International Cooperation Project Towards the Endogenous Development of Mathematics Education

東南・南アジア地域における小学校教師の持つ数学教育観が授業に与える影響の比較研究

（International Comparative Studies on Influence of Teachers' Views about Education on Mathematics Lessons at Primary Schools）

平成１６年度～１８年度科学研究費補助金
基盤研究（B）
課題番号　１６４０２０４５

研究成果報告書

平成１９年３月

研究代表者　馬場卓也
（広島大学　大学院国際協力研究科助教授）

はじめに

本報告書は、平成 16 年度～18 年度文部科学省科学研究費補助金(基盤研究 (B))による「東南・南アジア地域における小学校教師の持つ数学教育観が授業に与える影響の比較研究」の研究成果を取りまとめたものである。

国際教育協力の中でも理数科教育のそれは比較的経験が累積されている分野であるが、従来はカリキュラム開発に力点が置かれてきた。その活動の成果は教材、報告書などの形で見えやすかったものの、さらにその先の成果については必ずしも明確ではなかった。それに対して、現在の教育協力実践では、教室レベルでの変化が見られるように、教員教育に力点が移りつつある。このような状況を踏まえ、本研究では、教員や教授の文化的側面に注目し、数学教育協力の理論化に向けた基盤作りを目指している。

タイトルに含まれている「比較」について、本研究において厳密な数量的比較を目指すものではない。むしろ各国の数学教育の特徴を、質的な比較によって明らかにしようとしている。この質的特徴づけは、各国の専門家が数学教育開発を考えていく上で、基礎的資料となる。彼らは、各社会の伝統や文化と外的なモデルの双方に精通し、前者に適合するように後者を修正したり、再構成したりする。つまり内発的発展(鶴見、川田,1988)におけるキーパーソンの役割を果たす。さらに国境を越えた研究者集団の形成は、新しい教育開発協力のあり方を提示するものでもある。

本研究には、内外の大学・研究機関から多くの研究者が参加し、色々な形で本報告書の作成に貢献した。必ずしも全ての意見を反映できたわけではないが、最大限参考し研究の枠組みを構成した。また本研究を遂行する過程で、現地調査やワークショップの開催など、研究室の学生諸君にも様々な形で支援を頂いた。この場を借りて深甚なる謝意を表したい。

2007 年 3 月

広島大学大学院国際協力研究科
研究代表者　馬場卓也

【研究組織】(所属先 2007 年 3 月現在)

研究代表者　馬場卓也（広島大学・大学院国際協力研究科・助教授）

研究分担者　岩崎秀樹（広島大学・大学院教育学研究科・教授）

磯田正美（筑波大学・教育開発国際協力研究センター・助教授）

二宮裕之（埼玉大学・教育学部・助教授）

研究協力者　Alan J. Bishop (Professor Emeritus, Monash University, Australia)

Wee Tiong Seah (Lecturer, Monash University, Australia)

Norma Presmeg (Professor, Illinois State University, USA)

Dai Quin (Professor, Inner Mongolia Normal University, China)

Milagros D. Ibe (Professor Emeritus, University of the Philippines-Diliman, Dean, Graduate Studies, Miriam College Foundation, Philippines)

Maitree Inprasitha (Associate Professor, Khon Kaen University, Thailand)

Uddin MD Mohsin (Assistant Professor, Open University, Bangladesh)

A.H.M. Mohiuddin (Specialist, NAPE (The National Academy for Primary Education), Bangladesh)

J. Ghartey Ampiah (Lecturer, University of Cape Coast, Ghana)

Ernest Davis Kofi (Lecturer, University of Cape Coast, Ghana)

Narh Francis Kwabla (Tutor, Mount Mary Teachers' Training College, Ghana)

Bentry Nkhata (Lecturer, University of Zambia, Zambia)

加藤雅春(広島大学・大学院国際協力研究科・研究員)

桑山尚司(広島大学・大学院教育学研究科・助手)

植田敦三(広島大学・大学院教育学研究科・教授)

小原豊(鳴門教育大学・助教授)

丸山英樹(国立教育政策研究所・研究員)

中村聡(広島大学・大学院国際協力研究科・研究員)

金康彪(広島大学・大学院国際協力研究科博士号取得)

内田豊海(広島大学・大学院国際協力研究科・大学院生)

木根主税(広島大学・大学院国際協力研究科・大学院生)

小坂法美(広島大学・大学院国際協力研究科・大学院生)

Levi Elipane (埼玉大学・大学院協教育学研究科・大学院生)

【交付決定額(配分額)】

	直接経費	間接経費	合計
平成１６年度	2,300,000	0	2,300,000
平成１７年度	1,700,000	0	1,700,000
平成１８年度	1,800,000	0	1,800,000
総計	5,800,000	0	5,800,000

【研究発表】

1. 学会誌等

Mohsin, U. MD, Baba, T., "Analysis of Primary Mathematics in Bangladesh from Pupils' and Teachers' Perspectives - Focusing on Fraction", *International Journal of Curriculum Development and Practice*, 9(1), 2007, pp.37-53.

Baba,T., Iwasaki,H. "Intersection of Critical Mathematics Education and Ethnic Mathematics", *Journal of Inner Mongolia Normal University* 19, 2006, pp.77-84.

デイビス,E.K., 馬場卓也「The impact of In-service Teacher Training Through an Outreach Program on the Content Knowledge of Basic School mathematics Teachers in Ghana」, 『数学教育学研究』11, 2005 年, pp. 241-257.

馬場卓也、中村聡「バングラデシュ国初等理数科における教授的力量の評価枠組み構築に向けた授業の立体的考察」『国際教育協力論集』8(2),2005 年,pp.63-74.

2. 学会発表

中村聡、加藤雅春、馬場卓也「教育の質的側面の改善に対する授業分析の可能性について(II)ー授業の立体的分析による教授学習過程の記述可能性と課題ー」国際開発学会第 15 回全国大会、2005 年 11 月 26・27 日、神戸大学.

内田豊海、中村聡、馬場卓也「ザンビア基礎学校における生徒の分数概念獲得の到達段階について」国際開発学会第 17 回全国大会、2006 年 11 月 24・25 日、東京大学.

木根主税(2007)「開発途上国の数学教育における学習教授軌道の考察ーオランダ TAL プロジェクトと南アフリカ共和国 COCA プロジェクトの比較を通してー」全国数学教育学会第 25 回研究発表会,奈良教育大学.

内田豊海(2006)「国際教育協力としての教員センターに関する研究—ザンビアの事例を通してー」,第 42 回日本比較教育学会全国大会,広島大学.

金康彪(2005)「近代における日本と中国の数学教育交流に関する考察—書物の輸入による交流を通して」アジア教育史学会.

3. 本研究に関連する他の論文等

岩崎秀樹・山口武志(2005)「一般化分岐モデルに基づく教授単元の計画・実施・評価に関する研究」,『日本科学教育学会会誌 科学教育研究』, 29(2), 2005 年, pp.133-145.

清水静海、磯田正美、大久保和義、馬場卓也 監修(2005)『算数・数学授業研究』, 明治図書.

二宮裕之(2005)「算数・数学学習の評価に関する新たな視点」『日本数学教育学会誌』第87巻第8号, pp.13-20.

丸山英樹(2006)「トルコの教育改革ー欧州水準を目指した量的拡大と世俗主義維持の機能ー」,『国立教育政策研究所紀要第 135 集』, pp.137-151.

Maruyama, H., "Dynamics around Educational Research towards International Development Cooperation - Research Approach for All Lifelong Learners", Sasai, H. &

Maruyama, H. (Ed.), *Development Cooperation and Academism in Non-Formal Education - Sweden and Germany as case studies*, Study Group of International Cooperation in Non-Formal Education, Asia/Pacific Cultural Centre for UNESCO, 2005, pp.71-83.

桑山尚司・岩崎秀樹・馬場卓也，「東北タイにおける珠算による算数教育協力の現状と課題」，『日本教科教育学会誌』,26(4), 2004, pp. 29-38.

Uddin, M.M. "The Impact of In-service Teacher Training by Primary Training Institutes in Bangladesh (1): Focusing on Subject Knowledge, Pedagogical Skills and Attitudes of Mathematics Teachers" *Journal of Japan Academic Society of Mathematics Education*, 12, 2006, pp.201-214.

Praktipong, N, Nakamura, S., "Analysis of Mathematics Performance of Grade Five Students in Thailand Using Newman Procedure", *Journal of International Cooperation in Education*,9(1), 2006, pp.111-122

加藤 雅春、馬場 卓也、池田 秀雄「日本の大学（院）と JICA との連携に関する一考察～広島大学－JICA 連携特別教育プログラム制度の事例を踏まえて～」国際開発学会第 17 回全国大会、2006 年 11 月 24・25 日、東京大学.

中村聡「発展途上国教師教育分野における国際協力へ向けての基礎的考察(I)－ザンビア国 2 高等学校における理科・数学科教師の教授的力量観とその実践に関する事例を基に－」日本教師教育学会、山梨大学.

金康彪「近代中国の数学教育における日本の影響に関する研究(2)-算術教授法の導入を中心に」『全国数学教育学会誌数学教育学研究』, 11, 2005, pp.217-224 .

4. 国際協力シンポジウム 2007 にて研究成果を公表

2007 年 1 月 14 日　JICA 国際協力総合研修所国際会議場

Part I

Preface

Firstly I would like to congratulate all those people involved in this project – whether they have been directing it, operating it, participating in it, supporting it or have been involved in any other way. A developmental enterprise of this nature requires a great deal of energy on the part of many people. This is always the case whatever the size and scale of the educational endeavour.

This report is about the creation and execution of a developmental project in mathematics education involving six countries, each growing in its different way. This is clearly an enormous undertaking, and one can only admire the imagination and organisational abilities of those who have been responsible for it. To engage in development work in one country is complex enough, but to do this with six countries simultaneously demands our admiration.

Even more than this is the highly praiseworthy subscription to the concept and ethic of endogenous development, a process which has various significant characteristics. In relation to this educational project, these would include the local determination of the development choices and local control over the development process. Also endogenous development respects local values whilst exogenous development tends to ignore or subjugate them. Finally, endogenous development is founded mainly on locally available resources, and good schemes can revitalise and dynamise local resources which might otherwise be ignored or dismissed by others as being of little value.

In this report we can read about some of the special local features and history of mathematics education in the different countries, the questionnaire and test results from the students there, and some of the data coming from the videos of the classroom teaching. Whilst comparisons are made, and similarities and differences noted, there is no attempt to produce 'league tables' of 'winners' and 'losers', as happens in some international comparisons.

In an endogenous project, by contrast, the aim is by comparing situations, to identify specific problem areas, and to search for strengths on which the local development can build. By placing the research work in the hands of both local collaborators and research students from those countries, the project has sought to gain the best of both worlds. That is, the research students are in touch with the latest ideas in both research methods and mathematics education, while their local counterparts are in continuous contact with the local situation. Bringing these two together is the major strength and innovative part of this project.

As a Visiting Professor here, I am very honoured to be able to contribute in a small way to this project. I wish success to all those involved in it, and look forward to reading later reports of this promising developmental work.

Alan J. Bishop
Emeritus Professor
Centre for Science, Mathematics and Technology Education
Faculty of Education
Monash University
March 2007.

Table of Contents

Part Ⅰ

Part Ⅱ

Chapter 1

Research Background and Methodology

Chapter 1 Research Background and Methodology

1.1 Research Background and Objective

1.1.1 Background

Since the World Conference on Education for All at Jomtien, Thailand in 1990, international organizations and individual countries have been seriously engaged in promoting the participation of all children into school education and improving its quality. The beginning of this century is witnessing stronger as well as a greater number of initiatives toward this end, for example, Millennium Development Goals, UN Literacy Decade, and UN Decade for Sustainable Development..

As mentioned in the EFA Monitoring Report (2005), the attainment of EFA without quality education is a hollow victory. Although it is easy to recognize the importance of quality, the involvement of several interrelated factors and interpretations make it difficult to define and realize quality. We, however, are the agents that ensure quality education for all.

In this global context, since 2004, our research group has been engaged in identifying the status of mathematics education from the viewpoints of children as well as teachers in the participating countries through collaborative and joint research. In addition, we are developing a framework that can grasp the sociocultural aspect of mathematics education in these countries.

1.1.2 Endogenous Development

In this framework, we attach importance to the concept of "endogenous development" (Tsurumi, Kawata, 1989). Here, endogenous development implies development that is based on the following four conditions: (1) satisfaction of basic human needs, (2) cooperation within the community, (3) harmony with natural environment in the community, and (4) social change initiated within the community. Additionally, endogenous development is against the modernization theory, according to which a developing country merely follows the path of the developed countries in the process of modernizing society.

Subsequent to the adoption of such a stance of "endogenous-ness" in education development, we are confronted with many questions. Can we only consider basic human needs and not technical advancement? What is a community and how can we attain harmony with the natural environment in mathematics education development? Eventually, what are the kind of implications that we can deduce from this concept?

In this research, we do not discuss the definition of a community in depth because it involves multiple dimensions such as linguistic, geographical, and ethnical; moreover, a discussion on the definition of a community will require another volume. Thus, in some cases, we simply consider each participating country as a community, while in some other cases, we regard language-based ethnic groups as communities. We adopt this stance for the following three reasons.

(1) Historical development of mathematics education

The modernization movement in mathematics education during the 1960s in the USA influenced many countries in the world subsequently. Even after it was regarded as a failure in the USA, many other countries followed a similar path in some way or the other. During this period, many curriculum experts were dispatched to developing countries to teach the so-called modern curriculum.

This event is a pointer to some important issues such as the body that should take initiatives in curriculum development and the concept that the founding theory or philosophy should be based on. Although these questions are not easy to answer, we at least need to consider them before we begin the meaningful.

(2) Theoretical development of mathematics education

The 1980s witnessed the advent of a new research movement that focused on the sociocultural aspect of mathematics education. D'Ambrosio (1984) pointed out that, apart from formal mathematics that is learned in school, there exist mathematical activities within a culture, and during the ICME 5 conference, he coined a new word "ethnomathematics" that stimulated many researches in this field. On the other hand, Bishop (1991) pointed out that there are six universal mathematical activities, namely, counting, measuring, locating, designing, explaining, and playing.

Later, this report presents the arguments of Bishop and Presmeg on theoretical development with regard to this aspect of mathematics education.

(3) Agents that bring about change in mathematics education

Recently, there have been two important movements in educational development at large: international cooperative studies, such as TIMSS and PISA, and the globalization of university education through information technology.

Indeed, we do not, however, intend to completely exclude external influence. As Tsurumi expressed, through endogenous development and based on its own culture and tradition, a society modifies and recreates external models such that the change suits its own conditions (p.4). Therefore, it is very important to consider the relationship between external and internal factors. Further, it is with regard to this point that local experts play an important role as they are the agents who effect change in mathematics education in their respective countries.

Based on the abovementioned reasons, we need to at least start thinking about curriculum development, the type of mathematical activities, and the role of teachers and researchers. These topics are interrelated and constitute one whole when considered in the context of endogenous development of mathematics education. In this report, particularly, we pay considerable attention to teachers.

1.1.3 Objective

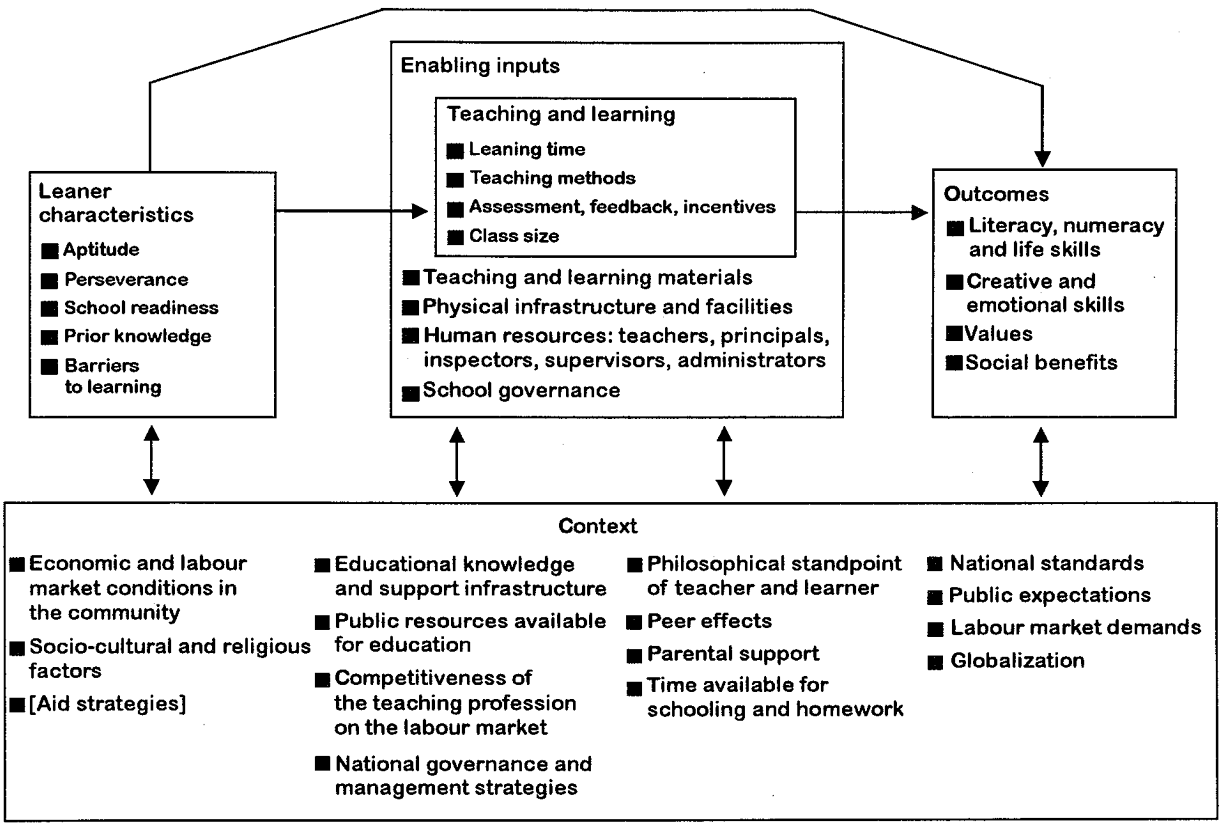
With this background, our research group has conducted an international comparative study for the period 2004–2006. Here, a comparative study does not imply a robust and quantitative comparison of countries to observe which one performs better. Instead, our analysis centers more on a qualitative comparison, where we characterize mathematics education in each country.

This characterization is utilized when the development of mathematics education is considered by local experts. In this sense, local experts play a crucial role in modifying and recreating external models based on their own culture and tradition such that the change suits the conditions prevalent in their society. Hence, it is important to grasp both external models and internal conditions thoroughly. It is in this regard that we can gain a deeper understanding of ourselves through a qualitative comparison of the participating countries. For this purpose, a reliable relationship among the local experts across various countries is extremely essential.

Thus, the ultimate goal of this project is to establish a professional and long-term relationship or network among the participating countries in the field of mathematics education, particularly with respect to curriculum development and teacher education.

As revealed in the following figure (UNESCO, 2005), the teaching and learning process is at the center of the complete framework for education quality. Therefore, it is extremely imperative to understand the status of teaching and learning before attempting to improve the quality of education, which has been a major task since the 1990 conference.

Figure 1.1: A framework for understanding education quality



Given the above, the following are the specific objectives of this project:

1. To compare and analyze the intended curriculum and the present status of mathematics classrooms in the participating countries
2. To exchange the views and information obtained on the issues revealed by (1), and to seek for alternative models for developing the mathematics classrooms through this process
3. Finally, to establish a professional relationship through this process of knowledge sharing among researchers in the participating countries for the endogenous development of mathematics education

1.2 Research Members

The list of the research project members both in Japan and overseas is provided below. Apart from the members on the list, there are three special advisors—renowned professors in this field—whose valuable contributions added another dimension to this report.

Members in Japan (as of March 2007)

NAME	Affiliation	Position	Role
Takuya Baba	IDEC, Hiroshima University	Associate Professor	Overall manager, Bangladesh
Masaharu Kato	IDEC, Hiroshima University	Researcher	Coordinator
Hisashi Kuwayama	Faculty of Education, Hiroshima University	Research Associate	Coodinator, Ghana
Hideki Iwasaki	IDEC, Hiroshima University	Professor	China
Atsumi Ueda	Faculty of Education, Hiroshima University	Professor	China
Masami Isoda	CREICED, Tsukuba University	Associate Professor	Thailand
Yutaka Ohara	Naruto University of Education	Associate Professor	Thailand
Hiroyuki Ninomiya	Faculty of Education, Saitama University	Associate Professor	Philippines
Hideki Maruyama	National Institute of Educational Policy Research	Researcher	Development of research tools, Ghana
Satoshi Nakamura	IDEC, Hiroshima University	Researcher	Zambia
Uddin MD Mohsin	Open University	Assistant Professor	Bangladesh
Toyomi Uchida	IDEC, Hiroshima University	Graduate Student	Zambia
Chikara Kinone	IDEC, Hiroshima University	Graduate Student	Japan
Narh Francis Kwabla	Mount Mary Teachers' Training College	Tutor	Ghana
Levi Elipane	Saitama University	Graduate Student	Philippines

Overseas members (as of March 2007)

NAME	Affiliation	Position	Role
Dai Quin	Inner Mongolia Normal University	Professor	China
Kang Biao Jin	IDEC, Hiroshima University	Graduate	China
Milagros D. Ibe	University of the Philippines-Diliman	Professor Emeritus, Dean, Graduate Studies, Miriam College Foundation	Philippines
Maitree Inprasitha	Khon Kaen University	Associate Professor	Thailand
A.H.M. Mohiuddin	NAPE (The National Academy for Primary Education)	Specialist	Bangladesh
J. Ghartey Ampiah	University of Cape Coast	Lecturer	Ghana
Ernest Davis Kofi	University of Cape Coast	Lecturer	Ghana
Bentry Nkhata	University of Zambia	Lecturer	Zambia

Special advisor (as of March 2007)

NAME	Affiliation	Position	Country
Alan J. Bishop	Monash University	Professor Emeritus	Australia
Wee Tiong Seah	Monash University	Lecturer	Australia
Norma Presmeg	Illinois University	Professor	USA

1.3 Contents of the Report

This report consists of four sections, namely, Research background and Methodology, Special Contribution, Country Report, and Discussion and Future Issues. In Special Contribution, research trends and issues have been intensively discussed.

With regard to the objectives, Country Report is the primary section of this report. It presents the data in the format that conforms to our prior agreement, which is as follows:

(Composition of Country Report)

History of Mathematics Education

Basic Information

- Schooling Age
- School Calendar and Examination
- Promotion System
- Medium of Instruction
- Class Organization
- Period of Survey
- Target Schools

Results from the First Year Field Survey

- Results of the Teachers' Interviews

Results from the Second Year Field Survey

- Results from the Teachers' Responses to the Questionnaire
- Results of the Teachers' Interviews

Discussion

Country Report first presents the basic data, such as teachers' qualifications and transfer systems, that we believe have influenced the quality of teaching. Subsequently, the results, which present the data collected through the field survey, and the discussion, which outlines some points of interest, are presented.

Discussion and Future Issues constitutes a very important part of this research. It provides a comprehensive overview of the entire research and presents some important issues for future activities pertaining to quality education. This cycle of projection, research (field survey), and discussion provides us with a foundation for our ultimate goal of the endogenous development of mathematics education.

Apart from these, some past researches have been reviewed and summarized in order to strengthen the discussion.

1.4 Research Methodology

1.4.1 Target Schools

Average schools from both urban and rural areas were chosen after consultation with local experts. Basically, we conducted the field survey in the same schools (for 2004 and 2005). In some countries, where language problem appeared to influence the result, an additional comparative survey was conducted (e.g., no explanation vs. explanation in local language).

1.4.2 Administration Procedure of the Questionnaire

It was necessary for the teachers to complete the questionnaire during the implementation of the Mathematics Achievement Test and the Interview targeted at the pupils.

Although the teachers' names were not mandatory in the questionnaire, researchers were expected to be aware of their names through their informal conversations and document the same. With respect to certain questions regarding the pupils in the questionnaire, the teachers were allowed to answer only those concerning their own pupils, and not pupils in the country as a whole.

After the completion of the questionnaire, the results were collected and examined by a collaborator and co-researcher from Hiroshima University. Further, the collaborator and co-researcher were asked for clarifications in case of any unclear description/writing in the results.

1.5 Plan of Activity

FIRST YEAR: To conduct a preliminary study in order to generally describe the status of mathematics education and its problems in each participating country.

(1) To contact the local expert

The following countries were selected: Ghana and Zambia in Africa; Bangladesh in South Asia; Myanmar, Thailand, and Philippines in Southeast Asia; and China in East Asia. In each country, we selected a curriculum developer or a teacher trainer. Apart from this, we presupposed that everybody could be contacted by email.

(2) To discuss the research methodology

We provided the first draft plan for methodology and then, for further refinement, discussed it with the local experts. Although, on the whole, we desired to describe the general features of mathematics education in each country, a comparative evaluation of the same was not our intention. We analyzed the points of similarity and difference. Further, in order to describe the average status of each country, we compared contrasting entities, e.g., urban vs. rural.

(Supplementary information: An "average" school in Bangladesh refers to a middle ranked school according to their ranking system in terms of enrolment rate, dropout rate, PTA, etc.)

(3) To conduct the preliminary study

We requested each local expert to conduct the preliminary study. They were allocated a budget, which enabled them to conduct the preliminary study, along with video shooting, translation of the questionnaires, execution of the survey, and its analysis.

(4) To have collaboration between Japanese and overseas members

The Japanese members visited different countries for collaboration and went on site to confirm the data collected.

(5) International workshop

An international workshop was held to discuss and share information and problems and to plan for the following year.

(6) Compilation of the report of the first year study

The results of the first year preliminary study were compiled as a report.

	2004				**2005**												**2006**		
	9	**10**	**11**	**12**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**
Preliminary Study3) & 4)		—	—	—	—	→													
Workshop 5)							★									★			
Report 6)									●										●

SECOND YEAR: To conduct the main study and to analyze and discuss the findings.

(1) To discuss the research methodology

Based on the discussion during the first international workshop, during the second year, the Japanese experts developed a draft of research tools and discussed it with the local experts with regard to the main study.

(2) To conduct the main study

The local and Japanese experts conducted the main study. The Japanese data was also collected for the discussion of the first year.

(3) International workshop

The workshop discussed and shared information and the results of the research.

(4) Compilation of the report of the second year study

THIRD YEAR: To write a summary report of the three-year activity and to plan for the next three years.

(1) To conduct a complementary study

If necessary, a complementary study will be conducted.

(2) To conduct an international seminar in Tokyo, Japan

(3) Compilation of the final report

References

D'Ambrosio, U. (1984), Socio-Cultural Bases for Mathematical Education, *Proceedings of 5th ICME*, Adelaide, Australia, 1-6.

Bishop, A.J. (1991), *Mathematical Enculturation: A Cultural Perspective on Mathematics Education*, Kluwer Academic Publishers.

Bishop, A.J. (1994), Cultural Conflict in Mathematics Education: Developing a Research Agenda, *For the Learning of Mathematics*, 14(2), 15-18.

Gerdes, P. (1985), Conditions and Strategies for Emancipatory Mathematics Education in Undeveloped Countries, *For the Learning of Mathematics* 5(1), 15-20.

Tsurumi, K. & Kawata, T. edits (1989) *Endogenous development*, Tokyo University Press (in Japanese).

UNESCO (2005) *EFA Monitoring Report*, Paris.

Chapter 2

Country Report

Chapter 2 Country Report

2.1 China

Dai Qin	Kang Biao Jin	Hideki Iwasaki	Atsumi Ueda
Inner Mongolia Normal University	Hiroshima University	Hiroshima University	Hiroshima University

2.1.1 History of Mathematics Education

(1) A summary of the ancient and modern history of mathematics education

It is not easy to summarize the history of mathematics education in China since it has more than 2500 years of important history. Therefore, this paper discusses only those transitions that are historically important. The transitions are as follows: mathematics education described in Liu Yi, mathematics education included in Jiu Zhang Suan Shu, mathematics education advocated in the Sui and Tang periods and its influence on foreign countries, mathematics education supported in Xi Cao Gang Mu that was written by Yang Hui (a mathematician in the Sung period), mathematics education recommended in Saun Fa Tong Zong in the Ming period, mathematics education supported in Euclid's Elements, the restructuring of mathematics education in the late Qing period, and mathematics education practiced in the Ming period.

1) Mathematics education described in Liu Yi

The number and numeration systems, or the ways of expressing numbers, were known to the Chinese even before the Zhou period (i.e., before BC 1100). It is believed that during this period, people engaged in mathematics education by word of mouth. The following were the two characteristics of mathematics education practiced during the Zhou period: (1) mathematics education was not regarded as education per se and (2) mathematical knowledge was possessed only by a few people, such as fortune-tellers, and it is they who played the role of passing the knowledge down to the next generation. Thus, mathematical knowledge was treated as a mysterious matter.

According to Liu Yi that existed in the Zhou period, general education consisted of Li (politeness), Yue (music), She (archery), Yu (manipulation of carriages), Shu (penmanship), and Shu (calculation). Shu was also known as Jiu Shu (nine branches of mathematics). Jiu Shu was the same as arithmetic. During the Chun Qiu Zhan Guo period, from BC 770 to BC 221, in line with the development of mathematics itself, educational contents in mathematics began to include more than just numbers and simple calculations. During the Han period, Jiu Shu represented all the mathematical knowledge possessed by the Chinese.

2) Mathematics education included in Jiu Zhang Suan Shu (nine chapters of arithmetic)

Jiu Zhang Suan Shu (nine chapters of arithmetic) was written around BC 100 and included nine chapters that were in accordance with the nature of mathematics, dealing with a total of 246 practical problems. Each chapter was published and made into a volume. The following are the chapters and their main contents.

