly agree), 3 (slightly disagree), and 4 (greatly disagree)

Table 8. Pupils' Activities in Their Mathematics Lessons

Pupils' Activities	Average				
	Combined	Rural		Urban	
		P4	P5	P4	P5
1. I practice adding, subtracting, multiplying, and dividing without using a calculator	2.0	2.9	1.1	2.0	2.3
2. I work on fractions and decimals	2.4	2.4	2.4	2.3	2.8
3. I measure things in the classroom and around the school	2.4	2.1	2.9	2.5	2.0
4. I make tables, charts, or graphs	2.5	2.2	3.0	2.3	2.8
5. I learn about shapes such as circles, triangles, and rectangles	2.2	2.1	2.6	2.5	1.7
6. I relate what I am learning in mathematics to my daily lives	1.6	1.6	1.6	2.0	1.3
7. I work with other students in small groups	2.5	2.0	2.5	3.2	2.0
8. I explain my answers	1.7	2.4	1.8	1.5	1.2
9. I listen to the teacher talk	2.6	1.3	2.1	3.2	3.5
10. I work problems on my own	1.5	1.8	1.8	1.6	1.0
11. I review my homework	1.5	1.7	1.6	1.3	1.4
12. I have a quiz or test	2.3	1.9	2.3	2.3	2.6
13. I use a calculator	3.8	3.6	3.6	3.9	3.9

Excludes "no response" and "impossible to interpret"

Legend: 1 (every or almost every lesson), 2 (about half of the lessons), 3 (some lessons), and 4 (never)

Table 9. Pupils' Attitude Towards School and Teacher

Pupils' Perceptions	Average				
	Combined	Rural		Urban	
		P4	P5	P4	P5
1. I like being in school	1.0	1.0	1.0	1.0	1.1
2. I think that students in my school try to do their best	1.2	1.4	1.2	1.3	1.0
3. I think that teachers in my school care about the students	1.1	1.1	1.3	1.1	1.0
4. I think that teachers in my school want students to do their best	1.1	1.3	1.4	1.0	1.0

Excludes "no response" and "impossible to interpret"

Legend: 1 (greatly agree), 2 (slightly agree), 3 (slightly disagree), and 4 (greatly disagree)

Table 10. Pupils' Activities before or after School (%)

	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or more hours	n.a.	Total
1. I watch TV	16.4	47.4	17.2	6.0	8.6	4.4	100
2. I play or talk with friends	9.5	32.8	29.3	11.2	14.7	2.5	100
3. I do jobs at home	6.0	25.9	31.0	16.4	16.4	4.3	100
4. I play sports	8.6	30.2	23.3	13.8	19.0	5.1	100
5. I read a book for enjoyment	5.2	25	30.2	22.4	13.8	3.4	100
6. I use computer	87.1	5.2	2.6	2.6	0.9	1.6	100
7. I do homework	1.7	41.4	26.7	8.6	20.7	0.9	100

Table 11. Distribution of Pupils' Family Member

	No. of people	3	4	5	6	7	8-	n.a.	Total
Rural School	P4	4.2	4.2	4.2	8.3	25.0	50.0	4.1	100
	P5	0	0	11.5	23.1	26.9	34.6	3.9	100
	Total	2.0	2.0	8.0	16.0	26.6	42.0	3.4	100

(4) Results of Achievement Test

Table 12. Distribution of Pupils' Score by Their Grade and School Location (%)

		Average	SD	Highest	Lowest
P4	Combined	43.6	9.10	65.8	21.9
	Rural	42.5	10.04	63.0	21.9
	Urban	44.3	8.51	65.8	31.5
P5	Combined	51.4	10.38	69.9	15.1
	Rural	53.7	8.49	69.9	35.6
	Urban	49.2	11.60	65.8	15.1

Table 13. Distribution of Pupils' Score by Their Grade and Gender (%)

		Average	SD	Highest	Lowest
P4	Combined	43.6	9.10	65.8	21.9
	Male	43.1	8.33	64.4	26.0
	Female	44.3	10.17	65.8	21.9
P5	Combined	51.4	10.38	69.9	15.1
	Male	54.0	8.27	69.9	31.5
	Female	48.3	11.82	67.1	15.1

Table 14. Average Rates of Correct Answers by Content Domain (%)

Content domain	P4		P5	
	Multiple-choice	Short-answer	Multiple-choice	Short-answer
Number	42.1	37.9	58.8	59.3
Number (Fraction)	50.2	24.2	60.8	40.7
Measurement	51.9	14.5	45.2	31.5
Geometry	52.0	40.9	51.4	46.3
Data	63.7	14.1	48.1	35.2
Patterns	54.8	67.7	70.7	53.7
Algebra	43.1	54.8	49.5	40.7

Table 15. Average Rates of Correct Answers by Content Domain in Each Location

Content Domain	Question Type	Ghana(P4, %)			Ghana(P5, %)		
		Total	Urban	Rural	Total	Urban	Rural
Number	Multiple Choice	42.1	42.8	41	58.8	57.7	59.9
	Short Answer	37.9	38.6	36.8	59.3	57.7	60.9
Number(Fraction)	Multiple Choice	50.2	46.6	56	60.8	56.6	65.4
	Short Answer	24.2	21.7	28.1	40.7	42.9	38.5
Measurement	Multiple Choice	51.9	59.4	39.9	45.2	44.9	45.5
	Short Answer	14.5	19.3	6.9	31.5	27.4	35.9
Geometry	Multiple Choice	52	48.7	57.3	51.4	59.8	47.1
	Short Answer	40.9	39	43.8	46.3	44.6	48.1
Data	Multiple Choice	63.7	65.8	60.4	48.1	55.4	40.4
	Short Answer	14.1	17.1	9.4	35.2	32.1	38.5
Patterns	Multiple Choice	54.8	51.6	60	70.7	67.1	74.6
	Short Answer	67.7	71.1	62.5	53.7	21.4	88.5
Algebra	Multiple Choice	43.1	40.8	46.9	49.5	36.6	63.5
	Short Answer	54.8	50	62.5	40.7	0	84.6

Table 16. Difference in Average Rates of Correct Answers by Content Domain between Grades and Locations

Content Domain	Question Type	P5−P4			Urban−Rural	
		Total	Urban	Rural	P4	P5
Number	Multiple Choice	16.7	14.9	18.9	1.8	−2.2
	Short Answer	21.4	19.1	24.1	1.8	−3.2
Number(Fraction)	Multiple Choice	10.6	10	9.4	−9.4	−8.8
	Short Answer	16.5	21.2	10.4	−6.4	4.4
Measurement	Multiple Choice	−6.7	−14.5	5.6	19.5	−0.6
	Short Answer	17	8.1	29	12.4	−8.5
Geometry	Multiple Choice	−0.6	11.1	−10.2	−8.6	12.7
	Short Answer	5.4	5.6	4.3	−4.8	−3.5
Data	Multiple Choice	−15.6	−10.4	−20	5.4	15
	Short Answer	21.1	15	29.1	7.7	−6.4
Patterns	Multiple Choice	15.9	15.5	14.6	−8.4	−7.5
	Short Answer	−14	−49.7	26	8.6	−67.1
Algebra	Multiple Choice	6.4	−4.2	16.6	−6.1	−26.9
	Short Answer	−14.1	−50	22.1	−12.5	−84.6

2.3.4 Results from the Second Year Field Survey

Table17. Schedule of the Second Year Field Survey

Date	Activity
21st/Nov/2005	Arrival at Accra and Move to Cape Coast (Japanese Researcher)
22nd	Reconfirmation of the Schedule and Activities
23rd	Mathematics Achievement Test on 5th Graders at Rural Primary School
24th	Mathematics Achievement Test on 5th Graders at Urban Primary School
25th	Interview to Pupils at Urban Primary School
26th	Departure for Japan (Japanese Researcher)
27th	N.A.
29th	Interview to Pupils at Rural Primary School (Local Team)

(1) Target Schools and Samples

For the second-year survey, we used the same sample pupils who were available for the schools that were sampled earlier. The collected data were administered to grade five pupils from the primary school, since they had just recently completed the grade four syllabus.

Table 18. Distribution of sample

	Grade	Total	Male	Female	Age
Urban School	5	35	11	24	11.74
Rural School	5	27	19	8	11.67
Total	-	62	30	32	11.71

(2) Results of Achievement Test

Table 19. Results of the Achievement Test (%)

	Average	Male	Female	Highest	Lowest
Urban School	45.37	52.82	41.96	73	15
Rural School	42.96	46.42	34.75	66	18
All	44.32	48.77	40.16	73	15

Table 20. Question-Wise Achievements of Pupils (%)

	Overall Total	Urban School			Rural School		
		Total	Male	Female	Total	Male	Female
Q1 (1)	87.1	85.8	100	79.3	89	94.8	75
(2)	84.7	90	95.5	87.5	77.8	79	75
Q2	9.7	17	37.5	8.5	0	0	0
Q3 (1)	80.9	85.2	69.7	92.3	75.3	82.5	58.3
Q3 (2)	77.2	66.7	84.8	58.3	90.7	91.2	89.7
Q4 (1)	79.4	71.4	90.9	62.5	89.9	90.75	87.5
Q4 (2)	0.8	1.4	0	2.1	0	0	0
Q4 (3)	21.0	27.9	43.1	20.9	12	11.9	12.5

Q5 (1)	50	40	36.5	41.5	63	68.5	50
Q5 (2)	3.2	5.5	0	8.5	0	0	0
Q5 (3)	24.2	28.5	36.5	25	18.5	21	12.5
Q6 (1)	69.2	72.5	87.5	65.6	64.9	64.5	65.6
Q6 (2)	56.7	65	71.6	62	45.9	59.9	12.5
Q6 (3)	20.0	26.8	29.5	25.5	11.1	9.9	14.1
Q7	12.9	14.3	13.8	14.5	11	13.3	6.25
Q8	14.5	11.4	18.1	8.4	18.5	26.4	0
Q9	53.2	51.5	63.7	45.8	55.5	63.2	37.5
Q10	26.6	27.2	31.8	25	26	31.5	12.5

(3) Results of Interview using Newman Procedure

Table 21. The pupils' Errors of Problem Solving Level in Urban and Rural Schools

Q. No.	Frequency of failure on Problem solving level									Explanation in English			No. of students with Correct Answer		
	I. Reading			II. Understanding of Question			III. Process								
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5(3)	3	2	5	4	5	9	7	6	13	3	1	4	4	2	6
Q6(1)	2	1	3	0	0	0	0	1	1	5	4	9	10	9	19
Q8	5	3	8	9	8	17	9	9	18	1	2	3	2	2	4

Table 22. The pupils' errors of problem solving level by their performance

Q. No.	Frequency of failure on Problem solving level									Explanation in English			No. of students with Correct Answer		
	I. Reading			II. Understanding of Problem			III. Process								
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5(3)	2	3	5	2	7	9	3	10	13	4	0	4	5	1	6
Q6(1)	1	2	3	0	0	0	0	1	1	6	3	9	10	9	19
Q8	3	5	8	7	10	17	8	10	18	1	2	3	3	1	4

2.3.5 Discussion

According to Table 5, pupils in an urban school possess more number of books at home than

Table 23. Pupils' mistakes in problem solving by location in Q5 (3)

Q5 (3)	Urban school	Rural School
Difficult words & Expressions	"choice" "greater than"	"choice" "greater than" "bracket"
Specific mistakes	- The pupil explained the question such that the answer is 1 when the question number is 1, and that the answer is 2 when the question number is 2. - Showing that "$\frac{1}{4}+\frac{1}{3}$" in their writing, and explain a wrong process of the calculation such that the answer is 2 because 1+1=2 in numerators. - The pupil happened to pick up one choice and could not explain why. - Showing the bar diagram which was divided into the wrong portions, and said "$\frac{1}{4}$ is greater than $\frac{1}{3}$."	- The pupil explained the question such that the answer is 1 when the question number is 1, and that the answer is 2 when the question number is 2. - Showing that "$\frac{1}{4}+\frac{1}{3}$" in their writing, and explain a wrong process of the calculation such as (2+3)/7. - Showing the bar diagram which was divided into the correct portions, but said "I added $\frac{1}{4}$ to $\frac{1}{3}$ to get 1." - Showing the bar diagram which was divided into the wrong portions, and said "$\frac{1}{4}$ is greater than $\frac{1}{3}$."

Table 24. Pupils' mistakes in problem solving by performance in Q5 (3)

Q5 (3)	High performance	Low performance
Difficult words & Expressions	"choice"	"choice" "greater than" "bracket"
Specific mistakes	- Showing that "$\frac{1}{4}+\frac{1}{3}$" in their writing, and said "I do not understand the question." - Showing the bar diagram which was divided into the wrong portions, and said "$\frac{1}{4}$ is greater than $\frac{1}{3}$."	- Showing that "$\frac{1}{4}+\frac{1}{3}$" in their writing, and explain a wrong process of the calculation such that the answer is 2 because 1+1=2 in numerators. - The students explained the question such that the answer is 1 when the question number is 1, and that the answer is 2 when the question number is 2. - The pupil happened to pick up one choice and could not explain why. - Showing the bar diagram which was divided into the correct portions, but said "I added $\frac{1}{4}$ to $\frac{1}{3}$ to get 1." - Showing the bar diagram which was divided into the wrong portions, and said "$\frac{1}{4}$ is greater than $\frac{1}{3}$." - Showing the bar diagram which was divided into the correct portions, but said "$\frac{1}{4}$ is greater than $\frac{1}{3}$."

Table 25. Pupils' mistakes in problem solving by location in Q6 (1)

Q6 (1)	Urban school	Rural School
Difficult words & Expressions	"ℓ" as litre "process" "container" - A pupil could not read the question. - A pupil read $\frac{1}{5}$ as one five, $\frac{2}{5}$ as two five	"ℓ" as litre "process" "container"
Specific mistakes		- $\frac{1}{5}+\frac{2}{5}=13$

Table 26. Pupils' mistakes in problem solving by performance in Q6 (1)

Q6 (1)	High performance	Low performance
Difficult words & Expressions	"process"	A pupil read $\frac{1}{5}$ as one five, $\frac{2}{5}$ as two five "ℓ" as litre "process" "container"
Specific mistakes		- $\frac{1}{5}+\frac{2}{5}=13$

Table 27. Pupils' mistakes in problem solving by location in Q (8)

Q (8)	Urban school	Rural School
Difficult words & Expressions	"Judy" "relationship" "terms" "weight" "bought" "George" "fraction" "5 kg of meat" "3kg of meat" - A pupil She could not read the question.	"Judy" "relationship" "terms" "weight" "bought" - It was difficult for the pupil to understand the whole question.
Specific mistakes	- "I did not understand the question." - "I had to add the two numbers, 5 kg and 3 kg." - "I subtracted 3 from 5 to get 2." - "I added 5 to 3 to get 8, and then divided 8 by 5, because I did that because the question required me to express the answer as a fraction."	- "I did not understand the question." - "I added George's meat to Judy's meat." - "I subtracted 3 from 5 to get 2." - "The question wants me to make 3 or 5 a fraction. So, the fraction for 3 is $1\frac{1}{2}$: You have to divide it into 2 to get its fraction. So, if 5 is to be in fraction, it will be $2\frac{1}{2}$.

Table 28. Pupils' mistakes in problem solving by performance in Q (8)

Q (8)	High performance	Low performance
Difficult words & Expressions	"Judy" "relationship" "weight" "bought"	"Judy" "relationship" "terms" "weight" "bought" "George" - It was difficult for the pupil to understand the whole question.
Specific mistakes	- "I subtracted 3 from 5 to get 2." - "I added 5 to 3 to get 8, and then divided 8 by 5, because I did that because the question required me to express the answer as a fraction." - "The question wants me to make 3 or 5 a fraction. So, the fraction for 3 is $1\frac{1}{2}$: You have to divide it into 2 to get its fraction. So, if 5 is to be in fraction, it will be $2\frac{1}{2}$.	- "I did not understand the question." - "I added George's meat to Judy's meat." - "I subtracted 3 from 5 to get 2."

2.3.5 Discussion

(1) Discussion about the first year survey

According to Table 5, pupils in an urban school possess more number of books at home than those in a rural school. Besides, the rate of possession of certain items listed in Table 6, with the exception of "desk and chair," suggests that pupils in an urban area are living in a better material condition. It appears that such conditions will reflect on the pupils' learning environment at home. The result concerning "desk and chair" should be examined again in other research attempts. Table 10 presents the result concerning pupils' activities in a day, and Table 11 indicates the result concerning the number of family members. These are also related to the pupils' learning environment.

The result provided in Table 7 shows that most pupils in Ghana have an affirmative view with regard to mathematics and their learning. Table 9 also shows the pupils' positive impressions regarding their schools and teachers. This may possibly lead the pupils to maintain a positive attitude toward learning mathematics even in the future. Some results, however, should be critically reexamined, since certain questions with negative sentences seem to be misunderstood by pupils. For example, the results do not appear to be significantly different despite the fact that the sentence in item 5 ("I am just not good at mathematics") appears to be almost contrary to that in item 1 ("I usually do well in mathematics").

Although Table 8 illustrates that the choices provided in certain questions may not be appropriate, the results obtained for items 8 ("I explain my idea") and 9 ("I listen to the teacher talk") might be suggestive of the way in which teaching is conducted in rural and urban areas. Therefore, in order to countercheck this tendency, it is necessary to conduct an analysis of the actual lessons conducted with regard to mathematics.

Table 12 provides the overall results of the scores obtained by pupils in the achievement tests. Since it appears to be a matter of course, the total score for grade five pupils was better than that for grade four pupils. With regard to some content domain, however, Table 14 reveals the results to be different. In addition, a contrary result can be found between the schools in the rural and urban areas. Although the result obtained for grade four pupils in the rural school was worse than that obtained for grade four pupils in the urban school, grade five pupils in the rural school scored better than those in the urban school. Based on an interview with the head teacher of the urban school, it was found that the class teacher for grade five had shown a relatively lower performance in the urban school. This was because at the time the teacher was enrolled for an upgrading course at Winneba University without study leave and hence was unable to give his pupils the undivided attention that they needed.

In Table 13, gender is not found to be a consistent tendency. Although a gender gap is often discussed in mathematics education, further research is required to make a thorough investigation in this regard.

With regard to Table 14, we identify some weak areas by grades and by question types. As regards the multiple-choice questions, the following are the weak areas for grade four pupils: (i) Number, (ii) Algebra, and (iii) Fraction. Even though grade five pupils demonstrate improved understanding of "Number" and "Fraction," worse results are obtained for other areas, especially "Measurement" and "Data." Better results are obtained for multiple-choice questions than for short-answer-type questions. Although the pupils demonstrate improvement in most areas, their weak areas in short-answer-type questions, such as "Measurement," "Data," and "Fraction," remain unchanged. In Table 15, barring some exceptions, similar results are observed in both rural and urban schools.

According to the curriculum test match performed by collaborators in Ghana, the following topics covered in the test are not included in the grade four syllabus: (i) subtraction and addition of decimals, (ii) ratio, (iii) identification of a point in space, (iv) the concept of parallel lines and perimeters, (v) identification of surfaces, edges, vertices, etc., of solids, and (vi) transformations. The lack of knowledge regarding these topics was partially but definitely reflected in the pupils' performances. It should be noted that even though some of the topics covered in the tests were unknown to the pupils, some of the grade four pupils answered the questions correctly.

Based on an interview with a grade four teacher, the following are found to be the two major reasons that grade four pupils could answer the questions that we deemed them to be incapable of answering. One reason is that the teacher is still implementing the old curriculum, which includes some of the topics that are no longer a part of the present curriculum. The other reason is that approximately 30% of his pupils attend extra classes and are always ahead of the class. These factors explain why some pupils in grade four performed better than grade five pupils. Moreover, it can be said that the learning content is definitely dependent on what teachers decide to teach. The teachers' decisions might be stronger than the restrictions imposed by the course of study, and this would explain some of the exceptions observed in Tables 15 and 16. Table 16 provides some inconsistent results. The expected order of grades four and five, or of rural and urban areas, would vary by content domains and by question types.

Data presented in Tables 14, 15, and 16 would be strongly affected by the difficulty level of each question item. With reference to data resource, some of the questions that were regarded as the most difficult ones by all pupils in grades four and five were Q2-2 in "Fraction," Q6-11 in "Measurement," Q6-2 in "Geometry," and Q2-1 and Q2-5b in "Data." Apart from these questions, Q1-7, Q3-8, Q5-7, Q4-2, Q4-3, and Q6-5 in "Number"; Q2-4 and Q4-9 in "Fraction"; Q3-9 and Q2-9 in "Measurement"; Q5-2 and Q4-7c in "Geometry"; and Q2-5a in "Data" were regarded as difficult by grade four pupils, although grade five students' understanding of these questions was somewhat improved. For further analysis of the matter, it is important to conduct an analysis on the category of mistakes noted in relation to these questions.

(2) Discussion about the second year survey

The result in Table 19 appears that there is no significant difference between the urban and the rural schools. However, there actually is, when the average is examined by gender. Since the average score of mail pupils is better than that of female pupils, the uneven numbers of male and female pupils keep the difference in the scores from sight. Though this tendency appears to be inconsistent with the results shown in Tables 12 and 13, we have already found a similar inconsistency by content domain in the first year.

In Table 20, the correct answers in some questions are remarkably low in percentage. The lack of pupils' proficiency in English and their accustomedness to the tests seemed to be the one of the reasons why the pupils could not answer some questions appropriately. In order to examine in what extent English proficiency and the accustomedness to tests affect the result of mathematics achievement tests, we conducted interview in Newman procedure.

We selected top-five and bottom-five pupils in accordance with the result of continuous assessment in the class where the achievement test was made in the second year survey. Table 21 shows in which stage the pupils made their mistakes by location. The interview reveals that the difference in the type of questions reflects on the steps in which the pupils made mistakes. In Q6 (1), apart from some difficult terms to read and understand, the number of those who could explain their strategy to solve the question in English is more than that in other questions. Some pupils who could not explain their solving process in English showed their idea in mother tongue. Even if the pupils could not understand some terms or the setting of the question in detail, they would be able to guess which operation to use from the term, "add". Though they could not explain why, many pupils could show their procedural knowledge about addition of fraction. On the other hand, in Q5 (3) and Q8, it was difficult for the pupils to understand the meaning of the questions and explain their solutions in English. Though these questions require the conceptual understanding of fraction, they could only show their guess-work in mother tongue due to difficulties in English expression and .the question settings. Table 22 also shows that this tendency is more remarkable for the bottom-five pupils than the top-five pupils in their daily assessment. It suggests that a linguistic support and a device for posing questions would be essential for those pupils especially in the questions which require understanding of mathematical concept.

In Table 23, there is almost no difference in students' error between the urban and the rural pupils. It appears to be difficult for pupils to understand the setting of Q5 (3), comparing two fractions and answer a symbol which is corresponding to the correct relation. The tendency is more obvious for the bottom-five pupils shown in Table 24. In addition, most pupils explain their solution in their mother tongue. Though some pupils who tried to draw the diagram for the comparison of two fractions, their diagram was not accurate. Surprisingly, a few pupils answered the fraction with a larger denominator is lager than that with a smaller denominator, even though their diagram for comparison was correct. The procedure in their solution did not seem to be logically structured, but just a routine work far from the context of the question.

The result in Table 25 shows the high rate of correct answer in Q6 (1). Many pupils seemed to be able to guess the intention of the question by the key term of "add", apart from full understanding of all the sentences. Table 26 shows that the bottom-five pupils also tended to answer the correct process of addition.

From Table 27, Q8 appears to be difficult to understand the situation. Since pupils did not seem to know the fraction as a ratio, the term of "relationship" did not link with fraction. As a result, some pupils who understand the term of "relationship" tried to show the relationship of two quantities as a difference, so that their process was subtraction. Those who understand only the term of "fraction" arranged a denominator and a numerator with 3 and 5 in an impromptu manner. Other pupils showed the addition of two fractions. It appeared that they seized on the operation of addition as an emergent escape root from the difficulty, since it is the simplest operation to remind. There is no difference by location, but by the daily performance. The top-five pupils tended to show the relationship at least. Except for only one pupil who could explain full understanding of the process in the solution, the difference between the top-five and the bottom-five pupils seems to depend on reading competence in English or the accustomedness to test. Those factors would affect the level of guess-works.

2.4 Philippines

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2.4.1 School Curriculum

The educational ladder in the Philippines is considered to be one of the shortest in the world. Elementary school education in this country is for a period of six years, after which the student can be admitted to secondary school, which lasts for four years. Together, ten years of elementary and secondary school comprise what is referred to as Basic Education.

2.4.2 Basic Information

(1) Schooling Age

A child enters the first year of elementary education at the age of six and is admitted to Grade 1 of elementary school. The child must be at least 5 years and 10 months before the commencement of the school year. The age of the child is calculated from his birth date. A majority of the public elementary schools still do not include preschool, i.e., below Grade 1. However, preschool is now offered—usually by private schools—and it is for the following ages and sequence:

Nursery – $3\frac{1}{2}$ to $4\frac{1}{2}$ years old

Kindergarten – $4\frac{1}{2}$ to $5\frac{1}{2}$ years old

A child who completes Kindergarten and meets the age requirement (6 years) may go on to Grade 1 in either a public or private school.

(2) School Calendar and Examination

The school year comprises 10 months. The Department of Education stipulates that the school year must have 200 to 205 school days. The school year usually begins in the first or second week of June in each calendar year and finishes in the last week of March or the first week of April of the following year. The end of the school year is partially influenced by the Catholic liturgical calendar. Usually, schools end before the Holy Week, although a few schools continue even after Holy Thursday of the Lenten season. The school calendar permits 2 weeks (12 to 15 days) for break from Christmas till New Year's Day. The summer vacation is for a duration of two months—April and May.

The year-end examinations are usually conducted from mid-March till close to Easter (this year, 2006, the school year ended on March 24). The National Examination for Grade 6 is conducted during January or February, if not earlier. The new school year begins on the first Monday in June in certain schools, otherwise on the second Monday.

Table 1. School calendar in (year)

Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
▨			Summer Break								▨

(3) Promotion System

There is almost 100 percent promotion in elementary school, except in the case of students who completely lack the skills required for the next grade. Retention or repeating a grade is more frequent in the higher grades (Grades 5 and 6) and at the secondary-school level, not in the first 4 grades in public schools.

In the initial grades, retention of students in the same grade has implications for denying a place to incoming students because schools have a shortage of places on account of the large population.

The students obtain numerical marks for each subject in the grade. The final grade is the average of the subject grades. In order to pass the grade level, a student must possess an average grade of 75 in all subjects. The promotion rate is usually 95 percent or higher in Grades 1 to 4 and 85 to 94 percent in Grades 5 and 6. There is seldom 100 percent promotion because of students who drop out or leave school during the school year.

(4) Medium of Instruction

There exists a bilingual policy on the medium of instruction from Grade 3 upward. Science, Mathematics, and English must be taught in English; Social Studies, Filipino Language, Art, Practical Arts, Health and Physical Education are taught in the National Language—Filipino. In Grades 1 and 2, Health Education is taught in lieu of Science. Formal study of English begins in Grade 3. However, certain schools—particularly private ones—begin teaching English in Grade 1; certain schools begin even as early as Kindergarten. Further, the medium of instruction, particularly in elite private schools, is English throughout; however, they have to teach Filipino (the national language) as a subject. Although there is a bilingual policy, it is not fully observed/implemented. The reality is that a large number of teachers are not entirely competent in English or Filipino; hence, there is a combination of languages and switching of codes between Filipino and English.

In Grades 1 and 2, the teacher may use the *lingua franca* or the commonly spoken local language, or the mother tongue of the children in the locality. However, overall, the national language is becoming widely used, if not entirely, at least side by side with English. It is possible that the mother tongue of the teacher is the same as that of the students.

(5) Class Organization

Grades 1 to 4 are usually class-based; they are self-contained, i.e., one teacher teaches all the subjects—she is the homeroom teacher. In Grades 5 and 6, certain teachers handle specific subjects, for example, Math, Science, and English. In the present elementary school curriculum, the four main subjects—Science, Mathematics, English, and Filipino—are each taught for 60 minutes per day, five days a week. A fifth subject—Makabayan—is an integration of Araling Panlipunan (Social Studies), Values, Art, Health and Physical Education, and Practical Arts. Each subject is taught in sessions of 30, 40, or 60 minutes, three or four days a week depending in the subject. Different grades in elementary schools remain in school anywhere from five to seven hours a day, depending on the grade level as well as space (rooms and grounds) in the school.

In the four main subjects, the number of lessons is dependant on the number of class sessions for the subject per week.

2.4.3 Results from the Second Year Field Survey

The Philippines was only able to take part in the Research Team in the second year.

(1) Survey Schedule

Although the counterpart researcher from Japan arrived in the Philippines on December 12, 2005 the field survey could not be conducted immediately because the schools were closed for Christmas Break the following week—school attendance was becoming irregular due to the impending holidays (Christmas is the most important religious celebration in Catholic Philippines).

In the meanwhile, the two researchers conferred and discussed the procedures for the study; necessary permissions were obtained from the two schools.

The actual survey and testing, after meeting with the principals of the schools, was scheduled in January 2006, as follows:

Table 2. Schedule of data collection

Date	Activity
12/01/2006	Data Collection in the Urban School (Quezon City)
13/01/2006	Data Collection in the Urban School (Quezon City)
16/01/2006	Data Collection in the Rural School (Apalit, Pampanga)
17/01/2006	Data Collection in the Rural School (Apalit, Pampanga)
23/01/2006	Departure of the researcher to Japan

The mathematics classes included in the survey were conducted for 60 minutes per day, from Monday to Friday and with the same number of lessons for every class meeting.

(2) Target Schools and Samples

Since over 80% of the enrollment in elementary schools in the Philippines is in public schools, the target schools in this study are public elementary schools; private schools are not included. Further, the schools in the survey included one from an urban area and another from a rural area, thereby providing a representation for both urban and rural schools. Moreover, the rural school must be one where the mother tongue of the students is not the national language

The schools selected were complete elementary schools, i.e., they have Grades 1 to 6, with over one section/class per grade. Grade 4 (or students aged ten or eleven) is the target grade in this study.

Two schools—one from Quezon City in the National Capital Region and the other from the province of Pampanga in a municipality 50km from Manila—were the testing sites.

Table 3. Location of schools

	School location
Urban Primary School	San Jose, Quezon City (in National Capital Region)
Rural Primary School	Apalit, Pampanga (50 km. from Manila)

The urban school is in the capital city, a short distance from the heart of the city. It is an average-ability school in an average-and-below-average-income community. The school also caters to students from three areas that are occupied by displaced families. This school has three sections for each Grade at the primary level.

The rural school is located in a town that is an hour away from Manila, when traveling in a bus, in an agricultural part of the country.

The two schools comprise six grade levels (1 to 6). Grades 1 to 4 are self-contained classes, implying that the teacher of the class teaches all subjects, except practical arts and home economics; these are taught by another teacher. Since only one class in Grade 4 was selected in each school, there are only two Grade-4 teachers included in the field survey.

The urban and rural schools have 20 and 16 classrooms, respectively. Every classroom has normal desks, each for two students. However, since the classes have over 50 students each, certain desks have 3 students seated together.

Table 4. Distribution of sample

	Grade	Boys	Girls	Age	Total
Urban Primary School	4	12	28	9 – 12 y/o	40
Rural Primary School	4	12	29	9 – 12 y/o	41
Total		24	57		81

(3) Results of Achievement Test

The Achievement Test comprised ten numbered items. Certain items included two or three questions; therefore, eighteen independent answers were scored.

The items were either multiple-choice, short-answer, or supply/constructed answers. The last item required the examinee to create a problem for which the answer was $\frac{2}{3}$. Thus, the Achievement Test also assessed the problem-posing ability of students.

The regular Grade 4 teacher of the class introduced the researcher (the one from Japan and his counterpart in the Philippines). The latter spoke to the students in a mixture of Filipino and English in order to put them at ease. She explained the testing and assured them that the scores in the test will not affect their grades in Science or Math.

First, the Filipino researcher asked the class to listen to the initial portion of the English instruction. She then asked the children to translate what they heard in their own words. The researcher either affirmed or modified the translation. In the latter part of the test instructions, the researcher asked the students to read one sentence at a time and translate what he/she read to Filipino. This was the researcher's method for ascertaining whether or not the children in Grade 4 are able to read the English problems and understand them.

The same procedure was followed in the rural school. Although the mother tongue of the children in this school was Pampango, they preferred to translate what they read to Filipino.

The test was assigned after it was ascertained that the anxieties of the students were laid to rest. There were two or three instances where certain students asked for reassurance with regard to what they were doing. However, their questions with regard to answers to the test were not answered by the researcher.

The testing in the urban school took 40 minutes (10:15 am—10:55 am). The first student to finish spent only 29 minutes answering the test. There were 40 students in the urban sample (28 girls and 12 boys) and 41 in the rural sample (29 girls and 12 boys).

In both schools, while the students were taking the test, the researcher from Japan interviewed the teacher in another room; the interview was completed in 20 minutes. In the rural school, both researchers interviewed the students.

In the urban school, the students had to be dismissed at 11:30 am and be back again at 1:30 pm. The interviews of the top and bottom five students (as nominated by teachers based on class achievement), were scheduled in the morning of the following day.

In the rural school, the students remained in school until 2:00 pm. Thus, the interviews were conducted from 1 to 1:30 pm on the same day. The researchers returned the following day to meet with other officials of the school.

A total of 81 students were tested—24 boys and 57 girls. Their ages ranged from 9 to 12 years (Table 1).

Table 5. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Urban School	57.25	53.75	58.75	90	20
Rural School	45.97561	44.16667	46.72414	95	15
All	51.54321	48.95833	52.63158	95	15

Table 5 shows that the average score of the samples from the urban and rural schools is 51.5%. It also shows that students from the urban school scored slightly better than their counterparts from the rural school. Further, girls outperformed boys by a small margin with regard to their performance in the assigned achievement test.

With regard to the range of scores, scores from the rural school had a slightly larger range as compared with those from the urban school.

Table 6. Question-wise achievements of pupils in percentage

	School in urban area			School in rural area		
	Combined	Boys	Girls	Combined	Boys	Girls
Q1 (1)	100.0	100.0	100.0	87.5	91.7	85.7
(2)	78.0	75.0	80.0	37.3	33.3	39.3
Q2	0.0	0.0	0.0	10.0	8.3	10.7
Q3 (1)	100.0	100.0	100.0	90.0	91.7	89.3
Q3 (2)	90.2	81.3	96.0	65.0	58.3	67.9
Q4 (1)	97.6	100.0	96.0	92.5	100.0	89.3
Q4 (2)	56.1	50.0	60.0	25.0	25.0	25.0
Q4 (3)	73.2	62.5	80.0	50.0	50.0	50.0
Q5 (1)	68.3	62.5	76.0	60.0	58.3	60.7
Q5 (2)	68.3	62.5	72.0	27.5	33.3	25.0
Q5 (3)	43.9	56.3	36.0	50.0	58.3	46.4
Q6 (1)	97.6	100.0	96.0	80.0	75.0	82.1
Q6 (2)	95.1	87.5	100.0	72.5	66.7	75.0
Q6 (3)	61.0	37.5	76.0	30.0	16.7	35.7
Q7	61.0	50.0	68.0	95.0	91.7	96.4
Q8	48.8	43.8	52.0	25.0	25.0	25.0
Q9	75.6	75.0	76.0	95	91.7	96.4
Q10	75.6	62.5	84.0	55.0	50.0	57.1

The percentage scores for each of the ten questions indicate that the items range from very difficult, where no correct answer was obtained, to very easy, where the score was 100 percent.

The following is a brief description of the performance of the students according to item.

Q_1 (1) The competence displayed was close to the level of mastery, particularly in the urban school where all the students provided the correct answer. With regard to the rural sample, the item was easy, but achievement was below 100%.

Q_1 **(2)** The item was correctly answered by merely 78% of the students in the urban school and 37.3% of the students from the rural school. Numerous students in the rural school *added* instead of subtracting the two fractions, perhaps thinking that Q1 (2) is of the same nature as Q1 (1).

Q_2 This item reads "Express the answer in a fraction: 3 ÷ 7 = ?" None of the students in the rural school answered this correctly. Merely 4 students in the urban school provided the correct answer—$\frac{3}{7}$. There were at least 20 different variations of the answer, indicating that the students did not understand the question. It was rather obvious that the students had not yet related the concept of *fractions* with that of *ratio*; they were also unaware that an indicated or expressed division can be expressed as a fraction.

Q_3 Answers to this question indicated that generally students understand and can illustrate $\frac{3}{4}$ on a bar that is divided into four equal portions; however, a large number of students were unable to closely approximate $\frac{1}{3}$ on an identical bar. Although certain students divided the bar into three portions, these were unequal. What they labeled as $\frac{1}{3}$ was incorrect; it was more like $\frac{1}{2}$ or $\frac{1}{4}$.

Q_4 In Q4 (1), almost all the students in the urban and rural schools were able to shade $\frac{1}{2}$ of a 2m bar. However, with regard to the next question, which asked them to shade $\frac{1}{2}$ m, merely 56% of the students in the urban school and 25% of those in the rural school provided the correct answer. In each school there was hardly any difference between the answers of the boys and girls.

In Q4 (3), as compared with students in the rural school, a higher percentage of students in the urban school (73.2% versus 50%) correctly shaded 1½ m of the bar.

Q_5 In item (1) of this question, 68.3% of the students in the urban school and 60% of those in the rural school correctly answered **B** (i.e., $\frac{4}{5}$ is greater than $\frac{2}{5}$. In item (3), 43.9% of the students in the urban school as compared with 50% of the students in the rural schools answered **B** (that is, $\frac{1}{3}$ is greater than $\frac{1}{4}$). In item (2), 68.3% and 27.5% of the students in the urban and rural samples, respectively, recognized that $\frac{3}{6}$ and $\frac{1}{2}$ are equal fractions.

Q_6 In item (1) the word <u>add</u> led almost all the students in the urban sample and 80% of the rural sample to provide the correct answer to $\frac{2}{5}+\frac{1}{5}$.

In item (2) under Q6, a high percentage of the students correctly subtracted $\frac{2}{7}$ from $\frac{5}{7}$.

In Item (3), merely 61.0% of the urban sample and 30.0% of the rural sample correctly answered the problem on the total length of six pieces of $\frac{1}{6}$m paper in a line.

Q$_7$ This item required that the decimal number 0.7 be changed to a fraction. Almost all the students in the rural school (95.0%) correctly wrote $\frac{7}{10}$, as compared with merely 61.0% of the students in the urban school.

Q$_8$ This item involves expressing the relation between 3kg and 5kg of meat. Merely half the students in the urban school and one-fourth of those in the rural school answered the problem correctly.

Q$_9$ The answers from students of the two schools indicated that they are able to recognize $\frac{1}{2}$ or one whole; and two wholes out of four implies $\frac{1}{2}$.

Q$_{10}$ Three-fourths (75.6%) of the urban sample as compared with merely 55.0% of the rural sample were able to formulate a sentence problem on fractions that leads to the answer $\frac{2}{3}$. A majority of the problems formulated involved adding two $\frac{1}{3}$s or taking away $\frac{1}{3}$ from one whole, or 1 from $\frac{5}{3}$.

(4) Results of Interview using Newman Procedure

The top five and the bottom five students in math (based on the performance in the test and nomination of the teacher) from each of the two schools were individually interviewed. Each interviewee was shown a copy of the test and sequentially asked questions with regard to Q5 (3), Q6 (1), and Q8.

The interview was conducted in a mixture of English and Filipino; however, the students were told that they can answer/explain in Filipino. The responses below are English translations of their Filipino answers.

The interview questions roughly followed the following sequence:

Do you remember the test you took this morning/afternoon?
I will show you the test and ask you to read particular items.

- Can you read Q5 (3) please? (The student reads the item aloud.)
- Is there any part (a word or expression) that you do not understand? Did you understand what you read?
- Do you remember what you answered in the test? What was your answer?
- How did you get that answer? (Please explain how you got the answer.)
- Are you sure about your answer? (Do you want to change it?)

Let us go to Q6 (1)
Do you understand the question?
Are there any terms you do not understand?
[Ask the interviewee questions in the same sequence]

Proceed to Q8

Now read Q8 aloud.
Do you understand what it is asking for?

The interviewee explained his answer to each question by showing a solution on a blank sheet of paper. The interviewer then asked follow-up questions in order to help the student clarify his answer or solution.

Table 7. The pupils' errors of problem solving level in urban and rural schools

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5 (3)	-	-	-	3	4	7	5	5	10	6	5	11
Q6 (1)	-	-	-	2	0	2	5	6	11	6	4	10
Q8	-	-	-	10	3	13	10	5	15	1	5	6

Table 8. The pupils' errors of problem solving level according to their performance

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5 (3)	-	-	-	0	7	7	0	10	10	10	1	11
Q6 (1)	-	-	-	0	2	2	2	9	11	8	2	10
Q8	-	-	-	5	8	13	6	9	15	4	2	6

From the interviews of students it was evident that reading was not much of a problem. Students from Grade 4 were able to read the problems, albeit slowly. A few words were indicated by certain students as new or unfamiliar. For example: bracket, relationship, and ratio. However, besides these words, everything in the test was judged readable by the students themselves.

It is evident from Tables 8 and 9 that it is in Q8 where a majority of the students from urban schools faced a difficulty, even with regard to understanding the concept itself. This was because the topic has not been discussed yet in their class. On the other hand, students from the rural samples also found Q8 challenging because the topic, according to the students, was merely discussed in passing—they have only recently begun studying the particular topic.

Table 8 also indicates that high performers from both the urban and rural schools completely understood the concept and process required for Q5 (3), and that a majority of them also understood the concept and process required for Q6 (1).

An observation of the errors of the students brings to light that the students' understanding of concepts does not imply that they can perform the mathematical process required to solve a problem. In the same manner, a student who was unable to correctly work out the mathematical solution to a problem may be able to provide the correct answer merely by guessing.

Table 9. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	"bracket" and "A=B"			
Any remarks regarding problem solving process	➢ The students considered only the denominators to identify which fraction is bigger. ➢ The students got confused with the instruction. ➢ The students guessed.	➢ The students considered only the denominators to identify which fraction is bigger. ➢ The students got confused with the instruction. The students guessed.		➢ The students considered only the denominators to identify which fraction is bigger. ➢ The students got confused with the instruction. The students guessed.
Specific mistakes	➢ $\frac{1}{4} = \frac{1}{3}$ ➢ $\frac{1}{4} > \frac{1}{3}$	➢ $\frac{1}{4} = \frac{1}{3}$ ➢ $\frac{1}{4} > \frac{1}{3}$		➢ $\frac{1}{4} = \frac{1}{3}$ ➢ $\frac{1}{4} > \frac{1}{3}$

Q5 (3):

The correct answer is 2 or **B** is greater than **A**.

The students' answers were as follows:

> "A is greater because 4 is greater than 3."
> "3 is greater than 4; hence, 1/3 is greater than 1/4."
> "Because $\frac{1}{3}$ is close to $\frac{1}{2}$, and $\frac{1}{4}$ is before $\frac{1}{2}$. I see it in a weighing scale."
> (A table weighing scale used in the market)
> "B is greater because 3 < 4 when we cross multiply."
> The illustration for the answer is $^{3}\frac{1}{4} \times \frac{1}{3}^{4}$.

(Other interviewees also showed the same cross multiplication, illustrating that $\frac{1}{3}$ is greater than $\frac{1}{4}$. They added that "3 × 1 is less than 4 × 1, or 4 × 1 is greater than 3 × 1.")

Certain students provided illustrations that depicted that 1 of 4 parts is smaller than 1 of 3 parts.

> "Since the numerators are both 1, they are the same. The one with the larger denominator is the smaller value."

Incorrect Answer: "1" or **A** is greater than **B**:

The incorrect answer was explained in the following manner:

“$\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3.”

“$\frac{1}{4}$ implies that the cake is cut into 4, and $\frac{1}{3}$ implies that the cake is cut into 3. Thus, there are more portions when dividing by 4 so $\frac{1}{4}$ is greater than $\frac{1}{3}$.”

“I just considered the denominators: since 4 > 3 so $\frac{1}{4}$ is greater than $\frac{1}{3}$.”