Chapter 1 「Fang Tian」 Square measure of plane figures and calculation of fractions

Chapter 2 「Su mi」 Proportions

Chapter 3 「Cui Fen」 Proportional distributions

Chapter 4 「Shao Guang」 Extraction of square roots

Chapter 5 「Shang Gong」 Calculation of volumes

Chapter 6 「Jun Shu」 Taxes and transportation

Chapter 7 「Ying Bu Zu」 Solutions to problems related to more than sufficient and problems of proportions

Chapter 8 「Fang Cheng」 Solutions to applied problems of simultaneous linear equations with multivariables

Chapter 9 「Gou Gu」 Applied problems of the Chinese Pythagorean theorem

Since the East Han period, Jiu Zhang Suan Shu played a unique educational role for 500 years and produced several distinguished mathematicians such as Liu Hong, Liu Hui, Zu Zhong Zhi, and Wang Xiao. For 500 years, mathematics education was, as it were, done by private sector and had the following characteristics.

First, practicality was an important aspect of mathematics education. Second, mathematics education provided enlightenment. Third, both the concepts of self-learning and learning from a mentor were respected in mathematics education. Fourth, Jiu Zhang Suan Shu came to be accepted as scripture and was also referred to as The Bible.

3) Mathematics education advocated in the Sui and Tang periods

In the Sui period, public education was revived and Guo Zi Xue, an institution, was established. This was the first institution that offered a course in mathematics. This course involved two doctors in mathematics, two assistance lecturers, and eighty students. The Tang period witnessed the establishment of Ming Suan Ke in Guo Zi Jian (earlier known as Guo Zi Xue) and ordinary people were allowed admission into it. Usually, the education system of Ming Suan Ke required seven years of learning. The following are some of the textbooks that were used at the time in Ming Suan Ke: Shi Bu Suan Jing, Jiu Zhang Suan Sho, Hai Dao Suan Jing, Suan Jing, Xia Hou Yang Suan Jing, Zhou Bi Suan Jing, Wu Jing Suan Shu, Zhui Shu, and Ji Gu Suan Jing. In all, Ming Suan Ke consisted of ten to thirty students.

Mathematics education in the Tang period greatly influenced Japan's education at the time. For instance, it is clear that yoro rei, the mathematics education system of Japan at the time, had borrowed the mathematics education system that was practiced during the Tang period.

4) Education supported in the Sung period

Although public mathematics education, which had existed for 300 years, was observed in the Sung period, it did not continue for a long time. Shi Bu Suan Jing, which was once used during the Tang period, was the mathematics textbook that was used during the Sung period.

It is significantly important to examine Xi Cao Gang Mu while discussing mathematics education in the Sung period Xi Cao Gang Mu was written by Yang Hui, an eminent mathematician, and is equivalent to a contemporary course in mathematics. Xi Cao Gang Mu was presented in a book entitled Chen Chu Tong Bian Ben Mo and included learning contents, scheme of work, major textbooks, methodologies, and important aspects of learning. This is the literature that stands out in the history of mathematics education and is regarded as China's first course of study in mathematics. While some Chinese scholars strongly support their contention that Xi Cao Gang Mu is the world's first course of study in mathematics, this contention is not necessarily valid.

5) Mathematics education recommended in the Jin, Yuan, and Ming periods

Although public mathematics education did not exist in the Jin period, private sectors are believed to have been involved in mathematics education. For example, Li Ye and some other mathematicians accepted pupils and taught them mathematics.

The emperors of Mongolia—Yuan, Monke, and Hubirai—admired mathematics. Monke learnt Euclid's Elements, and Hubirai had a mathematician teach his son mathematics. Those days, emperors demanded that bureaucrats make their pupils learn mathematics the same way as the Han Chinese did. During the Yuan period, even lower bureaucrats were expected to acquire some mathematical knowledge.

At this point, it is important to mention Zhu Shi Jie, a mathematician and mathematics teacher from the thirteenth and fourteenth centuries. Later on, his book "Suan Xue Qi Meng" had a great impact on the development of mathematics in Japan.

From 1369 onward, the Ming government expanded courses in mathematics education to the local cities and towns of the country. While this government revised the teaching contents and examinations of mathematics in 1692, it suspended public mathematics education in 1393 due to a case in which a mathematics student was believed to be involved.

Private mathematics education was very active in the Ming period. Cheng Da Wei wrote the book entitled "Suan Fa Tong Zong"; this became an important textbook for private education, significantly influencing other countries, especially Japan.

Although some mathematical knowledge was introduced from foreign countries in the Sui, Tang, and Yuan periods, it only provided a preview of the impact that such mathematical knowledge would have on China. In light of this, it can be said that the mathematics education practiced in this period was unique to China.

Toward the end of the Ming period, West European mathematics was introduced in China. All the six volumes of Euclid's Elements were translated and calculation with figures and mathematical tools were imported. These volumes exerted a wide influence and greatly shocked the intellectuals of the period.

For instance, Xu Guang Qi expressed his admiration for Euclid's Elements. The following is what he felt while admiring Euclid's Elements: "It has four 'no needs', i.e., no need to doubt, no need to assume, no need to try, and no need to correct. It has four 'impossibles', i.e., impossible to shorten, impossible to go against, impossible to degrade, and impossible to interchange sequence. There are three extremes and three feasible things. First, it seems extremely cloudy at first glance but in fact is extremely clear. Second, it seems extremely complex at first glance but in fact is extremely simple. Thus, it can make other extremely complex things simple. Third, it seems extremely difficult at first glance but in fact is extremely easy; therefore, this easiness can make other extremely difficult things easy. Since easiness comes from 'simple', all things become clear."

In addition, Xu Guang Qi emphasized that mathematics was the base of natural science and that the learning of Confucian was closely related to mathematics.

6) Education in the Qing period

At the beginning of the Qing period, mathematics education included abacus calculation, calculation with figures, and Chou Suan. Private mathematics education was commonly practiced in families and cram schools. For example, the family of Mei Wen Ding, a mathematician, was famous for practicing private mathematics education.

Kang Xi, an emperor during the Qing period, had a great interest in mathematics; therefore, he provided some reports on mathematics after learning East European mathematics and after conducting a survey. He edited Shu Li Jing Yun (three volumes, 1721), and this literature eventually became the mathematics textbook that was used in the late Qing period.

In the middle of the Qing period, foreign missionaries got involved in the mathematics education of China.

Mathematics education became popular at the time of the Opium War (1840); prior to that, this education was practiced only in church schools. One of the textbooks used for mathematics education in churches was "Shu Xue Qi Meng" written by A Wylie (1836–1887). This textbook was also imported to Japan.

In 1862, Tong Wen Guan and Suan Xue Guan were established in Beijing. Li Shan Lan became the principal of the latter. Suan Xue Guan followed the eight-year education system and the mathematics textbooks that it used, such as Euclid's Elements, were borrowed from East European countries.

At the end of the nineteenth century, several mathematics magazines were published. Suan Xue Bao was published in 1898. In 1902, Du Ya Quan issued Zhong Wai Suan Bao in Shanghai. In the same year, the Qing government promulgated the Ren Yin education system」. This education system of mathematics is similar to the present one. The Gui Mao education system was established in 1904. The Ren Yin and Gui Mao education systems imitated Japan's implementation regulations for the secondary school order (Meiji 32). Almost all the textbooks of mathematics during the period were translations of Japanese mathematics textbooks.

7) Mathematics education practiced in the Ming period

China's mathematics entered a new era during the Ming dynasty. The Ren Yin-Gui Mao system, which came into effect in 1913, was influenced by the mathematics education of both Japan and Germany. In 1922, Xue Xiao Xi Tong Gai Ge Ling was established as a result of the influences of Dewey's educational philosophy and the education system of the United States. It copied the 6:3:3 system that the United States had at the time. Since 1922, the United States' influence had become clearer. The following were some important curricula established at the time.

The Secondary Mathematics Curriculum (temporary) (1929)

The Secondary Mathematics Curriculum (permanent) (1932)

The Secondary Mathematics Curriculum (revised) (1936)

The Secondary Mathematics Curriculum (2nd revised) (1936)

(2) The history of mathematics education after 1949

On October 1, 1949, the People's Republic of China was established. Since then, China's mathematics education has been restructured and the following measures have been taken.

1) The phase of learning from the U.S.S.R.

In the early 1950s, the mathematics education practiced in the U.S.S.R. exerted a considerable amount of influence on the system practiced in China. The latter education system adopted a model that was similar to that of the former education system. Thus, in order to grasp the historical situation of that period, let us consider some of the public opinions that existed at the time.

It seemed as if the Chinese people had lost all rationalities under the influence of the U.S.S.R. For instance, some top mathematicians at the time expressed their opinions as follows.

Hua Luo Geng, in his book "How did I learn from the U.S.S.R.," said the following: "The present situations of the U.S.S.R., such as the happy lives of the people and the establishment of communism, show us the future of the motherland. ... The future of the U.S.S.R. brings to us the beauty of communism. This is our dream. It is not possible to not admire and adore the U.S.S.R. where people are friends as well as teachers who share happiness and misery and where we can understand each other as brothers of the same blood."

In his book "The 30 years' celebration of the great October Revolution," Chen Jian Gong remarked that the "U.S.S.R. is a communism country, which is ahead the times and our teacher. We have to learn all the new experiences of the U.S.S.R."

Further, Su Bu Qing, in "Learning the new scientific experiences of the U.S.S.R.," stated, "As a scientist and an educator, I strongly insist on learning scientific and technical thoughts from the U.S.S.R."

Since their opinions and actions were influential, they had a great impact on the educational policy and thinking of the ordinary people. In addition, the mathematics courses for secondary schools in the U.S.S.R. were translated and published in The Mathematics Journal of China. This had a direct impact on the creation of the Chinese mathematics course for secondary schools in that period.

In March 1950, the Ministry of Education published the draft of a guide on mathematics teaching materials and submitted the following three plan principles. First, the mathematics teaching materials have to be practical; mathematics must be combined with science. Second, the excessively abstract and difficult mathematics teaching materials in use must be omitted. Third, it is decided that the early secondary school mathematics course will include arithmetic, algebra, and plane geometry, whereas the late secondary school mathematics course will include solid geometry, high school algebra, and analytical geometry.

In March 1951, the Ministry of Education approved the secondary mathematics curriculum and submitted its policy on the compilation of textbooks. According to the policy, "Each subject must remain perfectly scientific and must persist in nationalism education. In addition to that, textbooks must be compiled in light of the practical demands of China and in reference to the basis of the textbooks of the U.S.S.R."

The first draft of the secondary mathematics curriculum defined the goals of mathematics teaching as follows.

(1) Knowledge of mathematics: Calculation of number and quantity, three-dimensional figure, knowledge of the general relations between them

(2) Scientific practice: Preciseness of number and reasoning, encouraging students to practice observation, analysis, induction, judgment, inference, and so on, considerably well, and helping them to foster the spirit of inquiry

(3) Dialectic thought: Cultivating students' dialectic thought through teaching changes in quantity and quality

(4) Applied skills: Making students master mathematical tools such as mathematical terms, signs, theorems, formulae, and methods, drawing figures precisely, solving practical problems of figures and number and quantity by applying acquired knowledge

In 1951, the draft of the secondary mathematics curriculum was revised and the following were its goals.

(1) To help students learn basic mathematics knowledge, skills, and methods that are required to solve various kinds of practical problems.

(2) When teachers teach mathematics, they must persist in executing the general duties of the new democratic education. In other words, they should strive to construct a worldview of dialectic materialism, cultivate the new nationalism and self-esteem of the nation, and build the personalities of their students and also help them to develop a strong determination.

(3) Junior high mathematics is a systematic course that includes arithmetic, algebra, geometry, and trigonometry.

(4) In order to assist the students of primary junior high and advanced junior high schools in their future courses, it is imperative that mathematics teaching provide mathematical knowledge that is closely related to general education.

The following were the goals of teaching stated in the secondary mathematics curriculum (1955).

(1) To help students learn the basic knowledge of arithmetic, algebra, geometry, and trigonometry, cultivate skills and methods that are required to solve various kinds of practical problems, and advance their logic and three-dimensional imagination.

(2) To educate students about ideas regarding communism when they teach mathematics. (Further explanation is omitted.)

In 1955, for the first time in the history of mathematics education in China, the secondary mathematics curriculum included a request for advancing students' logic and three-dimensional imagination. Needless to say, this, too, was the result of adopting the system followed by the U.S.S.R.

In the phase of learning from the U.S.S.R. (1950–1958), China's secondary mathematics textbooks were almost mere translations of the textbooks from the U.S.S.R. After some years of learning from the educational experiences of the U.S.S.R., tackling educational reform, and improving the teaching methods, several problems were found to exist inside the school systems and teaching methods.

2) The phase of the great educational revolution (1958–1961)

The great educational revolution began almost at the same time as the great advancement (1958); this revolution criticized the mathematics curricula, the schemes of mathematics education, and the textbooks, all of which had been used nationally and commonly. Accordingly, local attempts included the shortening of school systems, simplification of the curricula, and self-production of teaching materials.

In 1958, the reformation of mathematics education indicated that the teaching materials of secondary mathematics were poorly prepared, out of date, impractical, and repetitive. Following this, a draft for modernizing the reformation of mathematics education was written along with several functions. The following clauses were included in the draft:

(1) Mathematics teaching must contribute to the modernization of production and advanced science and technology. Therefore, secondary mathematics is required to provide modern mathematics knowledge to some extent.

(2) Mathematics teaching materials must bear reference to the rigid and theoretical systems.

(3) The quantity and difficulty of mathematics teaching materials must reflect the level of learning and students' ability to recognize. For instance, the easy and concrete mathematical concepts must be introduced before the difficult and abstract ones.

The restructuring of mathematics education in 1958 reduced the contents of plane geometry. However, it had more functions and an increased number of contents pertaining to plane analytical geometry, which relates equations to functions and graphs. This restructuring continued to face some problems. The evaluation and judgment of traditional teaching materials were carried out without careful consideration. Moreover, theories and practical experiences were lacking in adequately strong grounds. Consequently, the draft for the modern restructuring of mathematics education was not used.

3) The phase of rigidity, coordination, and fulfillment

From 1961 onward, under the policy of rigidity, coordination, and fulfillment, the secondary mathematics curriculum was revised (1961, 1963). In 1963, the draft of mathematics curriculum for full-time schools was embraced for the first time with the following three abilities: the ability to calculate, the ability to think logically, and the ability to use three-dimensional imaging. In reality, these three abilities were subsequent to the influence of the U.S.S.R. As a result of this draft, mathematics textbooks were compiled for twelve years of primary and secondary education. It is said that these textbooks were the best since the establishment of The People's Republic of China. The increased contents fitted well in the context of China's situation and played an active and a highly original role in the development of China's mathematics education. Unfortunately, the never-before-seen Great Cultural Revolution followed soon after.

4) The phase of The Great Cultural Revolution (1966–1976)

The Great Cultural Revolution (1966–1976) ruined mathematics education in China. It caused a major reduction in the number of contents included in secondary mathematics education. It lowered the level of secondary mathematics education in China, thus putting an end to the development of mathematics education.

Conventional mathematics education was rejected at the time of The Great Cultural Revolution because it was considered to be a part of capitalism, feudalism, and modification. The publication of universal textbooks written before 1966 was prohibited. Publishing companies were made to shut down in 1966 and reopen in 1972. Prior to political education, the policy of reducing the years of learning was implemented again and subjects were either integrated or abolished. This put a stop to the nationwide schemes of teaching mathematics and the different courses offered in mathematics, giving rise to a strong opinion that only mathematics textbooks that are compiled locally should be used.

5) The phase of gradual improvement (1976–2001)

Beginning 1976, China entered a new era with four modernizing establishments in communism. In 1978, the Ministry of Education promulgated the trial draft of the secondary mathematics curriculum for full-time schools with ten years of learning. The bill introduced was specific since it referred to foreign countries' experiences in terms of modernizing the movement of mathematics education. The bill requested education that could help students to learn the basic knowledge and improve their mathematical abilities.

The following were the general goals of the trial draft for full-time schools with ten years of learning.

(1) To help students learn the basic mathematical knowledge that is required for learning modern-day science and technology and is needed for participating in the revolution and establishment of communism.

(2) To cultivate the abilities of making precise calculations, thinking logically, and using three-dimensional imaging to a certain degree, which eventually led to the abilities of problem analysis and problem solving.

(3) To carry out political education through mathematics teaching.

(4) To encourage the spirits of revolution that help to learn mathematics for four modernizations and to cultivate students' viewpoints of dialectic materialism.

The Ministry of Education in 1982 promulgated the opinionated draft of the secondary mathematics curriculum for full-time schools and six years of learning. The general goals were almost the same as the ones published in 1978. However, it can be said there was a slight difference between them, specifically in the statement regarding the problem-solving ability. For example, the opinionated draft of 1982 included the application of mathematics, analysis of problems, and the fostering of the ability of problem solving, but the trial draft of 1978 did not include the application of mathematics. In addition, the opinionated draft of 1982 aimed to cultivate students' scientific behavior and a worldview of dialectic materialism; however, the trial draft of 1978 did not include logical thinking and the perception of space.

In 1992, the National Board of Education promulgated the trial edition of the secondary mathematics curriculum for nine years of full-time compulsory education. Although there was no difference between the trial edition and the first edition in terms of the general goals, there were some fundamental differences. One of the differences was that mathematics education was changed from education that is oriented toward examination to education that develops capacities. This shows the appropriate direction of mathematics education; moreover, mathematics education was the result of the integration of pure science and human science. Meanwhile, the National Board of Education compiled six sets of textbooks that could cater for the local needs. By doing so, the National Board of Education attained the goal of one guideline with many books, which implied the use of many textbooks under one course. These reforms are historically important because reforms such as the goals of courses, contents, and systems were meaningfully implemented.

The Ministry of Education in 1996 promulgated the secondary mathematics curriculum on an experimental basis for full-time schools. After three years of experimentation, in 2000, it promulgated the revised edition of the secondary mathematics curriculum for full-time schools. One of the features of these curricula was that the ability of logical thinking, which had been adopted since 1963, was replaced by the ability of thinking. This is also a clear indication that mathematicians' awareness of the roles of mathematics education had improved. In reality, mathematics could cultivate not only logical thinking but also scientific behavior with an inquiry into the mind and creativity.

The Ministry of Education promulgated the standard version of the secondary mathematics curriculum for full-time compulsory education in 2001 and another standard version for secondary education in 2003. This reformation clarifies the reforms that will be implemented and the suggestions that will be made; China' s mathematics education values human development and emphasizes the nature of mathematics education that can cultivate good abilities in people; it presents a fundamental philosophy that organically integrates knowledge, skills, methodology, behavior, and value. The table below shows the significance of this reform.

Table 1. A comparison of the general goals of the secondary mathematics curricula of compulsory education in 1992 and 2001

Articles	The general goals of the trial edition of the secondary mathematics curriculum ((1992) for nine years of compulsory full-time education	The general goals of the standard version of the secondary mathematics curriculum (2001) for full-time compulsory education
1	To help students grasp the basic knowledge and skills of algebra, geometry, solid figures, and statistics that are needed to live in the present society To participate in production and to proceed toward further education	To help students acquire mathematical knowledge, including mathematical facts and experiences of mathematical activities, basic procedures of mathematical thinking, and skills of application
2	To foster the ability of calculation and to develop the ability of logical thinking and the perception of space	To help students learn to observe and analyze the actual society through mathematical thinking To solve the problems of daily life and to increase awareness regarding the application of mathematics

3		To give students the opportunity to experience a close relationship among mathematics, nature, and human society
4	To cultivate fine personalities and the viewpoint of basic dialectic materialism	To make students be creative and practical, with adequate development of emotional behavior and general skills

Table 2. A comparison of the general goals of the secondary mathematics curricula of the late secondary education in 1996 and 2003

Articles	The general goals of the experimental secondary mathematics curriculum (1996) for full-time schools	The general goals of the standard version of the secondary mathematics curriculum (2003) for secondary education
1	To help students learn primarily the basic knowledge and skills of algebra, geometry, probability and statistics, and differential and integral calculus, which are required to pursue further education and to establish modern communism	To help students learn the things given below through experience To understand the fundamental mathematical concept and the nature of mathematical conclusion and to comprehend the background and application of concepts and conclusions as well as the goodness of mathematical thinking methods and conventional learning To help students experience the discoveries of mathematics and processes of creation through various voluntary methods of learning and investigation
2	In mathematics teaching, focus on students' mathematical abilities of submission, analysis, and solution to problems. To encourage students to be creative and applicable, cultivate abilities of inquiry, making mathematical models and mathematical communication, and develop students' practical ability of mathematics.	To increase the basic abilities of three-dimensional imagination, abstractness, generalization, demonstration, operation, and data analysis
3	To foster students' ability of mathematical thinking, including three-dimensional imagination, intuitive inference, induction and abstractness, signs, operation, deductive demonstration, and the construction of a system To train students to be able to think and judge the relations of a mathematical pattern and the quantity of objective phenomena.	To develop students' abilities of making, analyzing, and solving problems, abilities of expressing mathematics and mathematical communication as well as acquiring mathematical knowledge through self-learning
4		To encourage students to be creative and applicable To train students to be able to think and judge a slightly mathematical pattern in the real world
5		To increase students' interests in learning, build their confidence in mathematics, establish the spirit of inquiry and scientific behavior
6	To increase students' interests in learning, build confidence in mathematics, establish the spirit of inquiry and scientific behavior	To broaden students' mathematical horizons To gradually understand the scientific value of mathematics, the value of applying mathematics, and the cultural value of

	To help students understand the scientific and human scientific values of mathematics and establish a world view of dialectic materialism	mathematics To cultivate critical thinking, respect the rational spirit of mathematics, experience the beauty of mathematics, and establish a world view of dialectic materialism and historical materialism

With reference to Table 1, article 1 in the standard version includes the experiences of mathematical activities and the basic procedures of mathematical thinking, which did not appear in the trial edition. In article 2, the standard version discusses mathematical thinking, while the guidelines discuss logical thinking. The trial edition did not have article 3. In article 4, the standard version takes into account creativity and practice as well as the development of emotional behavior, but trial edition does not make the same consideration.

With reference to Table 2, article 1 of the standard version includes the mathematical concept, understands the nature of mathematical conclusion—which stems from the theory of knowledge (epistemology) in mathematics and science philosophy. Although the standard version helps students to experience the various discoveries of mathematics and the processes of creation through various voluntary methods of learning and investigation, the experimental curriculum does not sufficiently examine this matter. While article 3 in the standard version of the curriculum includes a request for students' ability to acquire mathematical knowledge by themselves, the experimental curriculum does not. Article 6 of the standard version includes requests to broaden students' mathematical horizons, for the cultural value of mathematics, to cultivate critical thinking, to respect the rational spirit of mathematics, and to experience the beauty of mathematics; however, these aspects were not sufficiently covered in the experimental curriculum.

The goals of the mathematics curricula were based on the former secondary curricula in the past, but the new educational philosophies mentioned above have been greatly influenced by the mathematics education of developed countries such as the United States and Japan.

2.1.2 Basic Information

(1) Teachers' Qualification

Teachers' qualification at each level of the education system is defined by 中華人民共和国教師法 (Teachers Law of the People's Republic of China), which was adopted in October 1993. In this law, Chapter 3 "Teachers' Qualification and Recruitment" defines "the implementation of teachers' qualification system" and states that "for the application to be a teacher, the following related academic backgrounds are necessary ... 'a graduate of an infant normal school or upwards for kindergarten teacher', 'a graduate of a secondary normal school or upwards for primary school teacher', 'a graduate of a specialized higher normal school, or other colleges or universities with two or three years' schooling or upwards for junior middle school teacher', 'a graduate of a normal college or other colleges or universities with four years' schooling or upwards for senior middle school teacher', 'a postgraduate or university graduate for a teacher in an institution of higher learning'." Regarding the establishment of the teacher qualification system, 'Teachers' Qualification Ordinance' has been implemented since December 1995, which certifies 'Teachers' Qualification Certificate'.

(2) School Calendar and Examination

In China, each school has two semesters—one from the beginning of September until the New Year by the Chinese lunar calendar (a day between the end of January and the beginning of February, differs by year) and the other from after the New Year to July. Further, since 1995, each school has been functioning on a five-day-per-week basis.

Schools in China have adopted the following semester system: the fall semester (September–June) and the spring semester (February–July). Further, these schools have been functioning on a five-day-per-week basis since 1995. The system of compulsory education is set at six years of elementary school and three years of junior high school. Although elementary schools are intended for children between six and twelve years of age, some of these schools are allowed to accept 7-year-old children as Grade 1 pupils.

Table 3. School calendar in (year)

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.

(3) Medium of Instruction

Besides "Han," a major ethnic minority group, there are fifty-five ethnic minorities. The Chinese government implements an education policy in which each ethnic minority group is expected to use both Mandarin Chinese (the language of Han) and its own language in school.

The sample schools of the first year are located around Beijing and those of the second year are around Hohhot (the capital of Inner Mongolia). Both these areas use the Han language (Chinese) in urban and rural areas.

(4) Class Organization

The daily class begins at 8 am, breaks for lunch for two hours, and ends at 5 pm. Each lesson lasts for forty-five minutes.

Table 4. School hours

	First shift						Second shift			
Hour	8	9	10	11	12	13	14	15	16	17
Grade		Grade I and II					Grade III, IV and V			

(5) Promotion System

There exists a prerequisite for examination for promotion in compulsory education, and so, students are required to study as much as possible. Yet, some students fail to proceed to the next grade.

2.1.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 5. Schedule of data collection

Date	Activity
1st Nov.	CA154 arrival at Beijing
2nd Nov.	Meeting on research Prof. Iwasaki, Li Jian Zhong, Jin ①Reconfirming research schedule ②Rewarding
3rd Nov	Collecting materials on mathematics to find out the present situation of Chinese mathematics education; "Teaching Plan for Primary Education(2004)," and "Curriculum of Mathematics for Primary education "etc.
4th Nov	Visit Primary School A (at Beijing) and research Grade 4 pupils. 10:00 Test part I, 11:00 Test part II, 11:50

	Recording classes (Fraction) 12:40 Lunch
5th Nov.	Visit Primary School B (at the province of Hebei) and research Grade 4 pupils. 9:00 Test Part I, 10:00 Test Part II, 11:00 Discussion, 12:00 Lunch, 14:00-15:00 Visit Primary School attached to Baoding normal school
6th Nov	9:50 Discussion at China National Institute for Educational Research

(2) Target Schools and Samples

Urban primary school: Founded in 1957, this school has fourteen teaching classes and two teaching buildings, which have special classrooms for music, art, and computer teaching. All the teaching classrooms are equipped with modern teaching facilities. The school has a 1,200-square-meter playground and has been distinguished by the Xuanwu District as a "key export-oriented school."

Rural primary school: The village has a population of 4,862, with the number of pupils being 362 and the enrollment rate of pupils being 100%. There are thirty-five teachers; of these, six teach mathematics.

Table 6. Location of schools

	School location
Urban Primary School	Xuanwu District, Beijing
Rural Primary School	Qingyuan County

(3) Results of Interview

Table 7. Responses from head teacher

	Interview items	Urban school	Rural school
[Problem]	What according to you is the most problematic aspect of teaching mathematics in your school?	To allow pupils to learn the most useful and necessary mathematics. Based on the "double basic teaching" system, allow different pupils to develop differently at the same time and help them understand that mathematics originates from life and is an important part of daily life.	1. Emphasize on teaching knowledge. 2. Emphasize on lesson standards 3. Emphasize on the knowledge provided in the textbook 4. Emphasize on accepting knowledge
	What are the actions taken by you as an administrator in order to deal with the problem?	Coordinate the requirements of mathematics teaching and teach different pupils differently. In order to develop specific discussion activities, everyone should make study play and introspection themselves teaching.	In order to develop with changing times, according to the drawback observed with regard to China's education, we are restructuring teaching methods and arranging for teachers to study new teaching theories and also put to use the new ways of teaching.
	Do you sometimes observe the lessons taught by	Yes, I observe lessons every day (except when I	Yes. I observe lessons on a daily basis.

	teachers?	am required to attend meetings)	
	What kind of advice do you give to young teachers?	To strengthen the study included in the theory, raise the level of thinking, participate in all kinds of training, raise the level of special competence, set short- and long-term targets for themselves, and make attempts to incorporate the initiative of young teachers in each activity planned by the school	Young teachers must adapt themselves in order to meet the needs of developing times. They should be able to strengthen their abilities and raise their quality in order to compete for a place at the school.
[In-service training]	Do you see any impact of the in-service course offered to teachers?	The impact is evident on teachers who build a good image of the occupation. The course played an important role in developing teacher's specializations, allowing them to meet the needs of time, combining theory with the practice that guides teachers to raise professional knowledge and form better technical ability with regard to the study condition.	Occupation training has a positive effect on teachers. Since they are very well aware of the fact that society is developing rapidly, they know that they will be superseded if they do not continue their studies.
	If a new training course is designed, what according to you will be the kind of training necessary for teachers in your school?	It is necessary for a teacher using teaching tactics to change the teaching ideas that have existed thus far, open the field of vision, guide other teachers to hold their teaching contents and raise their ability to study.	The following is required to strengthen a teacher's ability and to raise his or her quality: 1. Strengthen study. 2. Be aware of pupils and know the kind of pupils required in society. 3. Be aware of the drawback of traditional teaching and attempt to find ways to improve it. 4. Be aware of their own duties and love their occupation

Table 8. Responses from mathematics teachers

	Interview items	Urban school	Rural school
[Problem]	What do you regard to be the biggest problem in teaching mathematics in your school?	In my opinion, the biggest problem is that the ability of students to study varies between pupils due to the difference in knowledge and ability. The basics differ between pupils. Since the basics differ between pupils, it is difficult for teachers to take into account the situations of all the pupils while teaching. The basics differ between the pupils in a class, and hence, it is difficult for teachers to sustain the level of teaching contents. The basic knowledge and ability to understand is different for pupils; therefore, it is difficult for teachers to take into consideration these differences while teaching. The knowledge gained by pupils prior to school is different, and hence, it poses a difficulty for teaching students in a group.	I believe that in mathematics teaching, it is not sufficient for pupils to learn by themselves while studying. Language ability was weak when pupils answered questions in class. It is not sufficient for pupils to solve problems on their own. It is not merely enough for pupils to understand these problems easily and thoroughly. It is not merely sufficient for pupils to understand that which the teachers teach; such abilities are ill-suited for social development. Pupils lack the ability to solve problems on their own; this prevents them from applying what they have learned to reality. They are not fully able to understand and grasp what they have learned, thus finding it difficult to solve problems by themselves.