Similar explanations (misconceptions) were provided by other students in order to justify the answer “1” (A is greater than B).

Incorrect Answer “3”: **A = B**

“They're ($\frac{1}{3}$ and $\frac{1}{4}$) the same.”

“I provided the answer in haste.”

“I thought all the three answers [1, 2, and 3] should be selected so I answered 3.”

[he expected that all the 3 options must be selected at least once.]

Table 10. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	“$\frac{2}{5}$ l”	“$\frac{2}{5}$ l”	“$\frac{2}{5}$ l”	“$\frac{2}{5}$ l”
Any remarks regarding problem solving process	➢ Added both the numerator and denominator. ➢ Did ‘cross subtraction’. ➢ Could not distinguish between similar and dissimilar fractions. ➢ Does not know the concept at all.	➢ Added both the numerator and denominator. ➢ Could not distinguish between similar and dissimilar fractions. ➢ Does not know the concept at all.	Added both the numerator and the denominator.	➢ Added both the numerator and denominator. ➢ Did ‘cross subtraction’. ➢ Could not distinguish between similar and dissimilar fractions. ➢ Does not know the concept at all.
Specific mistakes	➢ $\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$ ➢ $\frac{2}{5}+\frac{1}{5}=\frac{3}{4}$	$\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$	$\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$	➢ $\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$ ➢ $\frac{2}{5}+\frac{1}{5}=\frac{3}{4}$

<u>Q6 (1)</u>:

When you add $\frac{2}{5}$ l of water to $\frac{1}{5}$ l of water in a container, how much water is in the container?

Those who provided the correct answer stated, “When the denominators are the same, add only the numerators."

Those who provided incorrect answers explained by stating the following:

"Add both numerators and denominators."

Another student stated: $\frac{2}{5}$ length + $\frac{1}{5}$ length = $\frac{3}{10}$ length. He mistook l for length that, on deeper probing, showed that the term "liter" was not yet introduced by the teacher.

One student answered: ***"Two five and one five of water is three five."*** Obviously, he drew from the concept of adding similar things and was unaware of how to use fractions. However, none of the students that were interviewed were familiar with the term "similar fractions." They simply referred to the fractions $\frac{2}{5}$ and $\frac{1}{5}$ as "proper fractions." Obviously this concept, not the ***similarity***, was what was emphasized by the teacher.

One low-performing student from the urban school provided the following solution:

$$\frac{2}{5} \times \frac{1}{5} = \frac{5-2}{5-1} = \frac{3}{4}$$

Answer: 3/4

This is obviously a wrong carry-over of the early introduction of the algorithm for comparing the values of two fractions.

Certain students, after reading the item, stated that they did not understand "l." (liter)

Table 11. Pupils' mistakes in problem solving in Q (8)

Q (8)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	"bracket", "kg", "bought", "Jody", "George"	"bracket", "kg", "bought"	"kg", "Jody", "George"	"kg", "bought", "George"
Any remarks regarding problem solving process	➢ Added, subtracted, or divided 5 and 3. ➢ Split 8 into 5 and 3. ➢ Does not understand the problem or the concept at all because it wasn't taken up yet.	➢ Added, subtracted, or divided 5 and 3. ➢ The topic was just taken up in passing. ➢ Guessed.	➢ Added or divided 5 and 3. ➢ Split 8 into 5 and 3. ➢ Does not understand the problem or the concept at all because it wasn't taken up yet.	➢ Subtracted or divided 5 and 3. ➢ Guessed
Specific mistakes	8	5/3, 1/3, 8	8 kg, 5/3	1/3, 2kg

<u>Q8:</u>

Judy bought 5kg of meat; George bought 3kg of meat. Write a fraction in the bracket to show the relationship between Judy's meat and George's meat in terms of weight.

"George meat is (   ) of Judy's meat."

Certain interviewees stated that they did not understand **kg**, **bracket**, and **relationship**, but provided answers nonetheless. Certain students were unable to read "George" correctly.

One student answered: "I divided 3 and 5 and got $\frac{3}{5}$"; obviously the student did not quite understand why he did this. He used the term "hinati," which means halved. ***"Hinati ko and 3 at 5 kaya** $\frac{3}{5}$."* If he actually halved 3 + 5 he should have arrived at 4. Perhaps he was unable to express in words his understanding of "relationship."

One student who provided the answer "8" explained, "I just added 5 and 3, I don't know what is asked for." He had answered 3 + 5 = 8.

Another student answered "greater" to indicate that Judy's meat compared with that of George is more.

> "I divided 5 by 3 or I just divided the two numbers to get a fraction."

One student admitted to not understanding the problem. His answer was "3 x 2 = 6." Obviously he subtracted 5 – 3 to obtain 2, and then multiplied 2 and 3.

From the answers to Q8 it is obvious that students have not yet grasped the concept of fractions as a ratio or relationship.

2.4.4 Discussion

In the Philippines, the concept of "fractions" is introduced as early as Grade 1; this is done through concrete objects, manipulates, representations, and pictures but limited to familiar unit fractions like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. In Grade 4, some attempt is made at addition and subtraction of similar and related fractions; fractional parts are introduced with concrete examples and situations in this grade.

Performance on each of the items was influenced by one or more of the following factors: Familiarity of the item format, illustration, and directions for recording the answer(s).

For example, with regard to Q1 (1) and Q1 (2), since both the items are under the heading Q1, certain students considered them both to be involving addition instead of one on addition and the other on subtraction.

Mixing multiple-choice items with supply-type items is not usually practiced in schools in the Philippines; compounded directions for one item are not practiced either, as done in Q_5(1, 2, and 3). Books in the country would have presented the item in the following manner:

Study each pair of fractions, answer
  "1" if the first fraction is greater
  "2" if the second fraction is greater
  "3" if the two fractions are equal

Another manner of presenting the item is to direct the student to write the symbol =, >, or < between the pair in order to indicate their relationship.

$$\frac{1}{4} \text{ — } \frac{1}{3}; \qquad \frac{2}{5} \text{ — } \frac{4}{5}$$

When dealing with fractions as a ratio, the phrase "write the answer as a fraction" is not usually used, as done in Q_2. Even the textbooks in the Philippines do not express it in this manner. Therefore, the low percentage of correct answers to Q_2 is largely due to the unfamiliar manner of wording the question; the same explanation applies to the low rate of correct responses for Q_8.

This question would have been easier to understand if it was worded as "What fraction shows the relationship between Judy's meat and George's meat in terms of weight?."

The sequence of Q_4 (1, 2, 3) and erroneous reading may have caused the large number of errors in Q_4 (1), Q_4 (2), and Q_4 (3). It is worth finding out if the percentage of correct answers would be higher if the sequence had been 1, 3, 2 or 3, 2, 1.

Q_6 (1) was rather easy because of the clue word "add" in the problem. Perhaps, in order to test real understanding, instead of using "add," which translates directly to "+," the phrase "put together" can be used.

Q_{10} is rather challenging; it actually tests the understanding of the concept of fractions. Answers should not be rated on grammar but on evidence of a sense of fractions and reality math.

Table 3 shows the frequencies and percentages of students who correctly answered each of the test items in the urban and rural schools, while Table 2 shows the comparative statistics with regard to the Achievement Test for the boys and girls as well as the combined group; this is presented separately for the urban and rural samples for each item. While the two schools were selected to represent students from two locations—one from an urban setting and another from a rural one—the levels of comprehension are similar for certain items or problems and differ in other questions. These suggest a difference in the teacher factor rather than a location factor.

The following are some observations from the data collected in this study:

1. In the urban sample, the students' ability to add or subtract similar fractions (Q1) is close to the level of mastery. The students' ability to apply this to word problems is not as high in the rural sample.
2. Ascertaining $\frac{3}{4}$ of a length is almost mastered; however, the ability to show $\frac{1}{3}$ when an object is divided into 4 equal parts is lacking.
3. In the rural sample, the concepts appeared to be better ingrained. Variations in Q6 were largely represented by including and excluding the unit of measure. If judged only in terms of comprehension of numerical addition or subtraction of similar fractions, it can be stated that this has been mastered. However, merely 24% of the students included the unit of measure "l" for "liter" in the answer. The same is true of the answer to the problem involving the subtraction of $\frac{2}{7}$m from $\frac{5}{7}$m. Merely 22 students answered $\frac{3}{7}$ m. The remaining, except one, provided the answer $\frac{3}{7}$. The lone exception answered $\frac{4}{7}$; this was obviously a careless error.
4. The third item in Q6 had four main variations of answers that indicate a little understanding of repeated addition of six $\frac{1}{6}$s. The four variations are 1m, 1, $\frac{6}{6}$ or 1, and $\frac{6}{6}$. These four variations together accounted for 61% of the rural sample.
5. In the urban sample, the answers were more varied. In both the rural and urban samples, a frequent incorrect answer was "$6\frac{1}{6}$," which most likely is the students' representation

of six $\frac{1}{6}$s, or confusion in writing $\frac{1}{6}$ x 6. Perhaps, the teachers do not sufficiently emphasize the writing of a multiplication phrase by using "×," as in ($6 \times \frac{1}{6}$), or using parentheses, as in $6\,(\frac{1}{6})$, to indicate a product.

6. The students have a good understanding of unit fractions, i.e., fractions with 1 as the numerator. However, without illustrations and concrete objects, the students are unable to visualize the relative sizes of two fractions.

7. In the problems formulated by the students in which $\frac{2}{3}$ must be the answer, at least four students had a number or word problem, where $\frac{1}{2} + \frac{1}{1} = \frac{2}{3}$. This approach was explained by the students as correct. Another was $\frac{3}{4} - \frac{1}{1} = \frac{2}{3}$, which confirmed the erroneous concept of addition and subtraction of fractions.

8. In Q2, which instructed the students to "express 3 ÷ 7 as a fraction," the 20 (approximately) variations of incorrect answers indicated that the students obtain the answers by operating on the two numbers in a random manner—for example, 7/3 , $3\sqrt{7}$, 3 x 7, 7 – 3—that provided the answers $2\frac{1}{3}$, 2 r.1 , 21, and 4. As explained earlier, the students are not familiar with this type of item possibly because the teacher has not emphasized the concept of a fraction as a ratio or quotient of two numbers.

2.4.5 Pupils' Errors and Misconceptions about Fractions

The misconceptions and causes of erroneous answers were discovered in the interviews, particularly with regard to the five low-performing students in each of the two schools.

Misconception 1: To add two similar fractions, add their numerators and denominators.

Q_1 (1): $\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$

Q_6 (1): $\frac{2}{5}l + \frac{1}{5}l = \frac{3}{10}l$

Q_1 (2): $\frac{5}{7} + \frac{2}{7} = \frac{7}{14}$

Misconception 2: To subtract two fractions, subtract the numerators and denominators.

Q_1 (2): $\frac{5}{7} - \frac{2}{7} = \frac{3}{0}$

Q_6 (2): Cut $\frac{2}{7}$m from $\frac{5}{7}$m

Answer: $\frac{3}{14}$

Misconception 3: The greater the denominator, the greater the value of a fraction. Hence, $\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3.

Misconception 4: $\frac{1}{4}$ is greater than $\frac{1}{3}$

because in $\frac{1}{4} \times \frac{1}{3}$, 4 × 1 is greater than 3 × 1.

Obviously, some impatient tutor taught the pupils the principle:

$\frac{a}{b} > \frac{c}{d}$ if a × d > b × c. However, the students did the multiplication in the incorrect sequence.

Misconception 5: To add two fractions, add all the terms

$$\frac{2}{5} + \frac{1}{5} = \frac{2+5}{1+5} = \frac{7}{6}$$

Certain students committed the above error. A few students also added the numbers diagonally.

e.g., $\frac{2}{5} + \frac{1}{5} = \frac{5+1}{2+5} = \frac{6}{7}$

This misconception appeared to have resulted from *Misconception 4* stated above.

Misconception 6: A unit fraction added several times is indicated as a mixed number.

e.g., Q_6 (3) $\frac{1}{6}$m taken six times is $6\frac{1}{6}$, which is interpreted as $6 \times \frac{1}{6}$.

Misconception 7: Students do not ascribe much importance to units of measurement like liters (l) and meter (m). They merely focus on the numbers that they operate on.

Finally, with the use of the Newman method for analyzing the errors committed by the students on certain items in the achievement test, it can be concluded that the understanding, or the lack of understanding of a student with regard to a concept, does not directly translate to their ability in working out the mathematical processes. Similarly, the students' ability with mathematical processes does not imply that they are able to record their answers correctly or incorrectly. This may be explained by the fact that students guess the correct answers, and are affected by the level of familiarity with the formulation of the questions.

Further, by analyzing the errors of the students and investigating the cause for the errors, their misconceptions became evident. Being able to perceive the misconceptions of students with regard to certain mathematical concepts will enable teachers to understand the manner in which they should approach and conduct their teaching, while being mindful of these misconceptions.

2.5 Thailand

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MASAMI ISODA — University of Tsukuba
YUTAKA OHARA — Naruto University of Education

2.5.1 School Curriculum

1) Historical List of Major Documents for Grades K-12

1960s	National Plan B.E. 2503 (A.D. 1960) enacted in the academic year 1961
1970s	Mathematics Curriculum developed by the Institute for Promotion of Science and Technology (IPST) A.D.1978
1980s	Revised mathematics curriculum at the senior high school level (A.D. 1981)
1990s	
1990	Revised elementary and junior high school curriculum (version A.D.1978)
1990	Revised senior high school curriculum (version A.D. 1981)
1999	National Education Act B.E. 2542 (A.D. 1999)
2000s	
2001	Curriculum for Basic Education B.E. 2544 (A.D. 2001)
2002	Amended National Education Act

2) Features of the Mathematics Curriculum for Basic Education of 2001

Standards for Basic Education

Content Area I: Number & Operations

Standard 1.1 Understanding various types of number expressions and using numbers in daily life

Standard 1.2 Understanding the effects of number operations and relations among various operations and using operations to solve problems

Standard 1.3 Using estimation to compute and solve problems

Standards 1.4 Understanding the number system and using the properties of number

Content Area II: Measurement

Standard 2.1 Understanding basic measurement

Standard 2.2 Measuring and estimating things

Standard 2.3 Solving measure problems

Content Area III: Geometry

Standard 3.1 Explaining and analyzing two- and three-dimensional geometrical figures

Standard 3.2 Visualization, spatial reasoning, geometric models in problem-solving

Content Area IV: Algebra

Standard 4.1 Explaining and analyzing various types of patterns, relations, and functions

Standard 4.2 Using expressions, equations, graphs, and mathematical models to represent various situations and interpret their meaning and implement them

Content Area V: Data Analysis & Probability

Standard 5.1 Understanding and using a statistical approach to analyze data

Standard 5.2 Using statistical methods and probability to estimate phenomena rationally

Standard 5.3 Using statistics and probability for decision-making and problem-solving

Content Area VI: Skills & Processes

Standard 6.1 Ability to solve problems

Standard 6.2 Ability to reason

Standard 6.3 Ability to communicate and represent mathematical meanings

Standard 6.4 Ability to connect mathematical concepts and other subjects

Standard 6.5 Creativity

2.5.2 Basic Information

(1) Schooling Age

Primary and secondary education in Thailand comprises six years in a compulsory elementary program and an additional six years in secondary school. A child must be enrolled in school by the age of seven and must attend until age 14.

The educational system is 6-3-3-4. Free public education is compulsory for all children from the ages of 6–15, providing nine years of compulsory education. Pre-school is available for children of ages 3–5, primary school for ages 6–11, lower secondary school for ages 12–14, and upper secondary for the ages of 15–17. Higher education is generally provided in a four-year program for the bachelor's degree.

(2) School Calendar and Examination

The first semester of the school year begins on May 16 and ends in the first week of October. After a three-week recess, the second semester begins on November 1, and continues until the second week of March. The long summer vacation is from the third week of March until May 15, and the cycle is repeated. Classes are held from Monday to Friday, during which time the students receive approximately six hours of instruction each day.

Schools vary in different parts of Thailand. Small rural schools are not as regimented as the large city schools nor do they possess as much technological equipment. Most schools require uniforms, depending on the affluence of the families. Students and their families show great respect for their teachers and appreciate the opportunity to learn.

Table 1. School calendar in 2004

<table>
<tr><th colspan="2">May</th><th>June</th><th>July</th><th>Aug.</th><th>Sep.</th><th colspan="2">Oct.</th><th>Nov.</th><th>Dec.</th><th>Jan.</th><th>Feb.</th><th colspan="2">Mar.</th><th>April</th></tr>
<tr><td>▨</td><td></td><td></td><td></td><td></td><td></td><td></td><td>▨</td><td></td><td></td><td></td><td></td><td></td><td>▨</td><td>▨</td></tr>
</table>

There is an end-of-term examination in each school. There is also a special national examination in Grades 3, 6, and 9 known as the National Test.

(3) Promotion System

The promotion system requires every student to sit for an annual exam in each school and pass the exam. The passing cut-off is 50% in each subject, without acquiring which the students are not permitted to proceed to the next grade.

(4) Medium of Instruction

The medium of instruction at the primary school level is Thai. The mother tongue for the children and the teachers are Thai.

(5) Class Organization

Each lesson in primary school is of a duration of 50 minutes.

Table 2. Class organization

Time (Min)	Thai Lesson
1	Teachers begin by asking students short-answer questions that lead into the day's lesson or questions related to earlier lessons; the teacher may also check homework by calling students to show their answers in front of the class.
10	Teacher distributes worksheet with similar problems. Students work independently.
20	Teacher monitors students' work, notices some confusion on particular problems, and demonstrates how to solve these problems. Typical for teacher to intervene at the first sign of confusion or struggle.
30	Teacher reviews another worksheet and demonstrates a method for solving the most challenging problem.
40	Teacher conducts a quick oral review of problems like those worked on earlier.
50	Teacher asks students to complete worksheets. Unusual to not assign homework.

A majority of the primary schools begin at 8:30 am and continue till 3:30 pm; lunch time is from 11:30–12:30 pm.

Table 3. School hours

Time	8.30-9.30	9.30-10.30	10.30-11.30	11.30-12.30	12.30-13.30	13.30-14.30	14.30-15.30
Subject	A	B	C	Lunch	D	E	F

2.5.3 Results from the First Year Field Survey

Thailand has adopted a five-day week and the school semester system. There are two semesters in a year—the first semester (May–September) and the second semester (November–March). Compulsory education is set at six years for elementary school and three for junior high school.

(1) Survey Schedule

Table 4. Schedule of data collection

Date	Activity
11th December 2004	Arrival at Center for Research in Mathematics Education (CRME), Khon Kaen University
12th December 2004	15.00 – 18.00 Meeting on research Assist. Prof. Maitree Inprasitha (Ph.D.), Assc.Prof. Yutaka Ohara, and CRME Center's Staffs
13th December 2004	Preparing materials for collecting data; Test I & II, VDO digital camera and so on.
14th December 2004	Visit Rural School in Khon Kaen and meeting with 4th grade students 09:00 Videotaping 4th grade classroom. (Fraction) 10:00 Interviewing the principal and mathematics teacher 10:30 Distributing questionnaires 11:00 Testing part I 12:00 Lunch 13:00 Testing part II
15th December 2004	Preparing materials for collecting data:Test I & II, VDO digital camera and so on. Leaving for Bangkok.
16th December 2004	Visit Urban School and meeting with 4th grade students. 09:30 Meeting with the principal and the administrators 10:20 Videotaping4th grade classroom.(Fraction) 11:10 Interviewing the principal and mathematics teacher
17th December 2004	Visit Urban School and Grade 4 students. 09:00 Distributing questionnaires 09.30 Testing Part I 10:30 Testing Part II
18th December 2004	Summary discussion at Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University.

(2) Target Schools and Samples

The following criteria were taken into consideration for the selection of sample schools: school location (urban, rural) and language used in schools and at home (standard Thai, local language). As a result, one urban primary school and one rural primary school were selected. In the urban school, students and teachers always use standard Thai language in classes, while in the rural primary school rural students and teachers occasionally use Isaan (local language) mixed with Thai language in the classes.

Table 5. Location of schools

	School location
Urban Primary School	Ratchaburana, Bangkok (Capital of Thailand)
Rural Primary School	Nong Rua District, Khon Kaen (About 440 kilometers far from Bangkok)

Urban Primary School: The selected urban school is situated in Bangkok, the capital of Thailand. It was founded in 1934 and conducts 24 teaching classes (Grades 1–6). The school is equipped with five teaching buildings, including special classrooms for music, arts, agriculture, and computers and an 8,800-square meter school area. The school is located in a good environmental area of Bangkok. The number of teachers and students are 45 and 828, respectively.

Rural Primary School: The selected rural school is situated in Khon Kaen Province in the northeastern part of Thailand. It is approximately 440 kilometers away from Bangkok. This school conducts 15 teaching classes (Grades 1–6). The school is equipped with three teaching buildings, including special classrooms for music, arts, agriculture, and computers and a 19,200-square meter school area. The number of teachers and students are 23 and 424, respectively.

Table 6. Distribution of sample

	Grade	Male	Female	Total
Urban Primary School	4	18	22	40
Rural Primary School	4	10	20	30
Total		28	42	70

Table 7. Distribution of students' age

Age (years)	No. of Pupils		
	Urban	Rural	Total
9	17	5	22
10	21	25	46
11	2	-	2
Average	9.6	9.8	9.7

(3) Results of Student Questionnaire

Table 8. Distribution of students' books (%)

	0-10 books	11-25 books	26-100 books	100- books	No response	Crossed out
Combined	68.57	22.86	7.14	1.43	-	-
Urban school	52.50	32.50	12.50	2.50	-	-
Rural school	90.00	10.00	0.00	0.00	-	-

Table 9. Distribution of students' home items (Yes, %)

	Calculator	TV	Radio	Desk	Quiet place	Dictionary	Books (Excluding text book)	Computer
Combined	80.00	97.14	77.14	48.57	42.86	47.14	71.42	28.57
Urban School	90.00	100.00	87.50	65.00	62.50	57.50	85.00	42.50
Rural School	66.67	93.33	63.33	26.67	16.67	33.33	53.33	10.00

Table 10. Students' attitude towards mathematics

Pupils' Perceptions	Average (%)		
	Combined	Urban	Rural
1. I usually do well in mathematics	1.34	1.48	0.97
2. I would like to do more mathematics in school	1.27	1.20	1.37
3. Mathematics is harder for me than for many of my classmates	2.76	3.08	2.33
4. I enjoy learning mathematics	1.37	1.28	1.50
5. I am just not good at mathematics	3.27	3.10	3.50
6. I learn things quickly in mathematics	2.01	2.03	2.00
7. I think learning mathematics will help me in my daily life	1.27	1.08	1.53
8. I need mathematics to learn other school subjects	1.91	1.78	2.1
9. I need to do well in mathematics to get into the university of my choice	1.19	1.15	1.23
10. I need to do well in mathematics to get the job I want	1.34	1.25	1.47

*Remark: 1 = Agree a lot, 2 = Agree a little, 3 = Disagree a little, 4 = Disagree a lot

Table 11. Students' activities in their mathematics lessons

Students' activities	Average		
	Combined	Urban	Rural
1. I practice adding, subtracting, multiplying, and dividing without using a calculator	2.13	2.05	2.23
2. I work on fractions and decimals	1.86	1.60	2.20
3. I measure things in the classroom and around the school	2.26	2.03	2.57
4. I make tables, charts, or graphs	2.17	1.78	2.70
5. I learn about shapes such as circles, triangles, and rectangles	1.63	1.45	1.87
6. I relate what I am learning in mathematics to my daily lives	1.81	1.65	2.03
7. I work with other students in small groups	1.91	1.88	2.30
8. I explain my answers	2.26	2.18	2.23
9. I listen to the teacher talk	1.23	1.15	1.33
10. I work problems on my own	1.81	1.80	1.83
11. I review my homework	1.84	1.78	2.93
12. I have a quiz or test	1.56	1.45	1.63
13. I use a calculator	3.43	3.53	3.30

*Remark: 1 = Every or almost every lesson, 2 = About half the lessons, 3 = Some lessons, 4 = Never

Table 12. Students' attitude towards school and teacher

Pupils' perceptions	Average		
	Combined	Urban	Rural
1. I like being in school	1.46	1.40	1.53
2. I think that students in my school try to do their best	1.39	1.33	1.47
3. I think that teachers in my school care about the students	1.21	1.23	1.20
4.I think that teachers in my school want students to do their best	1.13	1.05	1.23

*Remark: 1 = Agree a lot, 2 = Agree a little, 3 = Disagree a little, 4 = Disagree a lot

Table 13. Students' activities before or after school (%)

Items	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or more hours
1. I watch TV	4.29	58.57	20.00	7.14	10.00
2. I play or talk with friends	10.00	52.86	14.29	14.29	8.57
3. I do jobs at home	11.43	41.43	30.00	7.14	10.00
4. I play sports	11.43	35.71	30.00	12.86	10.00
5. I read a book for Enjoyment	27.14	38.57	17.14	5.71	10.00
6. I use computer	57.14	15.71	12.86	7.14	7.14
7. I do homework	7.14	45.71	32.86	10.00	4.29

Table 14. Distribution of students' family member

Family member	Pupils' Distribution (%)		
	Combined	Urban	Rural
2-3 persons	18.57	25.00	10.00
4-6 persons	44.29	35.00	56.67
7-9 persons	22.86	22.50	23.33
10-12 persons	7.14	10.00	3.33
More than 12 persons	5.71	2.50	3.33
Crossed out	1.43	-	3.33

(4) Results of Achievement Test

Table 15. Distribution of students' right answer (%)

	Average	Male	Female	Highest	Lowest
Combined	60.58	59.59	61.39	83.56	36.99
Urban school	38.40	40.14	37.53	67.12	16.44
Rural school	51.08	52.64	50.03	83.56	16.44

Table 16. Distribution of students' right answer according to age

Age (years)	Average right answer (%)		
	Combined	Urban school	Rural school
9	33.62	37.47	20.55
10	31.77	36.86	27.51
11	29.45	29.45	-

Table 17. Average rates of correct answers by content domain

	Multiple-choice		Short-answer		Total	
Content domain	No. of questions	Rates of correct answers (%)	No. of questions	Rates of correct answers (%)	No. of Questions	Rates of correct answers (%)
Number	19	56.62	9	49.29	28	52.96
Measurement	12	53.22	4	42.86	16	48.04
Algebra	4	47.14	1	24.29	5	35.72
Data	4	66.07	4	58.93	8	62.50
Patterns, relations and functions	5	53.72	1	47.14	6	50.43
Geometry	4	43.57	6	54.52	10	49.05
Total	48	53.39	25	46.17	73	52.87

Table 18. Average rates of correct answers of content domain by urban and rural school (%)

Content domains	Urban school			Rural school		
	Multiple-choice	Short-answer	Total	Multiple-choice	Short-answer	Total
Number	65.92	62.25	64.09	44.21	32.00	38.11
Measurement	58.75	50.83	54.79	45.83	32.22	39.03
Algebra	61.88	40.00	50.94	27.50	3.33	15.42
Data	77.50	69.38	73.44	50.83	45.00	47.92
Patterns, relations and Functions	61.50	60.00	60.75	43.33	30.00	36.67
Geometry	53.13	68.75	60.94	30.83	35.56	33.20

2.5.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 19. Schedule of data collection

Date	Activity
14th November 2005	Testing at the urban primary school Interviewing students at the urban primary school Interviewing teacher at the urban primary school
21st November 2005	Testing at the rural primary school Interviewing students at the rural primary school Interviewing a teacher at the rural primary school

(2) Target Schools and Samples

The following criteria were taken into consideration while selecting the sample schools: school location (urban, rural) and language used in schools and at home (standard Thai, local language). As a result, one urban primary school and one rural primary school were selected. In the urban primary school, students and teachers always use standard Thai language in classes, while in the rural primary school rural students and teachers occasionally use Isaan language (local language) mixed with Thai language in the classes.

Table 20. Location of schools

	School location
Urban Primary School	Ratchaburana, Bangkok (Capital of Thailand)
Rural Primary School	Nong Rua District, Khon Kaen (About 440 kilometers far from Bangkok)

<u>Urban Primary School</u>: The selected urban school is situated in Bangkok, the capital of Thailand. It was founded in 1934 and conducts 24 teaching classes (Grades 1–6). The school is equipped with five teaching buildings, including special classrooms for music, arts, agriculture, and computers and an 8,800-square meter school area. The school is located in a good environmental area of Bangkok. The number of teachers and students are 45 and 828, respectively.

<u>Rural Primary School</u>: The selected rural school is situated in Khon Kaen Province in the northeastern part of Thailand. It is approximately 440 kilometers away from Bangkok. The school conducts 15 teaching classes (Grades 1–6). It is equipped with three teaching buildings,

including special classrooms for music, arts, agriculture, and computers and a 19,200-square meter school area. The number of teachers and students are 23 and 424, respectively.

Table 21. Distribution of sample

	Grade	Boys	Girls	Age	Total
Urban Primary School	4	13	17	9.87	30
Rural Primary School	4	20	20	9.9	40
Total		33	37	9.89	70

(3) Results of Achievement Test

Table 22. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Urban School	43.2	42.07	44.19	75	14
Rural School	26.1	26.95	25.25	68	0
All	33.43	33.18	33.67	75	0

Table 23. Question-wise achievements of pupils in percentage

	School in urban area			School in rural area		
	Combined	Boys	Girls	Combined	Boys	Girls
Q1 (1)	90.00	85.71	93.75	20	20	20
(2)	70.00	50.00	87.50	16.25	7.5	25
Q2	15.00	14.29	15.63	10	15	5
Q3 (1)	93.89	92.86	94.79	68.75	66.67	70.83
Q3 (2)	93.33	92.86	93.75	61.25	56.67	65.83
Q4 (1)	83.33	78.57	87.50	74.38	83.75	65
Q4 (2)	3.33	0	6.25	11.25	12.50	10
Q4 (3)	9.17	12.50	6.25	0	0	0
Q5 (1)	48.33	25.00	68.75	46.25	45.00	47.50
Q5 (2)	18.33	25.00	12.50	12.50	20.00	5.00
Q5 (3)	3.33	7.14	0	2.50	0	5.00
Q6 (1)	77.92	74.11	81.25	31.56	29.38	33.75
Q6 (2)	36.25	39.29	33.59	19.38	25.00	13.75
Q6 (3)	13.33	9.82	16.41	15.63	20.63	10.63
Q7	0	0	0	0	0	0
Q8	20.00	35.71	6.25	20.00	15.00	25.00
Q9	46.67	57.14	37.5	21.25	25.00	17.50
Q10	26.67	10.71	40.63	6.25	7.50	5.00

(4) Results of Interview using Newman Procedure

Table 24. The pupils' errors of problem solving level in urban and rural schools

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5 (3)	1	3	***4***	0	0	***0***	10	10	***20***	0	0	***0***
Q6 (1)	3	4	***7***	0	0	***0***	1	7	***8***	9	3	***8***
Q8	3	2	***5***	5	2	***7***	10	8	***18***	2	1	***3***

Table 25. The pupils' errors of problem solving level according to their performance

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5 (3)	1	3	***4***	0	0	***0***	10	10	***20***	0	0	***0***
Q6 (1)	1	7	***8***	0	0	***0***	3	5	***8***	7	5	***12***
Q8	0	5	***5***	3	4	***7***	8	10	***18***	2	1	***3***

Table 26. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	"greater than" "bigger than"	Read 2 slash 5 for two over five "Priebtieb" (Compare) "Tela"(each) "Kaw"(item) "luak"(select)	Read 2 slash 5 for two over five	"Priebtieb"(Compare) "chamnuan"(number) "Tela"(each) "Kaw" (item) "luak"(select) "took"(correct) "suanyai" (most) "term"(fill) "yai kwa" (bigger than) " $\frac{2}{5}$ is two five"
Any remarks regarding problem solving process	$\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3, or when drawing a diagram, it also shows that $\frac{1}{4}$ is greater than $\frac{1}{3}$	$\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3 Or because four blocks from the diagram is greater than three blocks	$\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3, or when drawing a diagram, it also shows that $\frac{1}{4}$ is greater than $\frac{1}{3}$	$\frac{1}{4}$ is greater than $\frac{1}{3}$, because 4 is greater than 3.
Specific mistakes	-	-	-	-

Table 27. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	Say 2 or 5 for $\frac{2}{5}$, “saab” (know) “kaud” (bottle), Say “yahk-sai” (no meaning) for “yahk-sab” (to be curious)	Say 1 tup (slash) 5 for $\frac{2}{5}$ or say 2 jood (point) 5 for $\frac{2}{5}$ and some students can only read some words e.g., “Tha” (if) “Lit” (liter) “Nam” (water) “Nai” (in) “Pen” (is)	Say 1 tup (slash) 5 for $\frac{1}{5}$ “Krabaunkarn” (Process)	Say 2 or 5 for $\frac{2}{5}$, “sab” (know), “kaud” (bottle), Say “yahk-sai” (no meaning) for “yahk-sab” (to be curious), say 2 jood (point) 5 for $\frac{2}{5}$,There is a student who just only read that “tha” (if) “lit” (liter) “long” (to go down) “yahk-sab” (to be curious) “glai” (to become) and There is one student can only read that “nam” (water) “nai” (in) “pen” (is)
Any remarks regarding problem solving process	All of student know that if denominators are equal, taking plus numerators but do not understand “Why?” There is a student trying to explain by diagram but incorrect answer then they use addition of numerators	All of student know that if denominators are equal, taking plus numerators but do not understand “Why?” There is a student trying to explain by diagram but incorrect answer then they use addition of numerators	Most of the student's answer are $\frac{2}{5}$. Three student answer $\frac{3}{10}$. Student who get $\frac{3}{5}$ they do only addition between numerators student who get $\frac{3}{10}$ they do addition numerators and numerators,	One of the student answer 13 and explain by taking $\frac{2}{5} + \frac{1}{5}$ equal 13; 2 + 1 equal 3, 5+5 equal 10 then 10 plus 3 equal 13. And another also answer 13 but cannot explain.
Specific mistakes	-	-	-	-

Table 28. Pupils' mistakes in problem solving in Q (8)

Q (8)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	1. They wrongly read some words e.g.,. sampun (relation) call to pasompun (breed) and pookpun (bound)	1.One student can read only some words; ‘kilogrum”(kilograms) “namnak”(weight) “pen”(is)		1. They wrongly read some words e.g., “pasompun” (breed) o “pookpun” (bound) instead of “sampun” (relation) 2.One student can read only some words e.g., “kilogrum”(kilograms) “namnak”(weight) “pen”(is)
Any remarks regarding problem solving process	Urban school	Rural School	High performance	Low performance
Specific mistakes	1. Two students answer $\frac{3}{5}$ because it can not be written as $\frac{5}{3}$ because 5 bigger than 3. Thus, $\frac{5}{3}$ is not a fraction. 2. Most of the students answered that $\frac{5}{3}$.	1. One student answer $\frac{3}{5}$ because the question ask to write relationship between George's meat and Judy's meat then she write George's meat (3) and Judy's meat (5). 2. Other students answer that $\frac{5}{3}$ and 8; they take 5+3.	1. Most of students do not understand relationship between 3 and 5. 2. One student can write correct answer and explain for relationship between 3 and 5. 3. One student can write correct answer and explain that they cannot write fraction in term of $\frac{5}{3}$ because 5 is bigger than 3.	1. Most of the students do not understand the relationship between 3 and 5. 2. One student can write correct answer and explain that they cannot write fraction in terms of $\frac{5}{3}$ because 5 is bigger than 3. 3. They have to answer $\frac{5}{3}$ because 5 is bigger than 3.

2.5.5 Discussion

1) Analysis of questionnaire (1st year)

(1) Number of books or possession of items at home

According to Table 8, the highest percentage of books possessed in the urban and rural areas was in the range of 0 to 10 books (52.50% and 90.00%, respectively). The highest percentage of overall possession of books was in the range of 0 to 10 books.

It is evident from Table 9 that people in the urban area have a larger number of items at home than those in the rural area. In general, people in rural and urban areas possess TVs (Q. V-b, 97.14%), calculators (Q. V-a, 80.00%), and radios (Q. V-c, 77.14%).

(2) Students' attitude towards Mathematics

	1	2	3	4
I would like to do more mathematics in school	54 (77.14%)	13 (18.57%)	3 (4.29%)	0 (0%)
I enjoy learning mathematics	50 (71.43%)	14 (20.00%)	6 (8.57%)	0 (0%)
I think learning mathematics will help me in my daily life	55 (78.57%)	13 (18.57%)	0 (0%)	2 (2.86%)
I need to do well in mathematics to get into the university of my choice	57 (81.43%)	11 (15.71%)	0 (0%)	1 (1.43%)
I need to do well in mathematics to get the job I want	54 (77.14%)	10 (14.29%)	4 (5.71%)	2 (2.86%)

77.14% of students would like to do more mathematics in school, 71.43% of them enjoy learning mathematics, 78.57% of them are of the opinion that mathematics will help them in their daily life, 81.43% of them believe that mathematics is important for them to enter university, and 77.14% of them believe that mathematics is important for them to get the job they desire. This reveals that the attitude of Thai students toward mathematics is positive.

2) Analysis of test results

The most difficult questions for students were Q. 2-9 and Q. 5-9 and the rate of correct answers were merely 10%. Fractions, proportions, and decimal numbers were also difficult questions for Thai students. Further analysis will be required on the questions pertaining to fractions (Q.2-2, Q.2-4, and Q.4-9) that the Thai students were unable to solve. The rates of correct answers are as follows.

	Q.2-2	Q.2-4	Q.4-9
Urban School	17 (42.50%)	13 (32.50%)	15 (37.50%)
Rural School	0 (0%)	1 (3.33%)	4 (13.33%)
Average	17(24.29%)	14 (20.00%)	19 (27.14%)

There are certain differences in students' achievements between the urban and rural areas. This is caused by various factors. From among these, a few of the major factors are opportunity for attending cram school, competitive environment in urban schools, and access to information.

3) Other related factors in response to the test

(1) Students' experiences in written examination

A majority of the students are experienced in answering multiple-choice tests. Thus, when dealing with answering short-answer questions as in this test, a few students submitted a blank paper.

(2) Error in translation

There was an error in Q. 6-5 when translated into Thai. The meaning of the translated test was different from its English version. This made it difficult to check the correct answer. The test was modified by the researcher and reused again.

(3) Requirement of further explanation

The explanation provided in the test was not sufficient. For example, certain students selected the answer code as the number of their family members, although the number asked for was just before the answer code.

(4) Timing of the survey

There were certain topics that students were not taught during the period the data was collected. As a result, it became difficult for the students to answer the test.

(5) Language

The language used in classes was different in the urban and rural schools. The students in the urban school usually use standard Thai language in the class, which is similar to language used in the test. Local language has an influence in the interpretation of problems or tests. However, students in the rural school, who always use local language in daily life and occasionally in classes, faced some difficulty in the interpretation of the language used in the test.

(6) Gender

There was no noticeable difference between the results of male and female students.

4) Analysis of the Results of the Students' Achievement Test (2nd year)

The average score in the achievement test was 33.43%. The performance of the students in the urban school (average score 43.20%) was better than that of the students in the rural school (average 26.10%). The performance difference between the two groups was 17.1%

All students from both schools were unable to solve Q7. From the interviews with teachers and students, it was evident that the students have not been taught the particular topic as yet. A majority of the students could not solve Q4 entirely; however, they succeeded in answering Q4 (1). They solved the problem using the meaning of "a half" of the whole. Thus, they were able to shade $\frac{1}{2}$ of 2 metres. However, when they applied this idea with Q4 (2), they shaded 1 meter of the entire 2 meters instead of shading just $\frac{1}{2}$ of 1 meter.

A majority of the students also succeeded in solving addition and subtraction of fractions with equal denominators. However, all of them did not understand "why" they need not add or subtract the denominators. Data from interviews revealed that they have learned as such and insisted that it is what the teachers stated. They have a strong belief that they must follow the advice of the teachers because the teacher is much more experienced than them.

Table 22 presents the overall result of the students' scores in the achievement test. The average score was 33.43%. The result of the achievement result is not at the expected level. The performance of the students in the urban school (average score 43.2%) was better than that of those in the rural school (average score 26.1%). The performance difference between the two groups is 17.1%. If the results are examined separately, it is evident that both boys and girls from the urban school performed better than those from the rural school. However, there were a few exceptions. In Q4 (2), rural students obtained 11.25%, while urban students obtained only 3.33%; therefore, rural students outperformed urban students. Similarly, in Q6 (3), rural students obtained 15.63%, while urban students obtained a slightly lower score of 13.33%.

The question-wise analysis of students' achievement reveals that, in general, the performance of students is comparatively good in Q4 (1), Q1 (2), Q3 (1), and Q6 (1). It appears that to a certain extent students are familiar with these types of questions as they are similar to those assigned in the classroom test. In general, the most difficult problems were Q6 (3), Q4 (2), Q10, Q5 (2), Q5 (3), and Q5 (1).

It can be assumed that the most difficult questions for students from the urban school were Q7, Q4 (2), and Q5 (3), and the average scores were 0%, 3.33%, and 3.33%, respectively. On the other hand, the most difficult questions for students from the rural school were Q7, Q4 (3), and Q5 (3), and the average scores were 0%, 0%, and 2.5%, respectively. According to the teachers' interviews, the *question styles* of some of the abovementioned questions were not the same as the styles of the textbook or classroom tests. Teachers also reported that students are not accustomed to this type of a questionnaire.

The performance of students from the rural school in Q4 (2) and Q6 (3) was better than that of students in the urban school. The only reason behind this is that the mathematics teacher of this class is a network teacher of the teacher who attended the innovative workshop on teaching with open-approach method. These two teachers attempted to provide their students with the experience of expressing their ideas freely in class.

On the other hand, the performance of students from the urban school in Q2 is better than that of students from the rural school. However, the reasons for this are not clear. It is possible that students from the rural school lack conceptual understanding of this topic, which is apparent from their inability to relate division and fractions.

5) Analysis of the Results of Students' Responses to the Interview

We selected ten students for an interview on the basis of their examination results in mathematics. Five of them obtained a good score in mathematics and the other five obtained a low score.

Interview items for the students are divided into the following four categories: (a) reading, (b) understanding of concept, (c) process, and (d) specific errors (if any).

All students could were not successful in comparing $\frac{1}{4}$ and $\frac{1}{3}$ because they had not been taught this topic as yet. From their attempt, it was evident that they had a rather ***naïve concept*** of fractions, as follows. They explained that $\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater 3.

Another explanation was provided with the diagram of two circles that were divided into four and three equal parts—since four parts are more than three parts, consequently, $\frac{1}{4}$ is greater than $\frac{1}{3}$.

Data from the interview revealed that a majority of the students were able to solve problems similar to those they have learned earlier. However, when they are questioned "why did you do that?", their responses were something like this: "The teacher told us to do so." The students do not really understand "why" the strategy works. For example, when adding fractions with equal denominators, they recite that the result is obtained by adding the numerators and taking the denominators. They are unable to reason why only the numerators are added but not the denominators.

Table 24 present the levels of errors in problem-solving in urban and rural schools. According to the findings, all students were able to read Q5 (3), Q6 (1), and Q8, but none of them could understand the concept of Q5 (3) and Q8, and only a few were able to understand the concept of Q6 (1).

With regard to process skill, in Q5 (3), a majority of them were unable to show the correct process. In Q6 (1) and Q8, half of the students were unable to show the correct process. In comparison, a greater number of students from the urban school were able to solve problems than those from the rural school.

Table 25 shows the levels of problem-solving errors according to their performance. According to the findings, all students were able to read Q5 (3), Q6 (1), and Q8; none of them were able to understand the concept of Q5 (3) and Q8, and only a few of them were able to understand the concept of Q6 (1).

High-performers performed slightly better than the low performers, except in Q8 where the latter group committed a large number of errors. In comparison, the high performers were able to solve problems better than the low performers, except in Q8.

In Q5 (3), the students from both schools found it difficult to read the words "correct" and "appropriate." They committed a few errors in the process skill. For example, according to them, $\frac{1}{4}$ is greater than $\frac{1}{3}$ since 4 is greater than 3, and some stated that if a denominator is greater than other denominators then that fraction is greater than other fractions, etc.