	What kind of action do you take against such a problem?	I took the method that teaching separate separated the homework into difference, individual tutorial and tutorial each other between the pupils. I followed the method of teaching topics that are unrelated to the homework that is assigned. I choose separate topics to teach and to assign as homework; for the less bright pupils, I make up by teaching the relevant lessons. Make up by teaching the lesson to the less bright pupils. Teaching students in groups. Carry out exercises at separate levels. My teaching is separate from the questions put forth to different pupils. Further, I assign homework on different topics so that different pupils can learn what they should learn. In order to help each other, good pupils should be allowed to explain their ideas clearly so that they can help the less bright students.	In my mathematics classes, I allow pupils to solve problems as independently as possible. I permit pupils to speak and practice as much as possible in class. I allow pupils to practice numerous exercises in class in order to help them engage in quality education and increase - their comprehensive ability. In my class, I often give examples of what the pupils are aware of very well in their lives and allow them to solve the problem on their own, thus letting mathematics be understood more in terms of reality. I attempt to encourage pupils to indulge in quality education and to increase the number of exercises so that his or her ability to solve problems on his or her own is increased.

[Today's lesson]	What was the purpose of today's lesson?	Today's lesson delved on "area and the unity of area." The purpose of this lesson is to know the implication of an area by watching, carrying out the exercises, comparing, and counting. Pupils should be allowed to grasp the unity of area in practice so that they can develop the idea of an area's unity. In today's lesson, they learned how to multiply using a paper and a pencil for the purpose of calculation. The purpose of this lesson was to allow pupils to grasp multiplication between a single number and a large number and to develop good calculation habits. To review mixed and simple calculate of fraction in further more difficult calculate in third unite. To develop a pupil's ability to engage in nimble abstract thoughts and to summarize and strengthen the comparison of knowledge about parallelogram and trapezoids. The purpose is to develop the pupil's mode of thinking and grasping the law of resolving the problem and fostering a study interest. The pupils should be allowed to know "8" and grasp the manner of reading cardinal numbers.	Today's lesson aimed to put to practice what the pupils learned. Today's lesson aimed at learning how to put the lesson learned by pupils into practice. The purpose was to complete the teaching target for today, which is seven plus three equals ten. Allows pupils to understand the definition of a rectangle circumference and to grasp the way of calculation. Allow pupils to understand the definition of a circumference and to calculate the circumference of a square In order to increase the ability so solve problems.

	Do you think the goal was attained?	Yes, I think so; most pupils had gathered what the teacher had taught. All the pupils had mastered the law of calculation, but a few made some mistakes because they were unconscious of their mistakes. The target had achieved 85% in writing and had picked up the good habits of calculation. Yes, I think it was, but some pupils did not understand. Yes, I believe so. Ninety percent of the pupils had mastered what they had learned. Yes, I believe so.	Yes, I think so. Yes, I believe so. Yes, I would like to think so. According to me, most pupils understood. Yes, most pupils had understood and grasped what they had learned, with the exception of a few who needed teaching again. Yes, I believe so.

	What kind of lesson do you think is the best for you?	I believe that the best lesson requires that a careful study, of the teaching material be made, lessons be prepared, and the pupils' situations be known. It is necessary to make a careful study of the teaching materials and to prepare lessons. One should attempt to exchange feelings with the pupils in class and to guide them such that they learn to like mathematics. The aim was to make an excellent teaching plan along with the attempt to consider the problem from the pupils' perspective. Attempt to exchange feelings with the pupils in class in future to include the initiative made by pupils into play. Hold the teaching materials. Hold pupils. Hold the class. It is important for teachers to bring the pupils' initiatives into play and to foster the pupils' creativeness and ability with regard to study. Further, teachers should prepare lessons and should carefully study the teaching materials. To study the teaching materials and attempt to know the pupils. Combine theory with practice. To raise teachers' professional competence.	In my opinion, it is very important to allow the pupils to learn how to study. I consider it to be very important to allow the pupils to learn how to study. The teacher should play a leading role, bringing the pupils' initiative into full play in class. ④It is important to bring the pupils' initiative into play in study; the teacher and pupils should study mutually in class. To master what they learned and to learn to be happy from what they study. The pupils should be allowed to develop the ability to understand and master what they have learned and allow them to learn the ways in which they can study. It is important that the teacher and pupils engage in mutual study in class and to raise the pupil's initiative on study.

	What kind of teaching method would you like to practice?	I like teaching in perfect harmony and in a friendly atmosphere as well as in the setting where both the teacher and the pupils share a mutual understanding. I like teaching in perfect harmony and in a friendly atmosphere as well as in the setting where both the teacher and the pupils share a mutual understanding. The pupils should be allowed to teach each other in groups so that they can gain knowledge through different kinds of actions. Teaching in an atmosphere in which the teacher and pupils develop a mutual understanding. I like to practice happy teaching or teaching in an atmosphere in which the teacher and pupils share a mutual understanding and the pupils are allowed to gain knowledge when they are happy. I like to teach in an atmosphere of openness resulting in mutual pleasantness.	I wish that teachers and pupils participate more actively in a class setting. I would like to adopt a teaching method wherein the pupils and the teacher participate together in the class, helping pupils to study nimbly. I would like to practice a method in which the pupils and the teacher participate together in the class, helping pupils to study nimbly On base of considering the pupils' health to connect with pupil's future, society development, and mathematics attainment to unity between science, practical, and education. I like to engage in happy teaching, which allows both teachers and pupils to learn gladly while studying. On base of considering the pupils' health to connect with pupils' future, society development, and mathematics attainment to unity between science, practical, and education.
[In-service training]	Have you ever attended any training before?	Yes, I have attended training many times. Yes, I have. Yes, I have. Evaluation after lesson is a useful method. Yes, I have. Yes, certainly.	Yes, I have. Yes, I have attended training many times before. Yes, many times. Yes, I have attended training before. Yes, I have. Yes, I have attended training before.

	What kind of training did you attend before and do you think it benefits your teaching?	I believe it benefits teaching since it introduces different teaching methods and excellent teacher experiences. The training in which theory is combined with practice benefits teaching. Evaluation after lessons is a useful method. Observe lessons of other teacher and evaluate after lessons. Discuss lessons and evaluate after they are over. Training on the ability to use a computer. Study on teaching theory and new ideas. Training by specialists, training for new teaching theory and practice, training for special courses.	Continuous education training is useful. Continuous education training is useful. Continuous education training is useful. Training is closely connected to the following three factors: to encourage pupils; to develop moral education, intellectual development, physical training, and aesthetics; and to allows pupils to develop vivaciously. Training can help the present-day teaching methods that are closely related to the new teaching theory and teaching thought. Training is closely connected with the following three factors: to encourage the pupils; to develop moral education, intellectual development, physical training, and aesthetics; and to allow the pupils to develop vivaciously.

	In the event of a new training course, what kind of training do you think is necessary for improving your teaching methods?	I believe that it is necessary to increase teachers' professional knowledge and to combine theory with practice. It is beneficial to increase teachers' professional knowledge and to combine theory with practice. Allow the teacher to leave his or her her class and observe the lessons taught by the good teacher. Strengthen the training of professional technical ability. Strengthen the study of theory. The following three conditions should be considered to qualify a good teacher: Universality—Should be suited to teaching different classes. Specific subject—lay stress on strengthening the teaching of subject knowledge. In particular, organize special training based on different classes and teaching characteristics. It is better to give examples in order to explain how the new theory and idea can be incorporated into teaching in order to increase the quality of teaching. Training that involves a combination of theory and practice and training of solve requirement against the teachers.	I believe it is necessary for training to be based on the specific conditions of the pupils. I believe that it is more beneficial to raise the education quality than to increase the number of times that theory is applied to reality. In my opinion, it is beneficial to raise the education quality so that training can combine theory with practice. Not only to increase the teacher's knowledge but also to lay emphasis on the teaching theory and techniques. Not only to increase the teacher's knowledge but also to lay emphasis on the teaching theory and techniques. Not only to increase the teacher's knowledge but also to lay emphasis on the teaching theory and techniques.

(4) Results of Lesson Plan Analysis

Table 9. Responses of lesson observation checklists

Observation Items		Urban School					Rural School				
		0	1	2	3	4	0	1	2	3	4
Introduction	The teacher starts the class on time.					√					√
	The teacher made the objective clear.					√					√
	The objective suits to the level of children.					√				√	
	Relationship with the previous lesson is clear.					√					√
Development	The teacher gives supports to pupils who seem to have little understanding.				√					√	
	The teacher expresses appreciation for pupils' thinking attitudes.			√			√				
	The teacher assesses the pupils' comprehension during teaching and learning.				√			√			
	The teacher uses easy language.				√					√	
	The teacher uses an appropriate and familiar example to illustrate main concept.				√				√		
	The teacher creates friendly atmosphere.			√					√		
	The teacher accommodates discussion among pupils.			√						√	
	The teacher gives hands-on activity.			√							√
	The teacher enjoys teaching.			√			√				
	The teacher is impatient with wrong answer.				√		√				
	The teacher involves children to say opinions freely.				√		√				
	The teacher encourages children to display diverse opinions.				√				√		
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.			√							√
	The teacher combines individual work and group work appropriately.			√					√		
Summary	At the end of the lesson, the teacher summarizes the lesson.					√					√
	The teacher assigns homework at the end of lesson clearly.					√	√				
	The teacher explains about a connection between today's lesson and next lesson.	√					√				
General	The teacher prepares a lesson plan.					√					√
	The teacher prepares a plan for taking note on the blackboard.					√	√				

Describe objective of today' lesson. Urban: A. To understand and compare the fractions that have different denominators with numerator 1. B. To be able to compare fractions accurately. C. To acquire the ability to solve real problems in daily life. Rural: A. To recognize the fact that it can also compare the fractions that have numerator 1 by comparing the fractions which have different denominators with numerator 1. B. To raise the mutual consciousness between pupils and their observation ability and to simultaneously raise the idea-item virtue education by adjusting the notation of fraction.
Describe problems/activities. (No. of problems, relation among them, their difficulty level etc.) Urban: A. By dividing a cake into five equal pieces, draw a picture to show that three pieces are larger than one piece. B. To help understand a fraction with the same denominator but a larger numerator. Rural: The teacher instructed to fold a square paper and paint the parts equivalent to 1/2 and 1/3 as well as 1/4 and 1/6 with color pencils. Teacher instructed to draw a 10-cm segment and take the part equivalent to 1/6 and 1/10. However, it is unclear whether the pupils could correctly divide 1/3 of the paper or 1/6 of the segment.
Describe children's opinions Urban: Pupils expressed their thoughts positively. Rural: Pupils answered the question rather than express their understanding.
Assess who dominate solving problems during the lesson observed. Urban: Teacher asked many questions in order to make pupils think logically. Rural: Although there seem to be several pupil activities, the extent to which the pupils have understood the lesson is not clear.
Assess which of the followings is regarded as the most important in the lesson observed. Understanding concept/ mastering the procedure/ thinking mathematically/ applying to the daily life/ finding correct answer Urban: thinking mathematically Rural: mastering the procedure

(5) Results of Video Analysis and Classroom Observation Checklists

Since the content of video teaching was not identical to the content of the interview, the analysis in which the relations between the answers from teachers and teaching are given cannot be performed.

The academic degree of a mathematics teacher and his/her teaching experience were not investigated.

In China, the mathematics course standard, instead of the outline of mathematics teaching, was developed in July 2001. The mathematics course standard was implemented experimentally in 2005. It will be implemented widely from 2005 to 2010, after which it will be implemented officially. Based on the interview, we found that the textbook, based on the mathematics course standard and the outline of mathematics teaching, is used both in urban and rural schools.

2.1.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 10. Schedule of data collection

Date	Activity
18th November	Prof. Iwasaki, Jin, CA154, (Hiroshima)14:25-(Beijing)17:25 CA1116, (Beijing)20:20-21:25(Mongolia) ※Reconfirmation of the schedule and activities.
19th November	A.M. : Lecture by Prof. Iwasaki at the Normal school. P.M. : Round-table talks with the mathematics teachers.
20th November	Research on 5th graders at urban primary school. Round-table talks with the teachers at the school. Video-shooting.
21st November	Research on 5th graders at rural primary school. Round-table talks with the teachers at the school. Video-shooting.
22nd November	Round-table talks with the graduate students at Inner Mongolia Normal University. Lecture at vocational normal school.
23rd November	Meeting for the research contents and marking.
24th November	Move by train : 21:12—7:30(next morning).
25th November	Arrival in Beijing. Stay at the hotel, yanxiang.
26th November	Back to Japan. CA153 (Beijing)8:30 - (Hiroshima)1:30

(2) Target Schools and Samples

Total number of students in the urban primary school was 1405; of these, 280 belonged to various minorities. The total number of students in the rural primary school was 877; of these, 735 came from the outskirts of this area. This is the reality of a rural area surrounding an urban area with economic growth.

Table 11. Location of schools

	School location
Urban Primary School	Huhhoto
Rural Primary School	Suburb of Huhhoto

Table 12. Brief profiles of teachers

	Sex	How long you have taught	Subjects you teach
School in urban area	Female	28	Mathematics and society
School in rural area	Female	23	Mathematics

(3) Results of Teachers' Interview

Table 13. Teachers' responses to the questionnaire

Questionnaire Items	Average school in urban area	Average school in rural area
(1) Teacher's forecast of pupils' average score	—	80%
(2) Pupils' familiarity to the given test	Yes Reason/s: They like this kind of test	No Reason/s: We have never had such an activity before.

(3) Any questions which the pupils cannot solve.	Yes Reason/s: Although the pupils have preliminary knowledge of "Fractions," they find it difficult to solve some complex problems pertaining to "Fractions."	Yes Reason/s: Questions 3, 4, 6, and 8 have unit signs, but the pupils do not know how to use signs to express the units. They never learned this kind of expression.
(4) Difficulties in teaching "Fractions".	Reason/s: difficult	Easy Reason/s: We often use fractions in our daily life.
(5) The most difficult topic/s to teach in Grade 4.	Topic(s): Although there are many conceptions to teach in Grade 4, the most difficult topics for me are some operation principles of multiplication, especially when the pupils can relate the distributed principle of multiplication; however, it is difficult for the not-so-bright pupils to make use of such principles agilely. The change in decimal result from moving of the decimal point. Reason/s: 1. Since there are many types the distributed principle of multiplication, the flexibility offered to the pupils is insufficient. 2. It is difficult to study the change in the decimal result from moving of the decimal point, but it is better to grasp knowledge through the intensification exercise.	Topic(s): Get unknown x. Reason/s: Since it is new knowledge, the pupils accept it slowly.
(6) The easiest topic/s to teach in Grade4.	Topic(s): 1.The relationship between multiplication and division 2. The writing and reading of numbers over a hundred million.	Topic(s): Fractions Reason(s): Since fractions are small and used in actual life.
(7) Teacher's confidence in teaching "Fractions"	Confident	Confident
(8) Level of influence of the examination on teacher's teaching	Very little Description: I would like to continue teaching as before.	Little Description: The review process for fractions includes answering questions in order to motivate the pupils' study interest.
(9) Degree of difficulty for the pupils to learn "Fractions"?	Easy	Easy
(10) Points of difficulty for the pupils to learn the concept of "Fractions"?	1. It is difficult to understand and make use of the importance of "Fractions" and to develop the pupils' ability to solve practical problems. 2. It is difficult to solve some sentence problems of "Fractions."	Although it is not possible to averagely divide, something sometimes, the pupils desire to use fraction expressions for it.

(11) Existence of pupils' difficulty with the medium of instruction in learning mathematics	No	Yes Concrete example/s: The pupils are interested in medium, but teaching's effect is bad at last.
(12) Importance of learning "Fractions" with comparison to any other topics in mathematics.	Yes Reason/s: I think that "Fractions" is more important in math because there are some numbers that cannot be expressed with integers but can be expressed with fractions and decimals.	No Reason/s: We use integers and decimals more than fractions in actual life.
(13) Teacher's main point/s of concern to the pupils in teaching "Fractions"?	I believe it is important to emphasize on average division and the fraction unit	I like to stress on the ability of an integer to be divided averagely into some parts, of which either one or certain parts are considered.

- Teachers' Strategies in Teaching "Fractions"

Q(14) Describe how to teach the following question to the pupils?
"Which is longer 1/4m or 1/3m?"

Table14. Teachers' strategies in teaching fraction (1)

Average school in urban area	Average school in rural area
I would analyze by using the "Line Figure" and helping the pupils to understand that "1/4 m" is obtained by dividing 1 m into four parts; similarly, I will explain the concept of "1/3 m." Thus, the more the parts are divided, the smaller is each part.	Take two sheets of same-sized paper. Divide one sheet averagely into four parts and the other, into three parts. Following this, take out one part from each part and compare.

Q(15) Suppose you posed the following question to the pupils in a lesson. "What is a half of 2m?"Then a student answered, "It is 1/2m."How do you deal with such a student in class?

Table15. Teachers' strategies in teaching fraction (2)

Average school in urban area	Average school in rural area
First, I pointed out his mistake by using the "line figure." Second, I analyzed the study and solved the problem from the viewpoint of "fractions."	Help pupils to clearly understand the question and its meaning as well as clearly explain the relation between 1/2 and 1/2 m.

2.1.5 Discussion

(1) Analysis of Interview Items for Head Teacher

Table 16. The results obtained from head teachers with regard to the questionnaire administered during the mathematics teaching.

What do you think is the most problematic aspect of teaching mathematics in your school?	
Urban School	1. How to learn the useful and necessary mathematics based on the "double basic teaching" system? 2. How can different pupils develop differently at the same time and how can we determine whether or not mathematics originates from life and is used on a daily basis?
Rural School	1. One-sided teaching 2. Stress only on the test 3. Stress on teaching only knowledge

With regard to the questionnaire administered during mathematics teaching, the answers obtained from the head teachers of the urban school pertained to the manner in which useful and necessary mathematics can be learned based on the "double basic teaching (basic knowledge and basic ability)" system, the manner in which different pupils can develop differently at the same time, and the manner in which we can determine whether or not mathematics originates from life and is used on a daily basis. On the other hand, the answers obtained from the rural school indicated less stress on lesson standards and less stress on the knowledge provided in textbooks; it showed that the biggest problem encountered during mathematics teaching is the less emphasis on textbooks.

(2) Analysis of Interview Items for Mathematics Teacher

Table 17. The results regarding the questionnaire administered during mathematics teaching are obtained from mathematics teachers.

What according to you is the most problematic aspect of teaching mathematics in your school?	
Urban School	1. There are gaps in individual abilities because of the difference in knowledge. 2. There is a gap in each pupil's abilities and hence is difficult for the teacher to consider every pupil's learning during teaching. 3. The basic is different among the pupils in class; therefore, it is difficult for the teacher to adjust the level of teaching contents. 4. Prior to school, pupils' individual abilities and knowledge are different, and hence, it is difficult for teaching in-groups.
Rural School	1. I believe it is insufficient for pupils to complete their exercises and to study by themselves during the mathematics lesson. 2. Language ability is weak when pupils answer questions in class. 3. Pupils' abilities to solve problems are not fully developed. 4. The problem does not interfere with the activities of their daily life.

Concerning the questionnaire administered during mathematics teaching, the results obtained from the urban school showed that the ability to study differs among pupils because of the difference in knowledge and ability; in order to solve this question, it is important to teach individually. On the other hand, the results obtained from the rural school showed that it is not sufficient to allow pupils to complete exercises on their own in mathematics lessons. In real life, it is difficult to apply what they have learned.

What kind of action has been taken against the problem during mathematics teaching in urban and rural schools? Table 18 is prepared based on the interview results.

Table 18. What kind of action do you take against such a problem?

What kind of action do you take against such a problem?	
Urban School	① I took the method that teaching separate separated the homework into difference, individual tutorial and tutorial each other between the pupils. ② In order to help each other, the good pupils should be allowed to explain their ideas clearly so that they can help the less bright pupils.
Rural School	① To engage in quality education and to increase exercises in order to increasingly develop the pupils' abilities to solve problems on their own. ② In my class, I often give some examples of what the pupils are very well aware of in their lives and allow them to solve the problem on their own, letting mathematics to reach more closer to reality.

The urban school was often found to engage in "teaching separately, individual tutorial and separated the homework according to ability." With regard to the rural school, it is not enough to concretely engage in "quality education" and to "let pupils do exercises on their own in class."

Regarding the question whether or not the teaching objectives are achieved, all teachers from urban and rural schools agreed that they successfully achieved all the objectives of that day's lesson. For example, a teacher from the urban school answered, "All the pupils mastered the law of calculation, but a few made some mistakes because they were unconscious of such mistakes. The target had achieved 85% in writing and good habits of calculation.

With regard to the question what kind of lesson according to you is the best for you, the teachers from the urban school answered, "To make a careful study lesson plan" or "to make teaching materials" or "try to know pupils," etc. On the other hand, the answers that came from the rural schools were " the teacher should play a leading role" or "it is important to allow pupils to learn how to study," etc, based on the answers that were provided by both urban and rural school teachers, as stated in Table 19.

Table 19. What kind of lesson in your opinion is the best for you?

What kind of lesson in your opinion is the best for you?	
Urban school	① I believe that the best lesson requires that a careful study of the teaching materials be made, lessons be prepared, and the pupils' situations be known. ② It is important to make a careful study of the teaching materials and prepare lessons, exchange feelings with pupils in class, and guide them such that they learn to like mathematics. ③ One should attempt to make an excellent teaching plan with efforts to consider the problem from the pupils' perspective, exchanging feelings with pupils in class in order to include the pupils' initiative ④ Hold teaching materials, hold pupils, hold the class. ⑤ It is important for teachers to bring the pupils' initiative into play, fostering the pupils' creativeness and ability to study. Next, teachers should prepare lessons and make a careful study of the teaching materials. ⑥ To study the teaching materials and make an attempt to know the pupils. Combine theory with practice. To raise teachers' professional competence.

Rural school	① I believe it is very important that the pupils be allowed to learn how to study. ② The teacher should play a leading role, bringing the pupils' initiative into full play in class. ③ It is important that the pupils' initiatives be brought into full play in study so that they and the teacher can study mutually in class. ④ To master what they learn and to learn gladly while studying. The pupils should be allowed to develop the ability to understand and master what they have learned and to help them know how to study. ⑤ It is important that the teacher and pupils study mutually in class and raise the pupils' initiatives in study.

(3) Analysis of the Results from Teachers' Responses to the Questionnaire

We now analyze the results from the teachers' responses to the questionnaire in which they were asked to describe the manner in which they teach fractions. Table 5 shows that both urban and rural mathematics teachers wished to teach Q14 (Which is longer 1/4 m or 1/3 m?) in terms of fractional concepts. This implies that the introduction of fractional concepts in China is consistent with the mathematics teachers' responses to Q14.

With regard to responses to Q15, there was no concrete explanation on how students think or why they meet with a setback, as can be seen in Table16.

Acknowledgment

We are grateful to Ms. Li Chunlan and Ms. Wang xiaoxia, graduate students at Inner Mongolia Normal University, who helped us to write this report.

References

李迪，代欽：「中国数学史綱」/ 横地清，鐘善基，李迪編集：「中日近現代数学教育史」第四巻，ハンカイ出版印刷株式会社，2000 年。
魏群，張月仙：「中国中学数学課程教材演変史料」，北京：人民教育出版社，1996 年。
数学課程標準研制組：「普通高中数学課程標準解読」，江蘇教育出版者，2004 年。
中華人民共和国教育部：全日制義務教育数「学課程標準（実験稿）」，北京師範大学出版社，2001 年。
中華人民共和国教育部：九年義務教育全日制初級中学「数学教学大綱（試用修訂版），北京：人民教育出版社，2000 年。
中華人民共和国教育部：全日制普通高級中学「数学教学大綱」，北京：人民教育出版社，2002 年。
李迪著，大竹茂雄訳：「中国の数学通史」，森北出版株式会社，2002 年。
中国数学会編集，「中国数学雑誌」（第一巻第一期至第五期），中国全国自然科学専門学会連合会，1951－1952 年。
李其龍主編『教師教育課程の国際比較』2003。
北京教育科学研究・基礎教育教学研究中心編　九年義務教育六年制小学教科書『数学教学参考書』北京出版社，2001。

2.2 Thailand

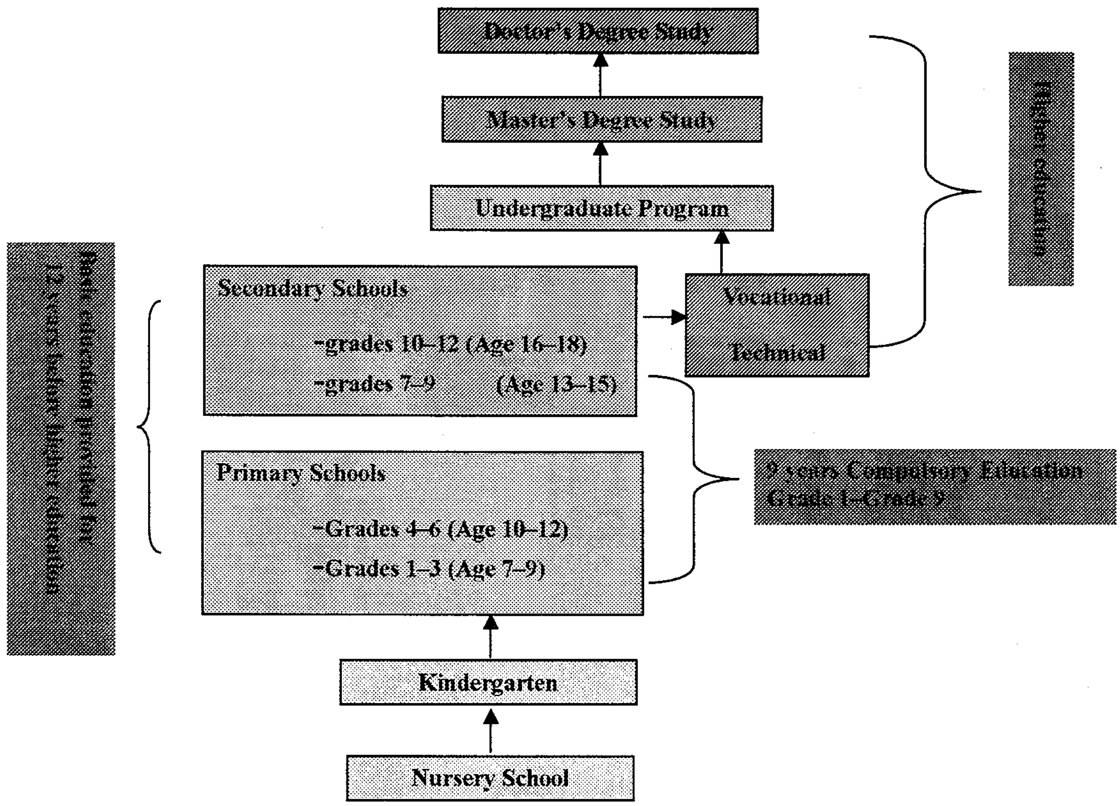
Maitree Inprasitha
Kohn Kaen University

Masami Isoda
University of Tsukuba

Yutaka Ohara
Naruto University of Education

2.2.1 History of Mathematics Education

The structure of formal education in Thailand is as follows:



The overall organization of education in Thailand is categorized into 3 types: 1) Formal education, 2) Non-formal education, and 3) Flexible education.

1) Formal Education

   - Education that is provided for 12 years before higher education is termed basic education.
   - Higher education is divided into two levels – lower-than degree level and degree level.

2) Non-formal education

The contents and curricula for non-formal education is that which is appropriate for individual groups of learners, responds to their requirements, and meets their needs.

3) Flexible education

Flexible education enables learners to learn by themselves according to their interests, potentialities, readiness and opportunities available from individuals, society, environment, media, or other sources of knowledge.

Education in Thailand comprises compulsory education for 9 years from Grades 1–9 for children who are 7–16 years of age.