In Q6 (1), the students of the rural school found it difficult to read the word "process." They committed a few errors in the process skill. For example, they added both the numerators and denominators in order to obtain the solution.

In Q8, the students of both schools found it difficult to read the words "relationship," "in terms of," and "bracket." They committed a few errors in the process skill. For example, they only wrote the answer—$\frac{5}{3}$—without showing the process.

To conclude, it appears that ***lack of sufficient orientation to various types of problems***, the so-called <u>**only**</u> textbook-dependent lesson plan, the tendency of copying problems from textbooks for a test, and lack of adequate stimulation for innovative thinking are the main obstructions for students in achieving a satisfactory level in mathematics learning.

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2.6.1 School Curriculum

The latest mathematics syllabus for Grades 1–7 in Zambia was published in 2003 and that for Grades 10–12 in 2002, although that for Grades 8 and 9 has not been renewed since 1983. The general aims of the syllabus for Grade 1–7 are as follows.

1. To equip the child with necessary knowledge and skills to enable him/her to live effectively in this modern age of Science and Technology and to contribute to the social and economical development of Zambia.
2. To stimulate and encourage creativity and problem solving.
3. To develop the mathematics abilities of a child to his/her full potential and assist him/her to study Mathematics as a discipline and to use it as a tool in various subject areas.
4. To assist the child to understand mathematical concept in order that he/she may better comprehend his/her environment.
5. To develop in the child an appreciation of Mathematics in the traditional environment.
6. To develop interest in Mathematics and encourage a spirit of inquiry.
7. To build up understanding and appreciation of basic mathematical concepts and computational skills in order to apply them in everyday life.
8. To develop clear mathematical thinking and expression in the child
9. To develop ability to recognize problems and to solve them with related
10. To develop and foster order, speed and accuracy.
11. To provide the child with the necessary mathematical knowledge and skills in order for him/her to be productive and self reliant.
12. To develop in the child a positive attitude towards production, entrepreneurialship and self-reliance.
13. To provide the necessary mathematical pre-requisites for further education.

(Mathematics Syllabus Grade 1-7, 2003, pp.5,6)

The mathematics syllabus in Zambia has the feature of being like a "spiral"; the same topics can be found in the syllabus of several grades and taught repeatedly in the period of 9 years of basic education. This appears to be the effect of the British Mathematics Curriculum that was adapted during the "Modernization of Mathematics Education."

2.6.2 Basic Information

(1) Schooling Age

According to Zambian policy, children are expected to begin schooling at the age of 7. However, in practice, the ages at which children enter school vary because in certain cases, children who reach the age of 7 are unable to find seats in schools, particularly in urban areas; while in certain cases parents are unable to raise money for school requirements and hence delay sending the child to school. In rural areas, certain children live at a substantial distance from a school and their parents only send their children when they are old enough to walk long distances to school. As a result of this, there is a wide range in terms of age of children within a class.

(2) School Calendar and Examination

The school year in Zambia begins in January and ends in December. The school year is divided into 3 terms, as shown in table 1, and each term has a duration of 3 months. There is a one-month break between each term.

Table 1. School calendar (year)

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
▨	1st Term			▨	2nd Term			▨	3rd Term		▨

At the end of Grade 7, students take a national examination and those who are able to attain the cut-off marks proceed to Grade 8. All items in the Grade 7 examination are multiple-choice and presented in English. Similarly, at the end of Grade 9, students take a national examination and those who qualify proceed to Grade 10. The plans are that in future, if there are sufficient number of seats available in Grade 8, students from Grade 7 must be promoted automatically to Grade 8.

(3) Promotion System

The school system in Zambia is changing from the system of 7 years of primary education, 2 years of junior secondary education, 3 years of senior secondary education, and 2–7 years of tertiary (higher) education to 9 years of basic education, 3 years of high school education, and 2–7 years of tertiary (higher) education.

This change is still underway; therefore, two school systems are coexisting at present. The speed of this school reformation is rather slow in rural areas; as a result of this, there are schools in these areas that continue to have only lower-basic (Grades 1–4) and middle-basic (Grades 5–7) grades, although they are known as basic schools.

Table 2. Education system in Zambia

Before Education Reform			
Primary School		Secondary School	
7 years		2 years (Lower)	3 years (Upper)
After Education Reform			
Basic School			High School
3 years (Lower)	4 years (Middle)	2 years (Upper)	3 years

(4) Medium of Instruction

The official language in Zambia is English, although there are approximately 72 tribes that have their own language (dialect), which is usually spoken in their homes. English is also the medium of instruction; therefore, teachers basically conduct lessons in every subject in English. However, students, particularly in the initial years of schooling, usually speak local languages outside classes and school. Thus, students—particularly in lower grades—face difficulty in understanding or speaking English. Occasionally, certain teachers use a local language to explain certain things to students; however, the problem is that certain students may not understand the local language and therefore will not be able to communicate with the teachers. In certain cases, the teacher may not know how to speak the main local language.

2.6.3 Results from the First Year Field Survey

(1) Survey Schedule

A one-man delegation visited Zambia from January 17, 2005 to January 27, 2005 in order to facilitate the field survey with his Zambian counterpart. This implies that data collection was conducted just two weeks after the commencement of the first term. The detailed schedule for data collection is tabulated as follows.

Table 3. Schedule of data collection

Date	Activity
16th / Jan / 2005	Arrival in Lusaka, Zambia
17th- 19th / Jan / 2005	Preparation of the survey Discussion with District Education Offices and Targeted Schools
20th – 27th / Jan / 2005	Data collection in two sample schools in Lusaka Province and their subsequent remedial work
28th / Jan / 2005	Departure from Lusaka, Zambia

(2) Target Schools and Samples

[1] Sampling procedures

From among the 9 provinces in Zambia, Lusaka—the capital province—was selected as a sample due to the time constraints for completing data collection. According to the Central Statistic Office, the extra-departmental organization under the Finance Ministry, an urban area is defined by 3 criteria—population size, economic activity, and facilities available in the area[1]. Taking into consideration these criteria, from among 4 districts in the province, Lusaka District was selected to represent the urban area and Chongwe District to represent the rural area.

For the selection of average schools in both urban and rural areas, the survey team requested each District Education Office (DEO) to recommend appropriate schools. From the discussion with District Education Standards Officers (DESOs), we selected one average government school each from both urban and rural districts. The criteria taken into consideration were as follows:

a. Ranking of the sample schools according to the results of National Final Exam,

b. Social and economical strata in the catchment area of the sample schools.

However, the survey team was unable to go through the actual data related to the abovementioned criteria.

For the selection of a sample Grade 4 class in the urban average school, it was reported that classes are not formed according to the results of the students. Thus, they are all assumed as being uniform and one sample Grade 4 class was selected since the timing of its lesson was able

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[1] In addition to this definition, an urban area must have a minimum population size of 5,000 people. The main economic activity of the population must be non-agricultural, such as wage employment. In addition, the area must have basic modern facilities such as piped water, tarred roads, post office, police station, health facility, etc.

to fit into the overall schedule for data collection. In the average rural school, there is only one Grade 4 class; thus, all the Grade 4 students have been targeted in this survey.

[2] Basic school information

As mentioned earlier, Zambia is currently transforming its school system from a 7-year primary and 5-year secondary education to a 9-year basic and 3-year high school education. Particularly in remote areas, there remain a few basic schools that accommodate only 4 grades due to a number of constraints. Due to these situations, there are various sizes of basic schools depending on the level of progress in their transformation; two of our sample schools are not exceptional.

The following table presents the details of the sample schools.

Table 4. Location of schools

	School location
Urban Primary School	2 km from the capital city, Lusaka
Rural Primary School	58 km from the capital city, Lusaka 13 km from District centre

[3] Size of samples and their distribution by school location, sex, and age

As mentioned earlier in sample procedure, we selected one class each from both urban and rural average primary schools. In the rural average school, there was only one Grade 4 class; therefore, we decided to include all the Grade 4 students in the class.

There were certain situations that made it difficult to determine the number of sample students in each school. Prominent among these situations were

a. Timing: The timing of our data collection overlapped with the registration period for both schools. This resulted in variability in the number of sample students. There were a few changes in the number of students due to the influx and outflux of repeaters from Grade 5 and to Grade 3, respectively.

b. Weather: The month of January is a part of the rainy season in Zambia. As a result, students' attendance actually depended on the weather. In the rural school, a number of students remained absent until the second day of our data collection due to the heavy rain in the morning.

Nonetheless, the following tables present the number and distribution of sample students by location, sex, and age. Due to the Government's policy of age restriction for school admission and existence of repeaters and dropouts, age distribution appears very wide and the peak is toward the older age group from the expected enrolling age of 10.

Table 5. Distribution of sample

	Grade	Male	Female	Total
Urban Primary School	4	29	21	50
Rural Primary School	4	16	17	33
Total	-	45	38	83

Table 6. Distribution of students' age

Age (years)	No. of Pupils		
	Urban	Rural	Total
8	0	0	0
9	1	1	2
10	3	3	6
11	8	14	22
12	7	2	9
13	20	10	30
14	9	1	10
15	2	2	4
Average	12.5 yrs	11.8 yrs	12.3 yrs

(3) Results of Student Questionnaire

Questionnaires were administered to the students by explaining the English version of each item in the questionnaire in Nyanja, which is the local language of instruction in the targeted districts. This method was agreed upon after observing that a majority of the targeted students did not possess sufficient reading comprehension in English. After having administered the questionnaire, the survey team also conducted a little remedial work individually with certain students in order to fill out the unanswered items or to confirm incorrectly marked items. In the rural school, the survey was conducted through individual interviews of the targeted students, and questionnaires were filled out by the Zambian counterparts. The followings are the major findings from the students' questionnaires.

[1] Frequency of use of English and Nyanja (local language of instruction in the area)

Unlike other participating countries, the survey team decided to question the frequency with which students use both the official and local languages of instruction in the targeted districts. In Zambia, in addition to the official language—English—7 major local languages have been selected as local languages of instruction in schools. Considering the composition of tribes in the concerned areas, one of these local languages was carefully decided upon and used in the schools. This arrangement can be said to be a sort of compromise as there are 73 tribes in the country. Therefore, it is possible that there are certain students whose mother tongue is different from the local language used in the schools and this situation may affect the achievement of students. In order to examine this influence, we had added an additional question to the questionnaire.

As shown in Table 5, only approximately 5% of all the targeted students answered that they either always or almost always speak English at home; all these students are from the rural school. This result may require some verification as it does not coincide with the achievement in mathematics tests.

On the other hand, a difference in the percentage of students who occasionally speak English can be explained by the difference in the number of opportunities these students have to interact with people other than their family members. It can be supposed that students in urban areas have a greater opportunity to interact with people around their house, in the market, etc.

With regard to the reasons behind the difference in the frequency in use of Nyanja between urban and rural schools, one reason is the uneven distribution of either fathers' or mothers' tribes in each school. 70–80 % of the students in the urban school have parents who are either Nyanja or its tribal cousin Benba; this figure is only approximately 30% in the rural school.

Similar to the case of speaking English, a difference in the level of interaction with people outside their houses can also contribute toward the difference in frequency of the use of Nyanja, as it is the most popular language of communication in public places in these districts.

Table 7. Pupil's distribution on frequency in use of English by location (%)

	Always	Almost Always	Sometimes	Never
Combined	1.2	3.7	65.9	29.3
Urban	0.0	0.0	76.0	24.0
Rural	3.1	9.4	50.0	37.5

Table 8. Pupil's distribution on frequency in use of Nyanja by location (%)

	Always	Almost Always	Sometimes	Never
Combined	42.7	25.6	30.5	1.2
Urban	44.0	34.0	20.0	2.0
Rural	40.6	12.5	46.9	0.0

Table 9. Pupil's distribution on their fathers' tribe by location (%)

	Nyanja	Bemba	Tonga	Lozi	Soli	Ndebele	Lenje	Others
Combined	41.0	20.5	2.4	7.2	4.8	12.0	4.8	7.2
Urban	50.0	32.0	4.0	12.0	0.0	0.0	0.0	2.0
Rural	27.3	3.0	0.0	0.0	12.1	30.3	12.1	15.2

Table 10. Pupil's distribution on their mothers' tribe by location (%)

	Nyanja	Bemba	Tonga	Lozi	Soli	Ndebele	Lenje	Others
Combined	46.3	13.4	11.0	4.9	1.2	7.3	4.9	11.0
Urban	58.0	16.0	16.0	6.0	0.0	0.0	2.0	2.0
Rural	28.1	9.4	3.1	3.1	3.1	18.8	9.4	25.0

[2] Possession of books and domestic items at home

Over 70 % of the targeted students in both urban and rural schools have 10 or less than 10 books at home. A majority of the targeted students in both schools have radios at home, while almost none of them have computers.

Table 11. Distribution of students' books (%)

	0-10 books	11-25 Books	26-100 books	100- books	No response	Crossed out
Combined	73.5	19.3	4.8	2.4	0.0	
Urban school	72.0	16.0	8.0	4.0	0.0	
Rural school	75.8	24.2	0.0	0.0	0.0	

Table 12. Distribution of students' home items (Yes, %)

	Calculator	TV	Radio	Desk	Quiet place	Dictionary	Books (Excluding text book)	Computer
Combined	39.8	56.6	91.6	39.8	50.6	34.9	56.6	2.4
Urban school	48.0	72.0	86.0	38.0	68.0	52.0	52.0	4.0
Rural school	27.3	33.3	100.0	42.4	24.2	9.1	63.6	0.0

Table 13. Students' attitude towards mathematics

Pupils' Perceptions	Average		
	Combined	Urban	Rural
1. I usually do well in mathematics	2.0	2.0	2.1
2. I would like to do more mathematics in school	1.5	1.4	1.7
3. Mathematics is harder for me than for many of my classmates	2.2	2.1	2.4
4. I enjoy learning mathematics	1.4	1.4	1.5
5. I am just not good at mathematics	2.5	2.1	3.2
6. I learn things quickly in mathematics	1.8	1.6	2.0
7. I think learning mathematics will help me in my daily life	1.9	1.6	2.4
8. I need mathematics to learn other school subjects	1.8	1.4	2.5
9. I need to do well in mathematics to get into the university of my choice	1.6	1.4	2.0
10. I need to do well in mathematics to get the job I want	1.7	1.3	2.0

The results presented in the above table reveal that students tended to have positive attitudes toward learning mathematics in schools as statements such as "I would like to do more mathematics in school," or "I enjoy learning mathematics" are valued positively. If we compare the perceptions of urban students with rural students, it may be stated that students in the rural school tended to be more neutral toward learning of mathematics than those in the urban school, particularly on questions regarding the meaning of learning mathematics in or outside school.

There is also an inconsistency in the urban students' perceptions toward their ability in mathematics. The positive statement of "I usually do well in mathematics," and negative ones such as "Mathematics is harder for me than for many of my classmates," and "I am just not good at mathematics" are equally perceived by students in the urban school.

[3] Frequency of students' activities in their mathematics lessons

The targeted students were asked to reveal the frequency of the following activities in their mathematics lessons according to f4 scales—1: every or almost every lesson, 2: about half of the lessons, 3: some lessons, or 4: never.

The table below shows that regardless of the location of the schools, a majority of the activities were not frequently conducted in class, except the following 3: "I practice adding, subtracting, multiplying, and dividing without using a calculator," I listen to the teacher talk," and "I work problems on my own."

Table 14. Students' activities in their mathematics lessons

Students' activities	Average		
	Combined	Urban	Rural
1. I practice adding, subtracting, multiplying, and dividing without using a calculator	2.1	1.3	3.3
2. I work on fractions and decimals	3.0	2.7	3.3
3. I measure things in the classroom and around the school	2.9	2.8	3.2
4. I make tables, charts, or graphs	3.1	2.8	3.5
5. I learn about shapes such as circles, triangles, and rectangles	2.2	1.7	2.9
6. I relate what I am learning in mathematics to my daily lives	2.5	2.0	3.4
7. I work with other students in small groups	2.6	2.0	3.5
8. I explain my answers	2.5	2.0	3.3
9. I listen to the teacher talk	1.3	1.4	1.1
10. I work problems on my own	1.8	1.7	1.9
11. I review my homework	2.2	1.8	2.9
12. I have a quiz or test	3.0	2.7	3.5
13. I use a calculator	3.4	3.0	4.0

[4] Students' perception of the school and teacher

Students were asked to rate their level of agreement with the following perceptions according to four scales—1: agree a lot, 2: agree a little, 3: disagree a little, or 4: disagree a lot.

The table below indicates that students in both locations have a positive opinion of their teachers and schools. On the other hand, students themselves believe that they are making an effort to do better in spite of their teachers' efforts and care.

Table 15. Pupils' perception towards school and teacher

Pupils' perceptions	Average		
	Combined	Urban	Rural
1. I like being in school	1.0	1.0	1.1
2. I think that students in my school try to do their best	1.9	2.0	1.8
3. I think that teachers in my school care about the students	1.1	1.1	1.1
4.I think that teachers in my school want students to do their best	1.1	1.2	1.1

[5] Time spent on other activities before or after school on a normal school day

According to the results shown below, approximately 40% of the students have no time to watch TV before or after school on a normal day. Almost 90% of the students do not have time to use the computer, and these figures are clearly related to the level of possession of these items.

On the other hand, over 40% of the students spend 1–2 hours to "play or talk with friends" and "do jobs at home," while 65–75% of the students spent less than two hours to "play sports, " read a book for enjoyment," and "do homework."

Table 16. Pupils' activities before or after school on a normal school day (%)

	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or more hours
1. I watch TV	39.8	25.3	21.7	6.0	7.2
2. I play or talk with friends	0.0	24.1	45.8	13.3	16.9
3. I do jobs at home	2.4	25.3	44.6	18.1	9.6
4. I play sports	26.5	41.0	19.3	4.8	8.4
5. I read a book for enjoyment	27.7	48.2	13.3	2.4	8.4
6. I use computer	89.2	6.0	2.4	0.0	2.4
7. I do homework	7.2	65.1	19.3	6.0	2.4

[6] Number of family members including students themselves

According to the survey, a majority of the students in both urban and rural schools have 4–9 members in their families. However, students from the rural school tended to have more members than those from the urban schools.

Table 17. Distribution of students' family member

Family member	Pupils' Distribution		
	Combined	Urban	Rural
2-3 persons	8.4	12.0	3.0
4-6 persons	31.3	34.0	27.3
7-9 persons	37.3	38.0	36.4
10-12 persons	14.5	10.0	21.2
More than 12 persons	8.4	6.0	12.1

(4) Results of Achievement Test

Unlike what was done for the questionnaire, only the instructions for the achievement test were explained to the students in their local language of teaching at the beginning of the test. Thereafter, no further explanations were provided to the students regarding the test in order to avoid providing any clues to the answers.

[1] Students' achievements according to location and sex

The average score in the mathematics achievement test in Zambia was 12%. An examination of the average scores according to the categories reveal that they are also rather low regardless of location and sex. In this sense, it may not have much meaning to make a comparison across categories; however, if we dare do so, it may generally be stated that the difference between location (urban vs. rural) is much larger than that between sexes (male vs. female), as shown in the table below.

In detail, students in the urban school were able to achieve almost twice more than those in the rural school. According to sex, girl students performed better than boy students, which was against our assumption.

Table 18. Distribution of students' right answer (%)

	Combined	Male	Female	Highest	Lowest
Combined	12.0	11.2	12.9	45.2	0.0
Urban school	15.1	14.4	16.0	45.2	0.0
Rural school	8.4	6.5	10.0	17.8	0.0

[2] Analysis of the results with regard to content domains

Similar to previous results in (1), the achievement of students is rather low across content domains. If the results are examined with regard to the type of questions, almost none of the students were able to answer the short-answer questions. With regard to multiple-choice questions, it appears that students were able to achieve slightly better in "measurement" and "algebra" in multiple-choice questions, although only a few students were responsible for this.

Table 19. Average rates of correct answers by content domain

	Multiple-choice		Short-answer		Total	
Content domain	No. of questions	Rates of correct answers (%)	No. of questions	Rates of correct answers (%)	No. of questions	Rates of correct answers (%)
Number	19	16.0	9	3.4	28	12.0
Measurement	12	20.5	4	0.0	16	15.4
Algebra	4	19.1	1	0.0	5	15.3
Data	4	16.8	4	0.3	8	8.5
Patterns, relations and functions	5	16.0	1	1.2	6	13.5
Geometry	4	12.4	6	3.7	10	7.2
Total	48	17.2	25	2.2	73	12.0

Table 20. Average rates of correct answers of content domain by urban and rural school (%)

Content domains	Urban school			Rural school		
	Multiple-choice	Short-answer	Total	Multiple-choice	Short-answer	Total
Number	20.1	5.1	15.3	11.2	1.4	8.1
Measurement	24.1	0.0	18.1	16.2	0.0	12.2
Algebra	21.7	0.0	17.4	16.0	0.0	12.8
Data	19.0	0.5	9.8	14.1	0.0	7.1
Patterns, relations and functions	21.3	2.2	18.1	9.7	0.0	8.1
Geometry	17.4	6.9	11.1	6.4	0.0	2.6

There was one aspect that was found strange in the multiple-choice questions. In a majority of the questions in the achievement test, 4 choices were provided. This implies that the rate of correct answers could be around 25%, even if students guess their answers. However, in all the domains, the rates were lower than 25%. A probable explanation for this is that almost 30% of the students, on an average, either did not provide any answers at all or their answers were not meaningful.

Discussion

From the performance of the students in this survey, it can be said that a majority of the students were neither ready nor sufficiently competent to take this type of achievement test. With regard to the results from the questionnaire, certain factors from the viewpoints of either mathematics education or others must be indicated. In Zambian case factors in external subject education appear to affect students' actions more seriously. Prominent among them are as follows.

[1] Language

Language appeared to be one of the biggest factors that resulted in the low performances of students. As revealed by the questionnaire, over 90% of the students do not speak English frequently at home. Besides, reading comprehension in English was not sufficiently good to understand the level of language used in both the questionnaire and test. However, it must not be concluded that only their mother tongue must be used due to a number of reasons.

For example, a Zambian scholar expressed that there has been no attempt thus far to interpret the contents of mathematics to local languages in the written form; the contents are written in English. It was analyzed that this fact may have resulted in a certain understanding gap between learning in English and other mother tongues.

A second example may be with regard to the level of development in reading. While observing a lesson in one of the schools, there were several students who copied what the teacher was writing on the blackboard into a series of letters without spaces, which did not form words. This may reveal that such students did not obtain any meaning from the sentences written on the board. In such a case, there would not be much difference in giving written tests either in the local languages or English.

[2] Students' experience or preparedness for writing the exam

It appears that a majority of the students at this level were not used to taking tests that had an enormous volume of question papers, which were given to individual students. They were also not familiar with questions such as multiple-choice, sentence problems, open-ended questions, etc. In addition to the above, it was found that there are a number of skills required either explicitly or implicitly for taking exams. These could be understanding the multiple-choice answers in the context of the questions, matching the answer numbers to the actual written answers, recognizing the answering spaces for open-ended questions, deciphering the meaning of pictures in the context of the questions, etc, to mention a few.

A little more care must be given to knowing the extent that the students have developed such skills in writing exams and how far they can be trained in the targeted grade.

[3] Existence of silent responders

According to the findings in 2.2, there were a number of students who did not provide any answers or those that were meaningful; this made the results less credible for ascertaining the difficult, easy, or favorite topics for the students in Zambia.

On the other hand, this fact can be utilized for analyzing students' tendencies or preferences in answering questions by comparing the rates of responses with other findings in this survey.

For example, there is a large variation in the rates of students' responses toward answering questions, which varies from 87% to 6% on the opposite ends of the spectrum. By comparing the type of questions according to the rates of responses with previous tests conducted by the school, the manner of solving questions, topics in the subject, a trend with regard to the experience of students with tests, their cognitive power, or preference of topics in mathematics may be ascertained.

Taking into consideration the abovementioned aspects, it is important to examine further the hindrances or perceptions that may impede the ability of the students to tackle questions not only from the viewpoint of mathematics education but also from the cognitive, cultural, and social viewpoints. This will, at a later stage, provide this research project with a few ideas to alter the current approach and adapt one that is more appropriate for ascertaining the real education scenario in countries such as Zambia.

2.6.4 Results from the Second Year Field Survey

(1) Survey Schedule

The survey was conducted during the period October 17–21, 2004; the details are provided in table 21.

Table 21. Schedule of data collection

Date	Activity
16th / Oct / 2005	Arrival to Lusaka, Zambia
17th / Oct / 2005	Discussion with Mr. Nkhata at University of Zambia
18th – 21st / Oct / 2005	Data collection in three sample schools
23rd / Oct / 2005	Departure from Lusaka, Zambia

(2) Target Schools and Samples

Until now, Urban School and Rural school A were selected as the target average schools. At this point, a comparison between the result of tests conducted with and without an explanation to the students in the local language will be provided. Rural School A has only one class in each grade; thus, an additional rural school must be included to make a comparison. Therefore, Rural School B was selected as an average rural school, located in Mazabuka District in the Southern Province of Zambia.

(Explanation in local language implies a translation, into the local language, of questions that are written in English, and an explanation regarding the manner in which to write the answers, particularly in the case of multiple-choice questions.)

Table 22. Location of schools

	School location
Urban Primary School	2 km from the capital city, Lusaka
Rural Primary School A	58 km from the capital city, Lusaka 13 km from District Centre
Rural Primary School B	24 km from District Centre

In the urban school, the test was conducted in 3 classes; however, it was observed that in one class, a teacher who was supervising the students provided them with a few answers. Therefore, the data pertaining to this particular class has not been included in the sample. Despite this, there were 2 classes out of which one was provided with an explanation of the test and the other was not. Rural school B had 2 classes, although these were combined and one teacher taught both classes at the same time. In spite of this, one class was provided with an explanation of the test in local language and the other class was not. Rural school A had just one class, and the test was conducted with an explanation.

Table 23. Distribution of sample

	Grade	Boys	Girls	Total
Urban Primary School	4	34	35	69
Rural Primary School A	4	19	20	39
Rural Primary School B	4	37	12	49
Total	-	90	67	157

The achievement test included topics that students were expected to know in Grades 3–7, as shown in table 24. However, there were 2 questions that were not covered in the mathematics curriculum for Grades 1–7 in Zambia.

Table 24. Curriculum coverage of the achievement test

Question no.	Grade when covered
Q1 (1), Q1 (2), Q3 (1), Q3 (2) and Q4 (2)	Grade 3
Q2,	Grade 4
Q4 (1), Q4 (3), Q6 (1), Q6 (2), Q6 (3) and Q7	Grade 5
Q5 (1), Q5 (2) and Q5 (3)	Grade 6
Q8	Grade 7
Q9 and Q10	Not covered in any grade.

(3) Results of Achievement Test

The performance of the students who took the test is presented below, beginning with the overall performance, followed by the question-wise performance.

[1] Overall performance

Table 25 below shows the overall performance of students who took the test.

Table 25. Pupils' general performance

	Average	Male	female	Explanation	Without
Urban area	11.2%	12.1%	10.4%	15.6%	8.2%
Rural area	6.9%	5.8%	9.0%	8.7%	2.7%
All	8.8%	8.2%	9.7%	12.3%	6.2%

As can be seen from the above table, in the urban school, boys performed better than girls. There was also a large difference within the urban area between the performance of students who received explanation and those who did not—those who received the explanation performed better than those who did not. However, in the case of students in the rural area, girls performed much better than boys; however, even in this case the performance of those who received explanations was superior to those who did not. It is also evident that students in the urban school performed better than those in the rural one.

[2] Question-wise performance of students

Table 26 shows the question-wise performance of the students in the achievement test.

Table26. Question by question performance of pupils

	School in urban area			School in rural area		
	Total	With explanation	Without explanation	Total	With explanation	Without explanation
Q1 (1)	0%	0%	0%	26.1%	42.3%	17.4%
(2)	1.5%	3.6%	0%	20.5%	30.8%	17.4%
Q2	1.8%	7.1%	1.2%	1.1%	0%	0%
Q3 (1)	24.7%	42.9%	12.2%	9.7%	10.9%	2.2%
Q3 (2)	16.7%	20.8%	13.8%	8.7%	1.3%	8.7%
Q4 (1)	47.8%	64.3%	36.6%	15.1%	18.2%	0%
Q4 (2)	1.5%	3.6%	0%	1.4%	0%	0%
Q4 (3)	0.4%	0.9%	0%	1.1%	0%	2.2%
Q5 (1)	42.8%	57.1%	32.9%	26.1%	50.0%	0%
Q5 (2)	20.3%	17.9%	22.0%	10.2%	19.2%	4.4%
Q5 (3)	8.0%	5.4%	10.0%	10.2%	15.4%	4.4%
Q6 (1)	0%	0%	0%	4.3%	10.6%	2.2%
Q6 (2)	0%	0%	0%	1.0%	3.4%	0%
Q6 (3)	0%	0%	0%	3.4%	5.8%	2.2%
Q7	12.3%	30.6%	0%	0%	0%	0%
Q8	10.1%	0%	22.8%	0%	0%	0%
Q9	21.7%	28.6%	17.1%	1.4%	5.8%	0%
Q10	10.1%	25.0%	0%	1.1%	0%	0%
Total	11.2%	15.6%	8.2%	6.9%	8.7%	2.7%

(There are 2 schools in the rural area and "total" includes both; however, "with explanation" and "without explanation" are the results just in one school that has 2 classes to compare with.)

[3] Levels of inability of students in solving problems assigned according to school location

After taking the test, students were also interviewed in order to identify the levels of their inability to solve the problems assigned to them. The inabilities were identified and grouped into 3 levels as follows: unable to read the question presented, unable to understand the question/concept in the question, and unable to carry out the process of finding the answer to the problem presented. The frequency of these inabilities in the two categories of schools for questions 5 (3), 6 (1), and 8 are presented in table 27.

It is evident from the above table that a large number of students in both the urban and rural schools were able to read the question presented to them. However, very few students demonstrated an understanding of the question, i.e., they were not able to translate the content of the question into the local language. With regard to working out (process) the solution to the problem given, merely 2 students in the urban school demonstrated mastery of the process for part 3 of question 5. The remaining students were not able to work out the solution. Table 8 also shows that merely 3 students provided the correct answer for part 1 of question 6. Although certain students (numbers in brackets) provided correct answers for part 3 of question 5 and for question 8, they did this accidentally without using the correct processes.

Table 27. Pupils' levels of inability to solve problems given

Q. No.	Frequency of occurrence of inabilities									No. of pupils with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	Urban (N=20)	Rural (N=30)	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural	Total
Q5(3)	17	27	44	1	3	4	2	0	2	0(1)	0(1)	0(2)
Q6(1)	16	27	43	2	2	4	0	0	0	2	1	3
Q8	17	27	44	3	3	6	0	0	0	0(1)	0(1)	0(2)

[4] Levels of ability of students for solving problems according to their performance

Table 28 shows students' levels of ability for solving questions according to their performance.

Table 28. The students' errors of problem solving level according to their performance

Q. No.	Frequency of failure on Problem solving level									No. of students with Correct Answer		
	I. Reading			II. Understanding of Concept			III. Process					
	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total	High performance	Low performance	Total
Q5(3)	19	25	44	4	0	4	2	0	2	0(2)	0	0(2)
Q6(1)	18	25	43	4	0	4	0	0	0	3	0	3
Q8	19	25	44	6	0	6	0	0	0	0(2)	0	0(2)

Table 29. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	Grater than 1/4, 1/3	Grater than 1/4, 1/3	Grater than 1/4, 1/3	They could not read any words.
Any remarks regarding problem solving process	1/4 is grater than 1/3, since 4 is grater than 3.	1/4 is grater than 1/3, since 4 is grater than 3. 1/4 is grater than 1/3, since 1+4=5, 1+3=4 and 5 is grater than 4.	1/4 is grater than 1/3, since 4 is grater than 3. 1/4 is grater than 1/3, since 1+4=5, 1+3=4 and 5 is grater than 4.	1/4 is grater than 1/3, since 4 is grater than 3. 1/4 is grater than 1/3, since 1+4=5, 1+3=4 and 5 is grater than 4.

Table 30. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	container	container	container	They could not read any words.
Any remarks regarding problem solving process	2/5+1/5=3/10 2/5+1/5=2+5+1+5 =13	2/5+1/5=3/10 2/5+1/5=2+5+1+5 =13	2/5+1/5=3/10 2/5+1/5=2+5+1+5 =13	2/5+1/5=3/10 2/5+1/5=2+5+1+5 =13

A majority of the students did not understand what the question implied so they merely guessed what needed to be done and added the numbers in the question.

Table 31. Pupils' mistakes in problem solving in Q8

Q8	Urban school	Rural School	High performance	Low performance
Difficult words & Expressions	Fraction Relationship Bought between	Fraction Relationship Bought between	Fraction Relationship Bought Between	They could not read any words.
Any remarks regarding problem solving process	5+3=8	5+3=8	5+3=8	Most of them did not answer anything

All the students did not understand the meaning of the question; therefore, it was explained in the local language. Then, they did understand the situation, which was that "Jody bought 5 kg of meat and George bought 3 kg of meat"; however, they did not understand what "the relationship between Judy's and George's meat in terms of weight" implied.

Discussion

1. Analysis of the Results of the Students' Achievement Test

Q1 (1) was an easy addition of fractions: 2/5 + 1/5. However, no students in the urban school answered correctly, and a large number of them wrote 3/10 as the answer. Thirteen students from among 157 added all the numbers that are shown there; this implies that they did 2 + 5 + 1 + 5 and got 13. Eighteen of them wrote 310 as the answer (3 may have been obtained from 2 + 1—the sum of the numerators, and 10 may have been obtained from 5 + 5—the sum of the denominators).

Q1 (2) was a subtraction of two fractions: 5/7 – 2/7. Thirty-one students provided the answer 3/0 and 16 of them answered 21, which is the sum of all the numbers there.

Merely 3 students provided the correct answer (3/7) to Q2 (Express the answer in a fraction—3 ÷ 7); 52 students wrote 2 or 2 remainder 1, which is the quotient of 7 ÷ 3.

With regard to sentence questions, a majority of the students appear to be unable to read or understand them well; this was true even in the classes that were provided explanation in the local language because a large number of students merely added the numbers provided in the questions, although they were not questions of addition. In the case of Q6 (3), the answer was 7, and in the case of Q8, it was 8.

A large number of students did not understand how to answer or what they were asked, so they wrote the answer at an inappropriate place, or what they wrote was incomprehensible.

In terms of the comparison between results of those students who were provided an explanation in the local language and those that were not, there were rather large differences. The greatest difference was that the students who were provided explanation in the local language knew how to write answers, particularly for multiple-choice questions; on the other hand, a majority of the students who were not provided any explanation failed to write the proper form of answers in the adequate place.

(4) Analysis of the Results of the Interview

It was rather difficult to communicate with the students in the interview because they did not understand English very well. A large number of students failed to read the questions aloud; certain students were unable to read even a single word. Students in the urban school could read more fluently as compared with students from the rural school.

Q5 (3) Which of the following is greater than the other? Write the correct choice (1, 2, or 3) in the parentheses.

1. A is greater than B.   2. B is greater than A   3. A = B

A: 1/4   B: 1/3   Answer (   )

A majority of the students believed that 1/4 was greater than 1/3 since 4 is greater than 3. They did not recognize that a fraction is a number, but just looked at each number shown there.

Q6(1) When you add 2/5l of water to 1/5l of water in a container, how much water is there in the container?

A large number of students were unable to read the question; however, they guessed what it stated and believed that the question asked for the 2 fractions to be added. Certain students did not even have a vague understanding of what they were supposed to do. When the meaning of the question was explained in local language they were able to understand it. However they were unable to add the fractions. Several students added not only the numerators, but also the denominators. Others calculated in the following manner: 2/5 + 1/5 = 2 + 5 + 1 + 5 = 13. Merely 3 students provided the correct answer.

When we asked the students to write the process of calculation on paper, several students were unable to use mathematical symbols like +, –, =, etc.; thus, the numerals appeared as 2 5 1 5 13 on the paper.

Q8 Judy bought 5 kg of meat. George bought 3 kg of meat. Write a fraction in the bracket to show the relationship between Judy's meat and George's meat in terms of weight.

George's meat is (   ) of Judy's meat.

None of the students understood the implication of the above question. When it was explained in the local language they were able to understand the quantity of meat that Judy and George bought; however, they were unable to understand what "relation" meant. They had not learned fractions in terms of proportions, and this question was too advanced for their current knowledge.

In addition, certain students who provided the correct answer on the test did not answer correctly in the interview. It was possible that they were upset because they had not been interviewed in this manner before.

Conclusion

Several students calculated a fraction in the following manner: 1/2 = 1 + 2 = 3. This implies that students do not consider a fraction to be a number but a combination of two numbers that is supposed to be added.

Further, a majority of them do not understand English very well. They may be used to attending class without studying, where they can merely guess what the teachers are saying.

Future research must be conducted in order to explore what is required to improve the understanding of students with regard to mathematical concepts.

Acknowledgment

We greatly appreciate the help rendered by Prof. Chistopher Haambokoma, professor in the University of Zambia, in writing this report.

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2.7 Japan

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2.7.1 School Curriculum

In the elementary schools of Japan, the topic "Fractions" is taught in Grades 4, 5, and 6. Table 1 presents the outline of the content development of fractions in the abovementioned grades.

Table 1. Outline of Content development of fractions in the Course of Study for Elementary School

Grade	Contents	Terms and symbols
4	➢ The meaning of fractions ➢ How to express fractions ➢ Fractions can be represented as a certain number of times unit fractions	➢ Denominator ➢ Numerator ➢ Mixed fraction ➢ Proper fraction ➢ Improper fraction
5	➢ Fractions of the same size ➢ Converting whole numbers and decimals into fractions, and representing fractions in decimals ➢ The result of dividing whole numbers can always be expressed as a single number using fractions ➢ Adding and subtracting fractions with the same denominators	(Nothing)
6	➢ The equivalence and size of fractions, and the methods how to compare their size ➢ Adding and subtracting fractions with different denominators ➢ Multiplying and dividing fractions	➢ Reduction to a common denominator

2.7.2 Basic Information

(1) Schooling Age

The school education system in Japan comprises elementary education for 6 years, junior secondary education for 3 years and senior secondary education for 3 years (6-3-3 system). Further, Japanese students enter elementary school at the age of 6, junior secondary school at the age of 12, and senior secondary school at the age of 15.

Table 2. Schooling age in Japan

Age: 6~12 (for 6 years)	Age: 12~15 (for 3 years)	Age: 15~18 (for 3 years)
Elementary School	Junior Secondary	Senior Secondary

(2) School Calendar and Examination

Generally speaking, a school year in Japan begins in the second week of April and ends in the fourth week of March. It is divided into 3 terms; there are school holidays after the completion of 2 terms, as shown in Table 3. (Recently, certain schools have begun introducing a 2-term system.)

Table 3. School calendar in (year)

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
▨ Term 3			▨ Term 1				▨	Term 2			▨

At the elementary school level in Japan, the end-of-term test or national examination is not conducted as a legal requirement. However, a type of evaluation test is conducted in order to assess students' achievement depending on each school or teacher.

(3) Promotion System

In Japan, elementary and junior secondary education is compulsory for a total of 9 years. All students can be automatically promoted during this period. This implies that all students in each class of elementary and junior secondary school in Japan are approximately the same age. After junior secondary school, there is an entrance examination for senior high school; students who want to enter senior high school must pass this examination.

The recent enrolment ratios of elementary and junior high schools and the progression ratio of senior high school are as follows:

Table 4. Enrolment and Progression ratio in Japan (2005)

School	Elementary	Junior Secondary	Senior Secondary
Enrolment or Progression Ratio	99.96%	99.98%	97.6%

Source: http://www.mext.go.jp/b_menu/toukei/002/002b/18/015.pdf

(4) Medium of Instruction

In Japan, the official language and mother tongue for both children and teachers is Japanese in general; therefore, the medium of instruction is also Japanese.

(5) Class Organization

In the elementary schools of Japan, each teacher takes charge of a particular class and teaches all subjects (class-specific teacher). In general, each lesson is of a duration of 45 minutes, and there are at least 23~27 lessons per week.

Table 5. School hours

<table>
<tr><td></td><td>▨</td><td colspan="9">First shift</td><td colspan="2">▨</td></tr>
<tr><td>Hour</td><td colspan="2">8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td colspan="2">15</td><td>16</td><td>17</td></tr>
<tr><td>Grade</td><td>▨</td><td colspan="9">All Grades</td><td colspan="2">▨</td></tr>
<tr><td>Period</td><td>▨</td><td>HR</td><td>1st</td><td>2nd</td><td>3rd</td><td>4th</td><td>Lunch Time
Cleaning</td><td>5th</td><td>6th</td><td>HR</td><td colspan="2">▨</td></tr>
</table>

2.7.3 Results from the Second Year Field Survey

(1) Survey Schedule

The survey in Japan was conducted according to the following schedule:

Table 6. Schedule of data collection

Date	Activity
06/03/2006	Data collection in the school, Onomichi-city, Hiroshima

(2) Target Schools and Samples

Onomichi city is one of the cities in Hiroshima prefecture and is located in the southeast of Hiroshima. The population of the city is approximately 150,000. An average school in Onomichi city was selected as the sample school based on an achievement test conducted by the education board of the city.

Table 7. Location of schools

	School location
Average Elementary School	On the top of a hill which is in Onomichi-city

Grade 4 was selected as a sample class for the survey. Although the actual strength of the class was 23 students, 4 were absent (due to illness) when the survey was conducted. The distribution of the sample for the survey was as follows:

Table 8. Distribution of sample

	Grade	Boys	Girls	Age	Total
Average Elementary School	4	5	14	10	19

Due to time limitations, the survey in Japan included only these 19 students studying in grade 4. However, all 19 students participated in the achievement test and were interviewed using the Newman Procedure by 7 graduate students of Hiroshima University.

(3) Results of the Achievement Test

From the result of the achievement test, the average score and the highest and lowest scores among the 19 students, 5 boys and 14 girls were found as follows:

Table 9. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Average School	60.7%	57.4%	61.9%	87.0%	15.0%

Figure 1 presents the individual achievements of the 19 students in decreasing order. As a result, J20, J01, J13, J22, and J11 were identified as *High Performers* and J06, J15, J09, J18, and J17 were identified as *Low Performers* for the analysis of the interview using the Newman Procedure. Incidentally, the scores of the high and low performers were over 75.0% and less than 45.0%, respectively.

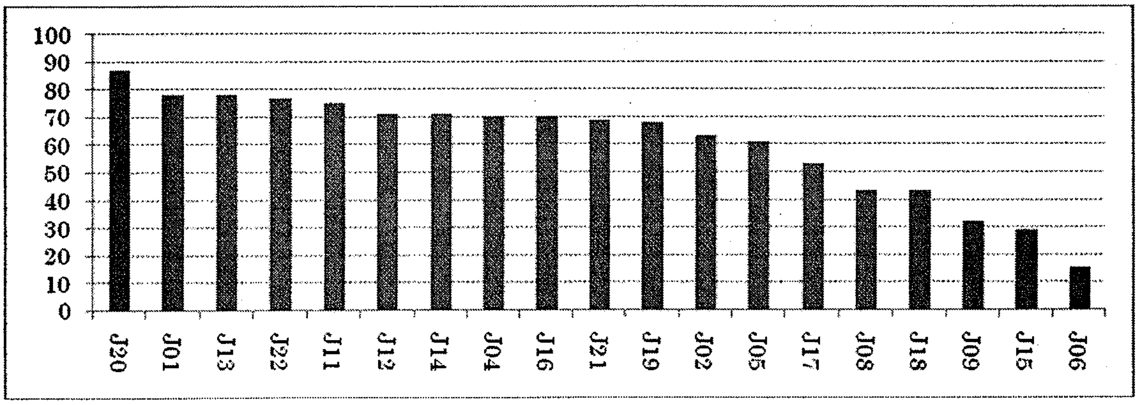


Figure 1. Individual Achievements of the Students

Table 10 shows the question-wise achievements of the students in percentage. In addition, the combined achievements are represented in decreasing order as bar graphs in Figure 2.

Table 10. Question-wise achievements of pupils in percentage

No.	Content	Average School		
		Combined	Boys	Girls
Q1 (1)	1/5+2/5	94.7%	100.0%	92.9%
Q1 (2)	5/7−2/7	78.9%	80.0%	78.6%
Q2	3÷7	5.3%	20.0%	0.0%
Q3 (1)	3/4m in Tape-Diagram	89.5%	100.0%	85.7%
Q3 (2)	1/3m in Tape-Diagram	79.8%	60.0%	86.9%
Q4 (1)	1/2 of 2m in Tape-Diagram	73.7%	40.0%	85.7%
Q4 (2)	1/2m in Tape-Diagram	52.6%	40.0%	57.1%
Q4 (3)	1½m in Tape-Diagram	73.7%	80.0%	71.4%
Q5 (1)	Ordering 2/5 and 4/5	97.4%	90.0%	100.0%
Q5 (2)	Ordering 3/6 and 1/2	26.3%	20.0%	28.6%
Q5 (3)	Ordering 1/4 and 1/3	26.3%	60.0%	14.3%
Q6 (1)	Add 2/5ℓ to 1/5ℓ	82.2%	77.5%	83.9%
Q6 (2)	Cut 2/7m from 5/7m	84.9%	75.0%	88.4%
Q6 (3)	6 pieces of 1/6m	47.4%	55.0%	44.6%
Q7	Change 0.7 to a fraction	50.0%	50.0%	50.0%
Q8	Relation 5kg and 3kg	5.3%	0.0%	7.1%
Q9	Representing "Half"	31.6%	30.0%	32.1%
Q10	Make sentence problem	50.0%	60.0%	46.4%

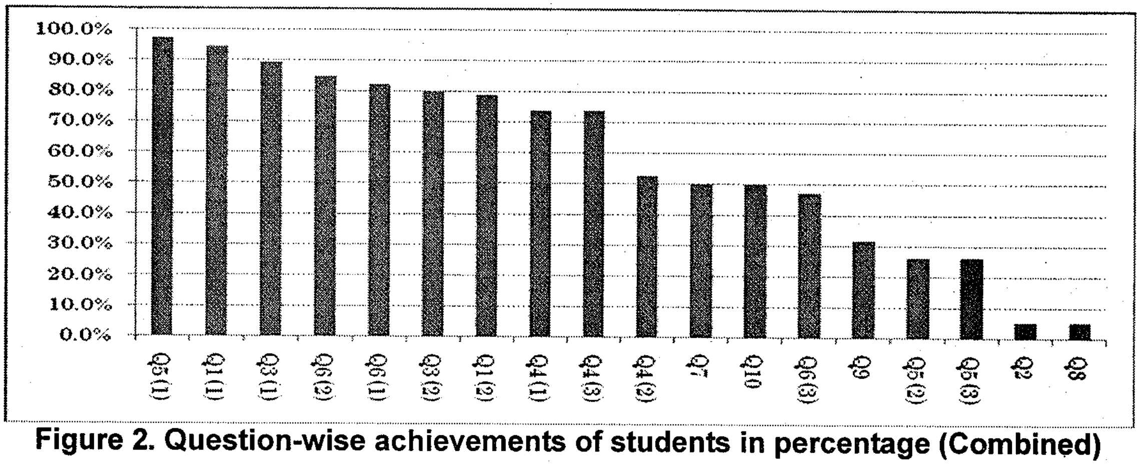


Figure 2. Question-wise achievements of students in percentage (Combined)

(4) Results of Interview using Newman Procedure

Before presenting the results of the interview, a comment on the given framework for the results of *Interview items for Students* is presented.

As is evident from reports on other countries, researchers were provided a common framework for presenting the results of the interview; this framework included *Frequency of failure on problem-solving level* with its three components i.e., *Reading*, *Understanding of Concept*, and *Process*. However, a difficulty was encountered in applying the results from the interview with Japanese students to the framework. It appeared that the interview items were not corresponding favorably with the abovementioned components.

Therefore, this report reconsiders the framework by comparing the interview items with the steps of the Newman Procedure.

The interview items were as follows:

i) *Ask them (the students) to read to you the sentence of the question. Can they correctly read it?*
ii) *If there is any difficult term/expression for them to read, describe it below.* (Note: This item asks interviewers to write the term/expressions down in the interview sheet, if any.)
iii) *Ask them to explain how they solved the question in words and/or any diagrams.*
iv) *Ask them to write their numerical expression to find the answer for the question.* (Note: This item was used only for Q6 (1), not for Q5 (3) nor Q8)
v) *Ask them to calculate the numerical expression. Can they correctly calculate it?* (Note: This item was used only for Q6 (1), not for Q5 (3) or Q8)
vi) *Describe any specific mistakes in the calculation, if there are any.* (Note: This item asks interviewers to write the errors down in the interview sheet, if any.)

As confirmed in Chapter 1, each stage in the Newman Procedure is defined as follows:

Table 11. Meaning of Each Step in Newman Procedure

Newman Procedure	Meaning of each step
Reading	Reading the problem
Comprehension	Comprehending what he/she read
Transformation	Carrying out the transformation from the words to selection of an appropriate mathematical 'model'
Process skills	Applying necessary process skills
Encoding	Encoding the answer

Taking into consideration the implication of these steps, interview items *i)* and *ii)* correspond to *Reading* in the Newman Procedure. If we regard the *mathematical "model"* in the Newman Procedure not only as symbolic models with mathematical symbols (such as numbers, variables, symbols of operation, etc.), but also as realistic models (such as length, mass, area, etc.), illustrative models (such as diagram, graph, etc.), linguistic models (explaining the rules of what students have learnt in previous mathematics lessons), etc., items *iii)* and *iv)* correspond to *Transformation*. Further, item *v)* corresponds to *Process skills*. Item *vi)* was included in order for researchers to analyze common errors committed by students. In addition, the reactions of students that correspond to *Encoding* can be found as the answer of each question in the achievement test. However, it appears that no interview items correspond to *Comprehension*, although *Understanding of Concept* in the given framework most likely corresponds to *Comprehension* in the Newman Procedure.

Therefore, Table 12 summarizes the correspondence between the stages of the Newman Procedure and the interview items.

As is evident from Table 12, instead of *Reading*, *Understanding of Concept*, and *Process*, *Reading*, *Transformation*, and *Process skills* have been used as the components of *Frequency of failure in the problem-solving level* in the following result tables.

Table 12. Correspondence between Newman Procedure and the *Interview Items for Pupils*

Newman Procedure	Question in the *Interview Items for Pupils*	Q5(3)	Q6(1)	Q8
Reading	*i) Ask them to read to you the sentence of the question. Can they correctly read it?* *ii) If there is any difficult term/expression for them to read, describe below.*	○	○	○
Comprehension	–	–	–	–
Transformation	*iii) Ask them to explain how they solved the question in words and/or any diagrams.* *iv) Ask them to write their numerical expression to find the answer for the question.*	○	○	○
Process skills	*v) Ask them to calculate the numerical expression. Can they correctly calculate it?*	×	○	×
Encoding	(We can assess this stage from the answer of the achievement test.)	○	○	○

The survey in Japan covered only 1 elementary school, as mentioned above; therefore, a comparison between urban and rural areas with regard to the students' errors in problem-solving was not conducted unlike the surveys in other countries.

The survey in Japan involved all 19 students in the interview; the results of the interview with these students are presented in Table 13 with the alternative framework.

Table 13. The pupils' errors of problem solving level in average school

Question No. and Contents		Frequency of failure on Problem solving level			No. of students with Correct Answers
		I. Reading	*II. Transformation*	*III. Process skills*	
Q5 (3)	Ordering 1/4 and 1/3	1	12	–	5
Q6 (1)	Add 2/5ℓ to 1/5ℓ	1	2	1	15
Q8	Relation 5kg and 3kg	1	17	–	2

Legend: The reason why the total number of the first and third rows (Q5 (3) and Q8) were not 19 is that a pupil failed putting correct answer despite making correct transformation in Q5 (3), and that a pupil put correct answer despite making wrong transformations in Q8.

As mentioned earlier, 5 high performers (J20, J01, J13, J22, and J11) and 5 low performers (J06, J15, J09, J18, and J17) were selected according to the achievement test. Table 14 shows the students' errors in the problem-solving level according to their performance.

Table 14. The pupils' errors of problem solving level according to their performance

Question No. and Contents		Frequency of failure on Problem solving level									No. of students with Correct Answers		
		I. Reading			*II. Transformation*			*III. Process skills*					
		High	Low	Total	High	Low	Total	High	Low	Total	High	Low	Total
Q5 (3)	Ordering 1/4 and 1/3	0	1	1	1	4	5	–	–	–	3	0	3
Q6 (1)	Add 2/5ℓ to 1/5ℓ	0	1	1	0	1	1	0	0	0	5	2	7
Q8	Relation 5kg and 3kg	0	1	1	4	4	8	–	–	–	0	0	0

Tables 15 to 17 show the specific responses of students from the interview according to the groups of high and low performers.

Table 15. Pupils' mistakes in problem solving in Q5 (3)

Q5 (3)	High performance	Low performance
Difficult words & Expressions	(Nothing)	J06, J15 Choice
Any remarks regarding problem solving process	J01: After changing the fractions into equivalent ones with the same denominators, the pupil compared the numerators. J11, J20: After expressing the fractions as diagrams, the pupils compared them. J13: The pupil said "if the fractions represented area, I think that 1/3 is larger." (In fact, he got wrong answer.)	J09: The pupil said, "if 1/3 is length, it is longer than 1/4. But if number, 1/3 is smaller than 1/4."
Specific mistakes	J13, J22: Compared only the denominators.	J08, J09, J15, J18: Compared only the denominators.

Table 16. Pupils' mistakes in problem solving in Q6 (1)

Q6 (1)	High performance	Low performance
Difficult words & Expressions	(Nothing)	J06, J18: Process to get the correct answer J06: Container
Any remarks regarding problem solving process	J13, J22: The pupils said, "In the question, there was the word "add". Then I thought there was the need to carry out the addition 1/5+2/5. But their denominators were same, so that I added only the numerators."	J08: The pupils said, "I added 1/5 and 2/5. But there is no need to change denominators, so that I added only the numerators."
Specific mistakes	(Nothing)	J18: Adding two numerators, and two denominators.

Table 17. Pupils' mistakes in problem solving in Q (8)

Q (8)	High performance	Low performance
Difficult words & Expressions	(Nothing)	J06: kg J09: Relationship in terms of weight
Any remarks regarding problem solving process	J01, J13: The pupils guessed the answers "3/5" based on the word "fraction" and the numbers in the sentence, believing that the numerator of a fraction is supposed to be smaller than the denominator.	J08, J09, J15, J18: The pupils guessed the answers "2", "8/5" "1/8" or "1/2" based on the word "fraction" and the numbers in the sentence, subtracting 3 from 5, adding 3 and 5, arranging numbers and so on.
Specific mistakes	J01, J13, J20, J22: Attached the unit "kg" to their answer "3/5".	

2.7.4 Discussion

(1) Analysis of the Results from Pupils' Achievement Test

Prior to analyzing the results of the achievement test, each question in the achievement test is compared with Table 1 in order to identify which questions students had already learnt and which they had not learnt at the time of conducting the test. As can be seen from Table 18, Questions 3, 4, and 9 had been taught previously; however, the other questions (except Question 10) were not previously taught.

Table 18. Questions' Coverage in Grade

No	Question	Coverage in Grade
Q1 (1)	1/5+2/5	Grade 5
(2)	5/7−2/7	Grade 5
Q2	3÷7	Grade 5
Q3 (1)	3/4m in Tape-Diagram	Grade 4
Q3 (2)	1/3m in Tape-Diagram	Grade 4
Q4 (1)	1/2 of 2m in Tape-Diagram	Grade 4
Q4 (2)	1/2m in Tape-Diagram	Grade 4
Q4 (3)	1½m in Tape-Diagram	Grade 4
Q5 (1)	Ordering 2/5 and 4/5	Grade 6
Q5 (2)	Ordering 3/6 and 1/2	Grade 5
Q5 (3)	Ordering 1/4 and 1/3	Grade 6
Q6 (1)	Add 2/5ℓ to 1/5ℓ	Grade 5
Q6 (2)	Cut 2/7m from 5/7m	Grade 5
Q6 (3)	6 pieces of 1/6m	Grade 5
Q7	Change 0.7 to a fraction	Grade 5
Q8	Relation 5kg and 3kg	Grade 6
Q9	Representing "Half"	Grade 4
Q10	Make sentence problem	X

Legend: X: It is not covered; the shaded questions are taught already.

At this point, the results of the achievement test are indicated again; the questions that had been previously taught and those that were not have been distinguished. Figure 3 shows the percentages of correct answers on the questions previously taught, and Figure 4 shows the ones on the questions not previously taught.

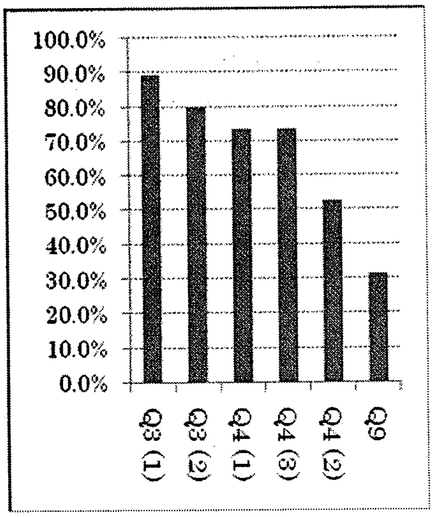


Figure 3. Percentages on Questions Taught

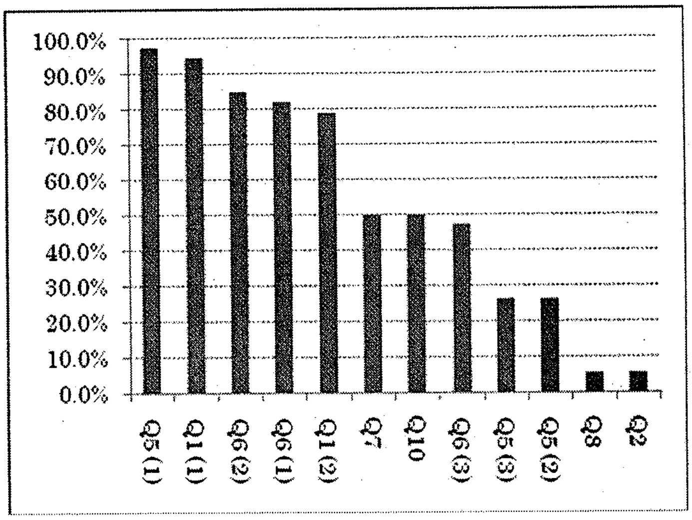


Figure 4. Percentages on Questions not Taught

From Figures 3 and 4, interestingly, it is evident that students were able to correctly answer certain questions that had not been previously taught, while they were unable to correctly answer questions that had already been taught. Therefore, it cannot be said that students will be able to solve questions that they have already been taught and will be unable to solve questions that they have not been taught previously.

Next, in order to identify the tendencies of students to answer different types of questions correctly or incorrectly, the questions that had been taught previously and those that were not, and those for which the percentages of correct answers were over 70% and under 30% were selected. Table 19 classifies these questions into 4 categories.

Table 19: Comparison of Pupils' achievements in percentage between questions taught and not taught, more than 70% and less than 30%

	More than 70%			Less than 30%		
Questions Taught	Q3(1)	3/4m in Tape-Diagram	89.5%	–		
	Q3(2)	1/3m in Tape-Diagram	79.8%			
	Q4(1)	1/2 of 2m in Tape-Diagram	73.7%			
	Q4(3)	1½m in Tape-Diagram	73.7%			
Questions not taught	Q5(1)	Ordering 2/5 and 4/5	97.4%	Q5(2)	Ordering 3/6 and 1/2	26.3%
	Q1(1)	1/5+2/5	94.7%	Q5(3)	Ordering 1/4 and 1/3	26.3%
	Q6(2)	Cut 2/7m from 5/7m	84.9%	Q2	3÷7	5.3%
	Q6(1)	Add 2/5ℓ to 1/5ℓ	82.2%	Q8	Relation 5kg and 3kg	5.3%
	Q1(2)	5/7−2/7	78.9%			

From Table 19, the following tendencies in the achievements of students can be indicated:

a) The students were able to solve questions that asked to represent lengths expressed in fractions as tape diagrams since they had learnt such content before.
b) The students were able to solve questions that comprised fractions with same denominators, although they had not learnt such content before.
c) The students were unable to solve questions that comprised fractions with different denominators since they had not learnt such content before.

d) The students were unable to solve questions that asked to represent the *Relationship* between two quantities (ratio, proportion) and quotient of division as fractions since they had not learnt such content before.

Comparing the above tendencies with the questions that were focused on in the interview using the Newman Procedure, it is evident that Q6 (1), Q5 (3), and Q8 correspond to items b), c), and d), respectively. By analyzing the students' responses to the questions that were not previously taught, we may be able to gain a deeper understanding of the learning or strategies of students in solving unknown problems.

(2) Analysis of the Results from Pupils' Responses to the Interview

In the interview using the Newman Procedure, the focus was on three questions—Q5 (3), Q6 (1), and Q8—that had not previously been taught to the students.

As is evident from Table 10, the percentages of correct answers in Q5 (3), Q6 (1), and Q8 were 26.3%, 82.2%, and 5.3%, respectively. This implies that the students could solve Q6 (1) well, while they were unable to solve Q5 (3) and Q8. According to Table 13, it can be stated that the reason why students were unable to solve Q5 (3) and Q8 was caused by the failure of *Transformation*. For example, 12 students failed in making a transformation in Q5 (3) and 17 in Q8. On the other hand, in Q6 (1), merely 2 students failed in making a transformation while the other 15 provided the correct answer, thereby succeeding in making a transformation.

However, rather interesting patterns in the process of *Transformation* can be observed in Table 14. According to Table 14, the high performers succeeded in making a transformation in Q5 (3) and Q6 (1) and failed in Q8. On the other hand, the low performers succeeded in making a transformation in Q6 (1) and failed in Q5 (3) and Q8. This implies that Q6 (1) was solved by both the high and low performers, while Q8 was not; Q5 (3) was solved by the high performers and not by the low performers.

Now, in order to analyze the students' strategies for solving questions that they had not previously learnt, the manner in which the transformation was made for Q5 (3), Q6 (1), and Q8 is analyzed.

At this point, the idea of Nakahara (1995) is introduced—he studied the *Representational System* in mathematics education. He set up 5 types of *Representational Modes—Realistic*, *Manipulative*, *Illustrative*, *Linguistic*, and *Symbolic*—in the system for a constructive approach in mathematics education as follows:

Table 20: 5 Types of Representational Modes (Nakahara, 1995)

Mode	Meaning
Realistic representation	Representation by using real world's situations and real things, including experiments with concrete and real things.
Manipulative representation	Representation by doing concrete manipulative activities, or by manipulating learning aids and concrete things which are modelled for learning mathematics.
Illustrative representation	Representation by using diagrams, figures, graphs and so on.
Linguistic representation	Representation by using language (mother tongue) or its abbreviated linguistic representations.
Symbolic representation	Representation by using mathematical symbols such as numbers, variables, symbols of operation, symbols of relation and so on.

Applying Nakahara's representational modes as a framework, the manner in which the high and low performers make a transformation is classified.

First, Table 22 shows the manner in which students make a transformation in Q6 (1).

In Q6 (1), 5 high performers and 2 low performers provided the correct answer. Two high performers (J20 and J11) and 1 low performer (J09) used realistic and illustrative representations as concrete models (imaging or drawing a container with water) in order to solve the question. Further, 3 high performers (J01, J13, and J22) and 1 low performer (J08) used linguistic representations expressing rules for translating the question into a mathematical expression (regarding the word "add" as addition) and for calculating addition of fractions (adding only numerators without changing the denominators).

In contrast, 3 low performers provided the wrong answers in Q6 (1) and 2 of them (J18 and J15) used linguistic representation like J01, J13, J22, and J08. However, they committed an error—adding not only the numerators but also the denominators. This implies that they merely concentrated on the word "add" without thinking of the implication of adding two fractions.

Two points emerge from this observation: i) If students have concrete images, such as realistic or illustrative models, they will be able to solve Q6 (1); ii) students who use linguistic representation, such as rules expressed by language, will not always be able to solve Q6 (1) unless they understand the meaning of adding fractions or have concrete images of addition of fractions.

Second, Table 23 presents the manner in which students make a transformation in Q8.

All the students failed to correctly answer Q8, although they displayed various linguistic representations. For example, since the question asked them to "write a fraction in the bracket," J20, J01, J13, and J22 merely arranged 2 numbers in the question without reason in order to make a fraction; J08, J09, and J15 added or subtracted the 2 numbers merely as a guess.

The reason why they provided wrong answers is obviously the lack of understanding of the meaning of the word *Relationship*. The concept of *Relationship* between two quantities (such as ratio and proportion) appeared too abstract and far from the real world and experiences of the students; therefore, they were unable to come up with any concrete images or real situations related to the concept. They merely attempted to apply "rules" or "knowledge" that they remembered or learned before without finding any appropriate connections between the question and such rules and knowledge.

Third, Table 24 presents the manner in which students made the transformation in Q5 (3).

In Q5 (3), 3 high performers provided the correct answer. Two of them (J20 and J11) used illustrative representations drawing lines or tape diagrams divided into 3 or 4 equal parts. Perhaps, they imaged the fractions 1/3 and 1/4 as lengths. Another student (J01) provided a linguistic representation explaining the method for conversion into an equivalent fraction with a common denominator. The student appeared to possess a deep understanding of the meaning of fractions; therefore, he was able to convert into any equivalent fraction of his choice.

On the other hand, 2 high performers and 5 low performers provided the wrong answer. Six of them provided a linguistic representation, such as comparing only denominators. The question asked to compare 1/3 and 1/4; however, the students merely thought that there was a need to compare the given "numbers (fractions)" in any case. Then, finding the numbers 3 and 4 in those fractions and comparing the numbers without considering what those denominators expressed, they decided that "1/4 was greater than 1/3."

Through this analysis, it is evident that the students had 2 strategies for solving questions that were not previously taught. The first strategy was to translate questions into more concrete situations or models that were familiar to them so that they could manage to deal with the questions. The second strategy was to apply mathematical rules and knowledge that they had previously acquired (or memorized) in solving questions. However, the second strategy can be further divided according to whether or not the students understand the rules and knowledge.

Skemp (1979) advocated that there were various levels of understanding, and two of them were *Instrumental understanding* and *Relational understanding*. According to him, *Instrumental understanding* implies the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works; *Relational understanding* implies the ability to deduce specific rules or procedures from more general mathematical relationships (Skemp, 1979).

Referring to Skemp's idea, applying mathematical rules and knowledge that the students have understood is related to *Relational understanding*, and applying mathematical rules and knowledge that the students have not understood is related to *Instrumental understanding*.

Therefore, we can categorize the students' strategies for solving questions that have not been taught previously into at least 3 categories, as follows:

Table 21: Pupils' Strategies for Solving Unknown Questions

Translation into more concrete models	Application of mathematical rules and knowledge	
	based on *Relational understanding*	based on *Instrumental understanding*

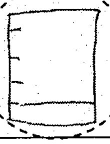
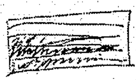
(3) Conclusion

From the above analyses, it has become evident that it is possible for students to solve not only those problems that they have previously learnt but also those that have not been previously learnt; in other words, they possess strategies for solving unknown problems. For example, although the questions regarding ordering, addition, and subtraction of fractions with same denominators were not previously taught to the students, numerous students solved such questions by translating them into more concrete models or applying acquired rules and knowledge.

Undoubtedly, the students faced difficulty in solving problems that comprised rather abstract content, such as expressing the *Relationship* between two quantities. In such a case, the students merely attempted to apply rules or knowledge that they did not understand.

However, considering students' strategies for solving problems and the characteristics of problems, it may be possible to seek a more fruitful learning and teaching method for students in mathematics education. Such challenges will be focused on in our future researches.

Table 22. Pupils' Ways of Making Transformations in Solving Q6 (1)

		Representation					Answer
		Realistic	Manipulative	Illustrative	Linguistic	Symbolic	
High	J20				Regarded the word "add" as addition.	$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$	3/5ℓ
	J01	*Transforming into concrete models*			Regarded the word "add" as addition. Added only numerators without changing the denominators.	$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$	3/5ℓ
	J13		*Applying rule based on understanding*		Regarded the word "add" as addition. Added only numerators without changing the denominators.	$\frac{1}{5}l+\frac{2}{5}l$ $=\frac{3}{5}l$	3/5ℓ
	J22				Regarded the word "add" as addition.	$\frac{1}{5}l+\frac{2}{5}l$ $=\frac{3}{5}l$	3/5ℓ
	J11	1/5ℓ is one of the 5 equal parts that 1ℓ is divided into.				$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$	3/5ℓ
Low	J08	*Transforming into concrete models*	*Applying rule based on understanding*		Regarded the word "add" as addition. Added only numerators without changing the denominators.	$\frac{1}{5}l+\frac{2}{5}l$ $=\frac{3}{5}l$	3/5ℓ
	J18				Regarded the word "add" as addition.	$\frac{1}{5}l+\frac{2}{5}l$ $=\frac{3}{10}l$	3/10ℓ
	J09	One of the 5 equal parts which the container is divided into.				$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$	3/5ℓ
	J15				Regarded the word "add" as addition. The denominator cannot increase.	$\frac{1}{5}+\frac{2}{5}$ $=\frac{1+2}{5+5}$	2/5ℓ
	J06	Didn't get the meaning of the question.		*Applying rule without understanding*			–

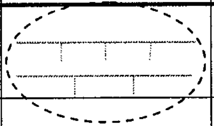
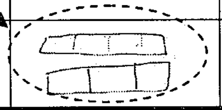
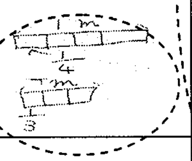
Legend 1: The shaded answers are wrong.
Legend 2: The italic sentences in the dotted frames are the comments of researcher.

Table 23. Pupils' Ways of Making Transformations in Solving Q8

		Representation					Answer
		Realistic	Manipulative	Illustrative	Linguistic	Symbolic	
High	J20				Compared 5kg and 3kg.		3/5kg
	J01				Believed that I had to put answer. Guessed 5/3 or 3/5.		3/5kg
	J13		*Applying rule without understanding*		Guessed 3/5kg seeing 5kg and 3kg. Believed the numerator must be smaller than the denominator.		3/5kg
	J22						3/5kg
	J11				Judy's meat was bigger than George's one.		About 2 times
Low	J08				Applied subtraction seeing 5kg and 3kg.		2 times
	J18				Just guessed		1/2
	J09		*Applying rule without understanding*		5+3=8 Guessed 8/5...		2kg
	J15				5+3=8 Believed that the answer must be a fraction and the numerator must not be 0		1/8
	J06	Didn't get the meaning of the question					–

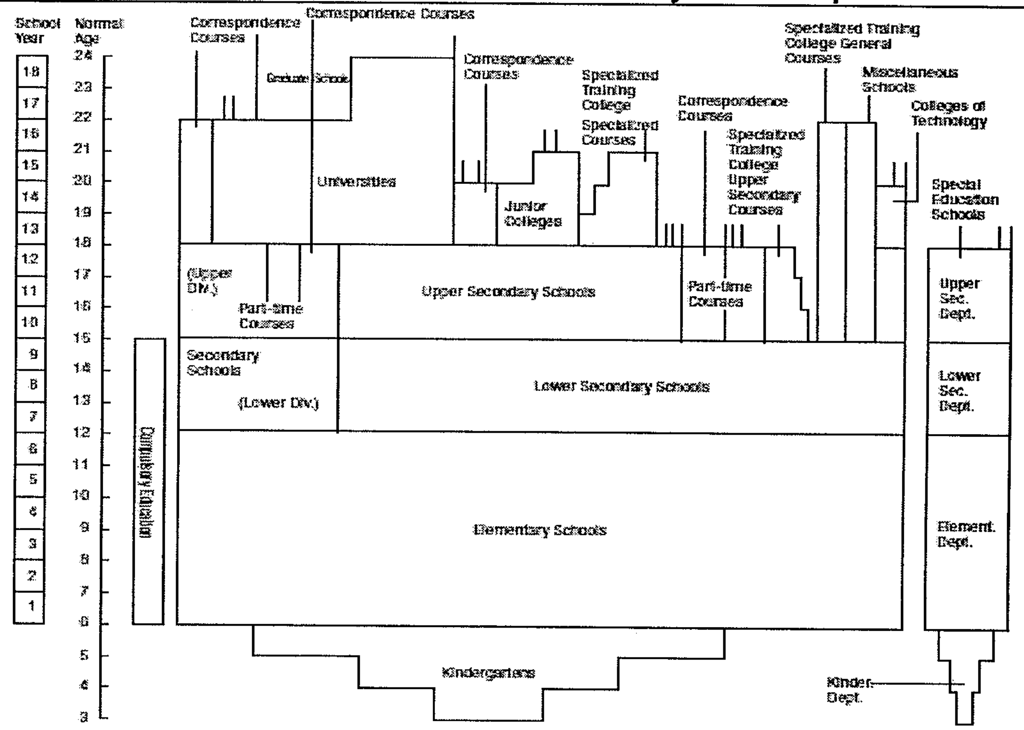
Legend 1: The shaded answers are wrong.
Legend 2: The italic sentences in the dotted frames are the comments of researcher.

Table 24. Pupils' Ways of Making Transformations in Solving Q5 (3)

		Representation					Answer
		Realistic	Manipulative	Illustrative	Linguistic	Symbolic	
High	J20						1/3>1/4
	J01	*Transforming into concrete models*		*Applying rule based on understanding*	Converted into equivalent fractions with a common denominator. Compared 3/12 and 4/12		1/3>1/4
	J13			*Applying rule without understanding*	Compared the denominators		1/4>1/3
	J22				Compared the denominators		1/4>1/3
	J11						1/3>1/4
Low	J08		*Applying rule without understanding*		Compared the denominators		1/4>1/3
	J18				Compared the denominators		1/4>1/3
	J09				Compared the denominators		1/4>1/3
	J15				Compared the denominators		1/4>1/3
	J06	Didn't get the meaning of the question					–

Legend 1: The shaded answers are wrong.
Legend 2: The italic sentences in the dotted frames are the comments of researcher.

Annex 1: Organization of the School System in Japan



Source: Japan Ministry of Education, Culture, Sports, Science and Technology (2005)

Annex 2: Content development of fractions in the Course of Study for Japanese elementary school

Grade 4	1. Objectives To enable children to understand the meaning of decimals and fractions and how to represent them. 2. Contents A. Numbers and Calculations To enable children to understand the meaning of fractions and how to express them. a) To use fractions in order to represent the size of reminders, and the size of the proportion that is formed by division into equal parts. To know how to represent fractions. b) To know that fractions can be represented as a certain number of times unit fractions. [Terms and symbols] Denominator, Numerator, Mixed fraction, Proper fraction, Improper fraction
Grade 5	1. Objectives (1) To enable children to deepen their understanding of the meaning of decimals and fractions, and how to express them. ··· To enable children to understand the meaning of adding and subtracting fractions, to examine these calculations, and to make use of these calculations. 2. Contents A. Numbers and Calculations (4) To enable children to deepen their understanding of fractions. To enable children to understand the meaning of adding and subtracting fractions with the same denominators, and to make use of these calculations in appropriate way. a) In simple cases, to notice fractions of the same size. b) To convert whole numbers and decimals into fractions, and represent fractions in decimals. c) To understand that the result of dividing whole numbers can always be expressed as a single number using fractions. d) To examine adding and subtracting fractions with the same denominator, and do these calculations.

	3. Points for consideration in dealing with contents (3) With regard to item (4) d) in "A. Numbers and Calculations", addition and subtraction as its opposite operation of two proper fractions should be included.
Grade 6	1. Objectives (1) To enable children to deepen their understanding of addition and subtraction of fractions, and to make appropriate use of these. To enable children to understand the meaning of multiplying and dividing fractions, to examine methods of calculating, and to make appropriate use of them. 2. Contents A. Numbers and Calculations (2) To enable children to deepen further their understanding of fractions, to understand the meaning of adding and subtracting fractions with different denominators, and to make appropriate use of these calculations. a) To understand that a fraction obtained by multiplying or by dividing the numerator and denominator of an existing fraction by the same number, has the same size as the existing fraction. b) To examine the equivalence and size of fractions, and organize the methods how to compare their size. c) To examine the methods of adding and subtracting fractions with different denominators, and do these calculations. (3) To enable children to understand the meaning of multiplying and dividing fractions, and to do these calculations appropriately. a) To understand the meaning of multiplication and division when the multipliers and divisors are whole numbers. b) To understand the meaning of multiplication and division when the multipliers and divisors are fractions, on the basis of an understanding of the calculations involved when the multipliers and divisors are whole numbers and decimals. c) To examine the methods of multiplication and division with fractions and decimals, and do these calculations. [Terms and symbols] Reduction to a common denominator 3. Points for consideration in dealing with contents (2) With regard to item (2) c) in "A. Numbers and Calculations", addition and subtraction of two proper fractions should be included. (3) With regard to item (3) in "A. Numbers and Calculations", calculations of mixed fractions should not be included. (4) With regard to item (3) b) in "A. Numbers and Calculations", simple cases should be handled, such as where the multipliers and divisors are fractions with a numerator of one.

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Chapter 3

Discussions and Future Issues

Chapter 3 Discussions and Future Issues

3.1 Discussions of Present Status of Mathematics Education

Although in this research robust numerical comparisons were not always possible because of the limitations of our samples, making comparisons within each country and characterizing differences among the participating countries were possible and valuable. The study of syllabi and textbooks to support the analyses was done in some cases.

In the process of comparison, four categories were created: students-related context, students and learning, language, and research method. Since "learning" is a focus of this report, students and learning are the natural one to address, while "students-related context" and "language" are somehow related to the preconditions of learning, and "research method" focuses on the methodology to approach these aspects of mathematics education.

Here we didn't include the "teachers and teaching" category, but this has been included in the second part of the joint study. In that part, we tried to uncover the status of teachers' teaching in mathematics. These two reports, two some extent, clarified the status of mathematics teaching and learning in each country.

3.1.1 Students related Context

The UNESCO Framework of Education Quality (UNESCO, 2004) emphasizes that context interacts with every part of the educational process. For students-related context we compiled the following list of factors that can determine the present status of students. These contexts affect students' learning directly and indirectly/explicitly and implicitly. The extent of each influence and the relationship among these interrelated factors differs in various countries.

- Teaching in school: curriculum, textbook, teacher's competency, teaching strategy, teacher's belief, and etc.
- Learning environment in school: supply of textbooks, teaching and learning resources, and etc.
- Family background: parents' educational background, economic state, parents' belief and value in education, learning resources at home.
- Learning environment in society: public resources available in education, opportunities for extra classes, private tutoring and self learning, and etc.
- Political and economical factors: national governance and management strategies, labour market demands, examination, career ladder, and etc.
- Sociocultural and religious factors: beliefs and values in society, and etc.

Though our focus and sample size were limited, we could find the following from country reports.

(1) Curriculum: syllabi and textbooks

All the participating countries have a national curriculum, but some introduced privatization in the production of textbooks, while in others the government produces the textbooks. In the syllabi and textbooks, the topics to be covered are more or less similar. In fact, Nebres (1988) noted the uniformity of mathematics curriculum despite cultural diversity.

As for the target topic of this research, fractions, some countries introduce it at very early stage such as grade 1, and others introduce it at a relatively later stage such as grade 4 when they judge that students have become mature enough. This means that there are two distinct approaches. One of them is to introduce the topic at an early stage and take time for the students to digest the concept. The other one is to wait for students' readiness and to deal with the topic in a concentrated manner.

(2) Family background: parents' background, learning resources at home

In some participating countries, there is a gap in educational resource at home between the urban area and the rural area (, though the quality control in a part of the result would be necessary in a few countries). The relationship between educational resource at home and students' achievement is, however, diversified among participating countries. Since the material conditions and requirements of "average school" also differ in each country, other research attempt is recommended to clarify the relationship among those factors.

(3) Learning environment in society: public resources available in education, opportunities for extra classes, private tutoring and self learning

From an informal interview and discussion in some participating countries, there is a gap in educational resource available in society between the urban area and the rural area. Educational resources in society include not only physical supply for teaching-learning materials but also accessibility to extra classes, private tutoring. Apart from such factors which directly affect students' learning, the frequency and the quality of opportunities for adult to exchange educational information would also have an effect upon students' learning.

(4) Political and economical factors: national governance and management strategies in education, and examination

Though the promotion system and the role of examinations are different in participating countries, this research has revealed that examinations are a very influential factor in each country both in relation to teaching and to learning. Teachers tend to teach only what will be tested, and they judge students' ability in terms of test results. Besides, some cases during the field survey were reported where students possibly cheated on the test questions and even where the teachers assisted the students in solving some questions. Although these were extreme cases, nevertheless they indicate the disproportionate importance of test scores.

(5) Sociocultural and religious factors: diversity of language and culture

From country reports in Chapter 2, the diversity with regards to language and culture is apparent in participating countries. We shall address the language issue separately in 3.1.3.

3.1.2 Students and learning

(1) Positive attitude towards mathematics, school, teacher, and themselves

In all the participating countries except for Japan, the results show the positive attitude of students towards mathematics, their school and teachers, and themselves. There is, however, a gap between such positive attitudes and actual achievement in mathematics in this survey.

(2) Performance comparisons within each country

Instead of comparisons between countries, comparisons within each country may help to reveal some of the following. Though the result should not be overgeneralized both within each country and within each survey area because of the limitation in the sample size, we shall sum up some findings for further research.

1) Disparities between urban and rural education

In most participating countries, the overall scores in the achievement tests often show superiority of urban students. Though the lack of educational resources would be one of the reasons which result in failure in rural areas, we can find another reality in the result by content domains or items. Since the urban students were not always superior to the rural students in every content domain or item, the research revealed the inconsistent strength and weakness by location in each country. Here, while we should not think little of physically worse condition in the rural area, the result implies the effect of teacher and teaching on learning is more crucial.

2) Gender gaps

From country reports in participating countries, gender in the achievement tests was not clear. The reason is that we could not find the consistent result in gender by countries, locations, content domains, and items as well as the gap between the urban and rural areas which is mentioned above.

3) Weak Domains and items

In the result over participating countries in the first year survey, the content domains where the students showed lower performance are generally "Geometry", "Algebra", and "Measurement". As for the result in the second year survey, the items with lower performance are Q2, Q3-2, Q4-2, Q4-3, Q5, Q6-3, Q7, Q8, and Q10.

The syllabus coverage against the contents in the achievement tests have been often argued throughout the research period, such that the low coverage would result in poor performance. However, the result of each item shows the case that students can obtain the correct answer in uncovered topics. The discussion with regards to students' weakness would make sense, only when we implement in-depth analysis on their mistakes as we did in Newman procedure.

(3) Students' readiness

Cognitive, affective and social aspects of students are all key factors. In some countries the ages of the students in the study varied greatly from the regular age of 9 (10) to as old as 15. This indicates the existence of grade levels being repeated, students dropping out of school, and delay in entering into primary school. In this research, we put emphasis on the present degree of achievements, neither maturity nor curriculum vitae.

3.1.3 Language

Students in some countries were not accustomed to the test practice in which each student is provided with a sheet of white paper for taking each test. Furthermore, an analysis of some answer sheets revealed that some students didn't understand the purpose of the questions. In some countries sentence problems are usually avoided on tests because they tend to be very difficult for the pupils. Such situation needs to be investigated in relation to the language fluency. Because of its unique position, we shall address the language issue separately here.

(1) Degree of acclimation to sentence problems

Students may not be accustomed to written examinations, even in cases where the language used is easy and relevant to the culture. So as to investigate such cases, an interview method, so called Newman procedure, was employed in this research. The Newman procedure allowed us to identify the exact levels where students were stuck in terms of solving sentence problems.

As a country report mentioned, the understanding or the lack of understanding with regard to a mathematical concept did not directly translate to their ability in working out the mathematical processes. Similarly, the students' ability with mathematical processes did not imply that they were able to record their answers correctly. This may be explained by the fact that some students could guess the correct answers, and are affected by the level of familiarity with the formulation of the questions. While we, of course, have to be more sensitive in wording and arrangements of questions, it seems that the accustomedness to the examination-oriented culture is presupposition for success in school education, which should be re-examined.

(2) Medium of instruction and students' understanding

The language condition is a notable precondition for education in most participating countries. In some countries the language used for providing instruction is the same language used in daily life, but in other countries this is not the case. The situation can become quite complex when there are various overlapping languages, such as the language of an ex-colonial government, a national language and local languages (Ex.: In Accra, Ghana they use at least the three languages of Ga, Asante and English). And also, language is not only an educational and psychological issue, but a political and economical issue as well.

With use of Newman procedure, we also took into consideration the point that language problems are classified as A-type problems (language fluency) and B-type problems (difference in cognitive structure) (Berry, 1988). Many teachers pointed to fluency problems among their pupils. A few teachers also identified conceptual gaps between English and local languages. This complex situation makes it much harder for students to express their ideas freely. There is a lot of literature on how difficult it is for students to attain a certain academic level and for teachers to manage the classrooms when the language used for teaching is different than the pupils' mother tongue (e.g. Fafunwa et al. 1989).

The interview in Newman procedure reveals that the characteristic of question reflects on the steps in which the pupils made mistakes. A kind of question reminded students of the procedural knowledge and skills, which some students could explain how to do it in English. Some pupils who could not explain their solving process in English showed their idea in mother tongue in the question. Though they could not explain why, many pupils could show their procedural knowledge. In other kind of question, which students were required to show the conceptual understanding, it was very difficult for students to understand the meaning of the question and explain their solutions in English. Though we can not say "B-type problems" existed in our survey too, it implies that there might be several kinds of interaction between question and the student attempting it. It also suggests that a linguistic support and a device in posing questions would be essential especially in the questions which require understanding of mathematical concept.

(3) Cultural appropriateness and social implication

Language conditions in some participating countries are very complex. Multi-linguistic and multi-cultural situations would demand which language(s) is/are to be used. In some cases there is no word in the language that can express the concept or the word has a completely different meaning. For example, some concepts such as "minus degree" in the first year survey may not be culturally appropriate if, for example, the students don't experience minus degrees in their daily lives. And also, the second year survey used the concepts of "sharing", "fractions", "half", "more than and less than" and "one-third or one-over-three". Such mathematics concepts should have been examined from the view of cultural appropriateness, since the students might have different meaning and usage in corresponding words in mother tongue.

The choice of which language to use in the classroom has not only educational implications, but social implications as well. This is because the acquisition of an international language gives students greater access to international information and greater opportunities for employment abroad. As a long-term issue, cultural appropriateness should be carefully discussed to broaden and deepen students' capability in mathematics.

3.1.4 Research method

We raised the following three points regarding the research method: selection of samples, linguistic influences and new types of problems.

(1) Selecting samples and combining research methods

We planned to obtain the best possible output despite the limitation of sample size. And the first task was to select an "average" school in order to describe mathematics education in a particular country. This was by no means an easy task to do because the criteria for determining an "average" school within each country were not uniform. All of the target countries have unique backgrounds including different socio-economic conditions and different requirements for schools and students. For example, an average school in Thailand has satisfied the minimum condition in terms of teaching materials and school infrastructure, but an average school in Zambia has fewer resources by comparison.

The research members recognized the importance of qualitative analysis after the field survey reports during the workshop because each report included rich and interesting data suitable for further comparative studies between countries and within individual countries. In order to view the subjects from different angles and to increase the validity of the research, we should combine not only different samples but we should attempt to combine different research methods as well. This combination of research methods is called "triangulation". There are a variety of combinations such as pre / post -lesson interviews, second / third-party observations and ideal / actual lessons.

(2) Linguistic and cultural influences on achievement

As mentioned earlier, linguistic and cultural influences on achievement tests are obvious; it would be applicable not only in this research project, but also in any other international survey. This kind of achievement tests should be followed by clinical interview method such as Newman procedure to identify the degree of influence from linguistic factors. In addition, by analyzing the errors of the students and investigating the cause for the errors, their misconceptions became evident. Being able to perceive the misconceptions of students with regard to certain mathematical concepts will enable teachers to understand the manner in which they should conduct their teaching.