College Admission

- Students who graduate from ***senior high schools*** have the right to enroll themselves for higher education through the entrance examination system
- Students who graduate from ***vocational or technical colleges (3 years courses)*** have the right to enroll themselves for higher education through the entrance examination system
- Students who graduate from ***vocational and technical and nursing colleges (5 years courses)*** have the right to enroll themselves for higher education through the entrance examination system

Historical List of Major Documents for Grades 1–12

- 1960s National Plan B.E. 2503 (A.D. 1960) enacted in the academic year 1961
- 1970s Mathematics Curriculum developed by Institute for Promotion of Science and Technology (IPST) A.D.1978
- 1980s Revised mathematics curriculum in senior high school level (A.D. 1981)
- 1990s
  - 1990 Revised elementary and junior high school curriculum (version A.D. 1978)
  - 1990 Revised senior high school curriculum (version A.D. 1981)
  - 1999 National Education Act B.E. 2542 (A.D. 1999)
- 2000s
  - 2001 Curriculum for Basic Education B.E. 2544 (A.D. 2001)
  - 2002 Amended National Education Act

Features of 2001 Mathematics Curriculum for Basic Education

Standards for Basic Education

- Content Area I: Number & Operations

Standard 1.1 Understanding various types of number expressions and using numbers in daily life

Standard 1.2 Understanding the effects of number operations and relations among various operations and using operations to solve problems

Standard 1.3 Using estimation to compute and solve problems

Standard 1.4 Understanding the number system and using the property of numbers

- Content Area II: Measurement

Standard 2.1 Understanding basic measurement

Standard 2.2 Measuring and estimating things

Standard 2.3 Solving measure problems

- Content Area III: Geometry

Standard 3.1 Explaining and analyzing two- and three-dimensional geometrical figures

Standard 3.2 Visualization, spatial reasoning, and geometric models in problem-solving

- Content Area IV: Algebra

Standard 4.1 Explain and analyze various types of patterns, relations, and functions

Standard 4.2 Use expressions, equations, graphs, and mathematical models to represent various situations and interpret their meaning and implement them

- Content Area V: Data Analysis & Probability

Standard 5.1 Understanding and using statistical approach to analyze data

Standard 5.2 Using statistical method and probability to estimate phenomena rationally

Standard 5.3 Using statistics and probability for decision-making and problem-solving

- Content Area VI: Skills & Processes

Standard 6.1 Ability to solve problems

Standard 6.2 Ability to reason

Standard 6.3 Ability to communicate and represent mathematical meanings

Standard 6.4 Ability to relate mathematical concepts and other subjects

Standard 6.5 Creativity

Sequence of Content Organization for Grade 1

14 Units for 150 hrs

- Unit 1 Counting Numbers 1–5 and 0 7 hrs
- Unit 2 Counting Number 6–10 9 hrs
- Unit 3 Addition 15 hrs
- Unit 4 Subtraction 14 hrs
- Unit 5 Counting Numbers 11–20 8 hrs
- Unit 6 Addition and Subtraction 19 hrs
- Unit 7 Linear Measurement 7 hrs
- Unit 8 Measurement (Weight) 6 hrs
- Unit 9 Measurement (Capacity) 6 hrs
- Unit 10 Counting Numbers 21–100 15 hrs
- Unit 11 Preparation for Geometry 4 hrs
- Unit 12 Time 5 hrs
- Unit 13 Addition and Subtraction (Less than 100) 19 hrs
- Unit 14 Implication Addition and Subtraction 13 hrs

Sequence of Content Organization for Grade 4

13 Units for 150 hrs

- Unit 1 Numbers greater than 100,000 7 hrs
- Unit 2 Addition and Subtraction 13 hrs
- Unit 3 Geometry 19 hrs

- Unit 4 Multiplication 15 hrs
- Unit 5 Division 21 hrs
- Unit 6 Statistic and Basic Probability 11 hrs
- Unit 7 Measurement 19 hrs
- Unit 8 Area 8 hrs
- Unit 9 Money 9 hrs
- Unit 10 Fractions 7 hrs
- Unit 11 Time 11 hrs
- Unit 12 Decimal 6 hrs
- Unit 13 Implication Addition, Subtraction, Multiplication, and Division 5 hrs

2.2.2 Basic Information

(1) Teachers' Qualification

In Thailand, the minimum required academic qualification for teachers is a bachelor's degree in education or related subjects with a Certificate in Teaching.

There are four categories of teachers in each school; principal, assistant principal, head of section (e.g. Mathematics, Science, etc.), and teachers.

Policy: Mathematics teachers must have graduated with bachelor's degree in mathematics or mathematics education.

Reality: Teachers for both rural and urban schools are graduates with bachelor's degree in elementary education.

(2) School Calendar and Examination

The first semester of the school year begins on May 16 and ends in the first week of October. The second semester begins on November 1, after a three-week recess and continues till the second week of March. There is a long summer vacation from the third week of March until May 15, after which the cycle begins again. Classes are held from Monday through Friday, during which time the students receive approximately six hours of instruction each day.

Schools vary in different parts of Thailand. Small rural schools are not as regimented as the large city schools nor are they very well equipped with technological equipment. Most schools require uniforms, depending on the affluence of the families. Students and their families show great respect for their teachers and appreciate the opportunity to learn. (http://www.school-portal.co.uk/GroupHomepage.asp?GroupID=66100)

Table 1. School calendar in (2004-2005)

May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	April

There is a term examination in each school at the end of each term. There is a special national examination in Grades 3, 6 and 9, known as the National Test.

(3) Medium of Instruction

The medium of instruction at the primary school level is Thai. The mother tongue for the children and the teachers are Thai.

(4) Class Organization

The duration of lessons in primary school is 50 minutes.

Table 2. Class organization

Time (Min)	Thai Lesson
1	Teachers begin by asking students short-answer questions that lead into the day's lesson or questions related to earlier lessons, the teacher may also check homework by calling students to show their answers in front of the class.
10	Teacher distributes worksheet with similar problems. Students work independently.
20	Teacher monitors students' work, notices some confusion on particular problems, and demonstrates how to solve these problems. Typical for teacher to intervene at the first sign of confusion or struggle.
30	Teacher reviews another worksheet and demonstrates a method for solving the most challenging problem.
40	Teacher conducts a quick oral review of problems like those worked on earlier.
50	Teacher asks students to complete worksheets. Unusual to not assign homework.

A majority of the primary schools in Thailand begin at 8:30 am. and continue up till 3:30 pm. lunch time is from 11:30–12:30 pm.

Table 3. School hours

Time	8.30-9.30	9.30-10.30	10.30-11.30	11.30-12.30	12.30-13.30	13.30-14.30	14.30-15.30
Subject	A	B	C	Lunch	D	E	F

(5) Transfer System

Teachers who want to be transferred to another school after being recruited to be a government servant can apply to the Director of Educational Area after the preliminary approval of the school principal.

(6) Reason to be a Teacher

In Thailand, teacher is considered to be a rather secure career as compared with other careers. The retirement age is 60, and all teachers receive pension after retirement.

2.2.3 Results from the First Year Field Survey

(1) Schedule of Survey

Table 4. Schedule of data collection

DATE	ACTIVITY
11th December 2004	Arrival at Center for Research in Mathematics Education (CRME), Khon Kaen University
12th December 2004	15.00 – 18.00 Meeting on research Assist. Prof. Maitree Inprasitha (Ph.D.), Mr. Yutaka Ohara, and CRME Center's Staffs
13th December 2004	Preparing materials for collecting data; Test I & II, VDO digital camera and so on.

14th December 2004	Visit Rural School (B School, Khon Kaen) and Grade 4 students 09:00 Videotaping classes (Fraction) 10:00 Interviewing the principal and mathematics teacher 10:30 Distributing questionnaire 11:00 Testing part I 12:00 Lunch 13:00 Testing part II
15th December 2004	Preparing materials for collecting data; Test I & II, VDO digital camera and so on. Leaving for Bangkok.
16th December 2004	Visit Urban School (W School, Bangkok) and Grade 4 students. 09:30 Meeting with the principal and administrator 10:20 Videotaping classes (Fraction) 11:10 Interviewing the principal and mathematics teacher
17th December 2004	Visit Urban School (W School, Bangkok) and Grade 4 students. 09:00 Distributing questionnaire 09.30 Testing Part I 10:30 Testing Part II
18th December 2004	Summary discussion at Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University.

(2) Target Schools and Samples

The following aspects were taken into consideration when selecting the sample schools: location of schools and language used and area from where students come. As a result, W School from the urban area and B School from the rural area were selected. In W School, students and teachers use standard Thai language in class, however, in B School in the rural area students and teachers occasionally use Isaan (local language) or Thai language in class.

Table 5. Location of schools

School	School Location
Urban School	Ratchaburana, Bangkok (Capital of Thailand)
Rural School	Nong Rua, Khon Kaen (About 440 kilometers away from the Capital of Thailand)

(3) Results of Interview

Table 6. Responses from head teacher

	Interview items	Urban school	Rural school
[Problem]	1-1) What do you think is the greatest problem in teaching mathematics in your school?	There are a large number of problems; in particular word problems and fractions are a common problem for most schools.	There are no mathematics teachers.
	1-2) What type of action do you take against such a problem as an administrator?	Encourage teachers to tackle this problem through collaboration.	Look for teachers that have a good attitude toward mathematics, and provide them training in teaching mathematics.

	1-3) Do you observe the lessons of teachers? YES or NO If YES, how often do you observe them?	No, because I have come here recently (December 2004).	Yes, occasionally.
	1-4) What type of advice do you give young teachers in your school?	Provide information regarding school policy, guide for actions and other related issues.	Life skills, government system, school culture and curriculum, and assign appropriate works to them.
[In-service training]	2.1) Do you see any impact of the in-service course offered to teachers? If yes, is it negative or positive? Please describe the impact.	Yes, after attending the workshop, they are asked to make a report on the implementation of what they gained from the workshop.	Yes, there are several positive impacts. We hold monthly meetings for teachers regarding several problems and attempt to implement what has been learned.
	2-2) If a new training course is designed, what sort of training do you think it must provide teachers in your school?	Construction of an integrated lesson plan which is an interesting issue for a majority of the teachers.	Every training course must have 3 phases. In the first phase, teachers must be made aware of theories or method. In the second phase, they must be taken to the classroom. The last phase must be for reflecting.

Table 7. Responses from mathematics teachers

	Interview items	Urban school	Rural school
[Problem]	1-1 What do you think is the greatest problem in teaching mathematics in your class?	Word problems – students are unable to tackle them.	A few students do not possess sufficient basic knowledge to connect to new knowledge.
	1-2 What type of action do you take against this problem?	If there are a few students facing this problem, I will talk to each of them clinically. However if there are a large number of them, I will use the blackboard for explanation.	Provide tuition to the students: in case the problem is serious. I occasionally hand them over to the Grade 1 teacher to teach them the basic for approximately 1 semester.
[Today's lesson]	2-1 What was the purpose of today's lesson?	For students to compute fractions with the same denominator.	To enable students to think differently and integrate their prior knowledge, such as that of symmetry and measurement, to deal with fractions. Moreover to understand the concept of fractions and use pictures to represent fractions.

	2-2 To what extent do you think the purpose was attained?	Approximately 80 % of the purpose was attainted.	The purpose was attained. Students were able to integrate their prior knowledge to create fractions, obtain greater understanding of fractions and use pictures to represent fractions.
	2-3 What do you think are the most important factors for a successful lesson?	Several factors are important–lesson plan, students, and teachers. The most important is teachers because they encourage students to enjoy the process of learning.	The prior knowledge of students is important; they may not succeed if they do not possess this knowledge.
	2-4 What type of teaching would you like to do?	I do not want to be restricted to any specific models. For example, today we emphasized the model that is rich with activities and instructional aids.	I would use several methods–substitutes for appropriateness–Open Approach method, descriptive teaching, and cooperative learning.
[In-service training]	3-1 Have you ever undergone teacher training after you become a teacher?	Yes, I have attended several training courses.	Yes, not only for mathematics but for several subjects. With regard to mathematics, I underwent a teacher training regarding critical thinking skills, It was useful and I was able to adapt this method into the Mathematics Club, in the hope of improving the thinking skill of students. Moreover, my school had organized a training session on creating lesson plans (Open Approach lesson plan). I too had participated and apply what I learnt in my classroom today.
	3-2 Which type of training, if you have undergone any before, do you think is useful for your teaching?	I had undergone a long-term workshop on educational evaluation that was implemented in the school.	Research methodology, evaluation and assessment, instructional method, technique in raising question. Especially raising question, if we raise a good question, we will get a lot of student's concept.
	3-3 If a new training course is designed, what type of training do you think is necessary for improvement of your lesson,	I think the best type of training is an integrating workshop.	Instructional method and making lesson plan. In my opinion, we should take many methods to teach, in so much as flexible for different students. Such as Open Approach method is suitable because of it's allow students to take action to solve problems and its funny activity.

(4) Results of Lesson Plan Analysis

Table 8. Responses of lesson observation checklists

		Urban school					Rural school				
Introduction		0	1	2	3	4	0	1	2	3	4
	The teacher starts the class on time					√					√
	The teacher made the objective clear					√					√
	The objective suits to the level of children					√					√
	Relationship with the previous lesson is clear				√					√	
Development	The teacher gives supports to pupils who seem to have little understanding			√						√	
	The expresses appreciation for pupils' thinking attitudes				√						√
	The teacher assesses the pupils' comprehension during teaching learning				√					√	
	The teacher uses easy language					√					√
	The teacher uses an appropriate and familiar example to illustrate main concept.				√						√
	The teacher creates friendly atmosphere				√					√	
	The teacher accommodates discussion among pupils		√								√
	The teacher gives hand-on activity				√						√
	The teacher enjoys teaching				√					√	
	The teacher is impatient with wrong answer				√			√			
	The teacher involves children to say opinions freely.		√								√
	The teacher encourages children to display diverse opinions			√							√
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.			√						√	
	The teacher combines individual work and group work appropriately.					√					√
Summary	At the end of the lesson, the teacher summarizes the lesson.				√					√	
	The teacher assigns homework at the end of lesson clearly.			√			√				
	The teacher explains about a connection between today's lesson and next lesson.	√					√				
General	The teacher prepares a lesson plan.	√							√		
	The teacher prepares a plan for taking note on the blackboard.			√					√		
Describe objectives of today's lesson		Students can compute fraction with the same denominator.					Meaning of fractions.				

Describe problems/activities (No. of problems, relation among them, their difficulty level etc.)	Three problems were given about denominator and numerator. Difficulty level suits for the ability level of the students.	One problem was give about denominator and numerator. Difficulty level suits for the ability level of the students because they can do themselves.
Describe children opinions	Not available	Not available
Assess who dominate solving problems during the lesson observed.	Teacher	Teacher, but students solving problem by themselves.
Assess which of the followings is regarded as the most important in the lesson observed. Understanding concept/mastering the procedure/thinking mathematically/applying to the daily life/finding correct answer.	Understanding concept.	Understanding concept.

Rating scale: 0-never, 1-seldom/to a little extent, 2-sometimes/to some extent, 3-often/to a considerable extent, 4-very often/to a great extent

(5) Results of Video Analysis and Classroom Observation Checklists

In the urban school, the teacher begins the lesson on time. The topic of the lesson is addition of fractions with the same denominator. The objective is for students to be able to compute fractions with the same denominator. The teacher reviews the meaning of fractions and exemplifies the addition of fractions with three examples. The teachers attempt to provide support to students who appear to have difficulties in understanding and appreciate the thinking attitudes of students. The teachers also use language that is easy to understand and teaching materials. Teachers assign worksheets to students on fractions and then pair of them takes addition fractions. As the students work on the activity, the teacher checks the answers and then each student presents their work to classmates. The lessons mostly comprise of the explanation of the teachers and a question and answer sequence. In the summary phase, the teachers summarize the lesson by questioning students.

The teacher begins the lesson on time in the rural school as well. The topic of the lesson is the meaning of fractions. The objective is to explain the concept of fractions to students and using pictures to represent fractions. The teacher introduces the materials to be used in the lesson–paper with a circle, triangle, and rectangle drawn on it. Students perform the assigned activity–dividing the paper into several equal parts–in groups and then present their work to their classmates. The lessons mostly comprise student activities. In the summary phase, the teachers summarize the lesson by relating the activity of students to the topic being taught.

2.2.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 9. Schedule of data collection

Date	Activity
14th November 2005	Testing at W School Interviewing students at W School Interviewing teacher at W School

21st November 2005	Testing at B School Interviewing students at B School Interviewing a teacher at B School

(2) Target Schools and Samples

The following criteria were taken into consideration for the selection of sample schools: location of schools (urban or rural), and language usage at schools and at home (standard Thai or local language). As a result, W School from the urban area and B School from the rural area were selected. In W School, students and teachers always use standard Thai language in class while in B School students and teachers occasionally use Isaan (local language) mixed with Thai language in class.

Table 10. Location of schools

	School location
Urban Primary School	W School, Ratchaburana, Bangkok (Capital of Thailand)
Rural Primary School	B School, Nong Rua District, Khon Kaen (About 440 kilometers far from Bangkok

W School: the selected urban school is situated in Bangkok, the capital of Thailand. It was founded in 1934 and conducts 24 teaching classes (Grades 1–6). The school is equipped with five teaching buildings, including special classrooms for music, arts, agriculture, and computers and an 8,800-square meter school area. The school is located in a good environmental area of Bangkok. The number of teachers and students are 45 and 828, respectively.

B School: The selected rural school is situated in Khon Kaen Province in the northeastern part of Thailand. It is approximately 440 kilometers away from Bangkok. This school conducts 15 teaching classes (Grades 1–6). The school is equipped with three teaching buildings, including special classrooms for music, arts, agriculture, and computers and a 19,200- square meter school area. The number of teachers and students are 23 and 424, respectively.

Table 11. Brief profiles of teachers

	Sex	How long you have taught	Subjects you teach
School in urban area	Male	1 year and 7 months	Mathematics, Computer, Vocation
School in rural area	Female	8 years and 4 months for teaching all subjects 4 years for teaching only mathematics	Mathematics

(3) Results of Questionnaire

Questionnaire items for mathematics teachers are divided into the following five categories (a) test-evaluation, (b) self-evaluation, (c) students-evaluation, (d) contents-evaluation, and (e) teaching-methodology.

According to the test-evaluation items, the prediction of teachers in urban and rural schools with regard to the average score of students on the given test is 50% and 40%, respectively. The prediction of teachers in both schools with regard to the performance of students is better than the actual scores of the students in the achievement test.

Teachers of both schools mentioned that students are unfamiliar with the pattern of the given test. The students have not yet been taught certain items that have been included in the achievement test. Teachers believe that the problems are presented in a difficult manner and the students have never solved such problems.

In response to the item on self evaluation, teachers of both schools stated that they find it easy to teach "Fractions". A few of the reasons for this are (a) content in this grade merely focuses on fractions with equal denominators, which are not so complicated, (b) there are few numbers to deal with when solving them, (c) teaching implies merely helping students to know how to divide numbers or things into equal parts. From the viewpoint of the teachers, teaching fractions implies following the steps demonstrated by them. However, they perceived teaching word problems on fractions as very difficult.

There are certain rather difficult topics to tcach in Grade 4, such as division and word problems, word problems related to addition, subtraction, multiplication, and division.

On the other hand, there are a few topics that are easy to teach in Grade 4, such as fractions and area. Some of the reasons for this are (a) students are able to visualize the diagram they have been taught, (b) the content is not very complicated, (c) teachers merely follow IPST (Institute for the Promotion of Teaching Science and Technology) textbooks.

The teachers of both schools stated that they are confident of teaching fractions. Despite this confidence the result of the achievement test does not indicate this. According to the teachers, there is a great influence of examinations on the teaching of teachers. Teachers are able to reflect upon their teaching techniques by analyzing examination results.

In response to the item on student evaluation, the teachers of both schools stated that it is easy for the students to learn fractions.

In response to the item on content evaluation, the teachers of both schools stated that fractions is an important topic among all other topics in mathematics. A few reasons behind this are as follows: (a) Students can relate fractions with other content areas such as addition, subtraction, multiplication, and division related to decimal numbers and word problems–if students can understand fractions well, it will influence understanding in other content areas; (b) learning fractions have an influence on daily life.

In response to the item on teaching methodology, with regard to how to teach the question "Which is longer $\frac{1}{4}$ m or $\frac{1}{3}$ m" teachers stated that they will instruct the students to compare by drawing a diagram or picture.

With regard to the question how to teach "What is half of 2m" teachers stated that they will show students 2m is 2 parts and half of it is 1. Then, they will show the students how to shade $\frac{1}{2}$ of 2 m.

Keeping in mind the strategies used by teachers from both schools a comparison between both schools in terms of students' understanding of fractions requires the use of some leading questions, waiting for students' responses, and providing a few examples to clarify certain difficult aspects.

(4) Results of Teachers' Interview

Table 12. Teachers' responses to the questionnaire

Questionnaire Items	Average school in urban area	Average school in rural area
(1) Teacher's forecast of pupils' average score	50%	40% In fact, I expected that they would obtain even lower than 40% because 10 students have problems with reading or

		interpreting. Regarding comparison of fractions, I think they are unable to do it. If I teach them this topic before you test the students, they could probably obtain 60%.
(2) Pupils' familiarity to the given test	Unfamiliar. In Grade 4, the curriculum encompasses only fractions with equal denominators but does not cover word problems and decimal numbers	Unfamiliar. Students have not learned using diagrams with fractions. If they are taught this, I think they can solve the problems because the problems are similar to the exercises that they have been assigned in class.
(3) Any questions which the pupils cannot solve.	Yes. Q2. A few students may answer in terms of division, instead of answering in terms of fractions. Q5. Students may not understand how to use variables to compare fractions. Q6. A few students were unable to interpret word problems in terms of fractions. Q7. All students have not learned about fractions in terms of decimal numbers. Q10. Students have not been drilled on word problems on fractions. Thus, they may not be able to pose or formulate problems.	Yes. Students have not learned yet. Q6–Q8 are word problems that they have not learned yet. Therefore, I think they are unable to solve these questions. However, I am not quite sure if this holds true for Q10. They could probably solve it.
(4) Difficulties in teaching "Fractions".	Easy. This is because content in this grade merely focuses on fractions with equal denominators, which are not so complicated: the students will be able to understand this.	Easy. Teaching fractions in this grade is easy for teachers and also easy for students to learn. It is also easier than other contents because there are fewer numbers to think about when solving them. Teaching means just helping students to know how to divide numbers or things into equal parts. However, it is difficult to teach word problems related to fractions.
(5) The most difficult topic/s to teach in Grade 5.	Division and Word Problems. Assigning word problems to students who do not understand division cause them to be confused with how to do addition, subtraction, and multiplication. Thus, we face the problem of how to drill the concept into the students.	Word problems related to addition, subtraction, multiplication, and division. The most difficult issue when teaching is dealing with word problems. Students are unable to deal with word problems. I do not know why they cannot solve word problems. I have been teaching for numerous

		years but fail to understand why students cannot handle these types of problems. In particular, word problems related to division are problematic for all grades.
(6) The easiest topic/s to teach in Grade 5.	Fractions. This is because students can visualize the diagram they have been taught. The content is not so complicated.	Fractions and area. These are the easiest topics among all the others. I merely follow the IPST textbooks. However, the most difficult topics are the ones related to word problems.
(7) Teacher's confidence in teaching "Fractions"	Confident.	I am confident, but not too much.
(8) Level of influence of the examination on teacher's teaching	Very influential. If our students do not perform well in tests conducted for further study or obtain lower ranks, we have to reflect on our teaching techniques.	Very influential. Occasionally, the contents of the test for further study exceeded what the students have learned. Moreover, the tests are occasionally written in a format that students are not familiar with.
(9) Degree of difficulty for the pupils to learn "Fractions"?	Easy.	Easier than other topics.
(10) Points of difficulty for the pupils to learn the concept of "Fractions"?	The difficulty in teaching fraction is how to help students understand dividing the given integers into equal parts. A majority of the students cannot understand this.	The obstacle is that fractions is a topic that is taught in Grade 4 and continues in Grade 5. The students do not understand the concept of "a half", this concept is not a trivial one. I also teach Grade 9. The students do not recognize that "a half" is $\frac{1}{2}$ because "a half" is an expression of spoken language. When they see $\frac{1}{2}$, they view it as a completely different thing.
(11) Existence of pupils' difficulty with the medium of instruction in learning mathematics	Existence of difficulty. Certain students lack basic mathematical knowledge. They have no intention or motivation to learn to think. We have to always encourage them to learn. Thus, teachers have to spend much of their time to closely monitor or regulate them.	No existence of difficulty. Learning with instructional medias is no problem. In particular, students like to view instructional VCDs. However, I wish to make the students develop a liking for mathematical games because certain games have a profound influence on students' understanding of mathematics.
(12) Importance of learning "Fractions" with comparison to any other topics in mathematics.	Important Students can connect fraction with other content areas such as addition, subtraction, multiplication, and division related to decimal numbers and	Important Learning fractions has a great influence on our daily lives. However, we seldom use fractions formally and use spoken language instead.

	word problems. Thus, if they can understand fractions well, it will influence their understanding in other content areas.	
(13) Teacher's main point/s of concern to the pupils in teaching "Fractions"?	Students must understand the meaning of "fractions" how to write fraction for integers, how to add and subtract fractions with equal denominators, and how to compare fractions.	In Grade 2, students must know the meaning of "fractions" In Grade 3, they must know how to divide "things" into "equal parts". In Grade 4, they must learn how to add and subtract fractions, as well as solve some word problems. In Grade 5, they must learn how to do multiplication and division.

2.2.5 Discussion

According to Table 12, teachers have strong belief that if they had taught their students the topics included in the test before it was administered, the students would have been able to solve the questions. One of the teachers stated that. "In fact, I expected that they would obtain lower than 40% because 10 students had problems with reading or interpreting the questions. I think they were unable to compare fractions ***if I had taught them this before testing, they may have obtained 60%.***" This statement reflected the teacher's viewpoint on learning mathematics. A teacher of the urban school also expressed his beliefs with regard to learning as follows "My students have not yet experienced ***being drilled to solve*** word problems on fractions. Thus, I do not think they can pose or formulate problems."

Undoubtedly both urban and rural school teachers expressed the idea that fraction was one of the easiest topics to teach. However, the students' scores did not reflect their confidence. In other words, urban and rural students obtained 43.2% and 26.1%, respectively, which is below the scores that the teachers expected their students to obtain. The interview revealed why they through fraction is easiest topic to teach they answered the question by stating "Fractions and area Fractions are the easiest topics among others. ***I just follow the steps given in the IPST[1] textbooks.***" This situation is consistent with what Inprasitha (1997) states "More than 90% of Thai teachers used mathematics textbooks as an instructional media. As a result, they tend to teach mathematical contents as appear on the pages of textbooks only." This reveals that teaching fractions by following steps provided in a textbook is not consistent with the level of understanding of the students' that is required by international tests like the one administered in this project. It must be noted that teaching mathematics according to the textbook is not harmful; however, mathematics textbooks in Thailand still focus on computational skills and not on process skills. As Inprasitha (1997) analyzed IPST mathematics textbooks in Grades 1–9 are "composed of drilling exercises and not related to real life situations."

In addition, the researcher noted from classroom observation that when students encounter a difficulty or run into trouble, the teacher helps them by providing suggestions or methods. This is apparent when students confront non-routine difficulties like what those encountered in this project.

Recently, the researcher himself has been dealing with another project to improve professional development through Lesson Study. However, it is too early to conclude what must be done to solve the problems of teaching and learning mathematics in Thailand. Participation in projects

[1] IPST is an abbreviation for Institution for Promoting Science and Technology. This organization takes responsibility for producing mathematics and science textbooks being used in all schools in Thailand.

like the present one causes the realization that some practical research questions, as provided below, must be raised:

1. How to shift mathematics teaching in Thailand, particularly from drill method to more effective ones?
2. How to effectively introduce the Lesson Study into Thai school culture based on these findings?
3. How to improve teacher education in Thailand based on international experiences like these projects?

References

Baba, T. (2005). *International comparative studies in influence of teachers' views about education on mathematics lesson at primary schools*. Hiroshima: Graduate school of International development and cooperation, Hiroshima University.

Boonyananta, S. and Kittivibul, N. (1996). *Synopsis of the Eighth National Education Development Plan (1997-2001)*. Office of the National Education Commission, Bangkok.

Inprasitha, M. (1997). Problem solving: a basis to reform mathematics instruction. *Journal of the National Research Council of Thailand, 29* (2), 221-259.

Komin, S., (1990). *The Psychology of the Thai people: Values and behavioral Patterns.* Bangkok: Research Center National Institute of Development Administration (NIDA).

The Institute for the Promotion of Teaching Science and Technology, Ministry of Education. (2003). *Mathematics Textbook for 4th grade.* Kurusapa Ladprao Publishing, Bangkok.

Thai Educational System. Retrieved January 10, 2005, from http://www.magma.ca/~thaiott/edsystem.htm

2.3 Philippines

Milagros D. Ibe
University of the Philippines

Hiroyuki Ninomiya
Saitama University

Levi Elipane
Saitama University

2.3.1 History of Mathematics Education

Education, in general, in the Philippines has undergone several stages of development from the pre-Spanish times to the present. As far as mathematics education is concerned, it may be divided into six periods: (1) the pre-Spanish period; (2) the Spanish period (1565–1898); (3) education under the American rule (1898–1923); (4) vocation-oriented science and mathematics education (1923–1948); (5) science and mathematics education reform period (1948–1973); and (6) institutionalization of science and mathematics education (1973–present). It must be noted that tracing the development of mathematics education in the Philippines goes hand in hand with the development of science education.

During the pre-Spanish period, education was informal, unstructured, and devoid of methods. Children were provided more vocational training by their parents and in the houses of tribal tutors and there was lesser focus on academics.

When the Spanish came, academic tutors were replaced by Spanish missionaries. Education was religion-oriented, and during the early stages of Spanish rule it was only accessible to the so-called "elites" of the society. It was only when the Educational Decree of 1863 was enacted that education was liberalized and a greater number of people in the Philippines obtained access to education. At least one primary school for boys and girls was established in each town under the purview of the municipal government. A normal school for male teachers was also established under the supervision of the Jesuits. Primary instruction was free and the teaching of Spanish was compulsory. Education during that period was described as inadequate, suppressed, and controlled (Estioko, 1994).

The signing of the Treaty of Paris between the United States and Spain in December 1898 ended the three centuries of Spanish rule in the Philippines and the Americans gained complete control of the Philippines.

The Americans, upon the recommendation of the Schurman Commission, immediately established a secularized and free public school system. Moreover, free primary instruction enforced by the Taft Commission deemed to train people for duties of citizenship and ordinary avocations of a civilized community. President McKinley, the president of the United States also suggested that primary education be provided in every part of the islands in the language of the people; however, in view of the many dialects spoken in the Philippines, he asked the Taft Commission to establish English as a common medium of communication and pay particular attention to its dissemination among people (Agoncillo and Guerrero, 1977).

A highly centralized public school system was installed in 1901 by the Philippine Commission by virtue of Act No. 74. However, this caused a problem with regard to the shortage of teachers, particularly that there were no Filipino teachers who could speak the English language. Hence, the Philippine Commission authorized the Secretary of Public Instruction to bring approximately 600 American teachers to the Philippines; these teachers were known as the Thomasites (after the name of the ship that brought them to the Philippines). However, the number of American teachers that were actually professionally qualified to teach Mathematics is unknown.

At the time, the objective of education emphasized the development of the three Rs—Arithmetic, Reading, and Writing—among students, thereby providing them with sufficient knowledge to be able to function in society. Science literacy was not a priority; however, numeric literacy remained a focus, making *arithmetic* a permanent feature in the elementary curriculum. With regard to the secondary curriculum, arithmetic was taught in the first year, Algebra in the second and third years, and Geometry and Trigonometry in the fourth year.