(3) New types of questions (problem posing)

Many developing countries are now exploring more student-centered lessons and the promotion of logical thinking skills. Problem posing was included in this context in order to see students' ability in this direction.

3.2 Future Issues

3.2.1 Revisiting endogenous development

While writing the final report we were able to identify some important points that need to be clarified further. In particular, "language problems" and "students-related context" have become the key words for pursuing a spirit of endogenous development. When we consider these two key words, it becomes clear that greater consideration of the research framework is needed. Therefore, for the short-term we compiled a list of the issues that we faced during the course of this study. This, in the long term, will form the starting point for our future research for the development of a comprehensive framework for mathematics education research in developing countries.

3.2.2 Future research

By the endeavor of this research group, we have to have a target to improve teaching and learning in each country although we should not be too short sighted.

(1) Way forwards

Endogenous development will continue to be a very basic presupposition for our future research. And this endogenous-ness should be considered in curriculum, teachers, and students. Endogenous development of curriculum is based upon the society's needs and may consider possibility of applying ethnomathematics into teaching and learning. Endogenous development of teaching may require reflection of trivial values and socio-mathematical norms within the school culture in each country and create new educational practice. Endogenous development of learning requires consideration of what students are to acquire at the end of school education where political, economical and cultural needs are intricately intervened with each other. All these endogenous-ness are interrelated to each other.

(2) Which points to focus

The majority of teachers have sufficient teaching experience. However, some of their predictions for student performance did not match the actual scores. This mismatch cast a question over the assumption that teachers were aware of students' background and weakness. The context and language of instruction are still the points to focus for future researches. The context was considered extensively in this research as follows.

- Teaching in school: curriculum, textbook, teacher's competency, teaching strategy, teacher's belief, and etc.
- Learning environment in school: supply of textbooks, teaching and learning resources, and etc.
- Family background: parents' educational background, economic state, parents' belief and value in education, learning resources at home.
- Learning environment in society: public resources available in education, opportunities for extra classes, private tutoring and self learning, and etc.
- Political and economical factors: national governance and management strategies, labour market demands, examination, career ladder, and etc.
- Sociocultural and religious factors: beliefs and values in society, and etc.

(3) Future research

Action research as a research method was proposed during the project. This is supported by the fact that this research involves many factors and their relation is ever complicated. So having thick description about present status and reorganizing conjecture is necessary. In any case, flexibility is required to adjust to the reality of each participating country.

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Postface

Postface

Firstly I would like to congratulate all those people involved in this project – whether they have been directing it, operating it, participating in it, supporting it or have been involved in any other way. A developmental enterprise of this nature requires a great deal of energy on the part of many people. This is always the case whatever the size and scale of the educational endeavour.

This report is about the creation and execution of a developmental project in mathematics education involving seven countries, each growing in its different way. This is clearly an enormous undertaking, and one can only admire the imagination and organisational abilities of those who have been responsible for it. To engage in development work in one country is complex enough, but to do this with seven countries simultaneously demands our admiration.

Even more than this is the highly praiseworthy subscription to the concept and ethic of endogenous development, a process which has various significant characteristics. In relation to this educational project, these would include the local determination of the development choices and local control over the development process. Also endogenous development respects local values whilst exogenous development tends to ignore or subjugate them. Finally, endogenous development is founded mainly on locally available resources, and good schemes can revitalise and dynamise local resources which might otherwise be ignored or dismissed by others as being of little value.

In this report we can read about some of the special local features and the questionnaire and test results from the students in the different countries. Whilst comparisons are made, and similarities and differences noted, there is no attempt to produce 'league tables' of 'winners' and 'losers', as happens in some international comparisons.

In an endogenous project, by contrast, the aim is by comparing situations, to identify specific problem areas, and to search for strengths on which the local development can build. By placing the research work in the hands of both local collaborators and research students from those countries, the project has sought to gain the best of both worlds. That is, the research students are in touch with the latest ideas in both research methods and mathematics education, while their local counterparts are in continuous contact with the local situation. Bringing these two together is the major strength and innovative part of this project.

As a special advisor, I am very honoured to be able to contribute in a small way to this project. I wish success to all those involved in this promising developmental work.

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Special Contribution

Special Contribution

1. Numeracy and Mathematics
– A Cultural Perspective on the Relationship

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1.1 Introduction - numeracy in a global cultural context

The concept of global citizenship has gained currency in recent years. The mass movement of people around the globe and the creation of regional entities such as the European Union and ASEAN have generated significant interest in the concept of global citizenship, as law-makers and curriculum policy advisors question the legitimacy of traditional notions of citizenship. These developments have tremendous implications for those of us who work in education, particularly in mathematics education.

In Australia for example, one must always be aware when considering educational issues, that it is predominantly a migrant country. A Report by the Public Affairs Section of the Department of Immigration and Multicultural Affairs, Canberra in 1996 states:

"Today nearly one in four of Australia's 18.5 million people was born overseas. In 1995–96 the number of settlers totalled 99 139. They came from more than 150 countries." In the particular cases of numeracy and lifelong education, I believe this recognition is essential for any educational initiatives to succeed. Even in Japan, which is rarely considered to be a multi-cultural society, the construct of 'culture' has strong meanings, and therefore important implications for numeracy education.

Whenever I am discussing educational issues I am always aware of three main constructs: curriculum, teaching, and learning. These are all of course embedded in social, cultural and political contexts which structure, support, and constrain them. When considering numeracy education, other people have tended to focus mainly on the learning aspects. So in this paper I wish to focus on the other two main areas which I feel are often neglected, and in particular on these ideas:

1.culturally, and socially - based numeracies, and

2.their relation to the mathematics curriculum.

There are clearly issues and challenges about defining and clarifying the nature and role of numeracy in today's society, and in particular its relationship with mathematics (e.g. FitzSimons et al., 2003) Is numeracy a part of mathematics? Is it a simple form of mathematics? Is it the same as mathematics? Is it radically different from mathematics? I will offer my own interpretation of the relationship, one which suggests some different ideas about a numeracy curriculum than those normally given, and one that recognises firstly that there are many numeracies in our societies, and secondly that numeracies are socially and culturally based.

1.2 The challenge of culturally-based mathematical knowledge.

In the last 20 years, two main strands of research have developed in mathematics education. The first was research on **ethnomathematics**, which has fundamentally changed many of our ideas and constructs in considering mathematics education (D'Ambrosio, 1985; Gerdes, 1995). The most significant influences have been in relation to:

Cultural roots. Ethnomathematics research has made us aware of the cultural starting points, and different histories, of mathematics. We now realise that Western Mathematics is but one form among many, and that Wasan was a special form of mathematics in ancient Japan.

Interactions between mathematics and languages. Ethnomathematics research has shown us that languages, and other symbol systems act as the principal carriers of mathematical ideas and values in different cultures and societies.

Human interactions. Ethnomathematics research focuses on mathematical activities and practices in society, and it thereby draws attention to the roles which people other than teachers and learners play in mathematics education.

Values and beliefs. Ethnomathematics research has made us realise that any mathematical activity involves values, beliefs and personal choices.

However the curriculum structures for mathematics teaching that we generally see in schools are typically culture and value free, and have evolved to suit the preparation of an elite minority of students who will study mathematics at university. When considering general education, and especially for those students who will never study mathematics at university, this elitist curriculum is highly inappropriate. It has contributed significantly to the widespread problems of alienation felt by many students, of all ages, towards mathematics.

But the problem is not just one of alienation, it is much more to do with the fact that a majority of the population has been debarred from any discussion or critique of the underlying mathematical constructs used in today's highly mathematically formatted and structured society (Skovsmose, 1994). Research therefore still needs to explore ways of making mathematics curricula more culturally and socially responsive, in order to encourage more societal participation at the higher levels particularly amongst cultural and social minority groups. I believe that a numeracy-based curriculum is one way to emphasise the societal and cultural aspects of mathematics.

The second trend in research which has developed in the past two decades, and which relates to this discussion, has demonstrated the significance for teaching, learning, and the curriculum, of the conceptual idea of **context**, that is the context within which the mathematical practice is situated. Indeed the construct of **"situated mathematical practice"** is now attracting a great deal of attention (see for example, Kirshner & Whitson, 1997; Zevenbergen, 2000). Much of this research work is based on the challenge coming from studies of mathematical activities occurring outside school (e.g. Abreu et al., 2002; Lave & Wenger, 1991) where the characteristics of the practice are markedly different from those in school.

These practices, many of which have been the subject of ethnomathematics research, are to my mind the roots of numeracy education. This idea demonstrates powerfully that the context of school is markedly different from the contexts outside school where numeracy practices occur. If we wish to develop numeracy curricula, how can we ensure that the school context doesn't ignore, or indeed over-ride, the outside-school numeracy contexts?

Bearing these two research trends in mind, the issue for researchers becomes: how can we develop a **numeracy-based mathematics curriculum** for schools, one which satisfies both the needs of a modern democratic society for having fully educated and politically contributing adults, and the needs of the mathematical research establishment which still wants competent, and in some cases brilliant, mathematicians?

1.3 Numeracies and mathematical theory

Numeracy is being defined in many different ways, particularly in its relation to mathematics, and particularly in the context of adult education (see for example, Bessot and Ridgway, 2000, and FitzSimons, 2002). There are those who define it as a part of mathematics, often as simple or basic arithmetic. Others see numeracy as far more than just arithmetic. Still others prefer to link numeracy to literacy rather than to mathematics (as UNESCO does), or to consider numeracy as mathematical literacy (Jablonka, 2003). A recent book from the USA focuses on Quantitative Literacy (Steen, 2001) although in the text the word 'numeracy' is often used in its place.

However I like to approach the issue differently. Firstly I believe we need a conceptualisation of numeracy which meets three criteria:

- it recognises the existence of many numeracies (Buckingham 1997, Tout, 2001)
- it clarifies the links between numeracies and mathematics, and
- it clarifies the educational task facing those responsible for numeracy education.

The first aspect of the relationship is to recognise the way that ethnomathematical ideas can be structured to enable them to be seen in relation to numeracy and to mathematics. In my original research (Bishop, 1988) I conceptualised a structure based on six fundamental and universal activities, which are found in all societies. These activities are the bases of the numeracies in the societies in which they appear. However they are also the bases of Western Mathematics, and this is the heart of the relationship we are seeking. Here are the six activities:

♦ **Counting**: This is the activity concerned with the question "How many?" in all its forms and variants eg. there are many ways of counting and of doing numerical calculations. The mathematical ideas derived from this activity are numbers, calculation methods, number systems, number patterns, numerical methods, statistics, etc.

♦ **Locating**: This activity concerns finding your way in the structured spatial world of today, with navigating, orienting oneself and other objects, and with describing where things are in relation to one another. We use various forms of description including maps, figures, charts, diagrams and coordinate systems. This area of activity is the 'geographical' aspect of mathematics. The mathematical topics derived from this activity are: dimensions, Cartesian and polar coordinates, axes, networks, loci, etc.

♦ **Measuring**: "How much?' is a question asked and answered in every society, whether the amounts which are valued are cloth, food, land, money or time. The techniques of measuring, with all the different units involved, become more complex as the society increases in complexity. Mathematical topics derived here are: order, size, units, measure systems, conversion of units, accuracy, continuous quantities, etc.

♦ **Designing**: Shapes are very important in the study of geometry and they appear to derive from designing objects to serve different purposes. Here we are particularly interested in how different shapes are constructed, in analysing their various properties, and in investigating the ways they relate to each other. The mathematical topics derived are: shapes, regularity, congruence, similarity, drawing constructions, geometrical properties, etc.

♦ **Playing**: Everyone enjoys playing and most people take playing very seriously. Not all play is important from a mathematical viewpoint, but puzzles, logical paradoxes, some games, and gambling all involve the mathematical nature of many activities in this category. Mathematical ideas derived here are: rules, procedures, plans, strategies, models, game theory, etc.

◆ **Explaining**: Trying to explain to oneself and to others why things happen the way they do is a universal human activity. In mathematics we are interested in, for example, why number calculations work, and in which situations, why certain geometric shapes do or do not fit together, why one algebraic result leads to another, and with different ways of symbolising these relationships. Mathematical topics derived here are: logic rules, proof, graphs, equations etc.

The next aspect of the relationship appears when we realise that the cultural history of Western Mathematics, which I shall continue to refer to by Mathematics with a capital 'M', (see Bishop, 1988) shows us that its essence is its **generality**. On the other hand the ethnomathematics literature indicates that numeracies are all about **particular practices**. To use the word 'practices' here is not just to refer to skills or algorithms, nor to minimise numeracy as just number work, but as with ethnomathematics, and the six activities above, numeracies include meanings, and conceptualisations also. No power relationship between Mathematics and numeracy is intended, as both are powerful forms of knowledge in their own contexts. It is just that the 'situated context' of Mathematics is the abstract world of the theorist, whereas the 'situated context' of numeracy is the pragmatic world of the ethnomathematical practitioner.

The relationship between them is therefore best understood to my mind as one between theory and practices, with **numeracies being the practices and Mathematics being the theory behind the practices**. Mathematics explains, theorises, and clarifies the rationales underlying those practices, and also gets applied through various developments of those practices. This conceptualisation of the relationship between the two forms of knowledge helps to construct a meaning for numeracy, which meets the three criteria above.

As we can see with the activity of 'explaining' above, the question "Why does any particular numeracy practice work?" is an important key to understanding the relationship, and in my view is the key to clarifying the educational task. Learners of all ages can bring many numeracy, and ethnomathematical, practices to their education, mainly from their families but also from their wider society contacts – friends, media, other adults etc. However the naive learner generally lacks any understanding of the underlying Mathematical theories that help to explain those practices. Without the Mathematical theories they lack the tools to understand, analyse and critique those practices. Without these tools they become trapped by those practices, as most adults currently are, without the understanding to question and develop alternatives.

There are many 'why' questions that can be asked of school students even within the current mathematics curriculum, such as these simple ones:

- Why do the many different practices for counting, addition, subtraction etc. all succeed? It is because of the underlying mathematical structures of those algorithms.

- Why do two negative numbers when multiplied together give a positive number? It is because of the wish to keep the rules of number theory consistent when dealing with positive and negative integers.

- Why can you divide one fraction by another by 'turning it upside down and multiplying'? Because multiplying the numerator and denominator of a fraction by the same number doesn't change the value of that fraction.

- Why does a three-legged stool always balance, while a four-legged one doesn't? Because 3 points determine a plane.

Of course it may seem to many that asking these 'why' questions is inappropriate when talking about the Mathematics curriculum, especially since the demise of proof from the school curriculum in many countries. Proofs are the ultimate mathematical explanation. But it is not necessary to learn proofs of theorems just for regurgitation at examination time. It is far more important to understand the role of the process of 'proving' in the development and education of the numerate and Mathematically literate student. 'Why' questions can be asked at any level of Mathematics and numeracy practice, and within Mathematics itself the answers to those questions will often be found by proving.

Thus my conceptualisation of the relationship between numeracies and Mathematics is that those numeracies are culturally, socially, and historically determined, and practically powerful. Mathematics on the other hand has developed culturally in a different way, and in a different context, as an extremely powerful way of theorising, explaining, and extending our knowledge of the world.

The implications for school mathematics curricula are many, but here are some of the main ones:

- These curricula need to recognise, and to be built around, the many numeracy practices which the learners have learnt at home and in the wider society, and which they bring to the education situation. Thus not all the curricular content can be determined centrally. Much must be determined locally within a national or regional framework. The six activities model can provide such a framework.
- These curricula need also to introduce learners to the many numeracy practices which powerful others in society practise and which can impinge on their lives in crucial ways. There are of course many of these in adult life, but the criterion should be whether, and how, they impinge on the lives of the learners.
- These numeracy practices need to be clarified, theorised, and critiqued in terms of the Mathematical theories behind them. This is to reverse the usual approach of teaching the Mathematical theory followed by its applications. The particular curricular challenge which this implication presents, is how to create sensible sequential designs which can help to structure that approach. This is a challenge of a high order.
- Generally, mathematical ideas should always be related to the various contexts and numeracy practices where they are used. The complaint is often voiced that mathematics is a meaningless and abstract subject, which reflects the need for this implication. If the relationship between the two were better understood by teachers, and then by the learners, perhaps we would not find so much alienation in our schools.

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2 The World Role of Culture in Mathematics Education[1]

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Mathematics education has experienced a major revolution in perceptions (cf. Kuhn, 1970) comparable to the Copernican revolution that no longer placed the earth at the center of the universe. This change has implicated beliefs about *the role of culture* in the historical development of mathematics (Eves, 1990), in the practices of mathematicians (Civil, 2002; Sfard, 1997), in its political aspects (Powell & Frankenstein, 1997), and hence necessarily in its teaching and learning (Bishop, 1988a; Bishop & Abreu, 1991; Bishop & Pompeu, 1991; Nickson, 1992). The change has also influenced methodologies that are used in mathematics education research (Pinxten, 1994). Researchers now increasingly concede that mathematics, long considered value- and culture-free, is indeed a cultural product, and hence that the role of culture—with all its complexities and contestations—is an important aspect of mathematics education.

Topics that are central in addressing this role of culture are those arising from and extending the notions of ethnomathematics and everyday cognition (Nunes, 1992, 1993). Various broad theoretical fields are relevant. Some of the theoretical notions that are apposite are rooted in—but not confined to—situated cognition (Kirshner & Whitson, 1997; Lave & Wenger, 1991; Watson, 1998), cultural models (Holland & Quinn, 1987), notions of cultural capital (Bourdieu, 1995), didactical phenomenology (Freudenthal, 1973, 1983), and Peircean semiotics (Peirce, 1992, 1998). From this partial list, the breadth of this developing field may be recognized. This paper does not attempt to treat the general theories in detail: The interested reader is referred to the original authors. Instead, some key notions that have explanatory power or usefulness are the central focus, which is *the role of culture in learning and teaching mathematics all over the world.* The seminal work of Ascher (1991, 2002), Bishop (1988a, 1988b), D'Ambrosio (1985, 1990) and Gerdes (1986, 1988a, 1988b, 1998) on ethnomathematics is still centrally relevant and thus is treated in some detail.

In the last decade, the field of research into the role of culture in mathematics education has evolved from "ethnomathematics and everyday cognition" (Nunes, 1992), although both ethnomathematics and everyday cognition are still important topics of investigation. The developments have rather consisted in a broadening of the field, clarification and evolution of definitions, recognition of the complexity of the constructs and issues, and inclusion of social, critical, and political dimensions as well as those from cultural psychology involving valorization, identity, and agency (Abreu, Bishop, & Presmeg, 2002). This paper in its scope cannot do full justice to political and critical views of mathematics education (see Mellin-Olsen, 1987; Skovsmose, 1994; Vithal, 2003, for a full treatment), but some of the *landscapes* from these fields—as Vithal calls them—are used to deepen and problematize aspects of the treatment of culture in mathematics education.

This paper has four sections. The first addresses an introduction to issues and definitions of key notions involving culture in mathematics education. The organization of the second and third sections uses the research framework of Brown and Dowling (1998). In this framework, resonating theoretical and empirical fields surround and enclose the central research topic, and the description involves layers of increasing specificity as it zooms in on details of

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[1] This paper is an abridged version of Presmeg's Chapter 11, "The role of culture in teaching and learning mathematics", in F. K Lester (Ed.), *Second handbook of research on mathematics teaching and learning*, pp. 435-458. Charlotte, North Carolina: Information Age Publishing, 2007.

the problematic and problems of the research issues, the empirical settings, and the results of the studies, only to zoom out again at the end in order to survey the issues in a broader field, informed by the results of the research studies examined, in order to see where further research on culture might be headed. Thus section 2 addresses theoretical fields that incorporate culture specifically in mathematics education; section 3 addresses salient empirical fields, settings, and some details of results of research on culture in mathematics education, and their implications for the teaching and learning of mathematics. Using a broader perspective, the fourth (final) section collects and elaborates suggestions for possible directions for future research on culture in mathematics education.

2.1 Definitions and Significance of Culture in Mathematics Education

Why is it important to address definitions carefully in considering the role of culture in teaching and learning mathematics? As in other areas of reported research in mathematics education, different authors use terminology in different ways. Particularly notions such as *culture, ethnomathematics,* and *everyday mathematics* have been controversial; they have been contested and given varied and sometimes competing interpretations in the literature. Thus these constructs must be problematized, not only for the sake of clarification, but more importantly for the roles they have played, and for their further potential to be major focal points in mathematics education research and practices. Further, especially in attempts to bridge the gap between the formal mathematics taught in classrooms and that used out-of-school in various cultural practices, *what counts as mathematics* assumes central importance (Civil, 1995, 2002; Presmeg, 1998a). Thus ontological aspects of the nature of mathematics itself must also be addressed.

(1) Culture

In 1988, when Bishop published his book, *Mathematical Enculturation: A Cultural Perspective on Mathematics Education* (1988a) and an article that summarized these ideas in *Educational Studies in Mathematics* (1988b, now reprinted as a classic in Carpenter, Dossey, & Koehler, 2004), the prevailing view of mathematics was that it was the one subject in the school curriculum that was value- and culture-free, notwithstanding a few research studies that suggested the contrary (e.g., Gay & Cole, 1967; Zaslavsky, 1973). Along with those of a few other authors (notably, D'Ambrosio), Bishop's ideas have been seminal in the recognition that culture plays a pivotal role in the teaching and learning of mathematics, and his insights are introduced repeatedly in this paper.

Whole books have been written about definitions of *culture* (Kroeber & Kluckhohn, 1952). Grappling with the ubiquity yet elusiveness of culture, Lerman (1994) confronted the need for a definition but could not find one that was entirely satisfactory. Yet, as he pointed out, culture is "ordinary. It is something that we all possess and that possesses us" (p. 1). Bishop (1988a) favored the following definition of culture in analyzing its role in mathematics education:

> Culture consists of a complex of shared understandings which serves as a medium through which individual human minds interact in communication with one another. (p. 5, citing Stenhouse, 1967, p. 16)

This definition highlights the communicative function of culture that is particularly relevant in teaching and learning. However, it does not focus on the continuous renewal of culture, the dynamic aspect that results in cultural change over time. This dynamic aspect caused Taylor (1996) to choose the "potentially transformative view" (p. 151) of cultural anthropologist Clifford Geertz (1973), for whom culture consists of "webs of significance" (p. 5) that we ourselves have spun.

This potentially transformative view assumes particular importance in the light of the necessity for negotiating social norms and sociomathematical norms in mathematics classrooms (Cobb & Yackel, 1996). The culture of the mathematics classroom, which was brought to our attention as being significant (Nickson, 1994), is not monolithic or static but continuously evolving, and different in different classrooms as these norms become negotiated. The mathematics classroom itself is one arena in which culture is contested, negotiated, and manifested (Vithal, 2003), but there are various levels of scale. Bishop's (1988a) view of mathematics education as a social process resonates with a transformative, dynamic notion of culture. He suggested that five significant levels of scale are involved in the social aspects of mathematics education. These are the cultural, the societal, the institutional, the pedagogical, and the individual aspects (1988a, p. 14). Culture here is viewed as an all-encompassing umbrella construct that enters into all the activities of humans in their communicative and social enterprises. In addition to a view of culture in this macroscopic aspect, as in these levels of scale, culture as webs of significance may be central also in the societal, institutional, and pedagogical aspects of mathematics education considered as a social process. Thus researchers may speak of the culture of a society, of a school, or of an actual classroom. Culture in all of these levels of scale impinges on the mathematical learning of individual students.

Notwithstanding these general definitions of culture, the word with its various characterizations does not have meaning in itself. Vithal and Skovsmose (1997) illustrated this point starkly by pointing out that interpretations of *culture* (and by implication also anthropology) were used in South Africa to justify the practices of apartheid. They extended the negative connotations still attached to this word in that context to suggest that ethnomathematics is also suspect (as suggested in the title of their article: "The End of Innocence: A Critique of 'Ethnomathematics'"). Some aspects of their critique are taken up in later sections.

How are *culture* and *society* interrelated? The words are not interchangeable (Lerman, 1994), although connections exist between these constructs:

> One would perhaps think of gender stereotypes as cultural, but of 'gender' as socially constructed. One would talk of the culture of the community of mathematicians, treating it as monolithic for a moment, but one would also talk, for example, of the social outcomes of being a member of that group. (p. 2)

Alan Bishop stated succinctly, on several occasions (personal communication, e.g., July, 1985), that *society involves various groups of people, and culture is the glue that binds them together.* This informal characterization resonates with Stenhouse's definition of *culture* as a complex of shared understandings, and also with that of Geertz, as webs of significance that we ourselves have spun. Considering the cultures of mathematics classrooms, Nickson (1994) wrote of the "invisible and apparently shared meanings that teachers and pupils bring to the mathematics classroom and that govern their interaction in it" (p. 8). Values, beliefs, and meanings are implicated in these "shared invisibles" (p. 18) in the classroom. Nickson saw socialization as a universal process, and culture as the *content* of the socialization process, which differs from one society to another, and indeed, from one classroom to another.

Part of culture as webs of significance, taken at various levels, are the prevailing notions of what counts as mathematics.

(2) Mathematics

As Nickson (1994) pointed out, "one of the major shifts in thinking in relation to the teaching and learning of mathematics in recent years has been with respect to the adoption of differing views of the nature of mathematics as a discipline" (p. 10). Nickson characterized this cultural shift as moving from a *formalist* tradition in which mathematics is absolute—consisting of "immutable truths and unquestionable certainty" (p. 11)—without a human face, to one of *growth and change*, under persuasive influences such as Lakatos's (1976) argument that "objective knowledge" is subject to proofs and refutations and thus that mathematical knowledge has a strong social component. That this shift is complex and that both views of mathematics are held simultaneously by many mathematicians was argued by Davis and Hersh (1981). The formalist and socially mediated views of mathematics resonate with the two categories of absolutist and fallibilist conceptions discussed by Ernest (1991). Contributing the notion of mathematics as problem solving, the Platonist, the problem-solving, and the fallibilist conceptions are categories reminiscent of the teachers' conceptions of mathematics that Alba Thompson, already in 1984, gave evidence were related to instructional practices in mathematics classrooms.

Both Civil (1990, 1995, 2002; Civil & Andrade, 2002) in her "Funds of Knowledge" project and in her later research with colleagues into ways of linking home and school mathematical practices, and Presmeg (1998a, 1998b, 2002b) in her use of ethnomathematics in teacher education and research into semiotic chaining as a means of building bridges between cultural practices and the teaching and learning of mathematics in school, described the necessity of broadening conceptions of the nature of mathematics in these endeavors. Without such broadened definitions, high-school and university students alike are naturally inclined to characterize mathematics according to what they have experienced in learning *institutional* mathematics—more often than not as "a bunch of numbers" (Presmeg, 2002b). On the one hand, such limited views of the nature of mathematics inhibit the recognition of mathematical ideas in out-of-school practices. On the other hand, if definitions of what counts as mathematics are too broad, then the "everything is mathematics" notion may trivialize mathematics itself, rendering the definition useless. In examining the mathematical practices of a group of carpenters in Cape Town, South Africa, Millroy (1992) expressed this tension well, as follows:

> [It] became clear to me that in order to proceed with the exploration of the mathematics of an unfamiliar culture, I would have to navigate a passage between two dangerous areas. The foundering point on the left represents the overwhelming notion that 'everything is mathematics' (like being swept away by a tidal wave!) while the foundering point on the right represents the constricting notion that 'formal academic mathematics is the only valid representation of people's mathematical ideas' (like being stranded on a desert island!). Part of the way in which to ensure a safe passage seemed to be to openly acknowledge that when I examined the mathematizing engaged in by the carpenters there would be examples of mathematical ideas and practices that I would recognize and that I would be able to describe in terms of the vocabulary of conventional Western mathematics. However, it was likely that there would also be mathematics that I could not recognize and for which I would have no familiar descriptive words. (pp. 11–13)

Some definitions of mathematics that achieve a balance between these two extremes are as follows. *Mathematics* is "the language and science of patterns" (Steen, 1990, p. iii). Steen's definition has been taken up widely in reform literature in the USA (National Council of Teachers of Mathematics [NCTM], 1989, 2000). Opening the gate to recognizing a human origin of mathematics, Saunders MacLane called mathematics "the study of formal abstract structures arising from human experience" (as cited in Lakoff, 1987, p. 361). According to Ada Lovelace, mathematics is the systematization of relationships (as described by Noss, 1997). All of these definitions strike some sort of balance between the human face of mathematics and its formal aspects. Going beyond Steen's well-known pattern definition in the direction of stressing abstraction, in a critique of ethnomathematics, Thomas (1996) defined mathematics as "the science of detachable relational insights" (p. 17). He suggested a useful distinction between *real mathematics* (as characterized in his definition), and *proto-mathematics* (the category in which he placed ethnomathematics). In the next section, Barton's (1996) characterization of ethnomathematics, which resolves many of these issues and clarifies this dualism, is presented along with some evolving definitions of ethnomathematics. (For details, the reader should consult Barton's original article.)

(3) Ethnomathematics

What is *ethnomathematics*? In his illuminating article, Barton (1996) wrote as follows:

> In the last decade, there has been a growing literature dealing with the relationship between culture and mathematics, and describing examples of mathematics in cultural contexts. What is not so well-recognised is the level at which contradictions exist within this literature: contradictions about the meaning of the term 'ethnomathematics' in particular, and also about its relationship to mathematics as an international discipline. (p. 201)

Barton pointed out that difficulties in defining ethnomathematics relate to three categories: epistemological confusion, "problems with the meanings of words used to explain ideas about culture and mathematics" (p. 201); philosophical confusion, the extent to which mathematics is regarded as universal; and confusion about the nature of mathematics. The nature of mathematics is part of its ontology, and because both ontology and epistemology are branches of philosophy, all of these categories may be regarded as philosophical difficulties. The strength of Barton's resolution of the difficulties lies in his creation of a preliminary framework (he admitted that it might need revision) whereby the differing views can be seen in relation to each other. His triadic framework is an "Intentional Map" (p. 204) with the three broad headings of *mathematics*, *mathematics education*, and *society* (cf. the whole day of sessions dedicated to these broad areas at the 6th International Congress on Mathematical Education held in Budapest, Hungary, in 1988). The seminal writers whose definitions of ethnomathematics he considered in detail and placed in relation to this framework were Ubiratan D'Ambrosio in Brazil, Paulus Gerdes in Mozambique, and Marcia Ascher in the USA.

As Barton (1996) pointed out, D'Ambrosio's prolific writings on the subject of ethnomathematics have influenced the majority of writers in this area. Thus on the Intentional Map, although D'Ambrosio's work (starting with his 1984 publication) falls predominantly in the socio-anthropological dimension between *society* and *mathematics*, some aspects of his concerns can be found in all of the dimensions. In his later work, he increasingly used his model to analyze "the way in which mathematical knowledge is colonized and how it rationalizes social divisions within societies and between societies" (Barton, 1996, p. 205). In his early writing, D'Ambrosio (1984) defined ethnomathematics as the way different cultural groups mathematize—count, measure, relate, classify, and infer. His definition evolved over the years, to include a changing form of knowledge manifest in practices that change over time. In 1985, he defined ethnomathematics as "the mathematics which is practiced among identifiable cultural groups" (p. 18). Later, in 1987, his definition of ethnomathematics was "the codification which allows a cultural group to describe, manage, and understand reality" (Barton, 1996, p. 207).

D'Ambrosio's (1991) well-known etymological definition of ethnomathematics is given in full in the following passage.

> The main ideas focus on the concept of *ethnomathematics* in the sense that follows. Let me clarify at the beginning that this term comes from an etymological abuse. I use *mathema(ta)* as the action of explaining and understanding in order to transcend and of managing and coping with reality in order to survive. Man has developed throughout each one's own life history and throughout the history of mankind *techné*'s (or *tics*) of *mathema* in very different and diversified cultural environments, i.e., in the diverse *ethno*'s. So, in order to satisfy the drive towards survival and transcendence in diverse cultural environments, man has developed and continuously develops, in every new experience, **ethno-mathema-tics.** These are *communicated* vertically and horizontally in time, respectively throughout history and through conviviality and education, relying on memory and on sharing experiences and

> knowledges. For the reasons of being more or less effective, more or less powerful and sometimes even for political reasons, some of these different *tics* have lasted and spread (ex.: counting, measuring) while others have disappeared or been confined to restricted groups. This synthesizes my approach to the history of ideas. (p. 3)

As in some of his other writings (1985, 1987, 1990), D'Ambrosio is in this definition characterizing ethnomathematics as a dynamic, evolving system of knowledge-the "process of knowledge-making" (Barton, 1996, p. 208), as well as a research program that encompasses the history of mathematics.

Returning to Barton's Intentional Map, the work of Paulus Gerdes is "practical, and politically explicit," concentrated in the *mathematics education* area of the Map (Barton, 1996, p. 205). Gerdes's definition of ethnomathematics evolved from the mathematics implicit or "frozen" in the cultural practices of Southern Africa (1986), to that of a mathematical movement that involves research and anthropological reconstruction (1994). The work of mathematician Marcia Ascher (1991, 1995, 2002), while overlapping with that of Gerdes to some extent, falls closer to the *mathematics* area on the Map, concerned as it is with cultural mathematics. Her definition is that ethnomathematics is "the study and presentation of the mathematical ideas of traditional peoples" (1991, p. 188). When Ascher (1991) worked out the kinship relations of the Warlpiri, say, in mathematical terms, she acknowledged that she was using her familiar "Western" mathematics. In that sense her ethnomathematics is subjective: The Warlpiri would be unlikely to view their kinship system through her lenses. Referring to mathematics and ethnomathematics, she stated, "They are both important, but they are different. And they are linked" (Ascher & D'Ambrosio, 1994, p. 38). In this view, there is no need to view ethnomathematics as "proto-mathematics" (Thomas, 1996), because it exists in its own right.

Finally, Barton (1996) found a useful metaphor to sum up the similarities and differences between the views of ethnomathematics held by these three proponents: "For D'Ambrosio it is a window on knowledge itself; for Gerdes it is a cultural window on mathematics; and for Ascher it is the mathematical window on other cultures" (p. 213). These three windows are distinguished by the standpoint of the viewer, and by what is being viewed, in each case. Although not eliminating the duality of ethnomathematics as opposed to mathematics (of mathematicians), these three distinct windows represent approaches each of which has something to offer. Taken together, they contribute a broadened lens on the role of culture in teaching and learning mathematics.

Several other writers in the field of ethnomathematics have acknowledged the need and attempted to define ethnomathematics. Scott (1985) regarded ethnomathematics as lying at the confluence of mathematics and cultural anthropology, "mathematics in the environment or community," or "the way that specific cultural groups go about the tasks of classifying, ordering, counting, and measuring" (p. 2). Several definitions of ethnomathematics highlight some of the "environmental activities" that Bishop (1988a) viewed as universal, and also "necessary and sufficient for the development of mathematical knowledge" (p. 182), namely counting, locating, measuring, designing, playing, and explaining. One further definition brings back the problem, hinted at in the foregoing account, of *ownership* of ethnomathematics. Whose mathematics is it?

> Ethnomathematics refers to any form of cultural knowledge or social activity characteristic of a social and/or cultural group, that can be recognized by other groups such as 'Western' anthropologists, but not necessarily by the group of origin, as mathematical knowledge or mathematical activity. (Pompeu, 1994, p. 3)

This definition resonates with Ascher's, without fully solving the problem of ownership. The same problem appears in definitions of everyday mathematics, considered next.

(4) Everyday Mathematics

Following on from the description of *everyday cognition* (Nunes, 1992, 1993) and important early studies that examined the use of mathematics in various practices, such as mathematical cognition of candy sellers in Brazil (Carraher, Carraher, & Schliemann, 1985; Saxe, 1991), constructs and issues are still being questioned. In this area, too, clarification of definitions is being sought, along with deeper consideration of the scope of the issues and their potential and significance for the classroom learning of mathematics.

Brenner and Moschkovich (2002) raised the following questions.

> What do we mean by *everyday mathematics*? How is everyday mathematics related to *academic mathematics*? What particular everyday practices are being brought into mathematics classrooms? What impact do different everyday practices actually have in classroom practices?" (p. v)

In a similar vein, and with the benefit of 2 decades of research experience in this area, Carraher and Schliemann (2002) examined how their perceptions had evolved, as they explored the topic of their chapter, "Is Everyday Mathematics Truly Relevant to Mathematics Education?"

All the authors of chapters in the monograph edited by Brenner and Moschkovich (2002) in one way or another set out to explore these and related questions. Several of these authors pointed out that it is problematic to oppose everyday and academic mathematics, for several reasons. For one thing, for mathematicians academic mathematics *is* an everyday practice (Civil, 2002; Moschkovich, 2002a). For another, studies of everyday mathematical practices in workplaces reveal a complex interplay with sociocultural and technological issues (FitzSimons, 2002). In the automobile production industry, variations in the mathematical cognition required of workers have less to do with the job itself than with the decisions of management concerning production procedures and organization of the workplace. Highly skilled machinists display spatial and geometric knowledge that goes beyond what is commonly taught in school: In contrast, assembly-line workers and some machine operators find few if any mathematical demands in their work, which is deliberately stripped of the need for decisions involving knowledge of mathematics beyond elementary counting (Smith, 2002). The complex relationship between use of technology and the demand for mathematical thinking in the workplace is a theme that is explored in a later section.

Another aspect that is again apparent in all the chapters of Brenner and Moschkovich's monograph is the importance of perceptions and beliefs about the nature of mathematics, both in the microculture of classroom practices (Brenner, 2002; Masingila, 2002) and in the broader endeavor to bridge the gap between mathematical thinking in and out of school (Arcavi, 2002; Civil, 2002; Moschkovich, 2002a). An essential element in all of these studies is the concern to connect knowledge of mathematics in and out of school. (This issue is revisited later.) Because of the difficulties surrounding the construct *everyday mathematics* the terminology that will be adopted in this chapter follows Masingila (2002), who referred to in-school and out-of-school mathematics practices (p. 38).

The developments described in this section parallel the genesis of the movement away from purely psychological cognitive and behavioral frameworks for research in mathematics education, towards cultural frameworks that embrace sociology, anthropology, and related fields, including political and critical perspectives. The following section introduces some relevant theoretical issues and lenses that have been used to examine some of these developments.

2.2 Theoretical Fields That Incorporate Culture in Mathematics Education

The notion of theoretical and empirical fields is drawn from Brown and Dowling (1998) and provides a useful framework for characterizing components of research. This section addresses some theoretical fields pertinent to culture in the teaching and learning of mathematics. Their instantiation in empirical studies is described in the next section. The reader is reminded again to consult the original authors for a full treatment of theoretical fields that are introduced in this section, which has as its purpose a wide but by no means exhaustive view of the scope of theories that are available for work in this area.

As suggested in Barton's (1996) sense-making article introduced in the previous section, in the last 2 decades there has already been considerable movement in theoretical fields regarding the interplay of culture and mathematics. One such movement is discernible in the definitions of ethnomathematics given by D'Ambrosio, Gerdes, and Ascher, as their theoretical formulations moved from more static definitions of ethnomathematics as the mathematics of different cultural groups, to characterizations of this field as an anthropological research program that embraces not only the history of mathematical ideas of marginalized populations, but the history of mathematical knowledge itself (see previous section). D'Ambrosio (2000, p. 83) called this enterprise *historiography.*

(1) Historiography

Moving beyond earlier theoretical formulations of ethnomathematics, its importance as a catalyst for further theoretical developments has been noted (Barton, 1996). D'Ambrosio played a large role and served as advisory editor in the enterprise that resulted in Helaine Selin's (2000) edited book, Mathematics Across Cultures: *The History of Non-Western Mathematics.* The chapters in this book are global in scope and record the mathematical thinking of cultures ranging from those of Iraq, Egypt, and other predominantly Islamic countries; through the Hebrew mathematical tradition; to that of the Incas, the Sioux of North America, Pacific cultures, Australian Aborigines, mathematical traditions of Central and Southern Africa; and those of Asia as represented by India, China, Japan, and Korea. As can be gleaned from the scope of this work, D'Ambrosio's original concern to valorize the mathematics of colonized and marginalized people (cf. Paolo Freire's *Pedagogy of the Oppressed* in 1970/1997) has broadened to encompass a movement that is both archeological and historical in nature, based on the theoretical field of "historiography" and visions of world knowledge through the "sociology of mathematics" (D'Ambrosio, 2000, pp. 85–87).

Although these antedated D'Ambrosio's program, earlier studies such as Claudia Zaslavsky's (1973) report on the counting systems of Africa and Glendon Lean's (1986) categorization of those of Papua New Guinea (see also Lancy, 1983), could also be thought of as historiography, as could anthropological research such as that of Pinxten, van Dooren, and Harvey (1983) who documented Navajo conceptions of space. Also in the cultural anthropology tradition, Crump's (1994) research on the anthropology of numbers is another fascinating example of historiography. More recent studies such as some of those collected as *Ethnomathematics* in the book by Powell and Frankenstein (1997) and the work of Marcia Ascher (1991, 1995, 2002) also fall into this category. Many of the cultural anthropological studies of various mathematical aspects of African practices, such as work on African fractals (Eglash, 1999); *lusona* of Africa (Gerdes, 1997); and women, art, and geometry of Africa (Gerdes, 1998), may also be regarded as historiography. As part of ethnomathematics conceived as a research program, this ambitious undertaking of historiography is designed to address lacunae in the literature on the history of mathematical thought through the ages. Much of the work of members of the International Study Group on Ethnomathematics (founded in 1985), and of the North American Study Group on Ethnomathematics (founded in 2003)—including research by Lawrence Shirley, Daniel Orey, and many others—could be placed in the category of historiography (see the list of some of the available web sites following the references).

Because historiography addresses some of the world's mathematical systems that have been ignored or undervalued in mathematics classrooms, it reminds us that there is *cultural* *capital* involved in the power relations associated with access in school mathematics. Some relevant theories are outlined in the following paragraphs.

(2) Cultural Capital and Habitus

The usefulness of the theoretical field outlined by Bourdieu (1995) for issues of culture in the learning of mathematics has been indicated in research on learner's transitions between mathematical contexts (Presmeg, 2002a). Resonating with D'Ambrosio's original issues of concern (although D'Ambrosio did not use this framework), Bourdieu's work belongs in the field of sociology. The relevance of this work consists in "the innumerable and subtle strategies by which words can be used as instruments of coercion and constraint, as tools of intimidation and abuse, as signs of politeness, condescension, and contempt" (Bourdieu, 1995, p. 1, editor's introduction). This theoretical field serves as a useful lens in examining empirical issues related to the social inequalities and dilemmas faced by mathematics learners as they move between different cultural contexts, for example in the transitions experienced by immigrant children learning mathematics in new cultural settings (Presmeg, 2002a). This field embraces Bourdieu's notions of *cultural capital, linguistic capital,* and *habitus*. Bourdieu (1995) used the ancient Aristotelian term *habitus* to refer to "a set of dispositions which incline agents to act and react in certain ways" (p.12). Such dispositions are part of culture viewed as a set of shared understandings. Various forms of capital are *economic capital* (material wealth), *cultural capital* (knowledge, skills, and other cultural acquisitions, as exemplified by educational or technical qualifications), and *symbolic capital* (accumulated prestige or honor) (p. 14). *Linguistic capital* is not only the capacity to produce expressions that are appropriate in a certain social context, but it is also the expression of the "correct" accent, grammar, and vocabulary. The *symbolic power* associated with possession of cultural, symbolic, and linguistic capital has a counterpart in the *symbolic violence* experienced by individuals whose cultural capital is devalued. Symbolic violence is a sociological construct. In that capacity it is a powerful lens with which to examine actions of a group and ways in which certain types of knowledge are included or excluded in what the group counts as knowledge (for examples embracing the learning of mathematics, see the chapters in Abreu, Bishop, & Presmeg, 2002).

(3) Borderland Discourses

The notion of symbolic violence leaves a possible theoretical gap relating to the ways in which individuals choose to construct, or choose *not* to construct, particular knowledge—mathematical or otherwise. Bishop (2002a) gave examples both of immigrant learners of mathematics in Australia who chose not to accept the view of themselves as constructed by their peers or their teacher—and of others who chose to accept these constructions. One student "shouted back" when peers in the mathematics class shouted derogatory names.

Discourse is a construct that is wider than mere use of language in conversation (Philips, 1993; Wood, 1998): It embraces all the aspects of social interaction that come into play when human beings interact with one another (Dörfler, 2000; Sfard, 2000). The notion of *Discourses* formulated by Gee (1992, his capitalization), and in particular his extension of the construct to *borderland Discourses*, those "community-based secondary Discourses" situated in the "borderland" between home and school knowledge (p. 146), are in line with Bourdieu's ideas of habitus and symbolic violence. Borderland Discourses take place in the borderlands between primary (e.g., home) and secondary (e.g., school) cultures of the diverse participants in social interactions. In situations where the secondary culture (e.g., that of the school) is conceived as threatening because of the possibility of symbolic violence there, the borderland may be a place of solidarity with others who may share a certain habitus. These ideas go some way towards closing the theoretical gap in Bourdieu's characterization of symbolic violence by raising some issues of individuals' choices, because individuals choose the extent to which they will participate in various forms of these Discourses (see also Bishop's, 2002b, use of Gee's constructs).

Gee's work was in the context of second language learning but is also useful in the analysis of meanings given to various experiences by mathematics learners in cultural transition situations. Bishop (2002a) used this theoretical field in moving from the notion of cultural conflict to that of cultural mediation, in analyzing these experiences of learners of mathematics. If one considers the primary Discourse of school mathematics learners to be the home-based practices and conversations that contributed to their socialization and enculturation (forming their habitus) in their early years, and continuing to a greater or lesser extent in their present home experiences, then the secondary Discourse, for the purpose of learning mathematics, could be designated as the formal mathematical Discourse of the established discipline of mathematics. The teacher is more familiar with this Discourse than are the students and thus has the responsibility of introducing students to this secondary Discourse. As students become familiar with this field, their language and practices may approximate more closely those of the teacher. But in this transition the borderland Discourse of interactive classroom practices provides an important mediating space.

The enculturating role of the mathematics teacher was suggested in the foregoing account. However, as Bishop (2002b) pointed out, the learning of school mathematics is frequently more of an acculturation experience than an enculturation. The difference between these anthropological terms is as follows. *Enculturation* is the induction, by the cultural group, of young people into their own culture. In contrast, *acculturation* is "the modification of one's culture through continuous contact with another" (Wolcutt, 1974, p. 136, as quoted in Bishop, 2002b, p. 193). The degree to which the culture of mathematics, as portrayed in the mathematics classroom, is viewed as their own or as a foreign culture by learners would determine whether their experiences there would be of enculturation or acculturation.

(4) Cultural Models

Allied with Gee's (1992) notion of different Discourses is his construct of *cultural models*. This construct, defined as "'first thoughts' or taken for granted assumptions about what is 'typical' or 'normal'" (Gee, 1999, p. 60, quoted in Setati, 2003, p. 153), was used by Setati (2003) as an illuminating theoretical lens in her research on language use in South African multilingual mathematics classrooms. Already in 1987, D'Andrade defined a cultural model—which he also called a *folk model*—as "a cognitive schema that is intersubjectively shared by a social group," and he elaborated, "One result of intersubjective sharing is that interpretations made about the world on the basis of the folk model are treated as if they were obvious facts about the world" (p. 112). The transparency of cultural models may help to explain why mathematics was for so long considered to be value- and culture-free. The well-known creativity principle of making the familiar strange and the strange familiar (e.g., De Bono, 1975) is necessary for participants to become aware of their implicit cultural beliefs and values, which is why the anthropologist is in a position to identify the beliefs that are invisible to many who are within the culture.

In the context of mathematics learning in multilingual classrooms, Adler (1998, 2001) pointed out aspects of the use of language as a cultural resource that relate to the transparency of cultural models. Particularly in classrooms where the language of instruction is an *additional language*—not their first language—for many of the learners (Adler, 2001), teachers must at times focus on the language itself, in which case the artifact of language no longer serves as a "window" of transparent glass through which to view the mathematical ideas (Lave & Wenger, 1991). In this case the language used is no longer *invisible*, and the focus on the language itself may detract from the conceptual learning of the mathematical content (Adler, 2001).

(5) Valorization in Mathematical Practices

If transparency of culture necessitates making the familiar strange before those sharing that culture become aware of the lenses through which they are viewing their world, then this principle points to a reason for the neglect of issues of valorization in the mathematics education research community until Abreu's (1993, 1995) research brought the topic to the fore. The *value* of formal mathematics as an academic subject was for so long taken for granted that it became a *given* notion that was not culturally questioned. Especially in its role as a gatekeeper to higher education, this status in education is likely to continue. But if ethnomathematics as a research program is to have a legitimate place in broadening notions both of what counts as mathematics and of which people have originated these forms of knowledge, then issues of valorization assume paramount importance.

Working from the theoretical fields of cultural psychology and sociocultural theory, Abreu and colleagues investigated the effects of valorization of various mathematical practices on Portuguese children (Abreu, Bishop, & Pompeu, 1997), Brazilian children (Abreu, 1993, 1995), and British children from Anglo and Asian backgrounds (Abreu, Cline, & Shamsi, 2002). As confirmed also in the research of Gorgorio, Planas, and Vilella (2002), many of these children denied the existence of, or devalued, mathematics as used in practices that they associated with their home- or out-of-school settings.

Valorization, the social process of assigning more value to certain practices than to others, is closely allied to Wertsch's (1998) notion of *privileging,* defined as "the fact that one mediational means, such as social language, is viewed as being more appropriate or efficacious than another in a particular social setting" (Wertsch, 1998, p. 64, in Abreu, 2002, p. 183). Abreu (2002) elaborated as follows.

> From this perspective, cultural practices become associated with particular social groups, which occupy certain positions in the structure of society. Groups can be seen as mainstream or as marginalised. In a similar vein individuals who participate in the practices will be given, or come to construct, identities associated with certain positions in these groups. The social representation enables the individual and social group to have access to a 'social code' that establishes relations between practices and social identities. (p. 184)

Thus Abreu argued strongly that in the learning of mathematics, valorization operates not only on the societal plane but also on the personal plane, because it impacts the construction of social identities. At this psychological level, the construction of mathematical knowledge may be subordinated to the construction of social identity by the individual learner in cases of cultural conflict, as suggested by Presmeg (2002a).

Abreu's ideas are embedded in the field of cultural psychology. Also taking the individual and society into account, another field that has grown in influence in mathematics education research in recent decades is that of situated cognition.

(6) Situated Cognition

The theoretical field of situated cognition explores related aspects of the interplay of knowledge on the societal and psychological planes. Hence it has been a useful lens in research that takes culture into account in mathematical thinking and learning (Watson, 1998). As Ubiratan D'Ambrosio by his writings founded and influenced the field of ethnomathematics, so Jean Lave in analyzing and reporting her anthropological research has influenced the theoretical field of situated cognition. From her early theorizing following ethnography with tailor's apprentices in Liberia (Lave, 1988) to her more recent writings, following research with grocery shoppers and weight-watchers (Lave, 1997), the notion of transfer of knowledge through abstraction in one context, and subsequent use in a new context, was questioned and problematized. In collaboration with Etienne Wenger, her theorizing led to the notion of cognitive apprenticeship and *legitimate peripheral participation* (Lave & Wenger, 1991; Wenger, 1998). In this view, learning consists in a centripetal movement of the apprentice from the periphery to the center of a practice, under the guidance of those who are already masters of the practice. This theory was not originally developed in the context of or for the purpose of informing mathematics education. However, in its challenge to the cognitive position that abstract learning of mathematics facilitates transfer and that this knowledge may be readily applied in other situations than the one in which it was learned (not born out by empirical research), the theory has been powerful and influential. The research studies inspired by Lave and reported by Watson (1998) bear witness to this strong influence.

In her later writings, Lave attempted to bring the theory of socially situated knowledge to bear on the classroom teaching and learning of mathematics. But issues of intentionality and recontextualization separate apprenticeship and classroom situations, although enough commonality exists in the two situations for both legitimately to be called *practices* (Lerman, 1998). Mathematics learners in school are not necessarily aiming to become either mathematicians or mathematics teachers. As Lerman pointed out in connection with the issue of voluntary and nonvoluntary participation, students' presence in the classroom may be nonvoluntary, creating a very different situation from that of apprenticeship learning, and calling into question the assumption of a goal of movement from the periphery to the center. At the same time, the teacher has the intention to teach her students mathematics, notwithstanding Lave's claim that teaching is not a precondition of learning. The learning of mathematics is the goal of the enterprise, and teaching is the teacher's job. In contrast, in the apprenticeship situation the learning is not a goal in itself, for example, in the case of the tailors the goal is to make garments efficiently. Thus, as Adler (1998) suggests, "It is in the understanding of the aims of school education that Lave and Wenger's seamless web of practices entailed in moving from peripheral to full participation in a community of practice is problematic" (p. 174).

Although the theory of legitimate peripheral participation may not translate easily into the classroom teaching and learning of mathematics, the view of learning as a social practice has powerful implications for this learning and has been an influence in changes that have taken place in the practice of teaching mathematics, such as an increased emphasis on communication (NCTM, 1989, 2000) in the mathematics classroom. Even more, situated cognition as a theory has given a warrant to attempts to bridge the gap between in- and out-of-school mathematical practices.

(7) Use of Semiotics in Linking Out-Of-School and In-School Mathematics

In the USA, the *Principles and Standards for School Mathematics* (NCTM, 2000) continued the earlier call (NCTM, 1989) for teachers to make connections, in particular between the everyday practices of their students and the mathematical concepts that are taught in the classroom. But various theoretical lenses of situated cognition (Lave & Wenger, 1991; Kirshner & Whitson, 1997) remind us that these connections can be problematic. There are at least three ways in which the activities of out-of-school practices differ from the mathematical activities of school classrooms (Walkerdine, 1988), as follows.

- The goals of activities in the two settings differ radically.
- Discourse patterns of the classroom do not mirror those of everyday practices.
- Mathematical terminology and symbolism have a specificity that differs markedly from the useful qualities of ambiguity and indexicality (interpretation according to context) of terms in everyday conversation.

A semiotic framework that uses chains of signification (Kirshner & Whitson, 1997) has the potential to bridge this apparent gap through a process of chaining of signifiers in which each sign "slides under" the subsequent signifier. In this process, goals, discourse patterns, and use of terms and symbols all move towards that of classroom mathematical practices in a way that has the potential to preserve essential structure and some of the meanings of the original activity.

This theoretical framework resonates with that of *Realistic Mathematics Education* (RME) developed by Freudenthal, Streefland, and colleagues at the Freudenthal Institute (Treffers, 1993). *Realistic* in this sense does not necessarily mean out-of-school in the real world: The term refers to problem situations that learners can imagine (van den Heuvel-Panhuizen, 2003). A theory of semiotic chaining in mathematics education resonates with RME in that the starting points for the learning are realistic in this sense. But more specifically the chaining model is a useful tool for linking out-of-school mathematical practices with the formal mathematics of school classrooms (an example of such use is presented in the next paragraph).

In brief, the theory of semiotic chaining used in mathematics education research, as it was initially presented by Whitson (1997), Cobb, Gravemeijer, Yackel, McClain, & Whitenack (1997), and Presmeg (1998b), followed the usage of Walkerdine (1988). Although she was working in a poststructural paradigm, Walkerdine found the Swiss structural approach of Saussure, as modified by Lacan, useful in building chains of signifiers to link a home practice, such as that of a daughter and her mother pouring drinks for guests, with more formal learning of mathematics, in this case the system of whole numbers. Walkerdine's chain had the following structure.

Signified 1: the actual people coming to visit;
Signifier 1 (*standing for* signified 1): the names of the people.
} **Sign 1**

The daughter is asked to raise a finger for each name.
Signified 2: the names of the guests;
Signifier 2: the raised fingers.
} **Sign 2**

The daughter is asked to count her raised fingers.
Signified 3: the raised fingers;
Signifier 3: the numerals, one two, three, four, five.
} **Sign 3**

In this process, signifier 3 actually comes to stand for all of the preceding links in the chain: Five guests are coming to tea. Note that each signifier in turn becomes the next signified, and that at any time any of the links in the chain may be revisited conceptually. As mother and daughter move through the three signs, the discourse shifts successively from actual people to their names, to the fingers of one hand, and finally to the more abstract discourse of the numerals of the whole number counting system.

In her distinctive style, Adler (1998) characterized the associated recontextualizing process as that of crossing a discursive bridge:

> That there is a bridge to cross between everyday and educated discourses is at the heart of Walkerdine's (1988) argument for 'good teaching' entailing chains of signification in the classroom where everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice. (p. 174)

Using this process, a teacher can use chains of signification as an instructional model that develops a mathematical concept starting with an out-of-school situation and linking it in a number of steps with formal school mathematics (Hall, 2000). Building on the work of Presmeg

(1998b) in his dissertation research, Hall taught three 4th-grade teachers to build semiotic chains appropriate to the practices of their students and the instructional needs of their individual classrooms. Using a similar semiotic chaining model, Cobb et al. (1997) reported on the emergence (using their *emergent* theoretical perspective) of chains of signification in one 1st-grade classroom. In addition to these sources, examples of mathematical chaining at elementary school, high school, and college levels may be found in Presmeg's (1998b, 2006) writings. In later research on building bridges between out-of-school and in-school mathematics, Presmeg (2006) preferred to use a nested triadic model based on the writings of Charles Sanders Peirce (1992, 1998). In addition to an *object* (which could be an abstract concept) and a *representamen* that stands for the object in some way, each nested sign has a third component that Peirce called the *interpretant*. This triadic model explicitly allows for learners' individual construction of meaning (through the interpretant) in a way that linear chains of signification can do only implicitly.

Attempts to facilitate teachers' use of children's out-of-school mathematical practices in their classrooms provide examples of practical uses of theoretical fields. This theme is continued in the next section.

2.3 Empirical Fields That Incorporate Culture in Mathematics Education

The last section dealt with some theoretical fields that have been useful, or have the potential to be useful, in research on the role of culture in mathematics education. In the current section, some associated empirical fields are discussed. Empirical fields entail broad methodologies of empirical research, but in addition to these, in this section specific methods of research and also the participants, data, and results of selected studies are introduced. These details of participants, time, and place in the research are termed the *empirical settings* (Brown & Dowling, 1998). Thus appropriate empirical fields and settings are the focus of this section, along with some results of research on the role of culture in mathematics education.

Because ethnography is the special province of holistic cultural anthropological research (Eisenhart, 1988), which has as a broad goal the understanding of various cultures, it is natural to expect that ethnography (sometimes in modified form) would be used by researchers who are interested in the role of culture in teaching and learning mathematics. This is indeed the case. However, researchers in mathematics education in general sometimes call their studies ethnography when they have not spent long enough in the field to warrant that term for their research (Eisenhart, 1988). For instance, valuable as studies such as that of Millroy (1992) are for mathematics educators to learn more about the use of mathematical ideas in out-of-school settings such as that of Millroy's carpenters in Cape Town, South Africa, the $5\frac{1}{2}$ month duration of her fieldwork might be considered brief for an ethnography, and most ethnographic studies in mathematics education have a shorter duration than Millroy's. For this reason, the field of such research is often more appropriately called *case studies* (Merriam, 1998), because they satisfy the criterion of being bounded in particular ways. If such studies are investigating a particular topic, such as ways of bringing out-of-school mathematics into the classroom, and using several cases to do so, they could be called *instrumental case studies* (Stake, 2000). Where the individual cases themselves are the focus, case studies are *intrinsic* (ibid.).

Most of the studies mentioned in this section may be regarded as instrumental case studies in the sense explained in the foregoing. Research that concerns in-school and out-of-school mathematical practices falls into two main categories. In the first category are studies that attempt to build bridges between informal or nonformal mathematical practices (Bishop, Mellin-Olsen, & van Dormolen, 1991) and those of formal school mathematics. In the second category are studies that link with formal mathematics education less directly but with the potential to deepen understanding of this education, by exploring the culture of mathematics in various workplaces. The studies in both of these categories are too numerous for a summary chapter such as this one to do full justice to the aims, research questions, methodologies and methods for data collection, and the subtleties unearthed in the results of such research. (Again, the reader is reminded to read the original literature for depth of coverage.) What follows is an introduction to the range of research in each of these two categories, with discussion of some of the empirical settings, a sampling of issues addressed, and some of the results.

2.4 Linking Mathematics Learning Out-Of-School and In-School

In the first category, studies that specifically investigated linking out-of-school and in-school mathematics are of several types. Some researchers interviewed parents of minority learners and their teachers to investigate relationships between home and school learning of mathematics. In England, Abreu and colleagues conducted such interviews for the purpose of investigating how the parents of immigrant learners are able to support their children in their transitions to learning mathematics in a new school culture (Abreu, Cline & Shamsi, 2002). Civil and colleagues in the USA interviewed such parents for the purpose of finding out which home practices of immigrant families might be suitable for pedagogical purposes in mathematics classrooms (Civil, 1990, 1995, 2002; Civil & Andrade, 2002). Related empirical studies in which researchers investigated the transitions experienced by immigrant children in their learning of mathematics have been reported by Bishop (2002a) in Australia and by Gorgorio, Planas, and Vilella (2002) in Catalonia, Spain.

In the next type of study in this category, research was conducted in mathematics classrooms to investigate the effects of bringing out-of-school practices into that arena. Using video recording as a data collection method, Brenner (2002) investigated how four teachers and their junior high school students went about an activity in which the students were required to decide cooperatively which of two fictitious pizza companies should be given the contract to supply pizza to the school cafeteria. In a different classroom video study, Moschkovich's (2002b) research addressed the mathematical activities of seventh-grade students during an architectural design project: Students working collaboratively tried to design working and living quarters for a team of scientists in Antarctica in such a way that space would be maximized while still taking into account the heating costs of various designs. These studies emphasized the point that through their pedagogy and instructional decisions in the classroom, teachers are an important component of the success (or lack of success) of attempts to use out-of-school practices effectively in mathematics classrooms. Beliefs of teachers about the nature of mathematics, what culture is, and its role in the classroom learning of mathematics are strong factors in teachers' decisions in this regard (Civil, 2002; Presmeg, 2002b).

Like the researchers in the preceding paragraphs, Presmeg (1998b, 2002b, 2006) and Hall (2000) recognized the lack of congruence of out-of-school cultural practices and those "same" practices when brought into the school mathematics classroom (Walkerdine, 1988). Their research used semiotic chaining as a theoretical tool (see previous section) in an attempt to bridge the gaps, not only between these practices in- and out-of-school, but also between mathematical ideas implicit in these activities and the formal mathematics of the syllabi that teachers are expected to use in their practices. The fourth-grade teachers with whom Hall worked built chains of signifiers that they implemented with their classes, starting with a cultural practice of at least some of their students and aimed at linking mathematical ideas in this practice in a number of steps with a formal school mathematics topic. Chains constructed by the teachers were designated as either *intercultural*-bridging two or more cultures, or *intracultural*-having a chain that remained within a single culture. Examples of the first type involved number of children in students' families, pizzas, coins, and measurement of students' hands, linking in a series of steps with classroom mathematical concepts. These are intercultural because the cultures and discourse of students' homes or activities are linked with the different discourse and culture of classroom mathematics, for instance, the making of bar graphs.

Manipulatives were frequently used as intermediate links in these chains. The intracultural type, involving chains that were developed within the culture of a single activity, was evidenced in a chain involving baseball team statistics. The movement along the chain could be summarized as follows:

Baseball Game ⟶ Hits vs. At Bats ⟶ Success Fraction ⟶ Batting Average.

It was not the activity that was preserved throughout the chain, but merely the culture of baseball (at least as it was imported into the classroom practice) within which the chain was developed. The need for interpretation at each link in the chain led to further theoretical developments using a triadic nested model that was a better lens for interpreting the results (Presmeg, 2006).

One study that is ethnographic in its scope and methodology has less explicit claims to link out-of-school and in-school mathematics, although implicit ties between the two contexts are present. This study is a thorough investigation of mathematical elements in learning the practice of selling newspapers in the streets by young boys (called in Portuguese *ardinas*) on the island of Cabo Verde in the Atlantic ocean (Santos & Matos, 2002). Using Lave and Wenger's (1991) theoretical framework of legitimate peripheral participation, Santos and Matos described the goal of their research as follows.

> Our goal was to look into the ways (mathematics) learning relates to forms of participation in social practice in an environment where mathematics is present but that escapes the characteristics of the school environment. Because we believe that culture is an unavoidable fact that shapes our way of seeing and analyzing things, we decided to look at a culturally distinct practice and that constituted a really strange domain for us: the practice of the *ardinas* at Cabo Verde islands in Africa. (p. 81)

In addition to interesting explicit and implicit mathematical aspects of the changing practice of the *ardinas* as they moved from being newcomers to full participants, methodological difficulties in this kind of research were foregrounded by Santos and Matos (2002):

> The fact that the research is studying a phenomena [*sic*] which was almost totally strange to us in most of its aspects, led us to realize that we had to go through a process which should involve, to a certain extent, our participation in the (*ardinas'*) practice with the explicit (for us and for them) goal of learning it but not in order to be a full member of that community of practice. This starting point (more in terms of knowing that there are more things that we don't know than that we know) opened that community of practice to us but also gave us consciousness that methodological issues were central in this research. (p. 120)

Many of the difficulties that Santos and Matos described with sensitivity and vividness are common to most anthropological research and thus also impact investigations of cultural practices in the field of mathematics education. They were required to become part of the culture sufficiently to be able to interpret it to others who are not participants, but at the same time not to become so immersed in the culture that it would be transparent-a lens that is not the focus of attention because one looks *through* it. They faced the difficulty that entering the group as an outsider might in fact change the culture of that group to a greater or lesser extent. On several occasions there was evidence that the mathematical reporting by key informants among the *ardinas* was influenced by the fact that the fieldworker was a Portuguese-speaking woman, whom these boys might have associated with their schoolteachers. Finally, the researchers recognized the not inconsiderable difficulties associated with practical matters concerned with collecting data in the street.

The investigation of the *ardinas*' mathematical thinking could be classified as an ethnographic workplace study. Thus this short account of this research provides a transition to the second aspect of this empirical section of the chapter, mathematics in the workplace.

(1) Culture of Mathematics in the Workplace

In the second category of studies in this section, the culture of mathematics in various workplaces was investigated intensively by FitzSimons (2002), by Noss and colleagues (Noss & Hoyles, 1996a; Noss, Hoyles & Pozzi, 2000; Noss, Pozzi & Hoyles, 1999; Pozzi, Noss, and Hoyles, 1998), and by several other researchers (see later). FitzSimons investigated the culture and epistemology of mathematics in Technical and Further Education in Australia, and how this education related or failed to relate to the cultures of mathematics as used in workplaces. Noss, Hoyles, and Pozzi examined workplace mathematics in more specific detail: *Inter alia* their studies addressed mathematical aspects of banking and nursing practices. A common thread in these studies is the *dematerialization* of the workplace: "As the seminal work of researchers Richard Noss and Celia Hoyles among others indicates, mathematics actually used in the workplace is contingent and rarely utilizes 'school mathematics' algorithms in their entirety, if at all, or necessarily correctly" (FitzSimons, 2002, p. 147).

The same point was brought out strongly, but contingently, in an investigation of mathematical activity in automobile production work in the USA (Smith, 2002). One result of Smith's study was that "the organizational structure and management of automobile production workplaces directly influenced the level of mathematics expected of production workers" (p. 112). This level varied from a minimal expectation on assembly lines-designed to be "worker-proof"-to "a surprisingly high level of spatial and geometric competence, which outstripped the preparation that most K–12 curricula provide" (p.112), e.g., as manifested by skilled machinists who translated between two- and three-dimensional space with sophistication and accuracy in creating products that were sometimes one-of-a-kind. Organization that used "lean manufacturing principles" (p. 124) to move away from assembly lines and give more autonomy to workers was more likely to enhance and require these highly developed forms of mathematical thinking. Workers are not usually allowed to decide what level of mathematics will be required in their work:

> The fact that the organization of production systems and work practices mediates and in some cases limits workers' mathematical expectations and their access to workplace mathematics is a reminder that the everyday mathematics of work is inseparable from issues of power and authority. In large measure, someone other than the production workers themselves decides when, how often, and how deeply they will be called on to think mathematically. (Smith, 2002, p. 130)

Other sources of insights into the mathematical ideas involved in practices of various workplaces are detailed in FitzSimons's (2002) book. These include the following:

> operators in the light metals industry (Buckingham, 1997); front-desk motel and airline staff (Kanes, 1997a, 1997b); landless peasants in Brazil ...(Knijnik, 1996, 1997, 1998); carpet layers (Masingila, 1993); commercial pilots (Noss, Hoyles, & Pozzi, 2000); ... draughtspersons (Strässer, 1998); semi-skilled operators (Wedege, 1998b, 2000a, 2000b) and swimming pool construction workers (Zevenbergen, 1996). Collectively, these reports highlight not only the breadth and depth of mathematical concepts encountered in the workplace, but underline the complex levels of interactions in the broad range of professional competencies as outlined above, where mathematical knowledge can come into play – when permitted by (or in spite of) management. (p. 72)

In addition to this broad range of reported research, in an early study Mary Harris (1987) investigated "women's work," challenging her readers to "derive a general expression for [knitting] the heel of a sock" (p. 28). The more recent mathematics education conferences have included presentations on the topic of the mathematics of the workplace (e.g., Strässer, 1998).

From the foregoing, the breadth of this field, the variety of practices that have been investigated from a mathematical point of view, and the scope of methodologies employed are apparent. Many of these studies may be characterized as investigations in ethnomathematics, for example, those of Masingila (1993) with the art of carpet laying, and Harris (1987). This overlap in classification also applies to research on mathematics in the world of work that is concerned with its artifacts and tools, and with its technology. The confluence of technology and culture in mathematics education is the topic of the last part of this section.

(2) Influences of Technology

A study that stands at the intersection of workplace mathematics studies and those that are concerned with the role and influence of technology in mathematics education is that by Magajna and Monaghan (2003). In their research they investigated both the mathematical elements and the use of computer aided design (CAD) and manufacture (CAM) in the work of six skilled technicians who designed and produced moulds for glass factories. The technicians specialized in "moulds for containers of intricate shapes" (p. 102), such as bottles in the form of twisted pyramids, stars, or guitars. The study is interesting because the mathematics used by the CAD/CAM technicians in various stages of their work (for instance, calculating the interior volume of a mould) was not elementary, and in theory school-learned mathematics could have been used. However, the researchers reported as follows.

> Although the technicians did not consider their activity was related to school mathematics there is evidence that in making sense of their practice they resorted to (a form of) school mathematics. The role of technology in technicians' mathematical activity was crucial: not only were the mathematical procedures they used shaped by the technology they used but the mathematics was a means to achieve technological results. Further to this the mathematics employed by the technicians must be interpreted within the goal-oriented behaviour of workers who 'live' the imperatives and constraints of the factory's production cycle. (p. 101)

An implication of results such as these, resonating with the conclusions of other workplace studies, is that there is no direct path from school mathematics to the mathematics used, sometimes indirectly, in various occupations, where the specific practices are learned on the job. It may not be possible then to gear a mathematics curriculum, even in vocational education (FitzSimons, 2002), to the specific requirements of a number of vocations simultaneously. However, as technology has developed in the last 6 decades, its influence has been felt in mathematics curricula of various time periods, as illustrated by Kelly (2003).

The research in this growing field is beyond the scope of this paper, but an example highlights the potential of graphing calculators and computers to change the culture of learning mathematics. The research of Ricardo Nemirovsky (2002) and his colleagues (Nemirovsky, Tierney, & Wright, 1998) demonstrates how the use of motion detectors and associated technology changes the culture of learning from one of fostering *formal* generalizations in mathematics (e.g., "all *x* are *y*"), to one of constructing *situated* generalizations. The latter kind of generalization is "embedded in how people relate to and participate in tasks, events, and conversations" (Nemirovsky, 2002, p. 250)—and it is "loaded" with the values of "grasping the circumstances and transforming aspects that appear to be just ordinary or incidental into objects of reflection and significance" (p. 251). Nemirovsky illustrated this kind of generalization in the informal mathematical constructions of Clio, an 11-year-old girl working with the problem of trying to predict, based on the graph created on a computer screen by a motion detector, where a toy train is situated in a tunnel.

In summary, then, technology, by entering all avenues of life, influences not only the mathematics of the curriculum and the ontology of mathematics itself, but also the culture of the future, through its children.

This section has examined in broad detail some of the empirical research relating to the role of culture in teaching and learning mathematics. Aspects that were considered included the linking of out-of-school and in-school mathematics learning, both from the point of view of comparing these different cultural practices and from the point of view of building bridges between cultural practices and the classroom learning of mathematics-of "bringing in the world" (Zaslavsky, 1996). A related strand that was outlined was the culture of mathematics in the workplace, and finally, some aspects of the cultural influence of technology on the learning of mathematics were suggested. Clearly in all of these strands ongoing research is needed and in progress. However, perhaps just as significantly, research is needed that will increase understanding of and highlight how these strands are related amongst themselves. For instance, as the use of technology both in the workplace and in the mathematics classroom changes the culture of learning mathematics in these broad arenas, hints of avenues to be explored in future work appear, for example in whether and how technology has the potential to bridge or decrease the gap between formal and informal mathematical knowledge. The theme of technology and culture in future research is elaborated, along with other significant areas, in the final section.

2.5 Future Directions for Research on Culture in Mathematics Education

The final section uses a wider lens to zoom out and consider areas of research on culture in mathematics education that require development or are likely to assume increasing importance in the years ahead. By its nature, this section is speculative, but trends are already apparent that are likely to continue. The influence of technology is one such trend (Morgan, 1994; Noss, 1994).

(1) Technology

Already in 1988, Noss pointed out the cultural entailments of the computer in mathematics education, as follows: "Making sense of the advent of the computer into the mathematics classroom entails a cultural perspective, not least because of the ways in which children are developing the computer culture by appropriating the technology for their own ends" (p. 251). He elaborated, "The key point is that children see computer screens as 'theirs', as a part of a predominantly adult culture which they can appropriate and use for their own ends" (p. 257). As these children become adults with a facility with technology beyond that of their parents (Margaret Mead's *prefigurative enculturation* comes into play as children *teach* their parents), the culture of mathematics education in all its aspects is likely to change in fundamental ways. Noss (1988) and Noss and Hoyles (1996b) put forward a vision for the role that computer microworlds have played and might play in the future of mathematics education. With accelerating changes in platforms, designs, and software, research must keep pace and inform resulting changes in the cultures of teaching and learning of mathematics in schools. An aspect of potential use for the computer stems from its ability to mimic reality in a world of virtual reality. Noting the complex relationship between formal and informal mathematics (ethnomathematics), Noss (1988) suggested, "I propose that the technology itself—specifically the computer—can be the instrument for bridging the gap between the two" (p. 252). This area remains one in which research is needed, both in its own right and in ways that the change of context of bringing the virtual reality of computer images into mathematics classrooms changes the culture of teaching and learning in those classrooms.

(2) Bridging Mathematics In-School and Out-Of-School

Resonating with the Realistic Mathematics Education viewpoint that *real* contexts are not confined to those concrete situations with which learners are familiar (van den Heuvel-Panhuizen, 2003), Carraher and Schliemann (2002) made a useful distinction between *realism* and *meaningfulness*:

> What makes everyday mathematics powerful is not the concreteness of the objects or the everyday realism of the situations, but the meaning attached to the problems under consideration (Schliemann, 1995). In addition meaningfulness must be distinguished from realism (D. W. Carraher & Schliemann, 1991). It is true that engaging in everyday activities

> such as buying and selling, sharing, or betting may help students establish links between their experience and intuitions already acquired and topics to be learned in school. However, we believe it would be a fundamental mistake that schools attempt to emulate out-of-school institutions. After all, the goals and purposes of schools are not the same as those of other institutions. (p. 137)

The dilemma, then, is *how* to incorporate out-of-school practices in school mathematics classrooms in ways that are meaningful to students and that do not trivialize the mathematical ideas inherent in those practices. This issue remains a significant one for mathematics education research.

Reported research (Civil, 2002; Masingila, 2002; Presmeg, 2002b) has shown that learners' beliefs about the nature of mathematics affect what mathematics they identify *as* mathematics in out-of-school settings. Students' and teachers' conceptions of mathematics as decontextualized and abstract may limit what can be accomplished in terms of meaningfulness derived from out-of-school practices (Presmeg, 2002b). And yet, abstract thinking is not necessarily antagonistic to the idea of reasoning in particular contexts, as Carraher and Schliemann (2002) pointed out, which led them to propose the construct of *situated generalization*. They summarized these issues as follows.

> Research sorely needs to find theoretical room for contexts that are not reducible to physical settings or social structures to which the student is passively subjected. Contexts can be imagined, alluded to, insinuated, explicitly created on the fly, or carefully constructed over long periods of time by teachers and students. Much of the work in developing flexible mathematical knowledge depends on our ability to recontextualize problems – to see them from diverse and fresh points of view and to draw upon our former experience, including formal mathematical learning. Mathematization is not to be opposed to contextualization, since it always involves thinking in contexts. Even the apparently context-free activity of applying syntax transformation rules to algebraic expressions can involve meaningful contexts, particularly for experienced mathematicians. (p. 147)

They mentioned the irony that the mechanical following of algorithms characterizes the approaches of both highly unsuccessful and highly successful mathematical thinkers.

Taking into account both the need for meaningful contexts in the learning of mathematics and the necessity of developing mathematical ideas in the direction of abstraction and generalization (in the flexible sense, not to be confused with decontextualization), at least two extant fields of research have the potential to address these issues in significant ways. The notions of *horizontal* and *vertical* mathematizing that have informed Realistic Mathematics Education research for several decades (Treffers, 1993) could resolve a seeming conflict between abstraction and context. In harmony with these ideas, recent attempts to use semiotic theories in linking out-of-school and in-school mathematics also have the potential for further development (Hall, 2000; Presmeg, 2006).

As Moschkovich (1995) pointed out, a tension exists between educators' attempts to engage learners in "real world" mathematics in classrooms and movements to make mathematics classrooms reflect the practices of mathematicians. Multicultural mathematics materials for use in classrooms have been available for some time (Krause, 1983; Zaslavsky, 1991, 1996). However, the tensions, the contradictions, and the complexity of trying to incorporate practices for which "making change" serves as a metonymy at the same time that students are "making mathematics" (Moschkovich, 1995) will engage researchers in mathematics education for some time to come. Bibliographies such as that compiled by Wilson and Mosquera (1991) will continue to be necessary, to inform both researchers and practitioners what has already been accomplished as the field of culture in mathematics education continues to grow.

(3) Teacher Education

In all of the foregoing areas of potential cultural research in mathematics education, the role of the teacher remains important. Noting that concrete and abstract domains in mathematical thinking are not necessarily disparate, Noss (1988) suggested,

> The key idea is that of focusing attention on the important relationships involved, a role in which—as Weir (1987) points out—the computer is rather well cast; but not without the conscious intervention of educators, and the careful development of an ambient learning culture. (p. 263)

Bishop (1988a, 1998b) was also intensely aware of the role of the *mathematical enculturators* in personifying the values that are inherent in the teaching and learning of mathematics. In the final chapter of his seminal book (1988a) he suggested requirements for the education-rather than the more restricted notion of "training"-of those who will be mathematics teachers, at both the elementary and secondary levels. He did not distinguish between these levels, for teachers at both elementary school and secondary school have important mathematical enculturation roles. Bishop (1988a) summarized the necessary criteria as follows.

> I propose, then, these four criteria for the selection of suitable Mathematical enculturators:
>
> - ability to 'personify' the mathematical culture
> - commitment to the Mathematics enculturation process
> - ability to communicate Mathematical ideas and values
> - acceptance of accountability to the Mathematical cultural group. (p. 168)

These ideas still seem timely; in fact the literature on discourse and communication has broadened in the decades since these words were written, to suggest that communication amongst all involved in the negotiation of the cultures of mathematics classrooms (teacher and learners) plays a significant role in the learning of mathematics in those arenas (Cobb et al., 1997; Dörfler, 2000; Sfard, 2000). *Language and Communication in Mathematics Education* was the title of a Topic Study Group at the 10th International Congress on Mathematical Education (Copenhagen, 2004), and the literature in this field is already extensive. But *how* to educate future teachers of mathematics to satisfy Bishop's four criteria is still an open field of research.

As Arcavi (2002) acknowledged, much has already been accomplished in curriculum development, research on teachers' beliefs and practices, and "the development of a classroom culture that functions in ways inspired by everyday practices of academic mathematics" (p. 27). However, open questions still exist concerning ways of using the recognition that the transition from out-of-school mathematical practices to those within school is sometimes not straightforward, in order to inform the practices of mathematics teaching. Arcavi (2002) gave examples of such questions.

> However, much remains to be researched. For example, is it always possible to smooth the transition between familiar and everyday contexts, in which students use ad hoc strategies to solve problems, and academic contexts in which more general, formal, and decontextualized mathematics is to be learned? Are there breaking points? If so, what is their nature? Studies in everyday mathematics and in ethnomathematics are very important contributions, not only because of their inherent value but also because of the reflection they provoke in the mathematics education community at large. There is much to be gained from those contributions. (p. 28)

Clearly ethnomathematics conceived as a research program (D'Ambrosio, 2000) has already permeated the cultures of mathematics education research and practice in various ways.

The influence, and the need for research that addresses the complexities of the issues involved, are ongoing.

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3. Teaching for numeracy
: Re-examining the role of cultural aspects of mathematics

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3.1 Numeracy in the school mathematics curriculum

Developments and advancement – scientific / technological or otherwise – in the world we live in have meant that over the years, we acquire or adapt new knowledge, new skills, new ways of doing the same tasks, new outlooks, new attitudes, and new perspectives. Likewise, the purpose of formal school education in general, and also institutional aims for the teaching and learning of mathematics in schools in particular, have evolved in new forms in response to new sociocultural demands. For school mathematics, one of the significant moves in the early 2000s in countries such as Australia, New Zealand, Sweden and United Kingdom is towards the teaching of mathematics for numeracy. This move acknowledges that general education for all implies that not all school students need to be trained to be mathematicians. Rather, general education for the masses has provided nations with the opportunity to educate their respective citizens so that they acquire the capacity to cope with the evolving changes in related demands in daily life. This is generally observed in most countries, although we are still reminded today that what topics constitute the study of mathematics in schools remain to be socio-politically regulated by different government systems. For example, the study of statistics had been removed from the secondary mathematics syllabi of schools in some countries.

It is in this climate that numeracy assumes a significant role in countries such as Australia, New Zealand, Sweden and United Kingdom. This term is also expressed differently in different cultures, such as 'numerical literacy', 'functional mathematics', and, in the United States, 'quantitative literacy'. In Australia, the notion of numeracy in schools was thrown in the spotlight in the 1999 *Adelaide Declaration on National Goals for Schooling in the Twenty-First Century*, in which Goal 2.2 states that students should have "attained the skills of numeracy and English literacy; such that every student should be numerate, able to read, write, spell and communicate at an appropriate level" with mathematics as one of eight key learning areas.

Numerous definitions of numeracy have been made, reflecting different approaches (Perso, 2006), and presenting "an enormous set of conceptual and pedagogical issues" (Coco, Kostogriz, Goos, & Jolly, 2006). In this chapter, the approach which appears to have been embraced by the intended curricula in many education systems (such as those in the UK and the different states within Australia) will be the reference for discussion. In this 'mathematical literacy' approach (Perso, 2006), the description made by the Australian Association of Mathematics Teachers (1997) is apt, in which "to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (p. 15). Likewise, Willis (1998) associates numeracy skills with the capability of one to conduct 'intelligent practical mathematical action in context'.

3.2 Numeracy in Victorian primary schools

In the state of Victoria, Australia, much of the primary school mathematics curricular activities has been influenced by the findings and recommendations of two state-level research projects in the early 2000s, namely, the Early Numeracy Research Project (Victoria Department of Education Employment and Training et al, 2002) and the Middle Years Numeracy Research Project (Victoria Department of Education Employment and Training et al, 2001). For example, students entering the first formal school year (i.e. Grade Prep) will have their individual numeracy standards assessed (and subsequently reported) through the School Entry Assessment, which takes the form of individualised interview sessions. While the school subject is formally called 'mathematics', the introduction of initiatives such as 'numeracy blocks' in lesson planning has meant that many primary school teachers in Victoria use the terms 'mathematics' and 'numeracy' interchangeably, a phenomenon which is also frequently observed in the UK. This is despite the fact that at the intended curriculum level, there is only one 'unforced' use of the term 'numeracy' in the chapter of the Victorian Essential Learning Standards for Mathematics (VCAA, 2005), which spells out the context of the discipline in the school system before the outcomes for each learning level are specified in subsequent chapters. Interestingly, that one inclusion may be interpreted to make an explicit distinction between mathematics and numeracy, where one aim of the curriculum is for students to "demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society" (p. 4). The other occurrences of the term 'numeracy' are in the last section (p. 9), which describes how student achievement is reported against the National Numeracy Benchmarks.

While numeracy in the early years of schooling is mostly (but not exclusively) concerned with equipping students with the necessary understandings (such as number sense and data sense) to facilitate more efficient learning in the middle years and beyond (such as how basic number sense aids in the development of algebraic thinking as described in Willis, 2000), the focus of this chapter is more on the numeracy lessons planned for the middle years of schooling (typically, the last two years of primary school education and the first two years of secondary school education) and beyond, where mathematical tasks in context allow students to apply what Willis (1998) calls the 'intelligent practical mathematical actions'. In this context, numeracy appears to be promoted in Victorian primary school classrooms in one of two ways. In the first way, mathematical concepts and/or processes are introduced to students, before opportunities for student applications and/or investigations are provided. In the second way, numeracy is delivered interdisciplinarily and contextualised in sessions where students work on investigations / projects to solve identified or pre-specified problems, often in groups. The necessary mathematical skills are acquired by students in context, and teachers are expected to direct student learnings of relevant, emerging mathematical concepts.

3.3 Does numeracy mirror daily life?

Both these teaching approaches are likely to not foster students' affective development in mathematics / numeracy, however. Given the reality of school lessons and the accompanying constraints, there will always be a limit to the authenticity and realism of mathematical problems in numeracy lessons. This may be especially so in the first of the teaching approaches, where the initial classroom learning of mathematical concepts and processes may stimulate students to ask for the reasons why they need to be learnt, or when they will ever be used in life. However, this does not mean that students experiencing the second, more holistic education approach would feel that what they learn in numeracy lessons are relevant to numeracy demands in daily life. The validity of this point is supported by observations that quite often, authentic numeracy activities are not really authentic, though it must be reminded at the same time that it is not attributed to teachers alone.

A few examples may be illustrative. In Victoria (as in most other educational systems in the world), rules for rounding decimal numbers ending with the numeral 5 are not the same as those used in similar operations in the Australian optical fibre industry (Fielding, 2003). Similarly, Victorian primary school students may learn to execute rounding to the nearest five cents in schools, yet children know that the cash registers of different retail outlets conduct the same operation for cash transactions in different ways (i.e. some round off, some round down, and others round up to the nearest five cents). The use of modals (such as 'might' and 'should') in numeracy questions might also stimulate some children to think that the price for several items of a particular commercial product needs not be proportional to their unit price (as is the case in real life), a situation which is likely to be assessed as their teachers to represent wrong computations!

This gap between what constitute school mathematics and daily/workplace mathematics remains not only because of the nature of schooling with its constraints and conditions, but also because very often the invisibility of mathematics in daily/workplace tasks has been 'black-boxed' paradoxically by the discipline itself. Latour (1999) refers 'black-boxing' to

> the way scientific and technical work is made invisible by its own success. When a machine runs efficiently, when a matter of fact is settled, one need focus only on its inputs and outputs and not on its internal complexity. (p. 304)

3.4 Emphasising the understanding of others' mathematics

Thus, introduction of numeracy in mathematics classrooms (at both the primary and secondary school levels) needs to be complemented by an explicit acknowledgement by schools and teachers that the very nature of lesson timetabling and also often of the intended curriculum mean that school mathematics will neither be academic mathematics, nor will it be daily/workplace mathematics. In education systems such as those found in Australia where numeracy is the aim for mathematics teaching in schools, teacher explicitness in highlighting that not all real-life situations can be reasonably modelled within the constraints of lesson planning is important in sustaining student interest and faith in the subject, and in emphasising instead the value in one's capacity to apply mathematical knowledge in daily life or in the workplace.