Further, since the commencement of the public school system in the country, the establishment of educational institutions for teachers has also been ascribed much attention. However, these institutions addressed teaching in general, and not science and mathematics education in particular. However, numeracy had always been considered a fundamental subject.

From 1923 onward, when vocational and industrial instruction was introduced in secondary schools, numerous American textbooks were already being adapted to Philippine requirements. There was an effort for the development of representative democracy; this marked the stage of vocation-oriented science and mathematics education in the Philippines (1923–1948). Science and Mathematics were considered to be important tools in the heavily vocational curricula. At this time, the students were being trained to be good citizens and skillful workers, not entrepreneurs, managers or inventors; much less, scientists and theorists. The focus was on rebuilding the economy post World War II.

This period was witness to several significant political events, such as the change from American governance to a Commonwealth regime, the Japanese occupation during World War II, the postwar reconstruction government, and the granting of independence by the United States. to the Philippines in 1946. Despite all these rapid, successive turn of events, the focus of education remained to accommodate school-age children at the elementary level, training them in vocational skills at the secondary level, and training teachers to teach. Mathematics remained a fundamental subject from the elementary grades onward.

During the reign of the Commonwealth government, the national language was introduced as a compulsory subject in high school beginning from the school year 1939–1940. Moreover, in 1940 the Secretary of Education authorized the use of local dialects as an auxiliary medium of classroom instruction in the primary grades. However, science and mathematics continued to be taught in English throughout this period.

From the average of a 1:30 teacher-student ratio during education under the American rule, the teacher-student ratio during this period increased to 1:40, on an average. There was also an acute shortage of qualified teachers, as a large number of teachers during this period were not college graduates.

There was a move to eliminate college undergraduates among the teachers; however, since over half of such teachers in several provinces were married and had families, a scheme was devised so they were able to continue their studies on Saturdays, or during their spare time.

With the end of World War II and the proclamation of independence in the Philippines in 1946, attention was focused on rehabilitation and reconstruction. This marked the beginning of another stage in the development of science and mathematics education in the Philippines, which was characterized by Educational Reforms.

In 1957, the Soviets launched SPUTNIK; this caused an alleged crisis in the educational system in the United States as the leader in technology and economy (Berends, 2004). The enthusiasm sparked by this event in the United States with regard to educational reforms also reverberated in the Philippines. Thus, the decade of the 60s witnessed an increase in the number of bodies of research, curriculum development, and teacher training.

In the mid-60s, changes in the mathematics curriculum were introduced in order to keep up with New Mathematics, the trend in the west at the time. The curriculum presented mathematics as a logical system of thinking and emphasized precision of language. Attention was ascribed to thinking processes instead of mechanical computation. Moreover, student activity and participation in problem-solving was maximized, and students were encouraged to think of alternative solutions. The most important factor in determining what was to be taught was its usefulness to society.

The reforms in curriculum that had begun in the mid-60s continued into the 70s. Textbooks and teachers' manuals, which emphasized inquiry learning, were developed. Since these new materials required new orientation with regard to teaching methods, in-service training, which was a serious requirement, was initiated for those teachers who were trained before the reforms were introduced. Therefore, workshops and seminars on strategies, equipment improvisation, and assessment procedures were conducted.

Further, during this period, science high schools were established along with regional science teaching centers. Moreover, science and mathematics teacher education programs offered new and improved courses.

The last period in the stages of development in mathematics education in the Philippines was the institutionalization of Science and Mathematics Education Programs that were introduced from 1973 onward. Efforts to improve science and mathematics education, which began during the previous period, were intensified in this period. Much emphasis was accorded to the type of curriculum material that was needed to be learned as well as the strategies and mode for teaching them. A number of curriculum revisions were undertaken. Mathematics material emphasized problem-solving and the relevance of this skill to daily life.

There was also great concern on the accessibility of education to all students as the government produced and distributed textbooks to public schools.

Qualifications of mathematics teachers began to be monitored and examined by education officials. As a result, passing government examinations and acquiring a license to teach have now become a requirement.

2.3.2 Basic Information

(1) Teachers' Qualification

Teachers in elementary schools are trained differently from those in secondary schools. Teachers of both levels have to be certified through the Licensure Examination for Teachers (LET), which is administered on the last Sunday of August every year by the Professional Regulation Commission.

Elementary school teachers must be graduates in the four-year Bachelor of Elementary Education degree program, and must possess a license to teach. The elementary school teacher does not have a subject specialization but is trained and competent in methods of teaching.

(2) School Calendar and Examination

The school year comprises ten months. The Department of Education stipulates that the School Year should have 200 to 205 school days. The school year usually begins in the first or second week of June in each calendar year and comes to a close in the last week of March or the first week of April of the following year. The end of the school year is partially influenced by the Catholic liturgical calendar. Usually, schools end before the Holy Week. A few schools continue even after Holy Thursday of the Lenten Season. The school calendar permits two weeks (12 to 15 days) for break from Christmas till New Year's Day. The summer vacation is for two months—April and May.

One school year is divided into four quarters. Each quarter ends with quarterly examinations. The end-of-school-year examinations are usually conducted from mid-March till close to Easter (for the year 2006, the school year ended on March 24). The National Examination for Grade 6 is around January or February, if not earlier. The new school year begins on the first Monday in

June in certain schools, otherwise on the second Monday

Table 1. School calendar in (year)

Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
			Summer Break								

(3) Medium of Instruction

There is a bilingual policy on the medium of instruction from Grade 3 upward. Science, Mathematics, and English must be taught in English; Social Studies, Filipino Language, Art, Practical Arts, Health and Physical Education are taught in the National Language—Filipino. In Grades 1 and 2, Health Education is taught in lieu of Science. Formal study of English begins in Grade 3. However, certain schools—particularly private schools—begin teaching English in Grade 1, some even as early as Kindergarten. Further, the medium of instruction, particularly in elite private schools, is English throughout; however, it is compulsory to teach Filipino as a subject. Although there is a bilingual policy, it is not fully observed/implemented. The reality is that a large number of teachers are not entirely competent in English or Filipino; therefore, there is mixing of languages and code-switching between Filipino and English.

In Grades 1 and 2, the teacher may use the ***lingua franca*** or the commonly spoken local language, or the mother tongue of the children in the locality. However, on the whole, the national language is becoming widely used—if not entirely, at least side by side with English. It is possible that the mother tongue of the teacher is the same as that of the students, but not necessarily so.

(4) Class Organization

Grades 1 to 4 are usually class-based; they are self-contained, that is, one teacher teaches all the subjects; she is the homeroom teacher. In Grades 5 and 6 certain teachers handle specific subjects like Math, Science, and English. In the present elementary school curriculum the four main subjects—Science, Mathematics, English, and Filipino—are each taught for 60 minutes per day, five days a week. A fifth subject—**Makabayan**—is an integration of Araling Panlipunan (Social Studies), Values, Art, Health and Physical Education, and Practical Arts. Each subject is taught in sessions of 30, 40, or 60 minutes, three or four days a week depending on the subject. Students in different grades in elementary schools remain in school anywhere from five to seven hours a day, depending on the grade as well as on the space (rooms and grounds) in the school.

With regard to the four main subjects, the number of lessons is dependant on the number of class sessions for the subject per week.

(5) Transfer System

A teacher is appointed by the Department of Education through the local office of the Department. He/she is assigned to a particular Division (province or city) or district, and a school. He may request a transfer to another school or municipality. If meritorious, the transfer is approved. However, requests for transfers are not common; therefore, teachers are less mobile as compared with other professionals. Once they become familiar with the culture of the school, they decide to stay rather than readjust to a new environment.

(6) Reason to be a Teacher

Teaching is a rather respected profession in the Philippines. Further, a degree course in elementary school teaching is easily available in all colleges and universities in the country. Teaching is less demanding as compared with other professions because teachers remain in school for merely six to seven hours per day, from Monday to Friday. Teaching as a profession blends well with family life, i.e., with being a wife and a mother, which is an important value among Filipinos. Teaching is now competitive in terms of salary with other professions; thus, a large number of Filipinos choose to become teachers.

2.3.3 Results from the Second Year Field Survey

(1) Survey Schedule

Although the counterpart researcher from Japan arrived in the Philippines on December 12, 2005 the field survey could not be conducted immediately because Christmas Break was commencing the following week. School attendance was becoming irregular due to the impending holidays (Christmas is the most important religious celebration in Catholic Philippines).

In the meanwhile, the two researchers conferred and discussed the procedures for the study; permissions were obtained from the two schools for the study.

The actual survey and testing, after meeting the principals of the schools, were scheduled in January 2006.

Table 2. Schedule of data collection

Date	Activity
12/01/2006	Data Collection in the Urban School (Quezon City)
13/01/2006	Data Collection in the Urban School (Quezon City)
16/01/2006	Data Collection in the Rural School (Apalit, Pampanga)
17/01/2006	Data Collection in the Rural School (Apalit, Pampanga)
23/01/2006	Departure of the researcher to Japan

(2) Target Schools and Samples

Since over 80% of the enrollment in elementary schools in the Philippines is in public schools, the schools targeted are public elementary schools; private schools are not included. Further, the schools in the survey included one from an urban area and the other from a rural area, thereby representing both urban and rural schools. Moreover, it was necessary that the rural school represent students whose mother tongue is not the National Language.

The selected schools were complete elementary schools, that is, they have Grades 1 to 6, with more than one section/class per grade. The target grade in this study is Grade 4 (or students aged 10 or 11 years).

Two schools—one from Quezon City in the National Capital Region and the other from the province of Pampanga in a municipality 50km from Manila—were the testing sites.

Table 3. Location of schools

	School location
Urban Primary School	San Jose, Quezon City (in National Capital Region)
Rural Primary School	Apalit, Pampanga (50 km. from Manila)

The urban school is located in the capital city, at a marginal distance from the heart of the city. It is an average ability school in an average-and-below-average-income community; it also caters to students from three areas settled-in by displaced families. There are three sections for each Grade at the primary level.

The rural school is located in a town that is a one-hour drive by bus from Manila in an agricultural part of the country.

Table 4. Brief profiles of teachers

	Sex	How long you have taught	Subjects you teach
School in urban area	Female	16 years	Science, Filipino, HEKASI (Social Studies) and Math
School in rural area	Female	14 years	All subjects in grade 4

(3) Results of Questionnaire

Table 5. Results of the achievement test (%)

	Average	Boys	Girls	Highest	Lowest
Urban School	57.25	53.75	58.75	90	20
Rural School	45.97561	44.16667	46.72414	95	15
All	51.54321	48.95833	52.63158	95	15

(4) Results of Teachers' Interview

Table 6. Teachers' responses to the questionnaire

Questionnaire Items	Average school in urban area	Average school in rural area
(1) Teacher's forecast of pupils' average score	80%	80% or 8 out of 10 Items are similar to those in periodic tests and quizzes.
(2) Pupils' familiarity to the given test	Yes. Same type of questions as in books and teacher tests	Yes. Same as in quizzes and Unit tests. Same as tests in textbooks.
(3) Any questions which the pupils cannot solve.	Yes. Q10 No example; not the usual type of questions in tests.	Yes, Q8 ; Q10 The way the question is worded is different ; misconceptions of pupils.
(4) Difficulties in teaching "Fractions".	"3" Difficult The pupils are not used to the subject matter.	"2" Easy With guidance of teacher, pupils will find it easy.
(5) The most difficult topic/s to teach in Grade 5.	Geometry Problems are different; pupils cannot identify figures.	Graphs Pupils don't know how to interpret data.
(6) The easiest topic/s to teach in Grade 5.	Simple addition and subtraction of whole numbers and similar fractions. These have been taught since Grade 1.	Operations (Addition) Because it is constantly taught.
(7) Teacher's confidence in teaching "Fractions"	"3" Confident	"4" Very Confident
(8) Level of influence of the examination on teacher's teaching	"1" Very much; assessment for my teaching.	"1" Very much Gives me feedback; usually positive.
(9) Degree of difficulty for the pupils to learn "Fractions"?	"2" Easy	"1" Very easy with proper guidance
(10) Points of difficulty for the pupils to learn the concept of	Difficulty : Pupil cannot visualize the figures or parts.	Difficulty : Adding or subtracting dissimilar fractions.

"Fractions"?		
(11) Existence of pupils' difficulty with the medium of instruction in learning mathematics	Yes. They are not used to English. They use it only in practice work.	No. Some difficulty with questioning techniques.
(12) Importance of learning "Fractions" with comparison to any other topics in mathematics.	Yes. It is needed in daily life.	Yes, because we use fractions in our daily life.
(13) Teacher's main point/s of concern to the pupils in teaching "Fractions"?	That they learn and understand the concept.	Sharing Patience Concrete illustrations.
(14) How to teach the following question to the pupils: "Which is longer, 1/4 m or 1/3 m?	Draw whole circles cut then into equal parts 1/3, ¼, etc. Teach it through paper folding, or cutting a string into fractional parts.	Use *cartolina* strips or paper ; Divide it into 4 or 3. Name the parts, and compare them. Go from concrete to abstract.
(15) How do deal with the student who answers that 1/2 m is half of 2 m.	Show by drawing. Ask the pupils to draw and illustrate 2m and 1/2m on the board before solving numerically.	Explain further by showing drawing and solutions. Measure off 1 m . Get ½ of the length; measure off 2 m. Get ½ of it. Compare the two.

2.3.4 Discussion

Since the classes in Grade 4 are self-contained (i.e., one teacher teaches all the subjects in the Grade), only two teachers were interviewed for the survey in the Philippines—one from the urban school and the other from the rural one.

Both teachers expected their students to be able to answer the test. They both predicted that the students will have an average score of 80%. However, it is evident from Table 5 that there is a large discrepancy between the expectation of the teachers and the actual performance of the students in the achievement test.

This large discrepancy in the perceived and actual performance of the students on the test draws implications on the manner in which the teachers understand and assess the mathematical learning of their students. Further, discrepancies with regard to the confidence of the teacher in teaching "fractions" and the performance of the students were also noted.

The difficulty of test items, as was perceived by the teachers and illustrated by the actual performance of the students, was influenced by factors such as familiarity of the item format to the students, the illustration, and directions for recording the answer(s). Hence, this draws implications on the need for teachers to possess the ability to be creative and innovative in thinking of various and alternative teaching approaches and materials for improving student learning. This may be addressed even at the time when prospective mathematics teachers are undergoing teacher training.

Moreover, both the teachers perceived that approximately half of the items will be easy for the students, and that the items were similar to those in the tests and quizzes given in the classroom and also in the textbooks. The two teachers believe that fractions is not a difficult topic for students. One stated that with guidance, patience, and utilization of concrete objects and illustrations, the students are able to understand fractions. One of the teachers stated that adding or subtracting similar fractions is as easy as adding (whole) numbers because the students have been practicing this since Grade 1. The topic that the teacher from the urban school identified as difficult to teach is <u>Geometry</u> because the students are unable to visualize or identify figures and shapes. The other teacher identified <u>Graphs</u> as difficult to teach because the students are unable to interpret them.

The difficulty experienced in teaching such topics, as explained by the teachers, is the unfamiliarity with the subject matter and the lack of prior knowledge required to progress with the topics. •On the other hand, easy topics are those that are constantly taught to the students from their initial years of learning.

In response to the question on how a students misconception that ½ of 2m and 1/2m are the same can be corrected, both teachers stated that they would help the students "visualize" by using 2m strips of paper and cutting these into two halves and half a meter in order to show the difference. •Teachers believe that enabling students to "visualize" provides them a better understanding. However, the teachers are restricted to the resources that are readily available to them.

References

Agoncillo, T. & Guerrero, M. (1977). *History of the Filipino People.* Quezon City, Philippines: RP Garcia Publishing Co.

Berends, M. (2004). In the Wake of "A Nation at Risk": New American Schools' Private Sector School Reform Initiative. *Peabody Journal of Education,* 79(1), 130-163.

Cutiongco, E., et.al. (2001). *One Hundred Years of Science and Mathematics Education in the Philippines.* Quezon City, Philippines: UP National Institue for Science and Mathematics Education Development and Pundasyon sa Pagpapaunlad ng Kaalaman sa Pagtuturo ng Agham, Ink.

Estioko, L. SVD (1994). *History of Education: A Filipino Perspective.* Manila: Logos Publications, Inc.

Republic of the Philippines Department of Education. *Historical Perspective of Philippine Educational System.* [online] http://www.deped.gov.ph/about_deped/history.asp

2.4 Bangladesh

Uddin Md. Mohsin
Open University

Takuya Baba
Hiroshima University

A.H.M. Mohiuddin
NAPE

2.4.1 History of Mathematics Education

During the pre-British period, 1 primary-level educational institution was established by the Muslim Nabab ruler for every 400 villagers. It was named the "Gurugriha" or "Maktab." It was run by a single teacher in a part of his/her house; there was no school building for running these schools. The education system was integrated rather than graded. The schooling duration was 4–5 years; however, this was extendable depending on the progress of the learner.

The subjects that were taught in these primary-level educational institutions were mathematics, mother tongue (Bangla), and religion. The content of mathematics was rather limited. The major contents related to effectively maintaining and handling daily accounts, acquiring skills in handling cash (sale and purchase in market), units of length and weight, etc.

During the British period in 1757, the East India Company invaded Bangladesh. The Wood's Education Despatch in 1854 was a significant attempt by the British rulers to modernize the education system in Bengal. Following the recommendation of Wood's Education Despatch, the Department of Public Instruction (DPI) was created in 1856 for the functioning and controlling of all primary-level educational institutions. There was no uniform curriculum or prescribed textbook for mathematics. The content of mathematics was limited to arithmetic. The major contents were addition, subtraction, multiplication, division, centesimal, etc. Students had to memorize multiplication tables and the units of weight, length, etc. Geometry was not taught at that time.

In 1947, Bangladesh became a part of Pakistan. In 1960, a commission was formed under the Ministry of Education of Pakistan in order to modernize mathematics education. The commission recommended a large amount of mathematics content for primary-level institutions. This included addition, subtraction, multiplication, division, fractions, division of fractions, decimals (addition, subtraction, and multiplication), division of decimal fractions, measurement (indigenous, British system), mean, unitary method, percentage, lowest common multiple, highest common factor, bar graph, preliminary concept of geometry, etc. In this period, a uniform curriculum and prescribed textbooks were introduced for all schools. An institutional teacher training system was also introduced for primary mathematics teachers and practice teaching was emphasized at the same time.

In 1971, Bangladesh became independent from Pakistan. The constitution states that primary education shall be the responsibility of the State (country). The State should adopt effective measures for the purpose of (a) establishing a uniform, mass-oriented, and universal system of education and extending free and compulsory education to all students to such a stage as may be determined by law, (b) relating education to the needs of the society and producing properly trained and motivated citizens to serve those needs, and (c) removing illiteracy within such time as may be determined by law. Soon after independence, a commission was formed in 1972 called the Dr. Kudret-e-Khuda commission; it recommended objectives, strategies, and action plans for creating a modern education system suited to the needs of an independent nation and compatible with the systems of neighboring countries. Under the recommendation of the commission, the curriculum and syllabus committee (in 1976) prepared the primary mathematics curriculum from the perspective of international mathematics education. A certain amount of new content was included in the curriculum in addition to the previous content. This included sets, use of mathematical signs and symbols, use of logic, number sentences, averages, budget, capital and expenditure, and the metric system.

Since 1992, a curriculum identified as the Essential Learning Continuum (ELC) with 53 competencies has been introduced at the primary education level. The ELC is a listing of competencies that serves as a guide to determine what to teach and assess among students at the primary level. Competency is defined as "the acquired knowledge, ability, and viewpoint when these could be applied in real life at the right time" (translated from the original by the author, in Bangla language). The students are expected to acquire these competencies in the five-year term of primary education; these competencies are referred to as the "terminal competencies of primary education."

From among the 53 competencies, 5 are mathematical. These are as follows: To gain the basic ideas of number and to be able to use them; to know the four fundamental operations and to be able to use them; to apply the simple methods of computation/calculation to problem-solving in everyday life; to know and use the units of money, length, weight, measurement, and time; and to know and understand geometrical shapes and figures.

2.4.2 Basic Information

(1) Teachers' Qualification

[For Primary School Teacher]

In Bangladesh, the minimum required academic qualification for female primary school teachers is Secondary School Certificate (S.S.C.; that is, 10^{th} grade) and that for male primary school teachers is Bachelors Degree or Higher Secondary Certificate (H.S.C.; that is, 12^{th} grade). Both male and female teachers are required to possess a Certificate in Education (C-in-Ed). Due to the increasing competitiveness of the job market, Bachelor and Master Degree holders also apply and accept teaching jobs in primary schools. This situation has practically resulted in recruiting teachers with higher qualifications at the primary level.

[For Secondary School Teacher]

There are 5 categories of teachers in each secondary school: headmaster, assistant headmaster, senior teachers, assistant teachers, and junior teachers. The minimum required academic qualification for secondary school teachers are as follows. The required academic qualifications for headmaster are second class Masters Degree with B.Ed. or its equivalent from a recognized university with 10 years experience in teaching/educational administration, or second class Bachelors Degree with B.Ed. and 12 years experience in teaching/educational administration, or Bachelors Degree with B.Ed. and 15 years experience in teaching/educational administration. With regard to the assistant headmaster, the required academic qualifications are second class Bachelors Degree with B.Ed. and 8 years teaching experience. In the case of senior teacher, the required academic qualifications are Bachelors Degree with B.Ed., or equivalent degree from a recognized university, or Kamil Degree (religious education) from a recognized Madrasah (religious school). The required academic qualification for assistant teacher is Bachelors Degree or Fazil Degree (religious education). In the case of junior teacher the required academic qualifications are S.S.C. with second division and C-in-Ed. for female candidates and H.S.C. with second division and C-in-Ed. or Bachelors Degree for male candidates.

(2) School Calendar and Examination

The school academic year begins from January and ends in December. In general, there are three school terms in Government Primary School (GPS): The first term (January–April); the second term (May–August), and the third term (September–December). Table 1 shows the school calendar in 2005; shaded portions represent long holidays. These holidays depend on the lunar calendar and vary with each year.

Table 1. School calendar in 2005

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.

There is no nationally established examination system from grades 1 to 4 at the primary level in Bangladesh. There is a term examination at the end of each term in individual schools. Moreover, there is a special national examination in grade 5 known as the primary scholarship examination. Every GPS is required to send students who are in the top 20 percent, and are completing grade 5, for this examination. Merely 31.7 percent of the examinees passed the scholarship examination in 2001 (Directorate of Primary Education, DPE, 2002). Table 2 shows the school calendar in 2005; shaded portions represent term examinations.

Table 2. School examinations

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.

(3) Medium of Instruction

The medium of instruction at the primary school level is Bangla. Bangla is the mother tongue for a majority of the people (e.g., Muslim and Hindu) and the national language (official language) in Bangladesh.

(4) Class Organization

Teachers at the primary level have to teach all subjects in different grades; there is no subject-specific teacher. The lesson duration at this level is 35 minutes.

A majority of the primary schools function with a two-shift schooling system. The first shift is from 9:30 a.m. till 12:00 noon and is for grades I and II; the second shift is from 12:30pm till 4:15 pm and is for grades III, IV, and V.

Schools that function with just one shift are from 9:30 am till 1:15 pm for grades I and II, and continue till 4:15 pm for grades III, IV, and V.

For two-shift schools, the weekly class load of a teacher is 46.6 lessons in rural schools and 43.5 lessons in urban schools. For one-shift schools, the weekly class load of a teacher is 39.5 lessons in rural schools and 38.5 lessons in urban schools (The National Academy for Primary Education, NAPE, 2002).

Table 3. School hours

	First shift						Second shift			
Hour	8	9	10	11	12	13	14	15	16	17
Grade		Grade I and II					Grade III, IV and V			

(5) Transfer System

There is no official transfer system of teachers in Bangladesh at the primary-school level. At the time of recruitment, the authorities ascribe greater priority to the candidates' own Upazila (region). Once they are appointed, the teachers seek appointment to a town school or to a school where there exists a good public transportation system. This is particularly true for female teachers as it is difficult for them to find accommodation in the village where they are supposed to serve.

In case there are problems among teachers in a particular school, the authorities transfer the particular teacher to another school.

(6) Reason to be a Teacher

There are certain reasons for which people in Bangladesh select the teaching profession. First, the attraction for this profession may be that teaching is recognized as a highly respected profession in the society and the working place is usually close to residential areas; this permits people to enjoy a dignified status. Second, lack of other job facilities may attract certain people toward teaching, despite the unsatisfactory salary. Lastly, another reason may be that teaching at the primary level is used merely as a platform for a better prospective job and career (according to Focus Group Discussion with teacher trainees, 2005).

2.4.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 4. Schedule of data collection

Date	Activity
25th November, 2004	Arrival in Dhaka, Bangladesh
26th November, 2004	Mymensing (NAPE)
27th &28th November, 2004	NAPE (Meeting for the research, questionnaire revision, correction etc.)
29th & 30th November, 2004	Data collection in Urban school, Mymensing
1st & 2nd December, 2004	Data collection in Rural school, Mymensing
3rd December, 2004	Return to Dhaka
4th December, 2004	Stay in Dhaka
5th December, 2004	Departure from Dhaka, Bangladesh

(2) Target Schools and Samples

In Bangladesh, GPSs are categorized into four levels—A, B, C, and D. This categorization is based on certain factors such as School Management Committee (SMC), Parent-Teachers Association (PTA), result of students, result of scholarship examination, playground, plantation, scout group, etc. A majority of the schools belong to category B.

From among 1,249 GPS, one average GPS (category B school) was selected from an urban area and one average GPS (category B school) was selected from a rural area in Mymensingh district (source: Primary Education Statistics in Bangladesh, 2001). It must be mentioned that Mymensingh is one of 64 districts in Bangladesh.

Table 5. Location of schools

	School location
Urban Primary School	Center of Mymensingh city
Rural Primary School	About 30 kilometers away from the main city

The selected urban school is situated in the center of Mymensingh city and the rural one is situated approximately 30 kilometers away from the main city. There are a total of 6 teachers in the urban school and 5 in the rural one. These teachers have to teach all subjects in different grades; there is no subject-specific teacher at the primary level.

There are a total of 6 rooms in both schools; 5 rooms are used as classrooms and the remaining as office-cum teachers' common room. The rooms used as classrooms are well decorated with colorful pictures and figures from textbooks such as social studies, science, mathematics, and geometry. Verses from the holy Quran and a selection of morals are also written on the walls inside the classrooms.

Table 6. Distribution of sample

	Grade	Male	Female	Total
Urban Primary School	4	14	32	46
Rural Primary School	4	15	14	29
Total		29	46	75

Table 7. Distribution of students' age

Age (Years)	No. of Pupils		
	Urban	Rural	Total
8	4	11	15
9	13	3	16
10	17	8	25
11	3	1	4
12	4	4	8
13	1	1	2
14	4	1	5
Average	10.2	9.7	10

(3) Results of Interview

(a) Result of interview for head teachers

Table 8. Responses from head teacher

	Interview items	Urban school	Rural school
[Problem]	1-1 What do you think is the biggest problem in teaching mathematics in your school?	High teacher-student ratio and absenteeism are the biggest problem in teaching.	Absenteeism is the biggest problem in teaching.
	1-2 What kind of action do you take against that problem as an administrator	Visiting students' home, addressing this problem in the SMC meeting, arranging mother rally.	Visiting students' home, engaging regular student to bring the absentee student into the school.
	1-3 Do you observe lessons by teachers? Yes or No. If yes, how often do you observe them?	Yes, 2 or 3 lessons in a month	Yes, 3 or 4 lessons in a month
	1-4 What kind of advice do you give to young teachers at your school?	Firstly, to know about the students' ability and then to take special care of the backward students.	Maintain good relationship with guardians and be punctual.
[In-service training]	2-1 Do you see any impact of in-service course offered to teachers? If yes, is it negative or positive? Please describe the impact a little more	Yes, it is a positive impact. For example use of teaching aid/materials properly, time management in effective way etc.	Yes, it is a positive impact. For example use of lesson plan, use of teaching aids/materials etc.
	2-2 What kind of training do you think is necessary for teachers in your school, if a new training course is designed?	On activity based teaching & learning and presentation of teaching aids.	Demonstration lesson based on classroom activities.

(b) Result of interview for mathematics teachers

Table 9. Responses from mathematics teachers

	Interview items	Urban school	Rural school
[Problem]	1-1 What do you think is the biggest problem in teaching mathematics in your class?	Students' absenteeism and lack of teaching aids.	Lack of home support and lack of attention to lesson.
	1-2 What kind of action do you take against that problem?	Preparing and collecting some teaching aids, visiting students' home, engaging regular students to bring the absent students.	Arranging meeting with guardian to motivate them and try to make the lesson joyful by using teaching aids and games.
[Today's lesson]	2-1 What was the purpose of today's lesson?	To understand what is fraction & simple fraction.	To understand simple fraction and proper fraction.
	2-2 How much do you think the purpose was attained?	I think almost all-purpose (aim) was attained.	95% of objectives were attained.
	2-3 What do you think are the most important factors for successful lesson?	Learning by doing, use of teaching aids, active participation of students.	Use of appropriate teaching aids, to make students attentive in the class.
	2-4 What kind of teaching would you like to do?	I would like to use Multiple Ways of Teaching & Learning (MWTL) in the classroom.	I would like to use Multiple Ways of Teaching & Learning (MWTL) in the classroom.
[In-service training]	3-1 Have you ever had a teacher training after you became a teacher?	Yes, MWTL	Yes, subject based training, MWTL.
	3-2 Which kind of training, if you had before, do you think is useful for your teaching?	MWTL training, I think it is useful for teaching.	Mathematics subject based training and MWTL, are useful in the mathematics teaching.
	3-3 What kind of training do you think is necessary for improvement of your lesson, if a new training course is designed?	Training should be focused on demonstration lesson on mathematics teaching.	Training should be focused on preparation, collection and preservation of teaching aids on mathematics teaching.

(4) Results of Video Analysis and Classroom Observation Checklists

(a) Result of video analysis of urban and rural school

Both urban and rural school teachers began the lesson on time and sang a song at the beginning with students in order to create a learning environment. The topic of the lesson was simple fractions. The lesson objectives were understandable to the student; however, the teachers did not create a relation with the previous lesson. Moreover, they did not explain the concept/meaning of fractions and their relationship with division (fractions and division are correlated).

The teachers attempted to provide support to students who appeared to have difficulties in understanding and appreciated the thinking attitudes of students. The teachers also used easy language and teaching materials and created a friendly atmosphere; however, they did not reflect on discussion among students in order to encourage their opinions, ask questions, and solve problems. It was observed that the teachers did not combine individual and group work appropriately.

The responses of the students were coded according to individual responses or a chorus reply. Teachers took excessive time to evaluate the classroom activities of students. In addition, they did not clearly explain the reasons of errors to the students. The lessons usually comprised explanations provided by the teachers and a question and answer sequence. The lessons appeared reiterative in nature rather than developmental to ensure progression in learning.

In the summary phase, the teachers summarized the lesson. They did not mention anything regarding homework and the connection between the day's lesson and the next lesson.

(b) Result of lesson observation checklists of both the urban and rural schools

The following lesson observation checklist has been evaluated by Mr. A.H.M. Mohiuddin, the specialist from the National Academy for Primary Education (NAPE). The results of the lesson observation checklists are presented in table 10.

Table 10. Responses of lesson observation checklists

		Urban school					Rural school				
Introduction		0	1	2	3	4	0	1	2	3	4
	The teacher starts the class on time					√					√
	The teacher made the objective clear					√					√
	The objective suits to the level of children					√					√
	Relationship with the previous lesson is clear			√					√		
Development	The teacher gives supports to pupils who seem to have little understanding			√						√	
	The expresses appreciation for pupils' thinking attitudes		√							√	
	The teacher assesses the pupils' comprehension during teaching learning				√		√				
	The teacher uses easy language					√					√
	The teacher uses an appropriate and familiar example to illustrate main concept.				√					√	
	The teacher creates friendly atmosphere				√					√	
	The teacher accommodates discussion among pupils		√					√			
	The teacher gives hand-on activity		√					√			
	The teacher enjoys teaching				√					√	
	The teacher is impatient with wrong answer				√				√		
	The teacher involves children to say opinions freely.			√						√	
	The teacher encourages children to display diverse opinions			√					√		
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.			√				√			
	The teacher combines individual work and group work appropriately.		√				√				

Summary	At the end of the lesson, the teacher summarizes the lesson.				√					√	
	The teacher assigns homework at the end of lesson clearly.	√					√				
	The teacher explains about a connection between today's lesson and next lesson.	√					√				
General	The teacher prepares a lesson plan.	√							√		
	The teacher prepares a plan for taking note on the blackboard.	√					√				
Describe objectives of today's lesson		To understand simple fraction.					To understand simple fraction and proper fraction.				
Describe problems/activities (No. of problems, relation among them, their difficulty level etc.)		Two problems were given about denominator and numerator. Difficulty level suits for the ability level of the students.					No mathematical problems were given to students but some of the students were taken to blackboard to solve a problem. Difficulty level suits for the ability level of the students.				
Describe children opinions		Not available					Not available				
Assess who dominate solving problems during the lesson observed.		Teacher dominates in solving problems.					Teacher dominates but students had considerable role in solving problem.				
Assess which of the followings is regarded as the most important in the lesson observed. Understanding concept/mastering the procedure/thinking mathematically/applying to the daily life/finding correct answer.		Understanding concept.					Understanding concept.				

Rating scale: 0-never, 1-seldom/to a little extent, 2-sometimes/to some extent, 3-often/to a considerable extent, 4-very often/to a great extent

2.4.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 11. Schedule of data collection

Date	Activity
9th November, 2005	Arrival in Dhaka, Bangladesh
10th November, 2005	Mymensing (NAPE)
11th &12th November, 2005	NAPE (Meeting for the research, questionnaire revision, correction etc.)
13th & 14th November, 2005	Data collection in Urban school, Mymensing
15th & 16th November, 2005	Data collection in Rural school, Mymensing
17th November, 2005	Departure from Dhaka, Bangladesh.

(2) Target Schools and Samples

In Bangladesh, government primary schools are categorized into four levels—A, B, C, and D. This categorization is based on certain factors such as School Management Committee (SMC), Parent-Teachers Association (PTA), results of students, result of scholarship examination, playground, plantation, scout group, etc. A majority of the schools belong to category B. For example, according to the JICA-Bangladesh project third pre-evaluation study report (2005) in Mymensingh district (2002–2003), 25.4% of the GPS belong to category A, 64.6% to category B, 9.9% to category C, and 0.1% to category D.

From among 1,249 GPSs, one average GPS (category B school) was selected from an urban area and one average GPS (category B school) was selected from a rural area in Mymensingh district (DPE, 2002).

Table 12. Location of schools

	School location
Urban Primary School	Center of Mymensingh city
Rural Primary School	About 30 kilometers away from the main city

The selected urban school is situated in the center of Mymensingh city and the rural one is situated approximately 30 kilometers away from the main city. There are a total of 6 teachers in the urban school and 5 in the rural school. These teachers have to teach all the subjects in different grades; there is no subject-specific teacher in these schools.

There are a total of 6 rooms in both schools, where 5 rooms are used as classrooms and the remaining as office-cum teachers' common room. The rooms used as classrooms are well decorated with colorful pictures and figures from textbooks such as social studies, science, mathematics, and geography. Verses from the holy Quran and a selection of morals are also written on the walls inside the classrooms.

Table 13. Brief profiles of teachers

	Sex	How long you have taught	Subjects you teach
School in urban area	*Male (1), Female (1)	14 years 5 months 29 years 10 months	All subjects Mathematics, Bangla
School in rural area	*Female (1) Female (1)	10 years 5 months 5 years 5 months	Mathematics, Social studies, Science Mathematics, English, Bangla

Legend: * At present mathematics class teacher at Grade 5, ( ) parenthesis denotes the number of teacher

Four teachers were selected for this research. Two of them were selected from the urban school and 2 from the rural school. Three teachers were female and 1 was male. All of them possess one-year training in C-in-Ed (Certificate in Education) training from Primary Training Institute (PTI). Further, they have been teaching in primary schools for a long time. They have to teach all subjects in different grades at the primary level.

(3) Results of Questionnaire

(a) Teachers' perception on the achievement test and teaching/learning of Mathematics and "fractions"

Table 14. Teachers' responses to the questionnaire

Questionnaire Items	Average school in urban area		Average school in rural area	
	1st person	2nd person	1st person	2nd person
(1) Teacher's forecast of pupils' average score	70 %	70%	33 %	40%
(2) Pupils' familiarity to the given test	No Reason/s: 1. Problems and solutions are not stated/explained in this way in the textbook. 2. The examination system/pattern is not in this way.	No Reason/s: 1. The textbook is not written in this way.	No Reason/s: 1. Pupils are not accustomed to this type of question as their classroom test.	No Reason/s: 1. Classroom test contents are almost the same as textbook contents.
(3) Any questions which the pupils cannot solve.	Yes Reason/s: 1. Question no. 4.3 is difficult for pupils to solve.	No	Yes Reason/s: 1. Question no. 5, I think the problem is presented in difficult way. Problem is not complete. 2. Question no. 8, Pupils never have solved such kind of problem.	Yes 1. Question no. 3, Pupils never have solved such kind of problem.
(4) Difficulties in teaching "Fractions".	Easy Reason/s:	Difficult Reason/s:	Difficult Reason/s: 1. Pupils can easily understand the integer but they find it difficult to understand the concept of fraction. 2. Practical use of fraction is difficult.	Difficult 1. Fraction related real/concrete materials are limited in the classroom teaching.
(5) The most difficult topic/s to teach in Grade 5.	Topic(s): Geometry, simple fraction and decimal fraction. Reason/s: 1. Lack of prior knowledge. 2. Lack of proper reflection of pupils' demand in the textbook.	Topic(s): Geometry, simple fraction and decimal fraction. Reason/s: 1. Lack of proper reflection of pupils' demand in the textbook. 2. Exercises are difficult to understand and to solve.	Topic(s): Highest common factor, least common multiple, measurement, simple fraction and decimal fraction. Reason/s: 1. Pupils cannot memorize the formula. 2. It is hard to explain about these topics	Topic(s): Highest common factor, least common multiple, measurement, simple fraction and decimal fraction. Reason/s: 1. Pupils cannot memorize the

			without having the real/concrete materials.	formula. 2. It is hard to explain about these topics without having the real/concrete materials.
(6) The easiest topic/s to teach in Grade 5.	Topic(s): Addition, subtraction, multiplication and division related problems; the highest number and the lowest number; average Reason/s: 1. Pupils are previously familiar with these topics. 2. Pupils are more interested towards well-known topics.	Topic(s): Addition, subtraction. Reason/s: 1. Pupils are more interested towards well-known topics. 2. These topics are not so difficult and therefore easy to understand.	Topic(s): Unitary method, simplification, average, percentage, time Reason/s: 1.These topics are more related to real life. 2. Easy to explain.	Topic(s): Unitary method, simplification, average. Reason/s: 1.These topics are more related to real life. 2. Easy to explain.
(7) Teacher's confidence in teaching "Fractions"	Confident	Confident	Confident	Confident
(8) Level of influence of the examination on teacher's teaching	Very much, Example/s: 1. Pupils' interest in examination decrease the tendency of absenteeism. 2. Creates the competitive attitude among the pupils	Much Example/s: 1. Gets an opportunity of self-evaluation. 2. Also increases the interest of learning.	Much Example/s: 1. Teachers are able to understand the weakness of teaching through examination result and then they try to recover the weakness in the next lesson.	Much Example/s: 1. Teachers are able to understand the pupils' level of attainment through examination result.
(9) Degree of difficulty for the pupils to learn "Fractions"?	Easy	Easy	Easy	Easy
(10) Points of difficulty for the pupils to learn the concept of "Fractions"?	1. To differentiate the types of fractions 2. To convert into fraction.	1. To convert into fraction 2. To fragment of fraction.	1. To find out the least common multiple. 2. To convert into equal denominator.	1. To convert into equal denominator. 2. To write the fraction on notebook.
(11) Existence of pupils' difficulty with the medium of instruction in learning mathematics	No	No	No	No
(12) Importance of learning "Fractions" with comparison to any other topics in mathematics.	Yes Reason/s: 1. If the idea of fraction is not clear at the initial level then it hampers the pupils' academic acquisition and practical life. 2. To solve the	Yes Reason/s: 1. To solve the mathematical problems the use of fraction is noteworthy. 2. Pupils are able to know about	Yes Reason/s: 1. Fraction is an inseparable part of mathematics. Integer can be formed by adding some fragmented parts	Yes Reason/s: 1. Pupils are able to know about larger, smaller, comparing things etc. through

	mathematical problems the use of fraction is noteworthy.	larger, smaller, comparing things etc. through fraction.	(e.g. 1= 1/3 + 1/3 + 1/3). That is why it is important.	fraction.
(13) Teacher's main point/s of concern to the pupils in teaching "Fractions"?	The main points are the concept of fraction, numerator and denominator, types of fraction, the relationship of numerator and denominator with types of fractions.	The main points are the concept of fraction, numerator and denominator, types of fraction.	The main points are the numerator and denominator, different types of fractions.	The main points are the concept of fraction, different types of fractions.

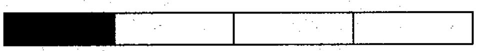
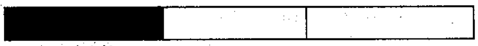
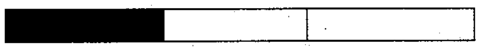
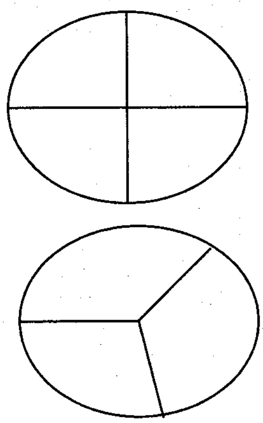
Legend: ( ) parenthesis denotes the number of respondent

(b) Teachers' Strategies in Teaching "Fractions"

Q (14) Describe how to teach the following question to the pupils?

"Which is longer 1/4m or 1/3m?"

Table 15. Teachers' strategies in teaching fraction

Average school in urban area	Average school in rural area
1. They will instruct the pupils at first to convert the above two fractions into the equal denominator fractions and then will show the pupils which one is longer. 2. Will explain it by using the following figures  1/4  1/3	1. Will explain it by using the following figures 1/4  1/3  2. Will explain it by using the following figures 

Q (15) Suppose you posed the following question to the students in a lesson. "What is half of 2m?" If the student answers, "It is 1/2m," how will you deal with such a student in class?

Table 16. Teachers' strategies in teaching fraction

Average school in urban area	Average school in rural area
1. Will explain the idea of half of 2m and 1/2m. 2. Will explain it by using the meter scale.	1. Will ask the student to take a 2-meter long meter scale and then instruct the student to divide it into two equal parts. Therefore, the student will be able to understand the correct answer.

2.4.5 Discussion

(1) Analysis of Interview Items for Head Teacher

Interview items for the head teacher were divided into 2 categories: school management and in-service teacher training.

According to the discussion of the interview of the head teachers (both in urban and rural schools), a high teacher-student ratio and absenteeism are the greatest difficulties in teaching mathematics in their schools. In order to overcome these difficulties, teachers visit students' homes, raise this problem to the School Management Committee (SMC), and engage other students to bring the absentee student to the school.

Usually, head teachers observe 2 or 3 regular lessons in a month and provide them with feedback. They also provide certain subsequent advice to young teachers. First, it is recommended that all the teachers are aware of the individual abilities of the students, and to take special care of the slower students. Moreover, teachers must also maintain a good relationship with guardians and explain to them their responsibilities toward children.

The head teachers of both schools agreed that the training of in-service teachers has a significant impact in classroom teaching. The modern technique of training has caused them to become self-motivated. As a positive effect, the teachers use teaching materials and lesson plans suitably, and are also able to manage time in an effective manner.

In response to the question on a training course in the future, they recommended courses on lesson demonstration related to classroom activities and presentation of teaching aids.

(2) Analysis of Interview Items for Mathematics Teacher

Interview items for the mathematics teacher were divided into the following 3 categories: school management, academic practice, and in-service teacher training.

In the discussion of the interview of the mathematics teacher, the teacher from the urban school argued that absenteeism of students and lack of teaching aids are the greatest difficulties in teaching mathematics. In order to overcome these problems, the teacher prepares and collects certain teaching aids, visits the homes of students, and engages regular students to bring absent students to school.

On the other hand, the teacher from the rural school stated that lack of home support and attention to lesson are the greatest difficulties in teaching mathematics. In order to overcome these problems, the teacher arranges a meeting with the guardians in order to motivate them, and attempts to make the lesson interesting by using teaching aids and games.

In response to the question on the day's lesson, teachers of both schools claimed that they were able to successfully attain all purposes of the day's lesson. They believe that the most important factors for a successful lesson are active participation of students, learning by doing, use of appropriate teaching aids, etc.

Teachers of both schools have undergone basic in-service training, which emphasizes MWTL and subject orientation subsequent to becoming a teacher; this is useful in mathematics teaching.

In response to the question on a training course in the future, they recommended courses on demonstration lessons for mathematics teaching, preparation, collection and preservation of teaching aids, etc.

(3) Analysis of Classroom Teaching Video and Lesson Observation Checklists

According to the results of video analysis and lesson observation checklists, it is observed that urban and rural schools have a large number of phenomena/conditions in common in terms of teaching and learning in the classroom.

Teachers in both schools begin the class on time and the objectives of the lesson are clear and suitable for the children; however, the relationship with the previous lesson is unclear.

Teachers provide little scope to the children to express their opinion freely, to display diverse opinions, to engage in learning by asking questions, solving problems, etc.

There are also certain weak areas in teaching and learning such as arranging discussion among students, assigning hands-on activity, combining individual and group work, expressing appreciation for the thinking attitudes of students, assigning home work, providing an explanation with regard to a connection between the day's lesson and the next, and preparing a plan for writing on the blackboard.

On the other hand, there are certain laudable aspects found in teaching and learning such as using easy language, using appropriate and familiar examples, illustrating the main concept, creating a friendly atmosphere, enjoying teaching, and summarizing the lesson at the end of the class.

The proportions of time allocation for each activity do not appear to be well organized. In addition, it was found that the teachers do not follow their lesson plans adequately. Consequently, the teaching-learning activities in the actual lesson failed to achieve the high expectation described in the objectives.

The analysis also revealed that the classrooms are mainly dominated by the explanation of the teachers and a question and answer sequence. Thinking/reasoning questions are extremely rare.

Through the above discussion it is evident that children are not involved in any creative or innovative activity. In most of the cases, there is a greater emphasis on memorizing instead of understanding the concepts and creative learning. Teachers are observed to dominate in problem-solving in the entire lesson.

In the interviews, teachers mentioned that they were able to attain all purposes of the day's lesson successfully; however, the perception of the third party is that the teachers have failed to attain their objectives of making the lesson clearly understandable to the children. The perception of teachers with regard to attainment does not guarantee good practice or achievement.

(4) Analysis of the Results from Teachers' Responses to the Questionnaire

Questionnaire items for mathematics teachers are divided into the following five categories: (a) Test-evaluation, (b) self evaluation, (c), evaluation of students, (d) evaluation of contents, and (e) teaching methodology.

According to the test evaluation items, the expectation of teachers in the urban and rural schools with regard to the average score of students on the given test are 70% and 33%, respectively. The high expectation of urban school teachers with regard to the performance of students does not reflect on the students' achievement test.

School teachers of both schools mentioned that difficulties and solutions in the textbook are presented in a different manner from that of the test. Classroom test contents are almost the same as textbook contents; therefore, students were not accustomed to the format of the test. They also noted that question numbers 4.3, 5, and 8 were difficult for students to solve. They believe that the problems were presented in a difficult manner and students have never solved such problems.

In response to the item on self-evaluation, teachers of both schools stated that they experience difficulties in teaching "Fractions." A few of the reasons behind this are (a) students can easily understand integers; however, they find it difficult to understand the concept of fractions, (b) practical use of fractions is difficult, (c) there is limited availability of fraction-related real/concrete materials in the classroom.

In general, teachers reported that there exist certain topics that are most difficult to teach in Grade 5, such as geometry, simple fractions, decimal fractions, highest common factor, and least common multiple. A few of the reasons for this are (a) lack of prior knowledge, (b) lack of adequate reflection of students' demand in the textbook, (c) inability of students to memorize the formula, etc.

On the other hand, there are certain topics that are easy to teach in Grade 5, such as addition, subtraction, multiplication, division-related problems, unitary method, simplification, and average. A few of the reasons for this are (a) students are previously familiar with these topics, (b) students are more interested in familiar topics, (c) these topics have a greater relation to real life, etc.

The teachers of both schools stated that they are confident of teaching fractions. In spite of this confidence, the scores of the students on the achievement result are unreasonably low. This gap may indicate that the perception of the teachers does not reflect the understanding of the students.

According to the teachers, examinations greatly influence their teaching. Teachers are able to understand the weakness of teaching through the results of examinations; they attempt to recover their weakness in the subsequent lesson. At the same time, they are able to understand the level of understanding of students through the result of the examination. The interest of students in examinations decreases the tendency of absenteeism, creates competitive attitudes among students, and also increases their learning interests, as claimed by the teachers.

In response to the item on the evaluation of students, the teachers of both schools stated that it is easy for students to learn fractions. They also mentioned that there are certain difficult aspects in learning the concept of fractions. Examples of this are types of fractions, finding the least common multiple, converting into equal denominator, finding a fragment of a fraction, etc.

In response to the item on contents-evaluation, the teachers of both schools stated that learning of fractions is an important topic among all other topics in mathematics. A few of the reasons behind this are (a) if the idea of fractions is not clear at the initial level then it hampers the students' academic attainment and practical life, (b) the use of fractions to solve mathematical problems is noteworthy, (c) fractions are an inseparable part of mathematics, and (d) integers can be formed by adding certain fragmented parts (e.g., $1 = 1/3 + 1/3 + 1/3$), etc. They mentioned that the main aspects of teaching "Fractions" is the concept of fractions, numerator and denominator, types of fractions, and the relationship of numerator and denominator with types of fractions. This is indicative of their limited idea of fractions.

In response to the item on teaching methodology, with regard the question on how to teach "which is longer 1/4m or 1/3m?" teachers stated that they will instruct the students to first convert the above 2 fractions into equal-denominator fractions and then will show the students which one is longer, or will explain it by using some figures or objects such as meter scale, circle, etc.

With regard to the question "how will you teach what is half of 2m?" teachers stated that they will explain the idea of half of 2m and 1/2m to the students in class, or will ask the student to take a 2-meter long meter scale and then instruct the student to divide it into 2 equal parts. As a result, the student will be able to ascertain the correct answer.

References

Directorate of Primary Education, DPE (2002). *Primary Education Statistics in Bangladesh-2001,* Ministry of Primary and Mass Education Division, GOB.

JICA-Bangladesh Project Report (2005). *The Third Pre-Evaluation Study Report,* Japan International Cooperation Agency, Bangladesh.

The National Academy for Primary Education, NAPE (2002). *To study the impact of one shift primary school on the learning achievement of children and management of school,* Mymensingh, Bangladesh.

2.5 Ghana

Hisashi Kuwayama, Hiroshima University
Ernest Kofi Davis, Univeristy of Cape Coast
J. Ghartey Ampiah, Univeristy of Cape Coast
Narh Francis Kwabla, Mount Mary Teachers' Training College

2.5.1 History of Mathematics Education

(1) Introduction

The history of mathematics education in Ghana dates back to the colonial era when castle schools were established. Under this school system, despite religious education being predominant, arithmetic was taught as a component of the school curriculum. The then school system aimed at enhancing commerce and converting the locals into Christians. The first section of this paper attempts to give an account of the history of mathematics education in Ghana by considering a brief history of traditional and formal education in Ghana as well as curriculum development and practices before and after independence in 1957.

- Brief History of Education In Ghana-Traditional Education

The education system existed in Ghana even prior to its colonization. At the time, education would take place within society; such education was not structured but sporadic. The elderly in society served as teachers. It was the collective responsibility of the elderly to help the young ones through the process of socialization.

Traditional education comprised the following two components: general education and specialist training. General education would take place before the onset of puberty. This was the stage at which children would be educated about the norms of society. At this stage, all the elderly in society served as teachers. Special training would begin after puberty. At this stage, the child would be taught an employable skill, which could be related to the parents' profession or trade, such as carpentry or carving. In those days, it was also common for parents to send their children to another person in order to learn the trade from that person. This type of education, however, had a major limitation in that it involved neither reading nor writing. Hence, major information would be lost upon the death of a specialist.

- Brief History of formal education in Ghana – Castle Schools

The history of formal education in Ghana dates back to the early fourteenth century. Formal education began in castles that were built by European merchants in Ghana during the colonial era. These castle schools were established in order to provide education for the molatto children. Subsequently, the children of prominent African merchants and chiefs were also admitted. These schools were meant for the elite in society.

In 1529, the Portuguese established the first castle school at the Elmina Castle in Ghana. In 1637, the Dutch began another castle school at the same venue when they defeated the Portuguese and drove them away. In 1872, the Dutch sold the castle to the English. In addition, the Danes established a school at the Christianborg Castle at Osu in Accra. The English established an English castle school at Cape Coast in 1688. With the exception of the castle schools established by the Portuguese, all other castle schools sent some local people abroad for studies. These local people returned as schoolteachers and priests.

With the influx of missionaries mainly from Europe and America, many more schools were established; however, formal/school education was still reserved for the elite in society. There were some missionaries who felt the need to extend formal education to all Ghanaians in order to help them read and understand the Bible and also encourage trade; these included the Wesleyan Missionary Society from England (1835), Basel Missionary from Scotland (1832), Bremen Missionary from Northern Germany (1847), the American Methodist Episcopal Zion (AME) missions from the USA (1898), the Anglican Mission (1904) from England, and the Catholic missionaries from Rome (1529).

These missionaries established schools at various levels of education, including primary schools, secondary schools, and teacher training colleges; the first teacher training college was established by Basel Missionary in 1848 at Akropong. Although castle schools began teaching reading, writing, and arithmetic, religious instruction remained a prominent feature of the education provided therein. Such schools began with prayers and also ended with prayers. Since the school system during the colonial era was an elite one, it can be inferred that mathematics was studied by the privileged in society.

Until the nineteenth century, the state did not interfere with the education system. The first attempt was made under Governor Hills in 1852. There were at least four ordinances regarding education during the colonial era (i.e., between 1850 and 1927). In 1902, due to the educational ordinance of 1882, the Governor introduced what was described as "payment by result." Teachers were paid according to the number of pupils who passed in each subject in the class. This system was abolished when Rodgers became the Governor in 1909. Grants for schools now came to depend on inspectors' reports on the quality of teaching in schools. Under his regime, English came to be used as the medium of instruction throughout the country and the government started to play an active role in teacher education. This led to the establishment of the first government teacher training college in 1909 in Accra. From 1919 to 1927, Governor Gorgisburg formed a committee, which included some blacks, to review the educational system. As a result, he introduced sixteen principles that guided Ghanaian education. The following are some of his principles: education must be thorough and comprehensive from the bottom to the top, equal opportunities must be provided for both boys and girls, coeducation must exist at all levels, character training must assume an important position in education, religious teaching must form a part of school life, the teaching staff must be of the highest possible quality. Some of his principles guide Ghanaian education even today (C. H. Ahiabele, 1980). Until this time, the state's role of improving the quality of formal education had improved tremendously. After independence, there were several modifications in the Ghanaian preuniversity educational system. A major reform reducing the period of preuniversity education from seventeen years to twelve years was implemented in 1987. Currently, all missionary schools are controlled by the state (MOE, 2000).

(2) Mathematics Curriculum Development and Practices before Independence (1957)

The mathematics curriculum development was always centralized. Curriculum development and practices were influenced by those of the colonial masters (the British). The entire school system was modeled after the British system. This section of the paper describes the development of mathematics syllabus and textbooks, mathematics teachers' preparations and practices, and the system of assessment prior to independence from the British rule.

- **Development of Mathematic Syllabus and Textbooks**

Until 1952, Ghanaian schools studied syllabuses meant for English children at all levels (B. A. Eshun, 1979). Mathematics syllabus and textbooks were developed and therefore authored by foreigners, mainly Europeans. These books were imported into the country from Europe. Some of the famous authors included La Combs from France who authored the book entitled "Arithmetic for primary schools." These books were used in grades one to three. Alexander Symon and George D. N. Millikin coauthored the mathematics books entitled "Arithmetic for schools." This book was used in grades four to seven. This was followed by Longman mathematics series, used from grades one to seven. At the secondary level, mathematics was categorized into three branches: arithmetic, algebra, and geometry. There were two mathematics syllabus: syllabus A, which consisted of a separate syllabus for algebra, geometry, and arithmetic, each of which was examined in a separate examination paper; and syllabus B, which was a unified syllabus for arithmetic, algebra, geometry, and trigonometry, all of which were examined in three separate papers—B1, B2, and B3 (B. A. Eshun, 1979). Europeans authored textbooks at the secondary school level; Clement B. Durrel is one of the famous authors. He authored a number of mathematics textbooks, including "Durrel certificate mathematics." It should be noted that before independence, none of the mathematics textbooks was written by Africans or Ghanaians either at the primary school level or at the secondary school level. The secondary schools followed the syllabus that was prepared by the University of Cambridge Local Examination Syndicate and took examinations that were conducted by the same body.

Some of the mathematics contents studied in secondary schools included factors, multiples, vulgar fractions, percentages, profit and loss, simple interest, etc. Topics such as statistics and probability and set theory didn't exist. The content of the textbooks was rather difficult, and the examples were far from students' experience. There were little or no learning activities provided for most of the topics. Students were required to considerably memorize and imagine things that they had never seen before. This made the teaching and learning of mathematics very difficult. At the primary school level, traditional school mathematics largely involved mechanical number facts and measurements with little or no application to daily lives (Mereku, 2004).

- Mathematics Teacher Preparation and Practices

As already noted, following the advent of formal education in Ghana, mathematics was taught by foreign expatriates mainly from Europe. With the influx of the missionaries, mainly from Europe and the USA, and the subsequent extension of formal education to a number of black individuals, Ghanaians were also given the opportunity to teach mathematics. These missionaries established teacher training colleges in the country, with the first being the Presbyterian Teacher Training College at Akropong (established by Basel Missionary Society from Scotland in 1848). Other missionaries followed suit and established several other teacher training colleges in the subsequent years. The Ghanaians looked up to the Europeans for the training of their teachers. The principals and tutors who taught at the colleges were Europeans.

At the primary school level, mathematics was learned mainly by rote. Teachers conducted oral drills with students; those with wrong answers were caned. Caning was used to force students to study mathematics in the 1950s. In the class, the teacher would read out arithmetic problems to the students. This forced many students to end their education prematurely (Gyang, 1979, cited in Mereku 2004).

According to B. A. Eshun (1979), the problem of poor teaching in mathematics existed due to the fact that teachers did not understand mathematics and those who did understand it did not teach it well; instead, they assumed that by merely "telling" and explaining the ideas or techniques involved in solving problems, their students will be able to understand mathematics.

- Assessment

Summative assessment was used at all levels during the preindependence era. Students were tested at the end of each term and finally at the end of grade seven at the elementary school level. At the secondary level, until 1951, secondary schools followed the syllabus that was prepared by the University of Cambridge Local Examination Syndicate and took examinations that were conducted by the same body. Since the same syllabus and examinations were taken by students in English grammar schools, the Africans and for that matter Ghanaians were regarded as external candidates and were awarded Cambridge school certificates for the first five courses ("O" levels). After the sixth course, successful candidates received Cambridge higher school certificates. This practice continued even after school certificates were replaced by a general certificate of examination.