How might this capacity be developed? Williams and Wake (2007) advocate that

> to make sense of workplace mathematics, outsiders need to develop flexible attitudes to the way mathematics looks, to the way it is 'black-boxed' .... Students had little awareness that the College mathematics they had learnt was itself idiosyncratically 'conventional', and that mathematical conventions might vary in time and place. (p. 338)

The intended curriculum can play a crucial contributing role here while still promoting the satisfaction of numeracy outcomes. In fact, it potentially adds depth to what it means to be functionally numerate in the society when students acquire the skill of "making sense of the mathematics of others" (Williams & Wake, 2007, p. 339). These 'others' would refer to members of a diversity of cultures – national, ethnic, workplace, religious, gender, for examples.

In this regard, Bishop's (1996) conception that cultures universally engage in six types of activities from which mathematical ideas evolve is a particularly useful one. At one point or another in time and space, all cultures find the need to count, locate, measure, explain, design, or play, and, Bishop (1996) argued, all cultures perform all these six mathematical activities. From the pedagogical point of view, this is more than enabling students (and teachers) to appreciate that different cultures perform mathematics tasks differently, that, for example, the Anglo-Australian parents emphasise the skill of counting in their young, whilst their Aboriginal-Australian counterparts traditionally emphasise the importance of their children to perceive their respective locations within the three-dimensional space. Without disputing that this in itself has rich mathematics pedagogical implications, the focus here is on the potential for students to understand mathematical situations.

Thus, making sense of how cashiers often provide the change for a cash transaction through 'adding up' rather than subtracting has the potential of enabling students to go beyond making sense and understanding. It also provides an opportunity for students to relate this change-giving method to the 'textbook' method (of using the decimal subtraction algorithm), to relate and understand more of the operations of addition and subtraction, to evaluate relative strengths and constraints, to explain how formal, school and workplace mathematics support one another.

Clearly, there is also the bonus advantage of fostering students' higher-order cognitive outcomes.

If a numeracy program seeks to expose students to real-life mathematics in the daily life or workplace, then one which identifies and discusses how mathematical processes are executed by 'others' can bring the mathematics out of the 'black-box', thereby making the underlying concepts and ideas more visible. It also acknowledges the limitation of the school mathematics framework in possibly discussing and examining selected mathematical practices of selected cultures, without appearing to advocate that the numeracy developed through school mathematics is sufficient in equipping the individual student to apply related concepts and skills to the world beyond the school gates. In fact, by highlighting the cultural aspects of mathematical practices in numeracy classes, student attainment and mastery of numeracy is situated within a cultural community of practice, which reflects the very essence of numeracy, that is, for effective sense-making and negotiation of demands arising from family, workplace, community and civis lives.

A numeracy approach which acknowledges the inevitable gaps amongst academic, school and daily/workplace mathematics can highlight to students that if mathematics is to be regarded as a form of language for communication, then different genres of this language evolve from the different contexts within which this language is used. Thus, school mathematics is one such genre, and the extent to which policy makers desire to inculcate numeracy in the young generation through school mathematics will be reflected in its closeness with the genres of daily/workplace mathematics. Most importantly, such an acknowledgement can address student sceptism abut the value of school mathematics, and the explicitness can foster student appreciation of the genre of school mathematics.

Students' cognitive 'movements' within and across the different genres of mathematics is also educationally valuable in that a meaningful appreciation and understanding of how 'others' do mathematics can promote positive development of their individual cultural intelligence [CQ]. First conceptualised by Earley and Mosakowski (2004), CQ extends the notion of emotional intelligence and is a measure of the extent to which an individual who is

> an outsider [to a particular culture] has a seemingly natural ability to interpret someone's unfamiliar and ambiguous gestures in just the way that person's compatriots and colleagues would, even to mirror them. (p. 139)

A numeracy curriculum which provides students opportunities to relate mathematical concepts and ideas to its various genres in different occupational, ethnic, gender or other categories of cultures is an exercise in interpreting these other cultures' unfamiliar ways of doing mathematics. This hidden curriculum of cultivating students' individual cultural intelligence extends beyond mathematics/numeracy learning, to the learning of other educationally desirable skills of social learning, such as interpersonal skills.

3.5 Closing remarks

The discussion above re-examines the role of the cultural aspects of mathematics in the teaching of numeracy in schools. It is expected to be relevant to educational systems in both developed nations and developing ones that seek to benefit from being part of the knowledge-production economies. Participation in these economies (as compared to agricultural, manufacturing, service and information economies) require more than ever before a workforce whose members are functionally, if not critically, literate and numerate. The argument made here is that achieving meaningful and sustained numeracy rates amongst a country's students may be supported by intended and implemented curricula which acknowledge the differences amongst academic, school, daily-life and workplace mathematical practices. By promoting student understanding of culturally-different ways of doing mathematics, the black-boxing of mathematical concepts and ideas in daily life and in the workplace can be reduced. This should optimise student progression through the school years with an increasing understanding and confidence in relating academic and school mathematics to the demands of tasks they encounter or will encounter in the respective personal and working lives they lead, with the additional potential of cultivating their individual cultural intelligence in the process.

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4. Minority Students and Teacher's Support
: Reviewing International Assessment Results and Cultural Approach

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4.1 Introduction

One of the most influential international comparative studies on education is the Programme for International Student Assessment (PISA) for recent years. The program has been started as a response to the need for cross-nationally comparable evidence on student performance by the Organisation for Economic Co-operation and Development (OECD) in 1997, and the first assessment was conducted in 2000. The results of PISA revealed wide differences in the extent to which countries succeed in enabling young adults to access, manage, integrate, evaluate and reflect on written information in order to develop their potential. At the same time, the results showed the common difficulties across countries and unique issues within them. Some countries recognize the significant variation of the performance level among their children and concerns about equity in the distribution of learning opportunities, although they have the equivalent of several years of schooling and sometimes despite high investments in education as the other better performed countries.

Some European countries found reasons why the disparity exists between student's high and low performances. Immigrant students have a lot of different backgrounds from natives as both personal concerns like a language and social issues such as economic conditions. The results of the international assessments introduce the clear difference, as quantitative data, and the governments have started to focus on students' socio-economic backgrounds for better education system. And meanwhile, teachers in a host country for the immigrant students are natives in many countries.

Our IDEC research project started to describe phenomena of mathematics education and classroom interaction in the wide range of countries. We are prone to imagine that instructions in classroom cause lower performance of the students in the developing countries because of its poor pedagogical approach such as chalk and talk. The results of the first and second year survey can be interpreted as a finding that the students who do not use a test language as their first language achieve lower academic performance. The author believes the language problem is an explicit aspect of cultures which students generally have in their classrooms because by and large they are always in multi-cultural environment and the interaction between a teacher and students must be complicated like those in European classrooms. Minority students need more supports especially from teachers because the teacher's role in classroom is rather a controller in the project countries and their outer views are limitedly sensitive to the backgrounds of the minority students.

In this review, the author summarizes the PISA results of immigrant students in mainly Europe with literature and analyzes the backgrounds of the immigrants from the cultural point of view. The author also tries to discuss the context of the classroom in developing countries in which local teachers might miss the cultural backgrounds of the minority students.

4.2 Findings from the Assessments Concerning the Immigrant Students

Many types of education assessments are available when we try to analyze student's performance today. Here we look at PISA and its related research because it is one of the most influential to education in the world today. Following the brief description of the world wide assessment, let us see the performance of immigrants and their language problem.

4.2.1 A Large-Scale International Assessment

PISA seeks to measure how the 15-year-old children, as the age of ending compulsory schooling in many countries, are prepared to use their knowledge and skills to meet real life challenges of today's knowledge society, rather than merely on the extent to which they have mastered a specific school curriculum. PISA is the most comprehensive and rigorous international program today for the participating countries/regions to assess their student performance and to collect data on student, family, and institutional factors that can help to explain differences in performance.

PISA has three cycles of its implementation procedure and three domains of student's knowledge and skills called "literacy,"(1) namely, reading, mathematical, and scientific ones. Each cycle covers all the domains but its focus is mainly on one of the three domains. The first PISA assessment was conducted, focusing on reading literacy, in 2000 and on mathematical literacy in 2003. More than a quarter of a million students and their school managers participated in the each. There will be the first final cycle for about 60 countries, including 30 OECD countries, on the scientific literacy in 2006. Besides the literacy, the participants also responded their family and economic backgrounds, their motivation to learn, their beliefs about themselves, and their learning strategies.

4.2.2 Lower Performance of Immigrant(2) Students in the Assessments

The results of the series of PISA provide us with the data the students whose parents are not native show lower performance than native students in many countries. The greatest gap, of 93 points in mathematics scores, is in Germany, and students themselves born outside the country tend to lag even further behind, in Belgium by 109 points.(3) While circumstances of different immigrant groups vary greatly, and some are disadvantaged by linguistic or socio-economic background as well as their migrant status itself, two particular findings are worrying for some countries. One is the relatively poor performance even among students who have grown up in the country and gone to school there. The other is that after controlling for the socio-economic background and language spoken at home, a substantial performance gap between immigrant students and others remains in many countries: it is above half a proficiency level in Belgium, Germany, the Netherlands, Sweden and Switzerland.(4)

Schnepf (2004) used the results of ten countries(5) which had participated in IEA's TIMSS and PIRLS in addition to PISA because all the three assessments collected information in the same format regarding immigration variables of their parents origin and language used at home.(6) She explained the immigrants achieved significantly lower test scores than natives in almost all countries and surveys. She concludes the promoting language education of immigrant students and decreasing school segregation are likely to have a positive outcome on students' performance, based on the three findings from her statistical analysis: 1) the immigrants' socio-economic background might be lower than natives and be immigrants' educational disadvantages in some countries; 2) immigrants' educational disadvantage might derive from their problems of integration into the host country; and 3) the process of selection/acceptation of immigrants to the host country is likely to impact upon their achievement results.

Ammermüller (2005) used the data from PISA extension study(7) in Germany and tried to find determinants that the results of the immigrant students were lower than native students. He found that the use of different language at home from German in classroom is typical characteristics for the immigrant students, in addition to the higher grade level of German students and more home resources as measured by the amount of books at home. These factors explained the test score gap would decrease by 40 to 49 %, if the immigrant students had had the same returns to student background as native students. However, he also found that differences in parental education and family situation were less influential.

Rangvid (2005) made an analysis of the immigrant students' performance in Denmark by using the results of PISA-Copenhagen which is based on international PISA study design. The PISA-Copenhagen results suggest similar performance between the native students and the students who have one immigrant parent, but the mean scores of the immigrant students who have both non-native parents are much lower. She also pointed out the immigrant students experienced lower teacher expectations and the peer composition at schools attended by immigrant students was potentially less conducive to academic achievement.

The results of the international assessments show that immigrant students achieve lower level of knowledge and skills than natives in many European countries. We can assume that the language proficiency of the immigrant students is one of the main reasons why their performance is lower than natives.

4.2.3 Teaching Limited English Proficient Students

Secada and Carey (1990) argue school practices might forge the links that the level of English language proficiency affects mathematics achievement. They believe that children enter school with a broad range of understandings about mathematical concepts. Instruction should build upon and develop the children's knowledge so that it would provide children's learning in mathematics and contribute to their confidence about their abilities. They describe one effort to accomplish this approach called Cognitively Guided Instruction (CGI).(8)

Although the CGI assessment focuses on the processes by which students get an answer and seems to be quite learner-centered approach to some countries in the IDEC project, the unique procedure relates with intercultural communication. To start assessment of student's thinking, a teacher should provide students with counters, pose a problem, and if the student responds, follow up by asking how he/she figured it out. The teacher then uses additional questions to help him/her clarify what it meant. Alternatively, if a student does not solve the problem correctly, the teacher has at least five options.(9) The point is to begin a conversation about the problem. Many limited English proficient students receive little encouragement to speak about their ideas and many girls are socialized to defer to boys. If teachers tend to call on students who answer first, the rest will often be left out of the conversation. Therefore, teachers need to reach out to these children and to be sure that they are included in the classroom's processes.

They recommend that students should be able to discuss how they are thinking about problems during mathematics. As a result, students learn mathematics from one another as well as from the teacher. This validates their thinking and students begin to recognize that both teacher and students are a source of knowledge in the classroom. They might share the backgrounds more.

4.3 Education for the Cultural Minority
: *integration as policy and personal level*

We have seen the lower performance of the immigrant students. In addition to the language issue, minority student's cultural backgrounds call for our attention to more influence in classroom interaction and their academic performance than language. European countries such as Germany, Denmark, and the Netherlands have received immigrants in past decades have emphasized the efforts for integration policy of the immigrants into the native societies as a solution. But the policymakers found the results of the assessments show the integration policy still faces challenges. At school level, many teachers understand immigrant students' poor achievement stems from their language problem and different cultural backgrounds.

Immigrants also have difficulties, however sometimes impossibility, to cope with different environments. Immigrant students live with host culture at school and original culture at home. Having focused on language, when their language ability is not enough to understand instruction in class, they not only feel less confident in learning lessons and participating classroom activities but also hesitate to ask what they do not understand and develop little concept of mathematics at last.

This section deals with cultural experience of the immigrant students, their cultural learning process, and possible supports by teachers. Descriptions can apply to some examples of IDEC project because this paper takes the problems of the immigrant students in mostly European countries as similar phenomena to the "cultural minority" groups.(10) Before proceeding, we should also remind ourselves of the risk of overgeneralization.

4.3.1 Cultural Experience of Immigrants

Immigrant students live in two or more different cultures(11) and are also in co-culture groups.(12) Parents' views and values affect students, especially in childhood prior to schooling so strongly that children have bias and feel uncomfortable when they first face different views from other students in school. For example, they experience the different manners of expression in low-context(13) culture in Europe and feel the large gap between their own in-group and other out-group cultures.(14) And meanwhile, the native students of dominant culture, or majority students, also feel difference of paralanguage such as gestures and facial expression when interacting with "strangers." As mental defense mechanism that they try to avoid the unfamiliarity and reduce uncertainty, they tend to stay with those who have similar backgrounds for their comfort through the interaction among students.

This grouping process is different from the psychological stage as gang age but rather based on mental state called ethnocentrism. Ethnocentrism is a universal tendency of the belief that one's own group or culture is superior to other groups or cultures. People generally interpret other cultures according to the values of their own. When the students consciously separate in- and out-groups, they attribute the other group's mistake, for instance, to the characteristics of the whole group but own group's to personal. A German female student sees a Turkish student wearing headscarf in physical education class, she tend to thinks all the Turkish girls wear it. But she does not think that all the German girls go to Sunday school because it depends on family's attitude to daughter. This is not a serious problem very much until students start to develop prejudice from the bias after cultural encounter because they are barrier to understanding others. The students need a mentor to follow up their observation and understanding the difference properly.

Both minority and majority groups generally have bias and possibilities of conflicts between the two. The host society is basically more affirmative and supportive to the majority, and the minority groups have to follow many of its social norms. The society enforces the minority integration or assimilation to be the citizen so that a strong reaction such as marginalization sometimes comes up from the minority group. It is understandable that a group of immigrant students seems culturally homogeneous to the majority students and native teachers because their appearance and attitude resemble each other. And their communication can be categorized as intra-cultural communication if we compare communication between explicitly different cultures. The large scale survey and analysis also categorize they are the same group; otherwise there are too many cultural-specific cases. When we easily categorize and judge them based on ethnocentric view, prejudice overcasts our eyes.

Here, we need to shift our views. What we think as intra-cultural communication can be actually intercultural when we see their experiences by *emic* approach. This view is originally from the standpoints of Pike (1967) who has generalized from phonetics and phonemics to what he has called the *"etics"* and *"emics"* of all socially meaningful human behavior. Whatever the names one may choose to call them, the concepts of *etic* and *emic* are indispensable for understanding the problems of description and comparison. *Etic* is pertaining to a concept that is culture-general and is therefore easy to describe when we examine cultures from outside. *Emic* is pertaining to a concept that is culture-specific and is therefore difficult to translate from one language to another when we examine a culture from the inside. In short, *etic* is external and *emic* is internal view.(15)

It might be much easier for us to perceive the various backgrounds of immigrant students or cultural minority if we try to observe from the inside.(16) Religious consciousness may be one of the biggest and typical issues which majority/minority students and teachers concern in classroom. Parker-Jenkins (1995) illustrates an example:

> "Muslim children are taught to respect and not question elders or those in authority and accordingly they may appear to be stereotypically passive and accepting. If a child does not question teachers it does not automatically denote lack of interest or intelligence but rather respect for their authority and position. Furthermore, the value placed on deference to authority, and *haya* or modesty within the Islamic consciousness, helps to explain the difficulties and contradictions some Muslim children may experience in participating in [British] state school activities which call for assertiveness and extrovert behaviour."(17)

4.3.2 Acculturation: Integration or Assimilation of the Immigrants

The immigrant students sometimes have no choice but follow the dominant group's norms. The students learn appropriate behaviors by living in a certain cultural context. This is called acculturation which could contain four types of strategies.

Herskovits and other American anthropologists (1936) proposed the following definition of acculturation. "Acculturation comprehends those phenomena which result when groups of individuals having different cultures come into continuous first-hand contact, with subsequent changes in the original cultural patterns of either or both groups."(18) Acculturation differs from cultural change in which the source of change is internal within the culture and either from enculturation.(19)

Berry (1992) categorizes acculturation strategies in which individual or group wishes to relate to the dominant society into four patterns: integration, assimilation, separation, and marginalization. They are conceptually the result of an interaction between ideas deriving from the culture change and the intergroup relations. In the former the central issue is the degree to which one wishes to remain culturally as one has been as opposed to giving it all up to become part of a larger society; in the latter it is the extent to which one wishes to have day-to-day interactions with members of other groups in the larger society as opposed to turning away from other groups and relating only to those of one's own group.

> When these two central issues are posed simultaneously, a conceptual framework [Fig. 1] is generated that posits four varieties of acculturation. When an acculturating individual does not wish to maintain culture and identify and seeks daily interaction with the dominant society, then the assimilation path or strategy is defined. In contrast, when there is a value placed on holding onto one's original culture and a wish to avoid interaction with others, then the separation alternative is defined. When there is an interest in both maintaining one's original culture and in daily interactions with others, integration is the option; here there is some degree of cultural integrity maintained, while moving to participate as an integral part of the larger social network. Integration is the strategy that attempts to "make the best of both worlds." Finally, when there is little possibility or interest in cultural maintenance, and then marginalization is defined.(20)

Although the officials try to promote integration of immigrants into the society, the immigrants sense its uncertainty and meanwhile the host local people are not interested in the promotion very much. Parker-Jenkins (1995) draws multiculturalism in Britain and details a phase of assimilation took place along with great restriction of immigration to the country in 1950s.

The author also interviewed German scholars about immigrants' integration in Germany and recognized some cases could be categorized as assimilation which possibly leads marginalization.(21) The immigrants or minority group of people follow the norms of the majority society with mental conflict.

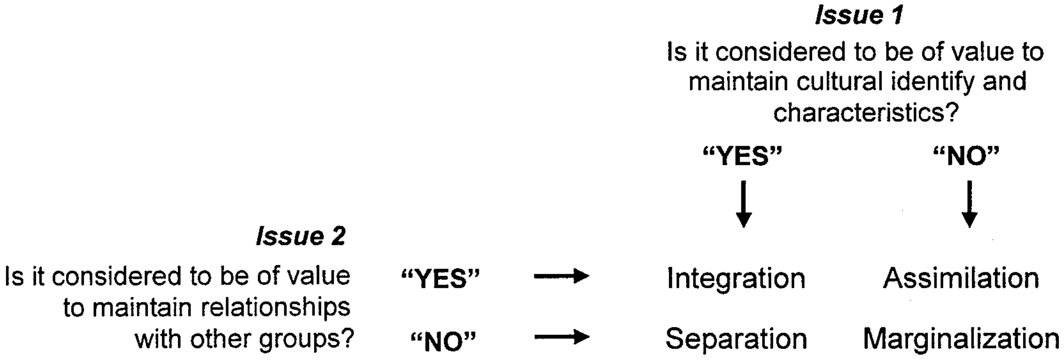

Fig. 1 Four varieties of acculturation (Berry, 1992)

4.3.3 Support from Teacher in Classroom

The minority students need supports from teachers and other students. Friends are the strongest supporter in and out of classroom but if no one wants to be a friend, then the immigrant students have only teachers to ask for help. Teachers become more important being today not only for the academic achievement of minority students but also for their coping strategy with the dominant culture. OECD recently operated a project of policy research on teachers in its member countries,(22) and its report introduces the importance of teachers:

> The demands on schools and teachers are becoming more complex. Society now expects schools to deal effectively with different languages and student backgrounds, to be sensitive to culture and gender issues, to promote tolerance and social cohesion, to respond effectively to disadvantaged students and students with learning or behavioural problems, to use new technologies, and to keep pace with rapidly developing fields of knowledge and approaches to student assessment. Teachers need to be capable of preparing students for a society and an economy in which they will be expected to be self-directed learners, able and motivated to keep learning over a lifetime.(23)

Teachers are not allowed to give lessons only for subjects anymore but need to keep learning in today's knowledge-based society. Communication is clearly important in classroom among students, and teachers actively receive feedbacks from them. In addition to receiving a message from them, teachers may have to imagine students' thoughts based on understanding on their backgrounds. It requires more efforts than before so that teachers might lose what to do first.

To imagine the student's needs, the first and explicit aspect of their backgrounds is their language. Their language proficiency and teacher's understanding of their language at home help both students and the teacher. The students' explanations may be tentative not only because of content mastery but also because of the language used in classroom.(24) Moreover, even when using their native languages, many students have difficulty in expressing their thoughts in as sophisticated a manner as teachers might like. Teachers must recognize students judge people who sound like they know what they are talking about have knowledge, while those who express themselves poorly do not have such knowledge. Teachers need to begin with what students understand and with how they can express their understandings. Therefore, mathematics teachers are required the knowledge of both subject and cultures today.(25)

To know more about cultures, Hofstede's framework can be helpful. Geert Hofstede (1984) categorizes four dimensions of values by surveying across 50 countries(26): 1) power distance, 2) uncertainty avoidance, 3) individualism and collectivism, and 4) masculinity and femininity. Power distance is a value regarding the degree to which a society accepts the fact that power in institutions is distributed unequally. If it is small, equality is highly sensed. Uncertainty avoidance is the attempt to understand the other person in a communication setting by increasing predictability; brought about by the high need to understand oneself and others in interpersonal situations. If a society has strong uncertainty reduction, it is less tolerance to deviant behaviors of others. Individualism gives more importance to individual's will than to group, and collectivism is vice versa. Masculinity refers to emphasis on heroism, argument, achievement and etc., while femininity prioritizes compassion to the weak, human relation, quality of life and etc. Hofstede compares working staff at country level, but teachers in classroom can identify themselves with the framework and locate the background of student's behaviors within culture and between co-cultures. This shift of viewpoint can be *emic* approach from the teacher's *etic* views.

When teaching side shifts its view to the inside, teachers would be more able to understand minority student's backgrounds besides language. As an example of Muslim students in Britain, Parker-Jenkins (1995) categorizes student needs into four:(27)

1) As religious/cultural needs: Time of school assemblies should be coordinated for daily prayers. School diet should be considered and understand the need of fasting. Dress code should be flexible, especially in physical education for girls, etc.
2) As curriculum needs: Sex education should be paid more attention, especially need for HIV issue. Language instruction for minor languages should be encouraged. Islamic dimensions should be promoted, etc.
3) As linguistic needs: Support for English language competence/acquisition, etc.
4) As general needs: Home-school links should be emphasized because enlisting parental support is vital to all attempts to accommodate Muslim needs within the school system. Muslim organizations are encouraged to become more proactive in attempting to bring about change within the educational system for school governance.

She conducted the survey on the above needs and practices of British schools and found the similarities and differences between the perceptions of head teachers in state schools and private Muslim schools over Muslim children's needs. The similarities were in the issue of religion and religious observance, and the differences were the "conceptualization of religion" in the children's lives, English language acquisition, effective home-school links, and balanced curriculum. She points out that English language acquisition is considered the major area of concern by state school administrators because it is the medium of instruction in British schools but opportunity for various groups to learn their own language need to be made for its cultural importance and the matter of parity of languages. She also pointed that curriculum as a "transmission of the culture" transmits minority cultures and the right balance between the indigenous and minority cultures is an area of concern. She suggested understanding the processes of assimilation and acculturation within multicultural education is pertinent through learning from each other and emphasized that teachers cannot be expected to translate educational theory into practice without adequate training and the selection and recruitment of teachers from minority backgrounds are needed.

4.4 Conclusion: Cultural Sensitivities of Teachers

In this review, we have seen the immigrants' low performance shown by the international assessments and cultural backgrounds, their cultural experiences and personal coping strategies, and teacher's room to learn to shift the view to the insider. We may summarize them in the following three.

4.4.1 Importance of teacher's role

The largest source of variation in student learning is attributable to differences of students' personal and cultural backgrounds such as their individual abilities, attitudes, family, and community. These factors are difficult for the outsiders to influence. On the other hand, factors concerning teachers and teaching are the most important influences on student learning. Teacher's education and experience influence on student achievement. But as OECD reports "a point of agreement among the various studies is that there are many important aspects of teacher quality that are not captured by the commonly used indicators such as qualifications, experience and tests of academic ability,"(28) we can focus more on teacher's potential to recognize the student backgrounds for their learning. In the IDEC project, we shall also remember more cultural-specific aspects, for example, the traditional African education(29) is based on oral communication as Reagan (2005) examines. The traditional education and training have its own contexts, and local teachers must be tolerance and sensitive to them. This is because "formal schooling played important roles in many non-Western traditions as one of fundamental distinctions."(30)

4.4.2 Cultural sensitivity

OECD (2005) introduces the five broad areas for the development of professional knowledge and expertise in teaching, as Shulman (1992) identified(31): teacher's behavior, cognition, content, character, and knowledge of and sensitivity to cultural, social, and political contexts and environments of the students. The fifth area treats teacher's sensitivity to cultural backgrounds of the minority students and it is what this paper finds the importance. Not only OECD but all teachers who have minority students already know classrooms are becoming increasingly diverse with students of different cultural and religious backgrounds. "Teachers are expected to work for social cohesion and integration by using appropriate classroom management techniques and applying cultural knowledge about different groups of students."(32)

Teachers are symbolized powerful authority in some cultural contexts. For this reason, teachers themselves are required for tolerance and sensitivity. Children with limited English proficiency generally have very difficult time learning in regular classrooms and easily lag behind leading to lack of confidence in their academic performance. Less confidence in language and communication negatively affect student's motivation to learn, and a teacher sees student's attitude and has low expectation so that the vicious cycle of impotence appears within the minority students in classroom. In the similar context, special attention should be paid to girls because they are not encouraged to express their thoughts more than boys and their performance is lower than male as shown by the international assessments. This relates with the concepts of empowerment of women and equal opportunity to access the basic education in the field of international development cooperation. Teachers have to be the all sensor in classroom in any cultures.

4.4.3 Inclusive atmosphere in classroom

The role of teachers becomes more important today, and they can create the cooperative atmosphere in classroom to promote more interaction among students. This is not only the cases in Europe but should also be African cases. Dei (2005) introduces the case of education policy in Ghana and emphasizes the inclusive education, even though Ghana has the goal of national integration, because postcolonial education in Ghana denies heterogeneity in local populations. He explains no attempts have been made to explore majority-minority relations in schooling or to draw comparisons with approaches to minority education in other pluralistic contexts. This situation missed the needs of students from minority backgrounds, and people take the difference negatively. For example, "girls are denied access to schooling precisely because of this marker of 'difference' that it they are different from boys, or more accurately (in terms of power, privilege and dominance), they are not boys."(33) He also introduces an interview result as reaction to the minority student: "For minority students it is very important to see that everybody can equally perform regardless of background. Specially here some people tend to look down on them, and you know, it hurts them psychologically. So, if in all places some of their community members serve as role models, it would really help them a lot."(34) When we observe a classroom in Ghana, we are hardly able, from the outside views, to understand the difference which the local people recognize.

Notes:

(1) The definitions of the each literacy are as follows (OECD, 2003):

- *Mathematical literacy*: an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.
- *Reading literacy*: understanding, using and reflecting written texts, in order to achieve one's goals, to develop one's knowledge and potential and to participate in society.
- *Scientific literacy*: the capacity to use scientific knowledge, to identify questions and to draw evidence-based conclusions in order to understand and help make decisions about the natural world and the changes made to it through human activity.

(2) There are various definitions of immigrants have been employed in the literature. Some take students born to one immigrant and one native parent as immigrants (Ammermüller 2005), other label only students with two immigrant parents as immigrants (OECD, 2001). As we are seeing the immigrant students' performance of PISA here, the immigrant students should be school children whose parents are not natives of the host country.

(3) OECD, 2003, p. 172

(4) See OECD, 2003, Table 4.2h, for more detail.

(5) Australia, Canada, France, Germany, the Netherlands, New Zealand, Sweden, Switzerland, the UK, and the USA.

(6) Her explanation shows the difference between PISA and TIMSS and the language influence to mathematics achievement: "We might expect that immigrants' educational disadvantage is smaller in subjects where language skills are generally of a lower importance like in math. However, achievement in math in PISA is measured by applying the 'life-skill' approach related to open-ended questions on wordy descriptions of 'real life' situations. Hence, it is not necessarily surprising, that immigrants do not fare better in math in PISA. This result stands in contrast to TIMSS: for all countries immigrants' educational achievement gap is significantly lower in TIMSS math than in TIMSS science. Hence, there seems to be the tendency that immigrants' educational disadvantage is smaller in technical subjects as long as achievement is assessed in a more curriculum based approach by using predominantly multiple-choice questions (Schnepf, 2004: 13)."

(7) It is so-called PISA-E in Germany which includes over 34,000 students compared to 5,500 students in the German OECD-PISA dataset. PISA-E data are detailed data on student performance for Germany and show 81% of the students have both German parents and 19% have one or both non-German parent. The data also show 40% of the 19% students speak another language than German at home.

(8) CGI is based on the following four interlocking assumptions: 1) Teachers should know how specific mathematical content is organized in children's minds; 2) Teachers should make mathematical problem solving the focus of their instruction of that content; 3) Teachers should find out what their students are thinking about the content in question; and 4) Teachers should make instructional decisions based on their own knowledge of their students' thinking.

(9) First, pose a similar problem. If the student needs more help, secondly, the teacher might simplify the language of the problem even further. The third option is to add localized context to a similar problem. The contexts added to story problem is to refer to the children's home cultures and backgrounds. Forth, the teacher translates the problem into the students' native languages. If all else fails, the teacher might try an easier problem which the students will be surely able to solve.

(10) The minority groups here should be those who have different cultural backgrounds in classroom. This includes the students who are not fluent in a test language and who can hardly understand local contexts in classroom. As the results of the first year IDEC survey in Ghana show English is a key competency for the students because the teachers reported the difficulty for the students were sometimes a very language problem. Even the Chinese case gives us a variety of the students' background for languages. As the author explained above, the language is one of explicit aspects of culture as a symbol of power and personal ability.

(11) Culture is defined in many ways in literature. Here we rather take it generally as "the deposit of knowledge, experience, beliefs, values, attitudes, meanings, hierarchies, religion, timing, roles, spatial relations, concepts of the universe, and material objects and possessions acquired by a group of people in the course of generations through individual and group striving (Samovar, L. and Porter, R., 1991)."

(12) co-culture: a group of people living within a dominant culture, yet having dual membership in another culture.

(13) A form of communication in which the explicit coded message contains almost all of the information to be shared. In contrast, little information is spoken out in a high-context culture. The four differences between high- and low-context are: 1) verbal message is importance and shared in low-context so that people need to observe the situation to gain the information; 2) people in high-context trust oral expression less than those in low-context; 3) people in high-context are more sensitive to non-verbal communication and are more reading the situation; and 4) people in high-context think anyone can achieve communication without words so that do not use words as much as those in low-context.

(14) in-group: the ground that one belongs to (relatives, clans, organizations); out-group: people who are not members of an individual's group.

(15) Pike wrote more about *etic/emic*: creation / discovery of a system; external / internal view; external / internal plan; absolute / relative criteria; non-integration / integration; sameness / difference as measured vs. systemic; partial / total data; preliminary / final presentation; and etc. In addition, *etic* approach is different from structurism which compares cultures and leads universality between structures in which events cause and effects. Researchers study cultures with the imposed *etic* but need to make it from *emic*. In other words, we need to pause our judgment as observing a different culture with *emic* approach and switch to derived *etic* as describing the culture in our language.

(16) However, the author admits his criteria for taking anthropological or psychological approach are not constant in this paper for the moment.

(17) Parker-Jenkins, 1995, p. 28

(18) Redfield, Linton, and Herskovits, 1936, p. 149

(19) Enculturation refers to the process by which the individual learns the culture of the belonging group. Socialization is also used for the same meaning. However, Herskovits (1948) defined enculturation as "the aspects of the learning experience which mark off man from other creatures, and by means of which, initially, and in later life, he achieves

competence in his culture" and socialization as "the process by means of which an individual is integrated into his society (p.38-39)."

(20) Berry, 1992, p. 278-279

(21) Interviews were conducted as semi-structured and informal way at the different time and venues as the author had opportunities to communicate with German scholars. The interviewees were: Ms. Inge Steinstraβer, Deputy Director, VHS in Bonn (Dec. 2, 2004); Dr. Heribert Hinzen and Dr. Werner Hutterer, IIZ/DVV (Dec. 19, 2005); and Dr. Raphäera Henze, Ministry of Science and Health of Hamburg (Jan. 11, 2006).

(22) "Attracting, Development and Retaining Effective Teachers" OECD Project conducted over the 2002 to 2004 period.

(23) OECD, 2005, p. 97

(24) In the studies we reviewed above, the results explain the language as "one aspect of the integration issue is the pupils' capacity to communicate in the language of the host country. Regression analysis showed consistently across surveys that speaking a foreign language at home decreases pupils' achievement greatly in all countries compared. (Schnepf, 2004, p.36)"

(25) Secada & Carey say "the instruction for students from culturally diverse backgrounds often does not take account of the everyday sources of their informal knowledge, i.e., the knowledge that they bring from home. ... teachers of LEP [limited English proficient] students not only need to focus on their students' informal understandings, but they also need to seek those understandings in ways and in settings that are other than commonly supposed."

(26) Later, more than 80 countries, including Arabic and African countries (Hofstede and Hofstede, 2005).

(27) Parker-Jenkins, *op. cit.* p. 118-120.

(28) OECD, *op. cit.*, p. 27

(29) Because scholars have tended to equate "education" with "schooling," and because they have consistently focused on the role of literacy and a literary tradition, many important and interesting - indeed, fascinating - traditions have been seen as falling outside of the parameters of "legitimate" study in the history and philosophy of education (Reagan, 2005, p.6).

(30) Reagan, 2005, p. 248

(31) "Shulman, L., 1991, Ways of seeing, ways of knowing, ways of teaching, ways of learning about teaching, Journal of Curriculum Studies, vol. 23, pp. 393-395." The author was unable to find descriptions of the five areas in the article.

(32) OECD, *op. cit.*, p. 98

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Literature Review

Literature Review

Prior Study of the Formation of Figural Concepts

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The Formation of figural concept is very important in geometry. Because the main point of learning geometry is that pupils acquire the ability of the abstract and the generalization for their daily life. In other words, through form the figural concepts, pupils can acquire them.

I describe the study of the formation of figural concepts as follow.

Sort of the Figural Concepts

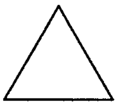
Matsuo (2000) told that mathematical concept is one of general concept and needs abstract and general thinking. The mathematical concepts include many concepts such as figural concepts. Also she divides the figural concepts into 3 parts as follow.

1. object concepts

   This is the concept which can be gotten through abstraction and generalization of the figure in the shape
2. related concepts

   This is the concept which is relationship between the figures.
3. any other concepts

Objected concept is very typical concept in geometry. Because for example I draw a triangle as follow.



As you can see, this is an equilateral triangle. It means that it can not describe all triangles. In other words, if you try to explain any shapes, you can not do draw. You should use only the linguistic methods. This is because object concept is very important and typical concept in geometry.

Formation of the figural concepts

Matsuo (2000) explained that when the concept is formed, the concept image and the concept definition are used. The concept image means the sets of mental pictures which is relation to the name of the concepts. The concepts definition means the linguistic formation which identifies the concepts. She explained that when the concepts are formed, both linguistic aspects and image aspects is needed. According to above, we can understand that the concept definition is common concept and the concept image is individual concepts.

Fischbein (1993) explained that while image is a symbol of the sense, concept is a symbol of sign and is used on the way of abstract thinking. In addition, the concept has universality. It means that the concept is almost same as the concept definition which is mentioned by Matsuo (2000) and the image is almost same as the concept image which is mentioned by Matsuo (2000).

Also Fischbein (1993) mentioned the relationship between the concept definition and the concept image. According to him, these are in different categories basically. It can be understood that the concept definition is very abstract and the concept image is very concrete. However he also told that we should not separate them but we should combine them in the situation which distills the essence from them. It means that through the relationship between them, the concept is formed deeply.

In addition, Fischbein (1993) mentioned that the image and the concept are related very much and sometimes conflicted. Also he told that the figural concepts are not growing automatically and the reason why geometry is very difficult is that the figural concept can not be formed naturally. In terms of above, we can realize that the relationship between the concept definition and the concept image is very important when the figural concept is formed. Besides he mentioned that the relationship between the experiments which pupils have and the lessons which the pupils have is very important and it should be intermittent.

The representation system of the figural concepts

When pupils learn, the representation is requisite. Now I describe the sort of representation. Nakahara (1995) expressed five representations in mathematics as follow.

1. Realistic Representation

   This is included the real world, the representation which is used the real, the experiments using the real or the embodiment figures.

2. Manipulative Representation

   This is the representation which is occurred the concrete manipulation, the concrete real which is modeled or changed by somebody. It means the representation which exhibits the active manipulate in the teaching tools.

3. Illustrative Representation

   This is the representation which is used the pictures, diagrams and graphs.

4. Linguistic Representation

   This is the representation which is used local language.

5. Symbolic Representation

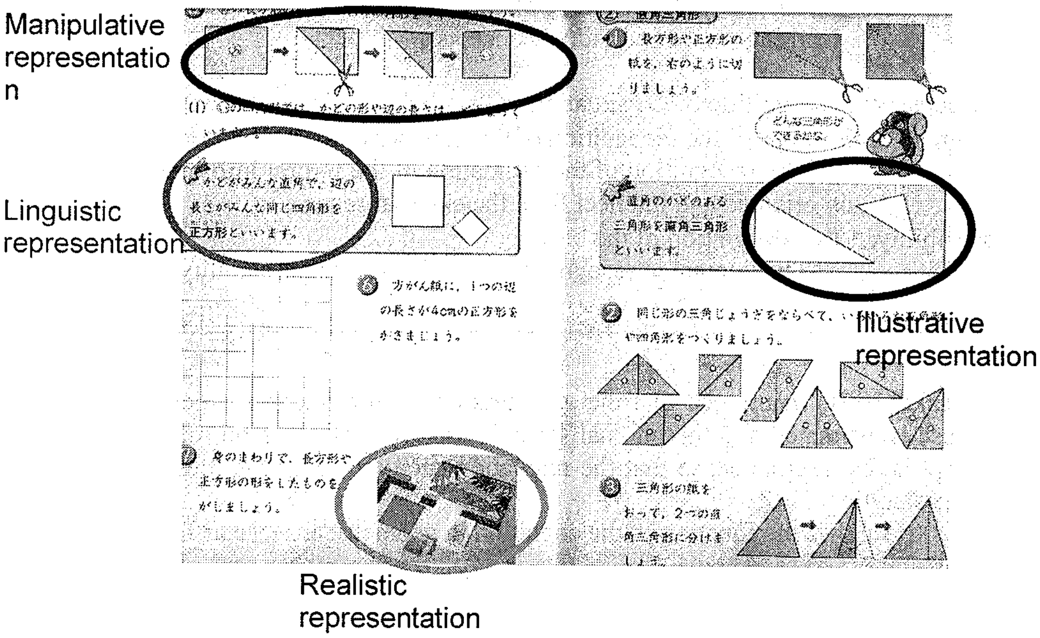
   This is the representation which is used figures, words and arithmetic operation

In basic education level, Realistic representation, manipulative representation, illustrative representation and linguistic representation are usually used. Symbolic representation is in the highest level. This is because it is very far from using in basic education level.

Besides Realistic representation, manipulative representation, and illustrative representation are named the affinity of the representation Linguistic representation and symbolic representation are named the rule of the representation.

The affinity of the representation (Realistic representation, manipulative representation, and illustrative representation) has the characteristic which full of the concreteness, the imaginable and the intuition. Because it includes the meaning in the visual (image). That is why its characteristic is very intimate. In other hands, the rule of the representation (Linguistic representation and symbolic representation)has the characteristic which full of the strictness and the objectiveness. Because it shows the detail. That is why its characteristic lucks of the concreteness, the imaginable and the intuition. By the mean of the above, the affinity of the representation is relation to the concept image and the rule of the representation is relation to the concept definition.

Nakahara(2001), *shougakusannsu3* osakashoseki



The sense of the geometry

Here is explained the sense of the geometry. It means that how pupils used the figural concepts and how pupils express. Kawasaki (2003) mentioned that the sense of the geometry is not only relation to the physical stimulates which are gotten from the environment but also how the pupils think it or how the pupils active. In addition, he explained that the sense of the geometry can be divided into two groups. The first is named the external sense, which is the geometrical recognition of the object (things and shapes) of the out side. Also it can be divided into two groups. One it is the mental manipulation which recognizes the facts naked. It is called the intuition of the visual. The other is the mental manipulation which recognizes the intimate meaning like the concepts and the philosophy. It is called the intuition of the intimate which tends to identify the meaning of the facts.

The second is named the internal sense, which recognizes the immanent condition of the objects (things, shapes) of the outside. Also it can be divided into two groups. One is the estimate of the value to recognize the goodness, which estimates the figural information and recognize on oriented and intellect from the external sense. The other is the sensibility, which is the immanent recognition of the sense or the feeling for the beauty of the figure.

External sense: the recognition of the objects of the outside.

- intuition of the visual : the recognition of the reality of the concrete things
  The target is the real things ・・・the real shapes
  The recognition of the figural element (shapes, sizes and positions) in visual
- intuition of the intimate: the recognition of the object in the philosophy.
  The target is the real things in the philosophy ・・・ general and universally shapes
  The recognition of the figural elements and the relation linguistically
  The intimate meaning of the shapes which are gotten by experiments or learning. (the individual concepts)

Internal sense: the recognition of the situation of the objects (things, shapes) of the outside

- estimate of the value: the recognition of the goodness
  The recognition of the estimation of the figural information and recognition on oriented and intellect from the external sense.
- sensibility: the recognition of the beauty
  The immanent recognition of the beauty or the stability of the shapes

Also Kawasaki (2001) mentioned that the goodness of the information which is identified by the external sense is estimated and the internal sense is used for getting good information. In addition he told that it is the internal sense that the regulation such as laying the tiles is recognized as the beauty or the stability.

Kawasaki (2001) shows the function of the cognition in geometrical sense as follow.

(1) recognition of the plane
  ① part and hole
    a. Gestalt ・・・When the shape is seen, the property or the structure of the shape is realized as a whole part.
    b. View and ground・・・View is the part (shape) which is cognized the first. And ground is the part (shape) which tends to be missed so that the impression is very shallow.
  ② composition and decomposition
    c. structure of the cognition ・・・This is the composition or the decomposition of the figures in cognition
    d. transformation of the cognition ・・・Congruent transformation(translation, rotations and symmetry) or similarity

(2) recognition of three dimension
    e. homeostasis ・・・ This is the manipulate which help the cognition through the realization of the figural chunk in homeostasis.
    f. depth perception ・・・ This is the cognition that the depth is realized by the figure rawn in the plane.