(3) Curriculum Development Practices after Independence

Shortly after independence, Africans and even Ghanaians began to play an increasing role in curriculum development and practices. This was due to the proliferation of many regional and subregional programs/projects in mathematics education. Notably among them were the African Mathematics Program (AMP), the West Africa Regional Mathematics Program (WARMP), and the Joint School Project (JSP). This section of the paper describes the development of mathematics syllabus and textbooks, mathematics teachers' preparations and practices, and the assessment in mathematics education after independence.

- Development of Syllabus and Textbooks

The first attempt to involve Africans in the organization of the mathematics syllabus and the subsequent writing of textbooks at the primary, secondary, and teacher training college levels was made in a town called Entebbe in Uganda in 1962. The government of the USA sponsored this project. The program was to be known as the African Mathematics Program (AMP). Many African mathematicians from universities, mathematics teachers from secondary schools, and training college tutors from Anglophone African countries as well as American and British experts were brought together to write the text using new ideas in mathematics education. The series of texts produced at the workshop came to be popularly known as the Entebbe mathematics series. These became the modern mathematics textbooks for primary and secondary schools as well as teacher training colleges (Lockard, 1968, cited Mereku 2004). The text followed a completely new syllabus. At the primary school level, for instance, while pupils studied only arithmetic during the precolonial era, after independence, topics such as set of numbers, number bases, clock arithmetic, chance, handling of data, vectors, place value, and rational numbers were introduced. In 1972, "New Mathematics for Primary Schools (NMPS)," a new mathematics scheme that was intended to make the learning of mathematics more interesting and meaningful for Ghanaian children, was officially introduced in the primary schools of the country (Lockard, 1972:9, cited Mereku 2004). Three years later, the textbooks were replaced by the Ghana Mathematics Series (GMS) scheme, which is being used even today as a matter of policy in Ghanaian schools.

The design of the GMS scheme was based on the teaching philosophies and models that have been described as the "new mathematics." All previous mathematics books were withdrawn following the introduction of the GMS. The GMS textbook was a product of the WARMP, a branch of the AMP. According to Mereku (2004), the GMS scheme had been criticized because its contributors were controlled by academicians rather than schoolteachers

The AMP was unable to achieve its goal to an appreciable level in Ghana mainly because while the sponsors of AMP organized a number of in-service training for training college tutors, no such training was organized for secondary school teachers. Consequently, the use of the text received better responses at the primary school and training college levels in Ghana. In addition, the text made considerable demands on teachers who would use them. Some of the materials were new to the teachers and they were also unfamiliar with some of the methods that were suggested for teaching the text.

The failure of the AMP, especially at the secondary school level, resulted in a new scheme called the JSP, initiated by the members of the Mathematics Association of Ghana (MAG). This idea came about after discussions on the new trends in mathematical curriculum and teaching methods at MAG's annual conference in April 1963 (B.A. Eshun, 1979). The project was aimed at producing a mathematics course for secondary schools at the West African School Certificate level (Lockard, 1969, cited Mereku 2004). The project was funded by the following agencies: The Nuffield foundation; the Centre for Curriculum Renewal and Education Development Overseas, London; and the Ministry of Overseas Development, London. Other agencies included the Institute of Education at the University of Ghana and the Ghana Ministry of Education.

A team comprising mathematics teachers from three secondary schools in Ghana—the Achimota school, the Mfantsipim school, and Prempeh college—as well as a lecturer from the University of Ghana, Legon, began the project vigorously as a subcommittee of the MAG and became self-directed by April 1965 with the appointment of an executive committee. By June 1971, the majority of the experimental work had been completed. The British Council, the MAG, and the Ghana Ministry of Education organized several in-service courses for teachers.

The executive committee was dissolved and the editors and the MAG became responsible for completing and reviewing the JSP. This has been revised three times since it was first published in 1970. JSP books were used in Ghana, Nigeria, Sierra Leone, and some countries in the Caribbean.

It must be noted that the AMP and JSP involved the development of curriculum materials in mathematics. The curriculum materials provided by the AMP resulted in a complete change in the content and treatment of topics in mathematics, whereas those provided by the JSP, which targeted only the secondary school level, did not show drastic changes in the content and treatment. It is believed that the fact that they came from two different projects created a situation wherein the major mathematics schemes used in basic and secondary schools lacked continuity. In other words, the GMS and JSP were not compatible. Consequently, most Ghanaian students found it difficult to switch to JSP at the secondary level after using the GMS scheme in basic schools.

- Mathematics Teacher Preparation and Practices after Independence

After independence, more teacher training colleges were established to cater for the increasing demand for formal education. In 1962, the University of Cape Coast, formerly the University College of Cape Coast, was established to deliberately train teachers for secondary schools as well as teacher training colleges. Currently, there are forty-two teacher training colleges; of these, four are private and the remaining are public training colleges. These training colleges prepare mathematics teachers for basic schools (primary and junior secondary schools). Two main universities are responsible for training teachers at the secondary and teacher training college levels. These levels of teacher training are now managed mainly by Ghanaians.

Although Ghana Education Service (GES) is trying several means to get all teachers trained, it is still common, owing to the shortage of teachers, to find untrained personnel recruited by GES to teach mathematics in Ghanaian schools, especially in the rural areas. Until the introduction of the Science, Technology, and Mathematics (STM) projects in 1998, Ghana did not have a system of regular in-service training for field teachers. In-service trainings were organized once in a while and mostly on ad-hoc basis. In one of the authors' life experiences as a basic schoolteacher for a period of three years, only one in-service training had been organized. However, following the advent of projects such as STM projects, the situation seems to be improving slightly. Currently, some schools have begun organizing school-based in-service training for mathematics teachers, especially in the project areas. In addition, there are many upgrading courses for teachers who wish to improve their competence in teaching the subject.

All said and done, the quality of teaching mathematics in schools is still not very good. Some teachers still find it difficult to cope with the content and methods of teaching mathematics. They continue to use the traditional approach of "show and tell," making them dispensers of knowledge rather than facilitators of learning. A research by the author in 2003 revealed that some of the teachers use the lecture method of teaching even at the primary school level (E. K. Davis, 2004). Hence, there is a need to reverse this trend through proper assessment, which can be attained through research.

- Assessment

Even after independence, the West Africa Examination Council (WAEC), which was established in 1952, continued to organize examinations on behalf of Cambridge (London). Following certain modifications in the WAEC, the council began to prepare its own syllabus for examinations. In the 1960s, the WAEC began conducting its own examinations for secondary schools along with those that it conducted on behalf of London. By 1969, the WAEC stopped organizing examinations on behalf of London. However, private candidates were allowed to opt for General Certificate of Examination (GCE) London.

As already mentioned, summative assessment was used in mathematics education even after independence. This type of assessment mainly measured the cognitive aspect of mathematics education. It tested mostly the recall of facts and mathematical calculations with very little critical thinking. Following the educational reform of 1987, the summative assessment gave way to continuous assessment. Under this assessment, students are assessed continually as they progress on the academic ladder. Such continuous assessment constitutes 30%, while the end-of-year examination constitutes 70% of the overall scores of students. Continuous assessment includes class exercises, class tests, and class assignments/projects, which enable teachers to measure domains other than the cognitive aspect.

However, there appear to be problems with the continuous assessment method at the moment. The major problem involves the quality of questions set by teachers and the fact that there is no correlation between the continuous assessment scores generated by school teachers and those obtained in the end-of-year/program examinations conducted by national examination bodies. Studies by E. K. Bartels (2003) at the teacher training level revealed that continuous assessment scores were generated without the use of proper procedure and were therefore unreliable.

(4) Characterization of Mathematics Education from Historical Perspective

This section of the paper takes into account the rationale for teaching mathematics and the general aims and objectives of teaching and learning mathematics at the primary, junior secondary, and senior secondary levels. A comparison between the general objectives of the present and past mathematics curricula at various levels of preuniversity education has also been made in this section.

- Aims and Objectives of Basic School Mathematics

The rationale for teaching mathematics at the primary school level: Mathematics at this level emphasizes knowledge and skills that will help pupils to develop the foundation for numeracy. Pupils are expected to be able to read and use numbers competently, reason logically, solve problems efficiently, and communicate mathematical ideas effectively to other people. The pupils' mathematical knowledge, skills, and competence at this stage should enable them to make more meaning of their world and should also develop their interest in mathematics as an essential tool for science, research, and development (MOE, 2001).

- General Aims

The general aims of teaching mathematics at the primary school level, as stated in the 2001 mathematics syllabus for primary schools, include the following:

- develop basic ideas of quantity and shapes
- develop the skills of selecting and applying criteria for classification and generalization
- communicate effectively using mathematical terms, symbols and explanations through logical reasoning.
- use mathematics in daily life by recognizing and applying appropriate mathematical

problem-solving strategies.

- understand the processes of measurement and acquire skills in using appropriate measuring instruments.
- develop the ability and willingness to perform investigations using various mathematical ideas and operations.
- work co-operatively with other pupils to carry out activities and projects in mathematics.
- see the study of mathematics as a means for developing human values and personal qualities such as diligence, perseverance confidence and tolerance, and as preparation for professions and careers in science, technology, commerce and a variety of work areas.
- develop interest in studying mathematics to a higher level

- **General Objectives**

The general objectives of teaching mathematics at the primary school level include the following: At the end of the instructional period, each pupil will be able to

- socialize and cooperate with other pupils effectively
- adjust and handle number words
- perform number operations
- make use of appropriate strategies of calculation
- recognize and use patterns, relationships and sequence, and make generalizations
- recognize and use functions, formulae, equations and inequalities
- use graphical representations of equation and inequalities
- identify/recognize the arbitrary/ standard units of measures
- use the arbitrary/ appropriate unit to estimate and measure various quantities
- collect, process and interpret data

- **Rational for teaching Junior Secondary School Mathematics**

The rationale for teaching mathematics at the junior secondary school (JSS) level: Mathematics at the JSS level in Ghana builds on the knowledge and competencies developed at the primary school level. Pupils are expected to be able to use mathematical ideas for investigating real life situations. The strong mathematical competence developed at the JSS level is a necessary requirement for effective study in mathematics, science, commerce, industry, and a variety of other professions and vocations for pupils ending their education at the JSS level and for those moving to senior secondary schools (SSS) and tertiary education and beyond (MOE, 2001).

- **General Aims**

The general aims of teaching mathematics at the JSS level, as stated in the 2001 mathematics syllabus for the JSS, include helping students to be able to do the following:

- develop basic ideas of quantity and space
- develop the skills of selecting and applying criteria for classification
- communicate effectively using mathematical terms, symbols and explanations through logical reasoning
- use mathematics in daily life by recognizing and applying appropriate mathematical

problem solving strategies

- understand the processes of measurement and acquire skills in using appropriate measuring instruments
- develop the ability and willingness to perform investigations using various mathematical ideas and operations
- work co-operatively with other pupils to carry out activities and projects in mathematics
- see the study of mathematics as a means for developing human values and personal qualities such as diligence, perseverance, confidence an tolerance, and as preparation for professions and careers in science, technology, commerce, industry and a variety of work areas
- use the calculator and the computer for problem solving and investigations in real life situations
- develop interest in studying mathematics at a higher level

- General Objectives

The general objectives of teaching mathematics at the JSS level include the following: Each pupil will be able to

- work co-operatively with other pupils and develop interest in mathematics
- read and write numbers
- use appropriate strategies to perform number operations
- recognize and use patterns, relationships and sequences and make generalizations
- recognize and use functions, formulae, equations and inequalities
- make and use graphical representation of equations and inequalities
- identify and use arbitrary and standard units of measure
- use the appropriate unit to estimate and measure various quantities
- relate solids and plane shapes and appreciate them in the environment
- collect, analyze and interpret data and find probability of events
- use calculator to enhance understanding of numerical computation and solve real life problems
- manipulate learning materials to enhance understanding of concepts and skills

A comparison of the general objectives of mathematics at the basic school level (primary and JSS) in 1988 and 2001 is presented in Table 1.

Table 1. Comparison of general objectives of basic school mathematics

Level of Education	1988	2001
Primary school	- to be able to use mathematics in daily affairs by recognizing situations that require mathematical solutions and appropriate techniques for solving them - to reason logically, develop the ability to give explanation based on clear reasons	- socialize and cooperate with other pupils effectively - adjust and handle number words - perform number operations - make use of appropriate strategies of

	when making decisions and to develop the skill of selecting and applying a criteria for classification - to be able to understand events and things as continually changing entities - to understand the process of measuring and develop an appreciation of the systems and instruments of measurement and to acquire skills in measuring - to develop the basic ideas of quantity, quantitative relationships and numbers and be able to apply them - to acquire knowledge of mathematical terms and symbols and be able to think and communicate, using these terms and symbols clearly and correctly - to develop an appropriate, dynamic, and systematic way of solving problems with some definite goals	calculation - recognize and use patterns, relationships and sequence, and make generalizations - recognize and use functions, formulae, equations and inequalities - use graphical representations of equation and inequalities - identify/recognize the arbitrary/ standard units of measures - use the arbitrary/ appropriate unit to estimate and measure various quantities - collect, process and interpret data
Junior secondary school	- pupils see mathematics as a unified body of knowledge and not as a collection of isolated topics - pupils develop a mathematical outlook by getting the opportunity to understand the world around them in mathematical terms, to express their ideas in mathematical language, to develop the ability to give clear and correct explanations and to make classifications and generalizations - pupils develop the understanding of basic Mathematical concepts and become aware of some application of mathematics - pupils see the study of mathematics as a means of inculcating human values and understanding the problems and behaviors of peoples	- work co-operatively with other pupils and develop interest in mathematics - read and write numbers - use appropriate strategies to perform number operations - recognize and use patterns, relationships and sequences and make generalizations - recognize and use functions, formulae, equations and inequalities - make and use graphical representation of equations and inequalities - identify and use arbitrary and standard units of measure - use the appropriate unit to estimate and measure various quantities - relate solids and plane shapes and appreciate them in the environment - collect, analyze and interpret data and find probability of events - use calculator to enhance understanding of numerical computation and solve real life problems - manipulate learning materials to enhance understanding of concepts and skills

(5) Aims and Objectives of Teaching Secondary School Mathematics

Mathematics at the secondary school level in Ghana is divided into two: core mathematics and elective mathematics. Core mathematics is compulsory for all students, while elective mathematics is an optional course that is taken mainly by students who pursue science. The author has decided to focus on the core mathematics syllabus since it is a compulsory course for all students.

- Rational for teaching core mathematics

A major concern of the SSS core mathematics syllabus is to consolidate the gains made in basic school mathematics and to raise the standards of attainment by providing for and stimulating students throughout the ability range. The core mathematics syllabus is based on the recognition that mathematics is not only a collection of concepts and skills to be mastered but also something that helps individuals to develop their abilities to explore, conjecture, and reason logically as well as to develop the ability to effectively use a variety of mathematical methods to solve routine and nonroutine problems in and outside school. In addition, the syllabus is intended to enable students to develop an understanding of, and interest in, the subject. Further, the syllabus enables students to gain more competence in the study of subjects such as commerce, science, and technology (MOE, 2003).

- General Aims

The general aims of teaching mathematics at the SSS level, as stated in the 2003 mathematics syllabus for SSS, include helping students to achieve the following:

- reason logically by developing the skills to select and apply criteria for classification and generalization;
- communicate effectively using mathematical terms and symbols and give explanations of situations based on logical reasoning;
- use mathematics in daily life by recognizing those situations that require mathematical problem-solving strategies and apply them;
- understand the processes involved in various systems of measurements using appropriate instruments;
- develop the ability and interest to perform investigations using various mathematics ideas and operations;
- see the study of mathematics as a means of developing human values, personal qualities, and psychosocial skills (e.g. diligence, perseverance, confidence, tolerance and analytical thinking);
- use the calculator and the computer as tools to enhance the development of mathematical ideas and for investigating real life situations and problem-solving;
- work co-operatively with other students to carry out activities and project in mathematics; see mathematics as relevant for the study of other school subjects and as a preparation for professions and careers that mathematics oriented;
- acquire a foundation appropriate to further study of mathematics and other disciplines;
- appreciate the connection among ideas within the subject itself and in other subjects, especially Science, Technology, Economics and Commerce.

- General Objectives

The general objectives of teaching mathematics at the SSS level include the following: By the end of the instructional period, students will be able to

- develop computational skills by using suitable methods to perform calculations;

- recall, apply and interpret mathematical knowledge in the context of everyday situations;
- develop the ability to translate word problems (story problems) into mathematical language and solve them with related mathematical knowledge;
- organize, interpret and present information accurately in written, tabular, graphical and diagrammatic forms;
- use mathematical and other instruments to measure and construct figures to an acceptable degree of accuracy;
- develop precise, logical and abstract thinking;
- analyze a problem, select a suitable strategy and apply an appropriate technique to obtain its solution;
- estimate, approximate and work to degrees of accuracy appropriate to the context;
- organize and use spatial relationships in two and three dimensions, particularly in solving problems;
- respond orally to questions about mathematics, discuss mathematical ideas and carry out mental computations;
- carry out practical and investigations work, and undertake extended pieces of work.
- use a calculator or computer to perform computations.

A comparison of the general objectives of core mathematics at the secondary school level in 1989 and 2003 is presented in Table 2.

Table 2. General objectives of the senior secondary school core mathematics

1989	2003
- give the student a broad view of the field of elementary mathematics in order to explore his interest and test his abilities, and - give the student the mathematical information and skills most likely to be useful to him in his vocational pursuits. More specifically, it must inculcate in the student: - the habit of creative thinking and the development of intellectual curiosity - the communication of thought through symbolic expression and graphs; - the development of the ability to make relevant judgment through the discrimination of values, and analysis of data; - the development of intellectual independence; - the development of skills for aesthetic appreciation and expression of the environment; - the fostering of cultural advancement through a realization of the significance of mathematics in mathematics in its own right and in its relation to	- develop computational skills by using suitable methods to perform calculations; - recall, apply and interpret mathematical knowledge in the context of everyday situations; - develop the ability to translate word problems (story problems) into mathematical language and solve them with related mathematical knowledge; - organize, interpret and present information accurately in written, tabular, graphical and diagrammatic forms; - use mathematical and other instruments to measure and construct figures to an acceptable degree of accuracy; - develop precise, logical and abstract thinking; - analyze a problem select a suitable strategy and apply an appropriate technique to obtain its solution; - estimate, approximate and work to degrees of accuracy appropriate to the context;

the total physical and social structure	- organize and use spatial relationships in two and three dimensions, particularly in solving problems; - respond orally to questions about mathematics, discuss mathematical ideas and carry out mental computations; - carry out practical and investigations work, and undertake extended pieces of work. - use a calculator or computer to perform computations.

On carefully comparing the objectives of the current syllabuses at the preuniversity level with those of 1988 for basic schools and of 1987 for SSS, we do not observe many changes in the scope of mathematics at the preuniversity level. For instance, at the primary school level, both the 2001 and 1988 syllabuses covered number, geometry (pages 23 & 59, primary school mathematics syllabus, 1988), measurement, statistics (ibid. pp 66), and problem solving and investigation with numbers.

The major difference is that the objective has been broken down further, i.e., it is now clearly stated. In addition, the aims of the current syllabus touch on the affective aspects such as the interest of the child, which the syllabus did not consider ten years ago. Finally, the current syllabus has been broken down into teaching syllabus with suggested teaching/learning activities.

(6) Conclusion

From the above discussions so far, one can deduce that mathematics education in Ghana has gone through three major stages. The first stage was the colonial era, wherein the Europeans controlled curriculum development and practices. There were no contributions made by Ghanaians. This was followed by the dispensation stage, wherein Africans as well as Ghanaians and other foreign expatriates came together to plan and develop curriculum materials (immediately after independence). The third stage was the dispensation where decisions on curriculum development and practices were carried out mainly by Africans and Ghanaians. However, in the present dispensation, one cannot rule out the contribution of the developed world through international cooperation in mathematics education.

2.5.2 Basic Information

(1) Teachers' Qualification

- Training teachers to meet the demand of the reforms

With effect from September 1987, Ghana adopted a 6-3-3-4 system of education. Under this structure, the educational system comprised six years of primary school, three years of JSS (which forms nine years of basic education), and three years of SSS. This constitutes twelve years of pre-tertiary education. Tertiary education consists of four years of university education or three to four years of training at polytechnics, teacher training colleges, and other training institutions in the fields of agriculture, health, etc. Since its implementation from 1987 onward, the Education Reform Programme has had a significant impact on the educational delivery process in the country. The areas of significant achievement relate to increased access to education, the redesigning of the curriculum toward greater relevance, the improvement of instructional effectiveness, and the training of teachers to meet the demands of the reform.

In response to the need for teachers to meet the requirements of the reform program, teacher education programs have been reformed and strengthened. With effect from 1988, intake into the four-year post-middle teacher training course was discontinued, and all intake into the teacher training colleges is now made into a three-year course that is reserved for ordinary level GCE and SSSC holders. The curriculum at teacher training colleges was also revised to reflect changes in the content and methods of basic education teaching. Teacher training colleges now train teachers to teach groups of subjects to match the reform program. While all trainee teachers, irrespective of their areas of specialization, study core subjects that are necessary for imparting relevant knowledge and skills in basic level schools, the in-service training for all categories of teachers and supervisors has been intensified to meet the demands of the reforms.

- PRE-SERVICE TEACHER TRAINING

Teachers are recruited for the following four levels of the educational system: the nursery and preschool level, the basic school level, the secondary/technical/vocational school level, and the tertiary level.

The preschool or nursery teachers training course lasts for three months. At the end of the training, teachers are awarded the Nursery Teachers' Certificate.

The Basic Level Teachers Certificate "A" Post Secondary Training course lasts for three years after which the Teachers Certificate "A" is awarded.

Teaching at the secondary level requires a diploma or degree that may be obtained after two to four years of training. Teaching at the teachers college level requires a post-diploma (obtained after training for two years following a diploma) or a degree (may be obtained after two to four years of training).

A master's or doctorate degree is obtained at universities. The training requires a maximum of two to three years. The holders of these degrees are generally employed at tertiary institutions.

- INSERVICE TEACHER TRAINING

In-service training courses are organized for teachers after they complete their preservice training. These courses are organized when certain problems arise in general in the course of classroom practice. The organizer first identifies the problem(s) through interaction with schools or through a committee that is set up to identify the problems that teachers face. Subject associations set up committees to assist members to improve themselves in certain areas in which they lack competence. The course also helps teachers to upgrade their knowledge and teaching methods and to discuss together the teaching syllabuses in order to ensure a common teaching approach. The organization of in-service training was accelerated during the new education reform process, and all teachers are currently benefiting from it. The in-service training courses are organized nationwide and involve teachers at the basic and SSS levels. It is obligatory for all teachers to attend the in-service training.

Further, teachers are provided with the opportunity to improve their professional status through normal progression via the ranks and study leave courses in academic/professional programs in institutions of higher learning. Continuing professional education through distance learning modules have been considered in the free Compulsory Universal Basic Education (FCUBE) program. All these facilities have the potential of raising the social and professional status of teachers, especially among the lower strata of the profession. For a teacher to qualify for study with pay, he/she should have served as a teacher for a minimum of three years after certification or since returning from the last approved course. In the case of rural teachers, a teaching period of two years is required for a study leave. This means that teachers who have vacated their posts and have been reinstated would qualify only after they have served for five years.

The subjects that make teachers consider the option of study leave with pay vary from year to year based on the needs analysis conducted by the human resources department and the district and regional directors of the GES. The GES council determines the number of applicants to be granted study leave with pay in a given year.

(2) School Calendar and Examination

The school year begins in September and ends in June/July; it is divided into three terms, namely, the first, second, and third terms. The first term lasts from September to December; the second term, from January to March; and the third term, from April to June/July.

Table 3. School Calendar

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
	2nd Term			3rd Term					1st Term		

Promotion from primary school to JSS takes place automatically. At the end of the third year of JSS, students take a national examination (BECE: Basic Education Certificate Examination) and the progression into SSS, technical institute, or national vocational training institute depends on the result of the BECE.

(3) Medium of Instruction

The Ghanaian language is usually the medium of instruction at the lower primary level (grades one to three); however, from upper primary to university levels, the medium of instruction is English (Language policy, 1973).

The controversial issue regarding the medium of instruction at the basic stages from grades one to three has the attendant problem of poor achievements since the larger bulk of pupils are in rural areas wherein English is far from being spoken or written on an everyday basis. At the lower primary level wherein the mother tongue is accepted as the medium of instruction, the lack of relevant materials and difficulties related to the ability of some teachers to communicate in children's native language—the children come from different ethnic areas—have been combined in order to render teaching at that level ineffective.

2.5.3 Results from the First Year Field Survey

(1) Survey Schedule

Table 4 shows the schedule of our field survey. With the help of a local team of Cape Coast University, we used questionnaires and video recordings of a lesson on fraction in an urban school between February 16 and February 23 and a second video shooting in a rural school on March 11.

Table 4. Schedule of the First Year Field Survey

13th/Feb/2005	Arrival at Ghana
14th	Moving to Cape Coast and Meeting
15th	Necessary Arrangements for the field survey
16th-23rd	Implementation of Questionnaires and Video Recording of a Lesson about Fraction in an Urban School
24th	Departure for Japan
11th/Mar	Video recording of a lesson about fraction in a rural School (by local team of Cape Coast University)

(2) Target Schools and Samples

Sample Procedure: Schools in Ghana are classified into three categories: schools A, B, and C. These categorizations were done based on the schools' performances in the performance monitoring test (PMT) and BECE results. Since this study focused on an "average school," two basic schools categorized as school B, one each from urban and rural areas, were randomly selected from a list of schools B that was obtained from the GES district office for the purpose of the study. The sample comprised two average schools, one each from urban and rural areas, in the Cape Coast municipal area in the Central Region of Ghana. Both schools were noted to be average in terms of fulfillment of equipment.

The urban school was located in the town of Cape Coast and had a staff strength of eighteen teachers, including five KG teachers, six primary school teachers, six JSS teachers, and two head teachers, one each for KG and primary/JSS, who were detached. The school comprised 300 pupils.

The rural school was located approximately 25 km north of Cape Coast and had a staff strength of fifteen teachers, including two KG teachers, six primary school teachers, six JSS teachers, and a head teacher who was detached. This school consisted of 180 students.

In both the schools, six classes were held at the primary school level with class teaching and three classes were held at the JSS level with subject teaching. Both the schools held just one class in each grade so that two teachers of grade four could be regarded as samples. The urban teacher had already undergone in-service teacher training. The rural teacher had not participated as a mathematics teacher in any training course, although he completed a mechanical engineering course at a polytechnic institute.

(3) Results of Interview

The result of the questionnaire for head teachers is shown in Table 5. The result from the rural head teacher could not be collected because she was absent.

Table 5. Responses from Head Teachers

Interview Items	Urban School
[Problem]	
[1-1] What do you think is the biggest problem in teaching mathematics in your school?	Lack of pupils' interest and textbooks. E.g. the textbooks do not correspond with syllabus.
[1-2] What kind of action do you take against that problem as an administrator?	I am arranging with a manufacturer whose correspond with the syllabus. The P. T. A. is aware of it.
[1-3] Do you observe lessons by teachers? (YES or NO) If YES, how often do you observe them?	Yes, everyday.
[1-4] What kind of advice do you give to young teachers at your school?	I comment on good/bad performances. I do not lord on them, but I advice them as a mother.
[In-service training]	
[2-1] Do you see any impact of in-service course offered to teachers? If yes, is it negative or positive? Please describe the impact a little more.	With the ideas gained, they are implementing them. E.g. on literacy and Maths etc.
[2-2] What kind of training do you think is necessary for teachers in your school, if a new training course is designed?	The new training should take off before I can know the type I should take to.

The result of the questionnaire for mathematics teachers is shown in Table 6.

Table 6. Responses from Mathematics Teachers

Interview Items	Urban School	Rural School
[Problem]		
[1-1] What do you think is the biggest problem in teaching mathematics in your class?	Inadequate Math textbooks and lack of teaching and learning materials	Lack of materials (textbooks)

[1-2] What kind of action do you take against that problem?	Photo copies of the page(s) to work with are made for pupils and I have reported the problem to the head teacher.	I have requested the head teacher to inform the Director.
[Today's Lesson]		
[2-1] What was the purpose of today's lesson?	Pupils should be able to write the different names of a fraction.	Should have an idea about one-half and one-quarter.
[2-2] How much do you think the purpose was attained?	Pupils contribute to the lesson and answer the questions that they are asked.	About 80%.
[2-3] What do you think are the most important factors for successful lesson?	Preparing well before teaching, thus having mastery over the subject matter.	Teacher's preparation and Children's preparedness
[2-4] What kind of teaching would you like to do?	Child/Pupil centered/activity type of teaching	(No comment)
[In-service training]		
[3-1] Have you ever had a teacher training after you become a teacher?	Yes.	No, never before I became a teacher. I read mechanical engineering at a polytechnic institute.
[3-2] Which kind of training, if you had before, do you think is useful for your teaching?	It was on the methods of teaching Math and I found it to be very useful.	Not applicable.
[3-3] What kind of training do you think is necessary for improvement of your lesson, if a new training course is designed?	I still think the methods of teaching Math are very necessary for the improvement of a lesson.	Upgrading course in education

(4) Results of Lesson Plan Analysis

[Lesson Plan by Urban School Teacher]

Week Ending: February 21–25, 2005

Subject: Mathematics

Reference: Maths Syllabus for Primary School, pp. 62–63

GMS Pupils' Book 4, pp. 79–81

Teaching Maths in Basic School, pp. 60–61

Day: Thursday (February 24, 2005)

Duration: 45 min

Topic/sub-Topic: Fraction/Equal Fraction

Objectives: By the end of the lesson, pupils will be able to

- write three different names for a fraction

RPK (Review of Pupils' Knowledge): Pupils have been taught to identify a common fraction and therefore they can identify a common fraction.

Teacher-Learner Activities:

(i) Introduction

Introduce the lesson by reviewing pupils' relevant previous knowledge of identification of a common fraction.

(ii) Sample Question

1/2, 2/3, 4/5, 5/6, 2/4, 5/8, 3/4

(iii) Expected Responses

Half, two-thirds, four-fifths, five-sixths, two-fourths, five-eights, three-fourths, etc.