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ANNEX

Research Tools

ANNEX: Research Tools

1. Questionnaire for the fisrt year survey (prototype)

[General Directions]

In this booklet, you will find questions about yourself. Some questions ask for facts while other questions ask for your opinions.

Read each question carefully and answer as accurately as possible. You may ask for help if you do not understand something or are not sure how to answer.

Some of the questions will be followed by a few possible choices indicated with a number. For these questions, circle the answer of your choice as shown in Examples 1, 2, and 3.

[Example 1]

Do you go to school?

Fill in one circle only

Yes -- [✓ 1]

No -- 2

[Example 2]

How often do you do these things?

Fill in one circle for each line

	Every day	At least Once a week	Once or Twice a month	A few times a year	Never
a) I listen to music	1	2	3	[✓ 4]	5
b) I talk with my friends	1	[✓ 2]	3	4	5
c) I play sports	1	2	[✓ 3]	4	5

[Example 3]

Indicate how much you agree with each of these statements.

Fill in one circle for each line

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) Watching movies is fun	1	2	[✓ 3]	4
b) I like eating ice cream	1	[✓ 2]	3	4

Read each question carefully, and pick the answer you think is best. There are no right or wrong answers. Fill in the circle the number of your answer. If you decide to change your answer, erase your first answer and then fill in the circle the number of new answer. Ask for help if you do not understand something or are not sure how to answer.

Thank you for your time, effort, and thought in completing this questionnaire.

[About You]

I.

When were you born?

A. Fill in the circle next to the year

Year

1. 1990
2. 1991
3. 1992
4. 1993
5. 1994
6. 1995
7. 1996
8. 1997
9. Other

B. Fill in the circle next to the month you were born

Month

1. January
2. February
3. March
4. April
5. May
6. June
7. July
8. August
9. September
10. October
11. November
12. December

II.

Are you a girl or a boy?

Fill in one circle only

Girl ---1
Boy ---2

III.

How often do you speak <language of test> at home?

Fill in one circle only

Always -- 1
Almost always ----------------------------------- 2
Sometimes -- 3
Never --- 4

IV.

About how many books are there in your home? (Do not count magazines, newspapers, or your school books.)

Fill in one circle only

None or very few
(0-10 books) --------------------------------------1
Enough to fill one shelf
(11-25 books) ------------------------------------ 2
Enough to fill one bookcase
(26-100 books) ------------------------------------3
Enough to fill two or more bookcases
(more than 100 books) ---------------------------4

V.

Do you have any of these items at your home?

Fill in one circle for each line

	Yes	No
a) Calculator	1	2
b) Television	1	2
c) Radio	1	2
d) Study desk/table for your use	1	2
e) A quiet place to study	1	2
f) Dictionary	1	2
g) Books to help with your school work	1	2
f) Computer (do not include TV/video game computers)	1	2

[Mathematics in School]

VI.

How much do you agree with these statements about learning mathematics?

Fill in one circle for each line

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) I usually do well in mathematics	1	2	3	4
b) I would like to do more mathematics in school	1	2	3	4
c) Mathematics is harder for me than for many of my classmates	1	2	3	4
d) I enjoy learning mathematics	1	2	3	4
e) I am just not good at mathematics	1	2	3	4
f) I learn things quickly in mathematics	1	2	3	4
g) I think learning mathematics will help me in my daily life	1	2	3	4
h) I need mathematics to learn other school subjects	1	2	3	4
i) I need to do well in mathematics to get into the university of my choice	1	2	3	4
j) I need to do well in mathematics to get the job I want	1	2	3	4

VII.

How often do you do these things in your mathematics lessons?

Fill in one circle for each line

	Every or almost every lesson	About half the lessons	Some lessons	Never
a) I practice adding, subtracting, multiplying, and dividing without using a calculator	1	2	3	4
b) I work on fractions and decimals	1	2	3	4
c) I measure things in the classroom and around the school	1	2	3	4
d) I make tables, charts, or graphs	1	2	3	4
e) I learn about shapes such as circles, triangles, and rectangles	1	2	3	4
f) I relate what I am learning in mathematics to my daily lives	1	2	3	4
g) I work with other students in small groups	1	2	3	4
h) I explain my answers	1	2	3	4

i) I listen to the teacher talk	1	2	3	4
j) I work problems on my own	1	2	3	4
k) I review my homework	1	2	3	4
l) I have a quiz or test	1	2	3	4
m) I use a calculator	1	2	3	4

[Your School]

VIII.

How much do you agree with these statements about your school?

Fill in one circle for each line

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
a) I like being in school	1	2	3	4
b) I think that students in my school try to do their best	1	2	3	4
c) I think that teachers in my school care about the students	1	2	3	4
d) I think that teachers in my school want students to do their best	1	2	3	4

IX.

In school, did any of these things happen during <u>the last one year</u>?

Fill in one circle for each line

	Yes	No
a) Something of mine was stolen	1	2
b) I was hit or hurt by other student(s) (for example, shoving, hitting, kicking)	1	2
c) I was made to do things I didn't want to do by other students	1	2
d) I was left out of activities by other students	1	2

[Things You Do Outside of School]

X.

On a normal school day, how much time do you spend before or after school doing each of these things?

Fill in one circle for each line

	No time	Less than 1 hour	1-2 hours	More than 2 but less than 4 hours	4 or More hours
	↓	↓	↓	↓	↓
a) I watch television	1	2	3	4	5
b) I play or talk with friends	1	2	3	4	5
c) I do jobs at home	1	2	3	4	5
d) I play sports	1	2	3	4	5
e) I read a book for enjoyment	1	2	3	4	5
f) I use computer	1	2	3	4	5
g) I do homework	1	2	3	4	5

[More About You]

XI.

Including yourself, how many people live in your home?

Fill in one circle only

2-3 persons --2
4-6 persons --3
7-9 persons --4
10-12 persons ---------------------------------------5
More than 12 persons-------------------------------6

XII.

A. Does your mother (or stepmother or female guardian) belong to <the major ethnic group> in your town?

Yes No
Fill in one circle only ------------------ 1 ------2

B. Does your father (or stepfather or male guardian) belong to <the major ethnic group> in your town?

Yes No
Fill in one circle only ------------------ 1 ------2

2. Achievement tests for the first year survey (prototype)

[Face Sheet for Part I]

Mathematics Achievement Test
Part I

a) This is a test on Mathematics.
The result which you achieve in this test will never affect your grade in school. Feel relaxed and never cheat.
Read the question carefully and answer what you think the best.

b) The time allocation is 41 minutes.

c) Some questions are multiple-choice type, and others short answer type.
All the questions have to be answered with clear writing.

d) Never use any calculator or multiplication table for your calculation.
You can use a ruler, if necessary.

e) When the question has some choices from (A) to (D) or (E), please encircle only one correct answer.

f) When the question needs a correct short answer, please write down the answer in the blank on a dotted line, or show your answer in an appropriate place in the question.

g) You have to write your process to get the answer in a space of each question.

School Name: ....

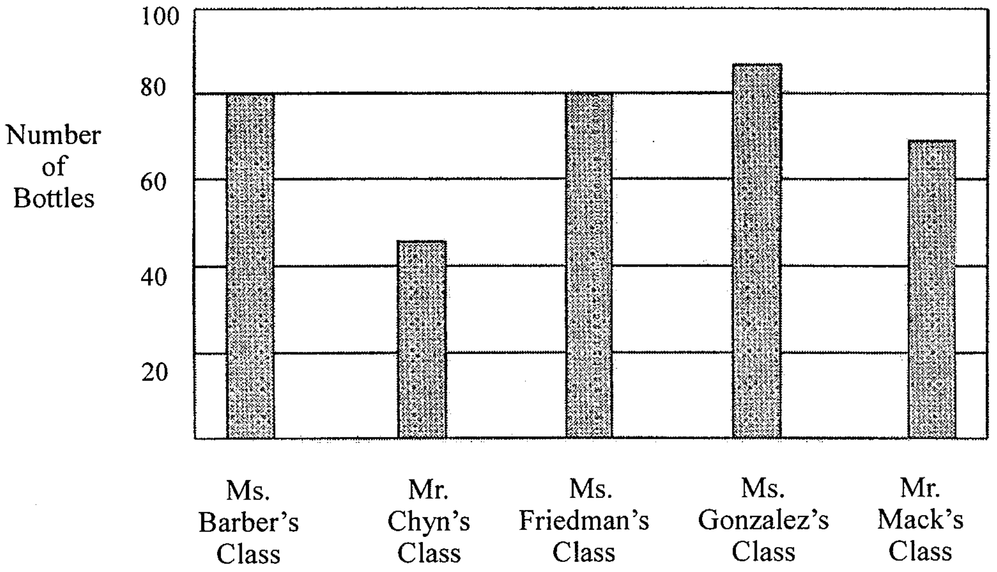
Class Name: ....

Sex (Please circle either): Male or Female

Age: .... years old

Your Name: ....

[1-1]
Central School had a bottle collection. Children in each class brought empty bottles to school. The principal made a bar graph of the number of bottles from five classes.



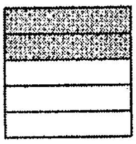
Which class collected 45 bottles?

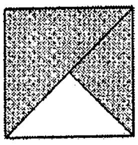
(A) Miss Barber's class
(B) Mr. Chyn's class
(C) Mrs. Friedman's class
(D) Mr. Mack's class

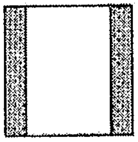
[1-2]
Jasmine made a stack of cubes of the same size. The stack had 5 layers and each layer had 10 cubes. What is the volume of the stack?

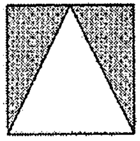
(A) 5 cubes
(B) 15 cubes
(C) 30 cubes
(D) 50 cubes

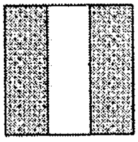
[1-3]
Which shows 2/3 of the square shaded?

(A)


(B)


(C)


(D)


(E)


[1-4]
It takes Chris 4 minutes to wash a window. He wants to know how many minutes it will take him to wash 8 windows at this rate. He should
(A) multiply 4×8
(B) divide 8 by 4
(C) subtract 4 from 8
(D) add 8 and 4

[1-5]
Here is a calendar for December.

DECEMBER						
S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Mary's birthday is on Thursday, December 2. She is going on a trip exactly 3 weeks later. On what date will she go on the trip?
(A) December 16th
(B) December 21st
(C) December 23rd
(D) December 30th

[1-6]
Which digit is in the hundreds place in 2345?
(A) 2
(B) 3
(C) 4
(D) 5

[1-7]
Which number would be rounded to 600 when rounded to the nearest hundred?
(A) 62
(B) 160
(C) 546
(D) 586
(E) 660

[1-8]

Which of these means $\frac{7}{10}$?

(A) 70

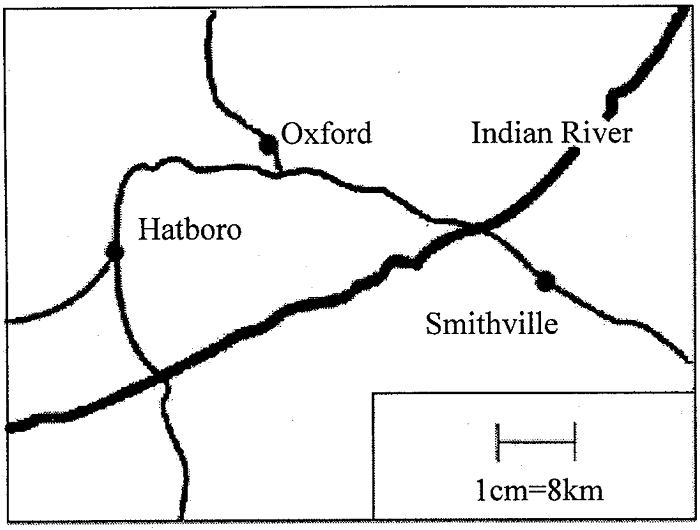
(B) 7

(C) 0.7

(D) 0.07

[1-9]

One centimeter on the map represents 8 kilometers on the land.



About how far apart are Oxford and Smithville on the land?

(A) 4 km

(B) 16 km

(C) 35 km

(D) 50 km

[1-10]

Which of these could be the weight (mass) of an adult?

(A) 1 kg

(B) 6 kg

(C) 60 kg

(D) 600 kg

[1-11]

Which of these is a name for 9740?

(A) Nine thousand seventy-four

(B) Nine thousand seven hundred forty

(C) Nine thousand seventy-four hundred

(D) Nine hundred seventy-four thousand

[1-12]

□ represents the number of magazines that Lina reads each week. Which of these represents the total number of magazines that Lina reads in 6 weeks?

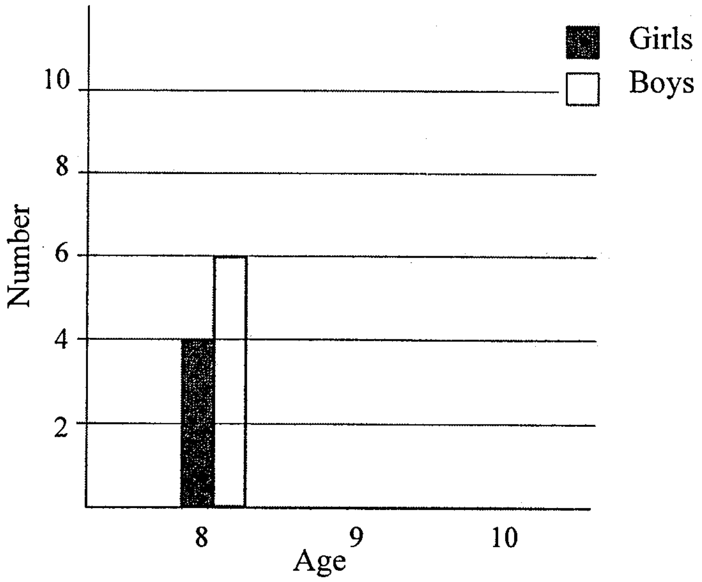
(A) 6+□
(B) 6×□
(C) □+6
(D) (□+□)×6

[2-1]

This table shows the ages of the girls and boys in a club.

Age	Number of Girls	Number of Boys
8	4	6
9	8	4
10	6	10

Use the information in the table to complete the graph for ages 9 and 10.



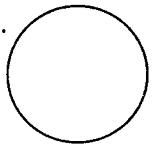
[2-2]

Sam said that $\frac{1}{3}$ of a pie is less than $\frac{1}{4}$ of the same pie.

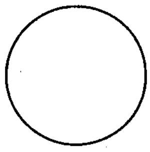
Is Sam correct? ....

Use the circles below to show why this is so.

Shade in $\frac{1}{3}$ of this circle.



Shade in $\frac{1}{4}$ of this circle.



[2-3]
These shapes are arranged in a pattern.

○△○○△△○○○△△△

Which set of shape is arranged in the same pattern?

(A) ★□★□★★□□★★□□

(B) □★□□★□□□★□□□□

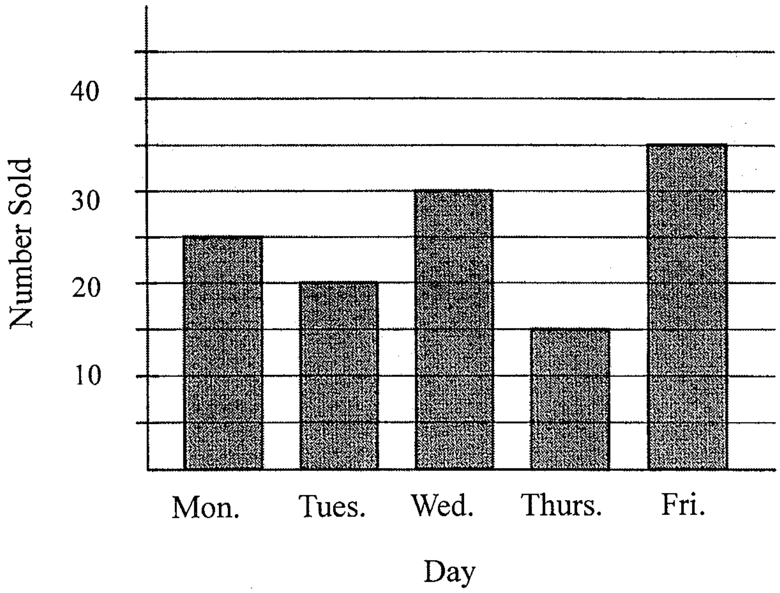
(C) ★□★★ □□★★★□□□

(D) □□★★□★□□★★□★

[2-4]
There are 600 balls in a box, and $\frac{1}{3}$ of the balls are red. How many red balls are in the box?

Answer : ....

[2-5]
The graph shows the number of cartons of milk sold each day of a week at a school.



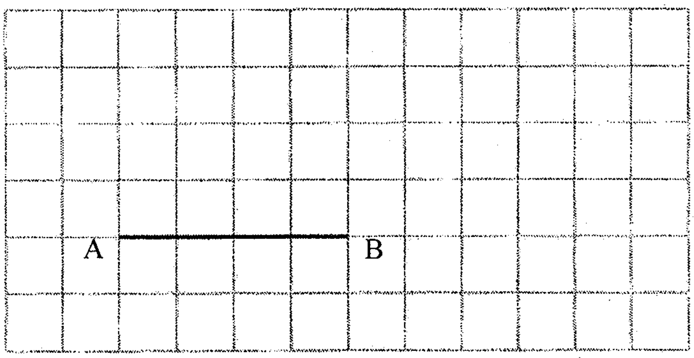
How many cartons of milk did the school sell on Monday?
Answer : ....
How many cartons of milk did the school sell that week? Show your work.
Answer : ....

[2-6]
Which of these could equal 150 milliliters?
(A) The amount of water in a cup
(B) The length of a kitten
(C) The weight of an egg
(D) The area of a coin

[2-7]
Lia is practicing addition and subtraction problems. What number should Lia add to 142 to get 369?

Answer : ....

[2-8]

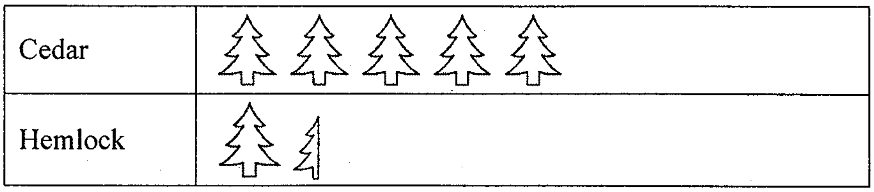


Draw a triangle in the grid so that the line AB is the base of the triangle and the two new sides are the same length as each other.

[2-9]
Mr. Brown goes for a walk and returns to where he started at 07:00. If his walk took 1 hour and 30 minutes, at what time did he start his walk?

Answer : ....

[2-10]
The graph shows 500 cedar trees and 150 hemlock trees.

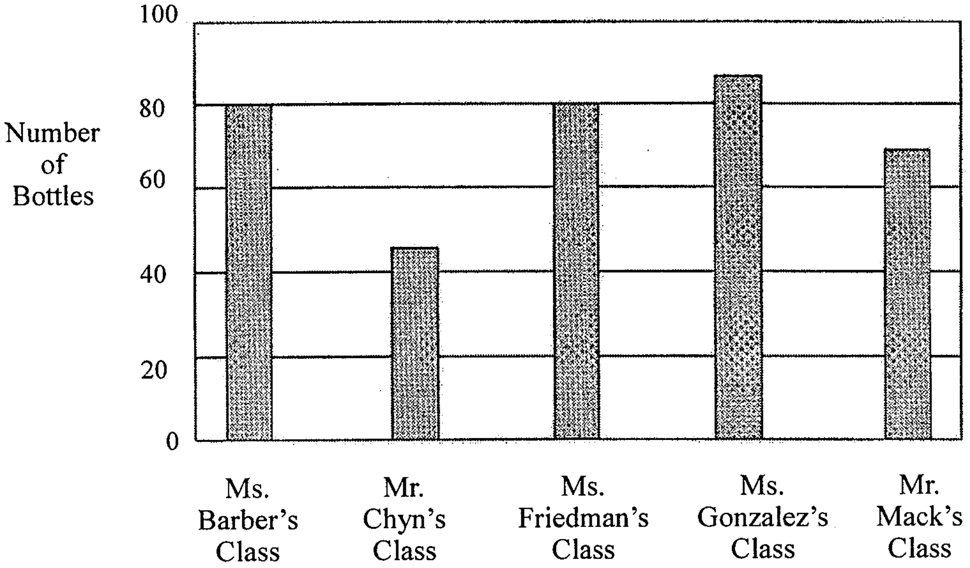


How many trees does each 🌲 represent?

Answer : ....

[3-1]

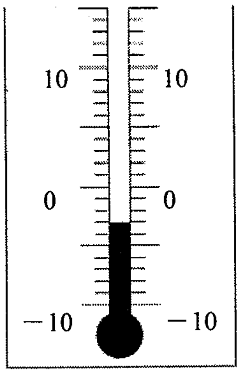
Central School had a bottle collection. Children in each class brought empty bottles to school. The principal made a bar graph of the number of bottles from five classes.



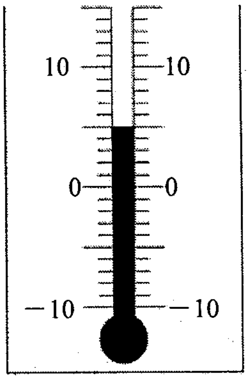
Which two classes collected exactly 80 bottles?
(A) Miss Barber's and Mrs. Friedman's classes
(B) Miss Barber's and Mr. Mack's classes
(C) Mrs. Friedman's and Miss Gonzalez's classes
(D) Miss Gonzalez's and Mr. Mack's classes

[3-2]

When Tracy left for school, the temperature was minus 3 degrees.



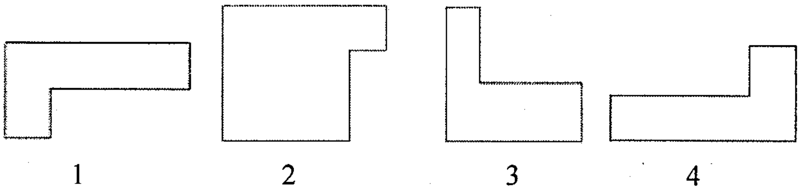
At recess, the temperature was 5 degrees.



How many degrees did the temperature rise?

(A) 2 degrees
(B) 3 degrees
(C) 5 degrees
(D) 8 degrees

[3-3]
Figures that are the same size and shape are called congruent figures.



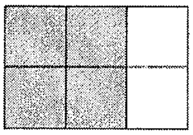
Which two figures are congruent?
(A) 1 and 2
(B) 1 and 3
(C) 1 and 4
(D) 3 and 4

[3-4]
Suject:

$$\begin{array}{r} 4.03 \\ -\underline{1.15} \end{array}$$

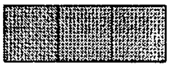
(A) 5.18
(B) 4.45
(C) 3.12
(D) 2.98
(E) 2.88

[3-5]
In this diagram, 2 out of every 3 squares are shaded.



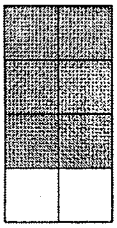
Which diagram has 3 out of every 4 squares shaded?

(A)

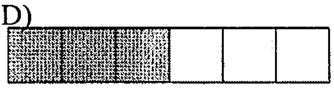


(B)

(C)



(D)



[3-6]
This chart shows temperature reading made at different times on four days.

TEMPERATURE					
	6 a.m.	9 a.m.	Noon	3 p.m.	8 p.m.
Monday	15°	17°	20°	21°	19°
Tuesday	15°	15°	15°	10°	9°
Wednesday	8°	10°	14°	13°	15°
Thursday	8°	11°	14°	17°	20°

What was the highest temperature recorded?
(A) Noon on Monday
(B) 3 p.m. on Monday
(C) Noon on Tuesday
(D) 3 p.m. on Wednesday

[3-7]
Janis, Maija, and their mother were eating a cake. Janis ate $\frac{1}{2}$ of the cake.

Maija ate $\frac{1}{4}$ of the cake. Their mother ate $\frac{1}{4}$ of the cake. How much of the cake is left?

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) None

[3-8]
What number equals 3 ones + 5 tens + 4 hundreds + 60 thousands?
(A) 6 453
(B) 60 453
(C) 64 530
(D) 354 060
(E) 604 530

[3-9]
What units would be best to use to measure the weight (mass) of an egg?
(A) centimeters
(B) milliliters
(C) grams
(D) kilograms

[3-10]
All of the pupils in a class cut out paper shapes. The teacher picked one out and said, "This shape is a triangle." Which of these statements MUST be correct?
(A) The shape has three sides.
(B) The shape has a right angle.
(C) The shape has equal sides.
(D) The shape has equal angles.

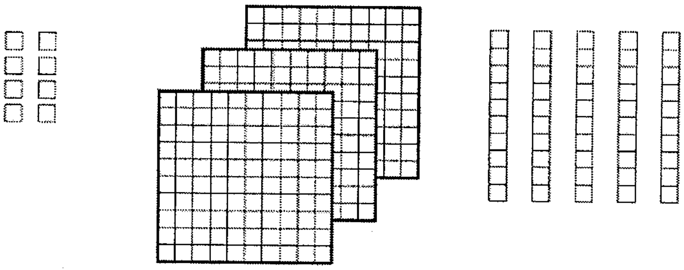
[3-11]

There are 9 boxes of pencils. Each box has 125 pencils. What is the total number of pencils?

(A) 1025
(B) 1100
(C) 1125
(D) 1220
(E) 1225

[3-12]

Each small square (□) is equal to 1. There are 10 small squares in each strip. There are 100 small squares in each large square.



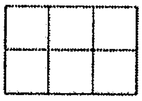
What number is shown?

(A) 16
(B) 358
(C) 538
(D) 835

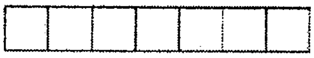
[3-13]

Which of these figures has the largest area?

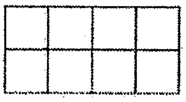
(A)



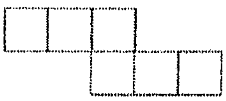
(B)



(C)



(D)



[Backsheet]

Thank You
for Your Cooperation!

The research is conducted by the Graduate School for International Development and Cooperation, Hiroshima University.
1-5-1 Kagamiyama, Higashi-Hiroshima, Japan 739-8529

All the test items are from TIMSS 2003 or TIMSS 1995.
TIMSS 2003 Mathematics Items Released Set for Fourth Grade.

TIMSS Mathematics Items: Released Set for Population 1 (Third and Fourth Grades), Reproduced from TIMSS Population 1 Item Pool.

[Face Sheet for Part II]

Mathematics Achievement Test
Part II

a) This is a test on Mathematics.
The result which you achieve in this test will never affect your grade in school. Feel relaxed and never cheat.
Read the question carefully and answer what you think the best.

b) The time allocation is 43 minutes.

c) Some questions are multiple-choice type, and others short answer type.
All the questions have to be answered with clear writing.

d) Never use any calculator or multiplication table for your calculation.
You can use a ruler, if necessary.

e) When the question has some choices from (A) to (D) or (E), please encircle only one correct answer.

f) When the question needs a correct short answer, please write down the answer in the blank on a dotted line, or show your answer in an appropriate place in the question.

g) You have to write your process to get the answer in a space of each question.

School Name: ....

Class Name: ....

Sex (Please circle either): Male or Female

Age: .... years old

Your Name: ....

[4-1]

Simon wants to watch a film that is between $1\frac{1}{2}$ and 2 hours long. Which of the following films should be chosen?

(A) a 59-minutes film
(B) a 102-minutes film
(C) a 121-minutes film
(D) a 150-minute film

[4-2]

What is the smallest whole number that you can make using the digits 4, 3, 9 and 1? Use each digit only once.

Answer : ....

[4-3]

$204 \div 4 =$

Answer : ....

[4-4]

Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?

(A) $130 + 78 = \square$
(B) $\square - 78 = 130$
(C) $130 \div 78 = \square$
(D) $130 - 78 = \square$

[4-5]

Here is a number sentence.

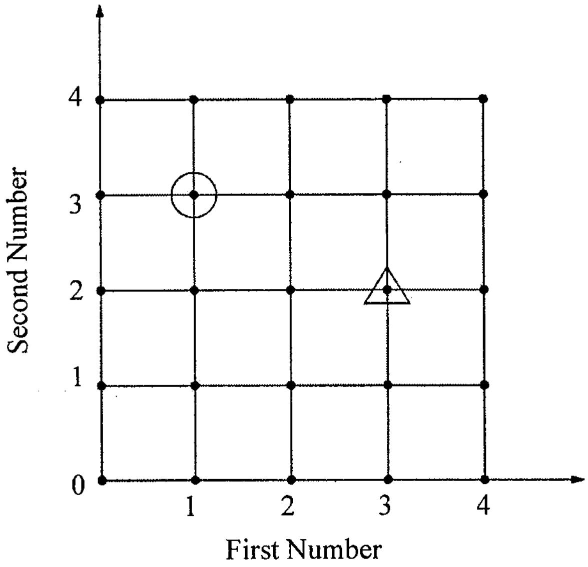
$4 \times \square < 17$

Which number could go in the □ to make the sentence true?

(A) 4
(B) 5
(C) 12
(D) 13

[4-6]

On this grid, find the dot with the circle around it. We can describe where this dot is by saying it is a First Number 1, Second Number 3.

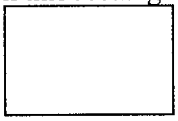


Now find the dot with the triangle around it. Describe where the dot is on the grid in the same way. Fill in the numbers we would use:

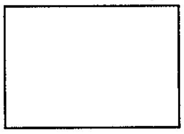
First number ________________ Second Number ________________

[4-7]

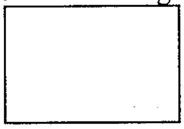
A. Draw 1 straight line on this rectangle to divide it into 2 triangles.



B. Draw 1 straight line on this rectangle to divide it into 2 rectangles.



C. Draw 2 straight lines on this rectangle to divide it into 1 rectangle and 2 triangles.



[4-8]

Additional Fact
4+4+4+4+4=20

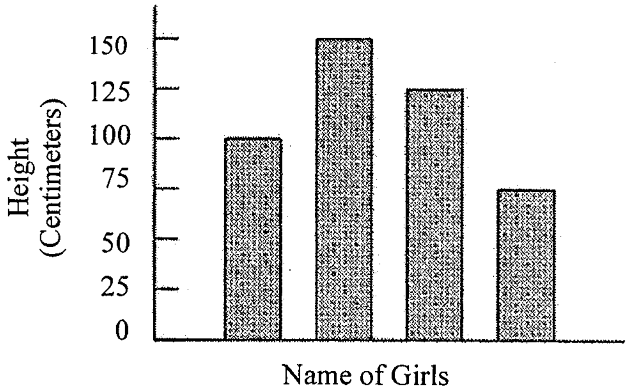
Write this addition fact as a multiplication fact.

__________×__________=__________

[4-9]
A teacher marks 10 of her pupils' tests every half hour. It takes her one and a half hours to mark all her pupils' tests. How many pupils are in her class?
Answer : ....

[4-10]
Ali had 50 apples. He sold some and then had 20 left. Which of these is a number sentence that shows this?
(A) □−20=50
(B) 20−□=50
(C) □−50=20
(D) 50−□=20

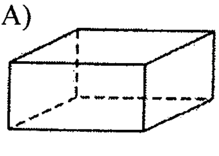
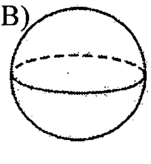
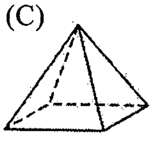
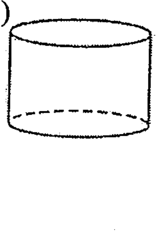
[5-1]
The graph shows the heights of four girls.



The names are missing from the graph. Debbie is the tallest. Amy is the shortest. Dawn is taller than Sarah. How tall is Sarah?
(A) 75 cm
(B) 100 cm
(C) 125 cm
(D) 150 cm

[5-2]
Here is a cone. Part of its surface is flat and part of its surface is curved.

Which of these solids also has both a flat surface and a curved surface?

(A)

(B)

(C)

(D)


[5-3]

Mark's garden has 84 rows of cabbages. There are 57 cabbages in each row. Which of these gives the BEST way to estimate how many cabbages there are altogether?
(A) 100×50=5000
(B) 90×60=5400
(C) 80×60=4800
(D) 80×50=4000

[5-4]

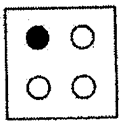
Which of these has the same value as 342?
(A) 3000+400+2
(B) 300+40+2
(C) 30+4+2
(D) 3+4+2

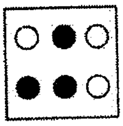
[5-5]

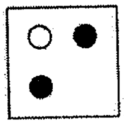
What is the sum of 2.5 and 3.8?
(A) 5.3
(B) 6.3
(C) 6.4
(D) 9.5

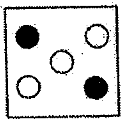
[5-6]

In which figure are one-half of the dots black?


(A)


(B)

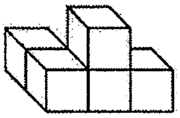

(C)


(D)

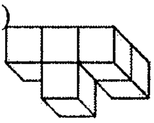
[5-7]

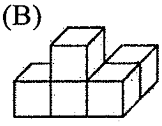
In Toshi's class there are twice as many girls as boys. There are 8 boys in the class. What is the total number of boys and girls in the class?
(A) 12
(B) 16
(C) 20
(D) 24

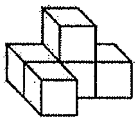
[5-8]
This figure will be turned to a different position.

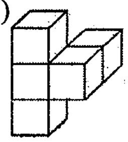


Which of these could be the figure after it is turned?

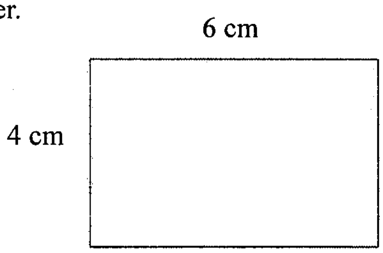
(A)


(B)


(C)


(D)


[5-9]
Here is a rectangle with length 6 centimeters and width 4 centimeters. The distance right around its shape is called its perimeter.



Which of these gives the perimeter of the rectangle in centimeters?
(A) 6+4
(B) 6×4
(C) 6×4×2
(D) 6+4+6+4

[5-10]
Which number sentence is true?
(A) 968<698
(B) 968<689
(C) 968>689
(D) 968=689

[5-11]
Here is a number pattern.

100, 1, 99, 2, 98, ☐ , ☐ , ☐

What three numbers should go in the boxes?
(A) 3, 97, 4
(B) 4, 97, 5
(C) 97, 3, 96
(D) 97, 4, 96

[5-12]
Which number is equal to eight tens plus nine tens?
(A) 17
(B) 170
(C) 1700
(D) 17000

[6-1]
Which pair of numbers follows the rule "Multiply the first number by 5 to get the second number"?

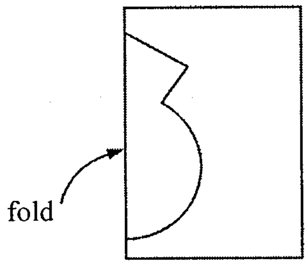
(A) 15 $\longrightarrow$ 3

(B) 6 $\longrightarrow$ 11

(C) 11 $\longrightarrow$ 6

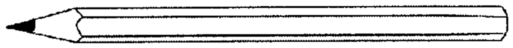
(D) 3 $\longrightarrow$ 15

[6-2]
Craig folded a piece of paper in half and cut out a shape.



Draw a picture to show what the cut-out shape will look like when it is opened up and flattened out.

[6-3]
About how long is this picture of a pencil?



(A) 5 cm
(B) 10 cm
(C) 20 cm
(D) 30 cm

[6-4]
Which of these is largest?
(A) 1 kilogram
(B) 1 centigram
(C) 1 milligram
(D) 1 gram

[6-5]
What is 5 less than 203?
Answer : ....

[6-6]
Henry is older than Bill, and Bill is older than Peter. Which statement must be true?
(A) Henry is older than Peter.
(B) Henry is younger than Peter.
(C) Henry is the same age as Peter.
(D) We cannot tell who is oldest from the information.

[6-7]
The daily start times for showing a movie are listed below:

Show	Start Time
1st	2:00 p.m.
2nd	3:30 p.m.
3rd	5:00 p.m.
4th	?

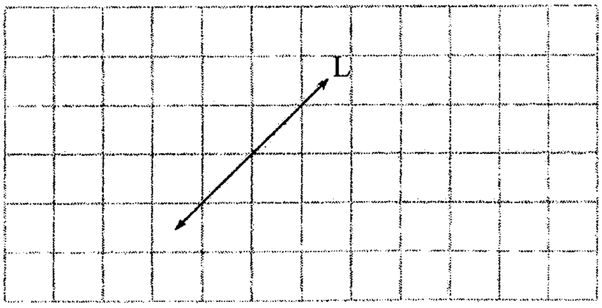
If this pattern continues, what is the start time for the 4th time?
(A) 5:30 p.m.
(B) 6:00 p.m.
(C) 6:30 p.m.
(D) 7:00 p.m.

[6-8]
These numbers are part of a pattern.

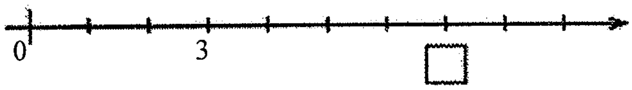
50, 46, 42, 38, 34, ...

What do you have to do to get the next number?
Answer : ....

[6-9]
On the grid, draw a line parallel to line L.



[6-10]



On the number line above, what number goes in the box?
Number in □ =..

[6-11]
How many millimeters are in meter?
Answer : ....

[6-12]
$37\times\square=703$
What is the value of $37\times\square+6$?
Answer : ....

[6-13]
Write a fraction that is larger than $\frac{2}{7}$

Answer : ....

[Backsheet]

Thank You
for Your Cooperation!

The research is conducted by the Graduate School for International Development and Cooperation, Hiroshima University.
1-5-1 Kagamiyama, Higashi-Hiroshima, Japan 739-8529

All the test items are from TIMSS 2003 or TIMSS 1995.
TIMSS 2003 Mathematics Items Released Set for Fourth Grade.

TIMSS Mathematics Items: Released Set for Population 1 (Third and Fourth Grades), Reproduced from TIMSS Population 1 Item Pool.

3. Achievement test for the second year survey (Prototype)

[Face Sheet]

Mathematics Achievement Test 2005

a) The result which you achieve in this test will never affect your grade in school. Feel relaxed and never cheat.
Read the questions carefully and answer what you think is the best.
b) The time allocation is 40 minutes.
c) Some questions are multiple-choice type, and others are short answer type.
All the questions have to be answered with clear writing.
d) Never use any calculator or multiplication table for your calculation.
You can use a ruler, if necessary.
e) You have to write your process to get the answer in the space of each question.

Your School Name: ....

Your Grade: ....

Sex (Please circle either): Male or Female

Age: .... years old

Your Name: ....

Q1. Work out the following calculations.

(1) $\frac{1}{5} + \frac{2}{5} =$

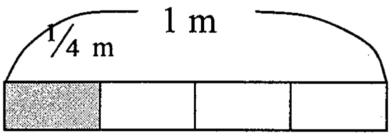
(2) $\frac{5}{7} - \frac{2}{7} =$

Q2. Express the answer in a fraction.

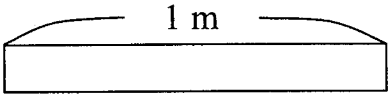
3 ÷ 7 =

Q3. The following figures show 1m of a bar. Shade a part of the figure to represent the following length.

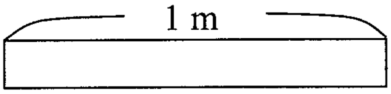
(Example) $\frac{1}{4}$ m



(1) $\frac{3}{4}$ m

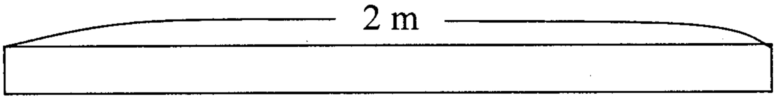


(2) $\frac{1}{3}$ m

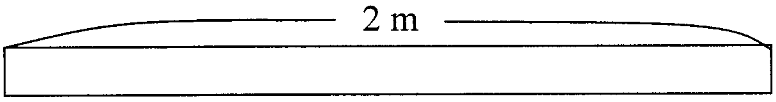


Q4. The following figures show 2m of a bar. Shade a part of the figure to represent the following length.

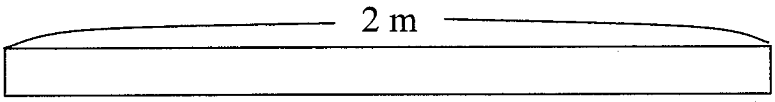
(1) $\frac{1}{2}$ of 2 m



(2) $\frac{1}{2}$ m



(3) $1\frac{1}{2}$ m



Q5. Which of the following is greater than the other? Write a correct choice (1, 2 or 3) in the bracket.

1. A is bigger than B.
2. B is bigger than A.
3. A = B

(1) A: $\frac{2}{5}$ B: $\frac{4}{5}$ Answer: ( )

(2) A: $\frac{3}{6}$ B: $\frac{1}{2}$ Answer: ( )

(3) A: $\frac{1}{4}$ B: $\frac{1}{3}$ Answer: ( )

Q6. Answer the following questions.
(Show your process to get the correct answer.)

(1) When you add $\frac{2}{5}$ ℓ of water to $\frac{1}{5}$ ℓ of water in a container,

how much water is in the container?

(2) When you cut $\frac{2}{7}$ m away from $\frac{5}{7}$ m of a string,

what is the remaining length?

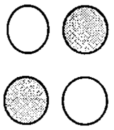
(3) When you arrange 6 pieces of $\frac{1}{6}$ m paper in a line, what is the total length?

Q7. Change the decimal number " 0.7 " to a fraction.
(Show your process to get the answer.)

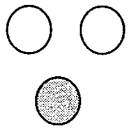
Q8. Judy bought 5kg of meat. George bought 3kg of meat. Write a fraction in the bracket to show the relationship between Judy's meat and George's meat in terms of weight.
George's meat is (   ) of Judy's meat.

Q9. Which of the following represents "half" *for you*? Circle all the possible choice(s) in which "half" of circle(s) is shaded.

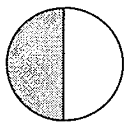
1.



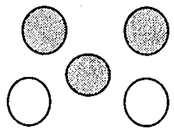
2.



3.



4.



Q10. Imagine and make a sentence problem in which the answer is $\frac{2}{3}$.

(You can express your sentence problem in words and/or any diagrams.)

Thank You Very Much for Your Cooperation!
The Graduate School for International Development and Cooperation,
Hiroshima University
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4. Interview items for the second year survey

[Interview Items for Pupils]

Select ten pupils on the basis of their latest end of term exam about mathematics. They should consist of the top five pupils who usually get a good grade in mathematics and the bottom five who are not good at mathematics.

Pupil's Name: ______________________________

Item 1: Inquire what choice they selected and why in Q5 (3).

i) Ask them to read to you the sentence of Q5 (3). Can they correctly read it?
YES NO

ii) If there is any difficult term/expression for them to read, describe it below.

iii) Ask them to explain how they solve the question in words and/or any diagrams.

iv) Describe any specific mistakes, if there are any.

Item 2: Inquire how they solved Q6 (1).

i) Ask them to read to you the sentence of Q6 (1). Can they correctly read it?
YES NO

ii) If there is any difficult term/expression for them to read, describe below.

iii) Ask them to explain how they solved the question in words and/or any diagrams.

iv) Ask them to write their numerical expression to find the answer for Q6 (1).

v) Ask them to calculate the numerical expression. Can they correctly calculate it?
YES NO

vi) Describe any specific mistakes in the calculation, if there are any.

Item 3: Inquire how they solved the Q8.

i) Ask them to read to you the sentence of Q8. Can they correctly read it?
YES NO

ii) If there is any difficult term/expression for them to read, describe it below.

iii) Ask them to explain how they solve the question in words and/or any diagrams.

iv) Describe any specific mistakes, if there are any.