(iv) Activities

Draw planes with shaded and unshaded portions on the board and help pupils to describe the shaded parts.

Guide pupils to find fractions that represent the same part of a given whole using

(a) paper folding

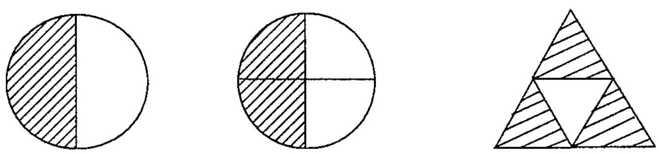
(b) fractional chart or fractional board. Cut or divide an A4 sheet into three equal parts. Fold one part into three.

(c) equal parts and shade two to represent 2/3. Divide the second strip of paper into six equal parts and shade four parts for 4/6. The third paper is to be folded into 6/9. Pupils are made to compare the three shaded sheets. By doing this, they will find out that all the shaded parts or regions are of equal size.

Teaching Learning Materials: Strips of paper and chalkboard illustration

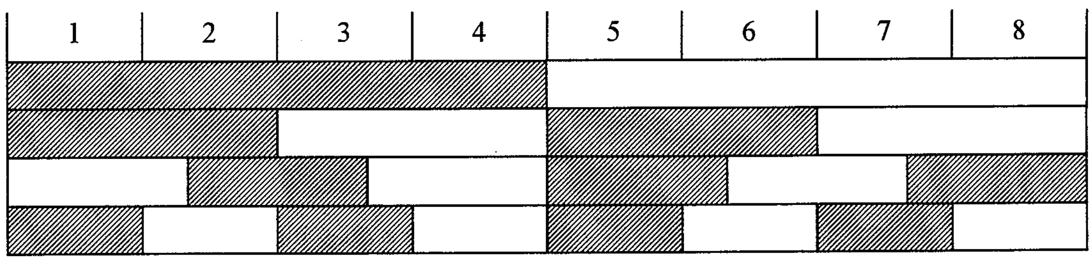
Core Points:

A fraction is a part of a whole object or a part of a set.



1/2 2/4 3/4

Fractional Chart/Board

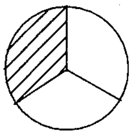
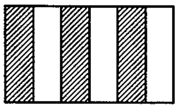


From the chart, one-half or half is the same as 2/4, 3/6, or 4/8.

1/2, 2/4, and 3/6 are different names for the same fraction.

Evaluation and Remarks:

What fraction is/are the shaded portion(s)?

Write down two equal fractions in addition to the following

(a) 1/2, ,

(b) 2/3, ,

[Lesson Plan by Rural School Teacher]

Day: Friday (March 11, 2005)

Duration: 60 min

Topic/subtopic: Fraction (one-half and one-quarter)

Objectives: By the end of the lesson, pupils will be able to divide one stick, one paper, and one orange into two and four equal parts.

RPK: Pupils have been sharing things together.

Teacher-Learner Activities:

i) Revising previous knowledge by asking a question on what they share.

ii) Remind the pupils that we use numbers called fractions when we talk about parts of whole objects or parts of a set.

iii) Teacher gives a demonstration on how to cut one stick into two equal parts.

iv) Teacher gives a demonstration on how to fold one paper into two equal parts.

v) Teacher gives a demonstration on how to divide one orange into two equal parts.

vi) Teacher invites some pupils in front of the class and shows them how to divide one orange into two equal parts.

vii) Teacher invites some pupils in front of the class and shows them how to fold one paper into two equal parts.

viii) Teacher invites some pupils in front of the class and shows them how to divide one stick into two equal parts.

Teacher allows pupils to fold the paper into two equal parts.

Teacher allows pupils to divide one stick into two equal parts.

Teacher goes around and ensures that pupils are doing the right thing.

Teacher shows the pupils how to fold the papers into four equal parts.

Teacher shows the pupils how to divide one stick into four equal parts.

Teacher shows the pupils how to divide one orange into four equal parts.

Teacher goes around and ensures that pupils are doing the right thing.

Teacher allows pupils to fold the papers into four equal parts.

Teacher allows pupils to divide one stick into four equal parts.

Teacher allows pupils to divide one orange into four equal parts.

Teacher asks pupils to complete the assigned exercise based on the topic provided in their exercise books.

Teaching Learning Materials: Blades, Scissors, Knives, Pencils, Pens, Exercises books, Papers, Sticks, Oranges, etc.

Core Points:

Idea of one-half and one-quarter

E.g., 1/2 1/2

1/4 1/4

Evaluation and Remarks:

i) Draw a circle and divide it into two equal parts.

ii) Draw a square and divide it into two equal parts.

iii) Draw a rectangle and divide it into two equal parts.

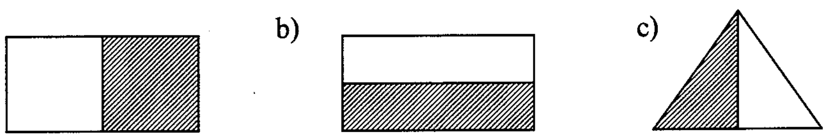
iv) Draw a triangle and divide it into two equal parts.

v) Draw a circle, divide it into four equal parts, and shade one quarter.

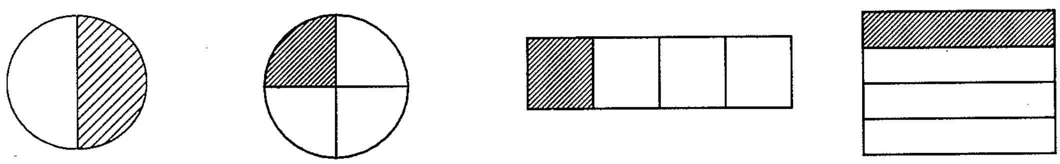
vi) Draw a rectangle, divide it into four equal parts, and shade one quarter.

vii) Pupils should answer the following questions.

a) b) c)



d) e) f) g)



(5) Results of Video Analysis and Classroom Observation Checklists

The result of lesson observation checklists is shown in Table 7.

Table 7. Lesson Observation Checklists

Observation Items		Urban School					Rural School				
		0	1	2	3	4	0	1	2	3	4
Introduction	The teacher starts the class on time.			√			n.a.				
	The teacher made the objective clear.				√				√		
	The objective suits to the level of children.					√			√		
	Relationship with the previous lesson is clear.			√				√			
Development	The teacher gives supports to pupils who seem to have little understanding.			√					√		
	The teacher expresses appreciation for pupils' thinking attitudes.			√				√			
	The teacher assesses the pupils' comprehension during teaching and learning.			√					√		
	The teacher uses easy language.			√						√	
	The teacher uses an appropriate and familiar example to illustrate main concept.			√						√	
	The teacher creates friendly atmosphere.			√						√	
	The teacher accommodates discussion among pupils.			√				√			
	The teacher gives hands-on activity.					√				√	
	The teacher enjoys teaching.			√					√		
	The teacher is impatient with wrong answer.		√					√			
	The teacher involves children to say opinions freely.			√				√			
	The teacher encourages children to display diverse opinions.		√					√			
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.				√				√		
	The teacher combines individual work and group work appropriately.		√					√			
Summary	At the end of the lesson, the teacher summarizes the lesson.			√				√			
	The teacher assigns home work at the end of lesson clearly.	√									√
	The teacher explains about a connection between today's lesson and next lesson.	√					√				
General	The teacher prepares a lesson plan.					√					√
	The teacher prepares a plan for taking note on the blackboard.	√					√				

Describe the objective of today' lesson.

Urban: To understand "Equal fraction" through activities, although the objective is different from the one in the lesson plan.

Rural: To be able to divide one paper, one stick, or one orange into two and four equal parts.

Describe problems/activities (No. of problems, the relation among them, their difficulty level, etc.)

Urban: An activity to fold three strips of paper and shade some of the divisions to represent 1/2, 2/4, and 4/8. The same activity to represent 1/2, 2/4, and 6/9. The relation between the first and the second activities did not appear to be clear. The teacher's conclusion of the second activity was mathematically incorrect. It seemed difficult for pupils to divide one sheet of paper into three equal parts.

Rural: Activities to divide one paper, one stick, and one orange into two or four equal parts, such as teacher's demonstration, pupils' demonstration, and exercises on the board. The technical terms regarding fraction were learned after completing the activities. If the lesson aimed solely at improving pupils' skills to divide things into two or four equal parts, it seemed rather easy for pupils.

Describe children's opinions Urban: Upon completion of the activities, teacher asked pupils to answer what they saw. They replied that the portions representing 1/2 and 2/4 are the same size as 4/8. Some pupils answered only with regard to the appearances of each paper strip, such as "a paper was divided into two equal parts and one out of two was shaded." Rural: Pupils were often involved in activities and exercises on the board, but they were not asked to express their own opinions except for answering the exercises.
Assess who dominate solving problems during the lesson observed. Urban: Pupils were actively working well, but the class in solving problem seemed to be dominated by the teacher. The teacher seemed to induce the single answer presupposed in his/her mind. Rural: Pupils were actively working well. The class focused on skills, not on solving problems.
Assess which of the following is regarded as the most important in the lesson observed. Understanding concept/mastering the procedure/thinking mathematically/applying to daily life/finding the correct answer Urban: Aiming to understand the concept but actually finding the correct answer. Rural: Mastering the procedure

Rating scale: 0-never, 1-seldom/to a little extent, 2-sometimes/to some extent,
3-often/to a considerable extent, 4-very often/ to a great extent

2.5.4 Results from the Second Year Field Survey

(1) Survey Schedule

Table 8. Schedule of the Second Year Field Survey

Date	Activity
21st/Nov/2005	Arrival at Accra and Move to Cape Coast (Japanese Researcher)
22nd	Reconfirmation of the Schedule and Activities
23rd	Implementation of Questionnaires for 5th Grade Teacher at Rural Primary School
24th	Implementation of Questionnaires for 5th Grade Teacher at Rural Primary School
25th	Data Processing and Discussion
26th	Departure for Japan

(2) Target Schools and Samples

As regards the urban school, the same sample teacher is used for data collection. However, since another sample teacher moved from the same rural school, the new teacher cooperated with our field survey in the school. We administered the data collection for two teachers of grade five at the time, since they just recently completed the grade four syllabus.

The conditions of sample schools remained unchanged. Table 9 presents the brief profiles of the sample teachers.

Table 9. Brief profiles of teachers

	Sex	How long you have taught (yrs)	Subjects you teach
Urban School	Male	4	All subjects
Rural School	Male	16	All subjects

(3) Results of Questionnaire

Table 10 shows the teachers' perceptions regarding the achievement test and teaching/learning of Mathematics and "fractions."

Table 10. Teachers' Perception

Questionnaire Items	Urban School	Rural School
Teacher's forecast of pupils' average score	54%	40%
Pupils' familiarity to the given test	Yes **Reason/s**: The reason being that some students from the University of Cape Coast usually come around to conduct such a test.	Yes **Reason/s:** Because pupils have been learning fractions by cutting oranges and cardboards with shaded squares and angles.
Any questions which the pupils cannot solve.	Yes **Reason/s**: Questions 6 and 8 because of the way those questions were set. I think at the given level the question should have been straightforward.	Yes **Reason/s:** Because some pupils will find it difficult to understand.
Difficulties in teaching "Fractions".	Easy **Reason/s**: I feel so because fraction is a practical topic that we unconsciously come across and perform as part of daily life. The materials used in teaching it are not difficult to obtain.	Easy **Reason/s**: It is easy because pupils have been using consecrate objects in class to perform fractions.
The most difficult topic/s to teach in Grade 4.	(No answer)	The most difficult topic for teaching in grade 4 is the estimation and verification of the lengths of given line segments.
The easiest topic/s to teach in Grade4.	(No answer)	The easiest topic for teaching in grade 4 is an equivalent fraction. It is easy because pupils understand it very well.
Teacher's confidence in teaching "Fractions"	Confident	Very Confident
Level of influence of the examination on teacher's teaching	Very much **Description**: I see it since it evaluates my work as a teacher and also tells me the extent to which children or pupils understand my teaching.	(No answer) **Description**: It affects my teaching because the last two periods are wasted.
Degree of difficulty for the pupils to learn "Fractions"?	Difficult	Easy
Points of difficulty for the pupils to learn the concept of "Fractions"?	Because pupils do not link it to their everyday activities. They also see it as a difficult topic.	The difficulty lies in the pupils' understanding of the questions.
Existence of pupils' difficulty with the medium of instruction in learning mathematics	Yes **Concrete example/s**: The difficulty arises when the teacher decides to use English throughout the instructional period without blending it with the local language.	Yes **Concrete example/s:** Some pupils can never understand the questions since they do not understand the English language.
Importance of learning "Fractions" with comparison to any other topics in mathematics.	Yes **Reason/s:** I think fraction is an important topic among multiplication and division.	Yes **Reason/s:** It is an important topic among other topics. It helps pupils to know how to

	Fractions have a bearing on these two topics.	measure things.
Teacher's main point/s of concern to the pupils in teaching "Fractions"?	(a) Recognize fraction as a number (b) Equal fractions (c) Comparing fractions (d) Fraction as a division (e) Addition and subtraction of fraction	The main point for teaching fractions is the cutting of oranges, drawing circles or diagrams, shading of diagrams and angles.

Tables 11 and 12 show the teachers' strategies in teaching "Fractions" in Q14 and Q15, respectively.

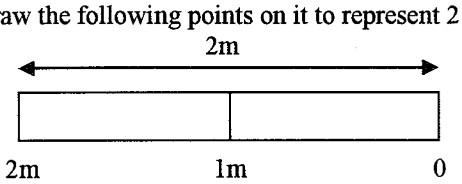
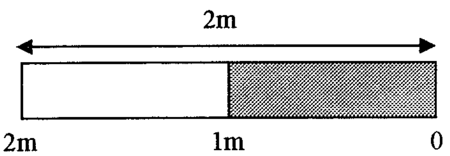
Q14: Describe how to teach the following question to the pupils?, "Which is longer 1/4m or 1/3m?"

Q15: Suppose you posed the following question to the pupils in a lesson. "What is a half of 2m?"Then a student answered, "It is 1/2m."How do you deal with such a student in class?

Table 11. Teachers' Srategies in Taching "Fractions" (1)

Urban School	Rural School
I use concrete materials such as the method of folding paper into equal parts. Ask pupils to take two rectangular sheets of paper of equivalent shapes. Ask them to fold one into four equal parts and then shade one part to represent 1/4. The other paper is then folded into three equal parts And one part is shaded to represent 1/3. The two papers are then kept next to each other and the shaded portions are compared to determine whether 1/3 is bigger than 1/4. This indicates that 1/3 > 1/4 and that 1/3 m is longer than 1/4.	(1) Fractions using units (2) Measurements using units (3) Measure a 1 m line on the ground and divide it into four. Then, sketch. Divide into three, and sketch again.

Table 12. Teachers' strategies in teaching "Fractions" (2)

Urban School	Rural School
I will ask the pupil to take a strip of paper and draw the following points on it to represent 2 m.  I will then ask the pupil to fold the strip of paper into two equal halves or parts and ask him/her to shade one half. 	The child lacks the skills of applying fractions to find parts of whole numbers. Ex) 1/2 of 4 = 1/2 × 4/1 = 1×4/2×1 = 4/2 = 2 The teacher should re-teach the child in order to ensure that the child will be able to answer the question correctly.

After shading, the pupil will recognize that the shaded portion or area corresponds to 1 m. Hence, the pupil will learn that the half of 2 m is 1 m.	

2.5.5 Discussion

(1) Difference in Cognition about Teaching and Learning among Teachers

Since the sample size is very small in this field survey, we must not overgeneralize the result. Here, we would simply like to describe some points regarding the difference among teachers for further discussion.

We had three sample teachers: a young rural teacher with a few years' teaching experience, who had no experience of any preservice or in-service teacher training after his graduation in mechanical engineering at a polytechnic institute; a young urban teacher with four years' teaching experience, who has been through preservice and in-service training several times; and a middle-aged teacher with fifteen years' teaching experience, who had been through in-service training several times.

From the questionnaire, lesson plan, and lesson observation in the first-year survey, we can find the difference in cognition about teaching and learning between the young rural teacher and the young urban teacher. Owing to his experience of in-service training, the teacher in the urban area seemed to regard a "good lesson" as "learner-centered." The structure of his lesson plan seemed to show his idea of a "good lesson" and his actual lesson and answers to the questionnaire were based on his experience. For example, his answer included praises for pupils' contribution to the lesson. The urban teacher intended to teach the mathematical concept through activities apart from whether or not it could be fully attained. On the other hand, the rural teacher regarded both knowledge and skills as the objectives of his lesson. In his class, activities were followed by chorus-teaching of technical terms about fractions.

The second-year questionnaire survey shows the confidence of the middle-aged teacher in teaching fraction. For him, faction is an easy topic to teach and learn, although his forecast of pupils' average score is 40%. It seemed that he guessed the low average score of 40% because pupils would not be able to sufficiently understand the question in English. For the young urban teacher, fraction is an easy topic to teach but difficult to learn. He found it difficult to note that pupils could not relate fraction to their everyday activities.

This does not imply that the former teacher is better than the latter. In Japan, "fractions" are perceived as the first topic in abstract mathematics. Though we could not find such cognition in their answers, the former teacher would have a rigid style in teaching fraction without treating it as abstract mathematics. We will never be able to say that cognition and treatment of teaching-learning contents in Japan is always effective in Ghana. Instead, we can say that further clinical research as well as practical discussion among Ghanaian teachers is essential to investigate teachers' cognition of the mathematical concept in the context of teaching and learning.

(2) Weak Points in Teaching: Gap among Self-cognition, Actual Teaching, and Learning

Although all the three teachers demonstrated a positive aspiration regarding education and a will to develop their teaching skills for pupils, we should point out the gap between their confidence and actual lessons. Their common weak points were failure to arrange a discussion among pupils, combine individual and group works, express appreciation for pupils' thinking attitudes, and explain the connection between today's lesson and the next. These weaknesses were evident when pupils expressed their ideas, which were different from what the teacher expected them to answer. For example, while the lesson plan by the urban teacher was intended to conduct a learner-centered lesson, he actually dominated the pupils' discussion. Further, it could be observed that he used up time before summing up the lesson. The missing summary makes the objective of those lessons vague. This appears to have been due to insufficient consideration of the relation between the objective and activity in their lessons and of the relation among the previous day's, today's, and the following day's lessons.

Both considerations relate to how pupils can broaden and deepen their experiences of daily life and concepts of mathematics (fraction). Although the lesson on that day might have developed in the following lesson, it did not appear that the teachers consciously constructed their lessons to broaden or deepen pupils' activity/naïve concept into a mathematical one. It would lead to the gap between their expectations in teaching and pupils' actual understanding in learning. In this sense, it is essential to conduct further research on how pupils construct the mathematical concept in a linguistically/culturally-mixed situation in order to build the foundation of teaching.

References

Ahiabele-Addo C. H. (1980). Education Reform and Failure in Ghana (1920-51). In S. T. Bajah et al (Eds), West African Journal of Educational vol, xxxi, #3, pp 107-134.

Bartels E. K. (2003). The Practice of Continuous Assessment in Teacher Training Colleges in Ghana. In J. Anamuah-Mensah et al, Journal of Educational Development and Practice (JED-P), vol1, #1, pp 59-73.

Core mathematics syllabus for senior secondary schools (2003). Issued by Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

Davis E. K. (2004). The Effectiveness of In-service Teacher Training Program for the Improvement of Basic School Mathematics in Ghana. A case study of the Outreach Program Conducted by the University of Cape Coast. Unpublished master's thesis submitted to the Hiroshima University, Japan.

Eshun B. A. (May, 1979). Mathematics Curricula and Textbooks: The West African Experience. A paper presented at the Inter-African Seminar on Teaching of Mathematics in African Primary and Secondary Schools-held in Accra, Ghana.

Mereku K. (2004). Mathematics Curriculum implementation in Ghana Second Edition. Accra, Ghana ,Danjoe Production (printers).

Mathematics syllabus for junior secondary schools (1988). Issued by Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

Mathematics syllabus for junior secondary schools (2001). Issued by Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

Mathematics syllabus for primary schools (1988). Issued by Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

Mathematics syllabus for primary schools (2001). Issued by Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

Ministry of education of Ghana (2000). Reforms of the 1980s. Online at: http//www.Ghana.edu.gh/past/eightiesReforms.html (04/06/27).

Suggested core mathematics syllabus for senior secondary schools (1989). Issued by

Curriculum Research and Development Division (CRDD), Ghana Education Service, Accra, Ghana.

2.6 Zambia

Touomi Uchida, Hiroshima University
Satoshi Nakamura, Hiroshima University
Bentry Nkhata, University of Zambia

2.6.1 History of Mathematics Education

• Introduction

This paper provides a brief overview of the changes in mathematics education in Zambia from right before political independence till date. The major focus is on mathematics at the primary/basic school stage, where the initiatives have mostly been local.

• Mathematics Education in the Colonial Era

Zambia gained its political independence from Great Britain in 1964. The general aims of the Primary School Course in the colonial era were stated as follows:

"To develop

a) the highest possible standards of individual conduct and social behavior;
b) permanent literacy in one vernacular language and in English;
c) sufficient skill in arithmetic to meet everyday requirements with speed and accuracy;
d) an understanding of the immediate environment and how other people live in other places;
e) the acquisition of some practical skills." (MOE, 1962: p. ii)

The syllabus provided guidance to teachers and included the following aspects:

(1) The recommended text books cover the syllabus at the required level; however, teachers will have to outline their own terms and schemes of work. They must always be attentive to everyday examples and remain up to date with Post Office charges, costs of food, time-tables, etc.

(2) The teacher must be mindful of the following three aims:

(a) To teach "meaning" so that children learn to reason for themselves.

(b) To give children a sound basis for further study (e.g., a necessary preparation for secondary school mathematics).

(3) It is important that teaching methods be suited to the student's individual capabilities. This cannot be achieved by class teaching alone. In the initial stages, the students will often require individual attention; group teaching methods are strongly recommended at a later stage.

(4) The teacher must

(a) teach one thing at a time;

(b) ensure that he explains all processes;

(c) see that the facts of addition, subtraction, multiplication and division are systematically drilled and so thoroughly mastered that, by the end of the course, they can be assigned without hesitation.

Accuracy and neatness are most important in written work. When these are established, the children must be encouraged to work as fast as possible. In mental exercises, speed and accuracy must be the main aims. Mental exercises must be used to practice problems that the children will encounter in their daily lives.

(5) Great care should be taken over the correct layout of work. ... The following points should also be noted:

(a) correct and uniform figuring

(b) all working to be shown as an integral part of the sum and writing calculations in the margins should not be permitted.

(c) the correct use of the sign of equality

(d) answers must be clearly distinguishable. This is best done by underlining the answer with a neat penciled line. Units must always be given when required, e.g., 20 cows.

(e) Each sum should be neatly ruled off in pencil. A ruler should also be used for drawing lines in the calculation.

(f) The use of erasers or razor blades to erase mistakes should be discouraged.

The philosophical undertones in the "guidance" to teachers may be questioned; however, at least, considerable effort was expended in providing teachers guidance with regard to the manner in which to streamline the general intentions of the syllabus.

- **Educational Reforms of 1977**

In 1997, the Government of the Republic of Zambia announced fundamental changes to the educational system in the country. At this stage, the national ideology of the country was firmly entrenched in what the then President of the Republic of Zambia termed as the philosophy of *Zambian Humanism*. This was a version of socialism. From an educational viewpoint, it involved forging closer linkages between schooling and the world of work, so that those that complete school could become self-reliant individuals. The changes in the syllabuses toward the fulfillment of the new educational vision were released in 1983. The new syllabus for Mathematics was known as the Basic Education Mathematics Syllabus: *Grades 1--9*. The document on the syllabus states in its introduction "when constructing the aims of this syllabus, special consideration was given to the present social needs, and the traditional applications of the subject in addition to the Mathematical requirements for other subjects." The syllabus emphasized essential knowledge and skills that would aid in self-reliance.

The aim of the basic Mathematics Curriculum was to strengthen the link between schooling and preparation for working life. Students who were taught according to this curriculum were not only to acquire knowledge and skills that would enable them to become productive, but also those that would enable them to become self-reliant by the time they complete Grade 9.

A number of working skills in Mathematics were identified. These were as follows:

(1) Productive skills—defined in terms of the following features:

(a) It can be taught.

(b) It can be improved with practice and feedback.

(c) It can be applied in a variety of different skills that are usually combined to form a smooth sequence of actions directed toward a particular outcome.

(2) Social and life skills—activities and understanding required to manage one's personal life successfully.

(3) General skills—grouped into two as follows:

(a) Attitudes to work; Confidence; Commitment; Motivation; Common sense

(b) Basic knowledge of working life

-Understanding the role and purpose of work in relation to society.

-Understanding the difference between work and employment.

-Knowledge of occupation categories.

-Understanding basic economic processes at the individual, occupational, and national level.

-Knowledge of basic technological and industrial processes.

-Knowledge of self employment, such as cooperatives.

The aims of the syllabus were stated as follows:

(1) To equip the child to live effectively in this modern age of Science and Technology and enable him/her to contribute to the social and economical development of Zambia.

(2) To stimulate and encourage creativity and problem-solving.

(3) To develop the Mathematical abilities of a child to his/her full potential, and assist him/her to study Mathematics as a discipline and use it as a tool in various subject areas.

(4) To assist the child to understand mathematical concepts in order that he/she may better comprehend his/her environment.

(5) To develop in the child an appreciation of Mathematics in the traditional environment.

The specific objectives of the syllabus were as follows:

(1) To develop an interest in Mathematics and encourage a spirit of enquiry.

(2) To build up understanding and appreciation of basic mathematical concepts and computational skills in order to apply them in everyday life.

(3) To develop clear mathematical thinking and expression in the child.

(4) To develop the ability to recognize problems and solve them with related mathematical knowledge and skills.

(5) To develop and foster speed and accuracy.

(6) To provide the child with necessary mathematical knowledge and skills for him/her to be productive and self-reliant.

(7) To develop in the child a positive attitude toward production and self-reliance.

(8) To provide necessary mathematical pre-requisites for further education.

An exhaustive list of "productive skills" was also specified. These were as follows:

"PRODUCTIVE SKILLS IN MATHEMATICS

The pupils should be able to

(1) Classify objects and numbers according to a given condition.

(2) Demonstrate an understanding of number concept and numeration.

(3) Perform the four basic operations on numbers and measures.

(4) Demonstrate skills in measurement in appropriate units.

(5) Estimate and approximate numbers and measures.

(6) Translate verbal data into symbols and vice-versa.

(7) Identify plain and solid shapes and acquire an understanding of their basic properties and special relationships.

(8) Draw and construct geometrical shapes and solids.

(9) Collect, classify, tabulate, represent, and interpret data.

(10) Solve problems involving fractions, ratios and proportions, average and percentages as applied to numbers and measures.

(11) Solve problems involving household, social and commercial arithmetic.

(12) Solve problems involving measurements (length, area, volume, capacity, mass, money, time and speed, time and distance).

(13) Identify different types of symmetry and draw symmetrical figures.

(14) Read and draw compass bearings and use them in scale drawings and map reading.

(15) Use appropriate mathematical language.

(16) Construct and use graphs.

(17) Perform algebraic operations." (CDC, 1983)

The abovementioned details regarding the syllabus have been provided in order to identify points of departure between lofty statements of policy and ultimate implementation. Policy appears to have been left to implement itself. None of the ideals envisaged in the policy document were realized. With regard to the syllabus, the content basically remained unchanged. For example, it can be questioned what is *productive* about the specified "productive skills"?

- **Educating Our Future**

The next policy change in education came in 1996 in a document that was entitled *Educating Our Future* (MOE, 1996). The document filled the void left by the lack of implementation of the Education Reforms of 1977 and another document that was introduced in between—Focus on Learning (MOE, 1991), which only "focused" on Primary Education and ignored all other aspects of education. Certain salient aspects of *Educating Our Future* particularly relevant to the topic at hand were as follows:

- a recognition of the importance of mathematics and science education to national development;
- a directive that initial literacy would be in the language children were most familiar with, in this case, the local languages. (Up until now, English was the official medium of instruction at all levels of schooling. Local languages were often used in teaching but more from the point of view of common sense rather than a policy perspective.)

Educating Our Future was followed by the *Basic School Curriculum Framework* document (MOE, 2000) that aimed at providing a framework for implementing the new education policy. Again, fundamental changes for the provision of basic school education were articulated in this document. This implied impending reforms in the basic school curriculum in the following respects:

- More learning time: 20 hours of lesson time per week in Grade 1; 25 hours in Grade 2; 27 in Grades 3 and 4; and 30 in Grades 5 to 7.
- Concentration on fewer subjects
- Basic literacy and numeracy were to be accorded the status of being the highest prioritized competencies.
- Localized curriculum—in addition to the one that was centrally defined
- HIV/AIDS awareness and protection
- Life skills of various categories
- Outcomes-based curriculum
- Continuous assessment methods were to be a constant feature in all teaching and learning.

- **Syllabus of 2003: Numeracy and Mathematics**

The translation of the Curriculum Framework document came in the syllabus of 2003, which is currently the governing syllabus. The introduction of the syllabus document states the following:

"The new curriculum is outcomes based and focuses on results rather than on goals, aims and objectives. It places emphasis on observable and measurable skills, knowledge and values to be acquired by learners at specified levels of their schooling. The new curriculum emphasizes learner centeredness and provides for increased learner-teacher contact time, different ability groups and use of a familiar language for initial literacy.

Continuous Assessment is another prominent feature of the new curriculum. This allows for regular monitoring of individual learning process, diagnosis of learning difficulties and provision of remedial teaching.

The outcomes-based approach recognizes that learners do not attain the outcomes through a set of prescribed learning experiences in one Learning Area. They attain them through exposure to a wide range of experiences and varied content drawn from all Learning Areas.

This syllabus aims at enabling the learners to acquire mathematical knowledge and develop skills necessary for application in their everyday lives."

Further, the syllabus document provides the following general learning outcomes

"By the end of Grade 7, learners should be able to:

- Develop mathematical knowledge and skills.
- Communicate mathematical ideas effectively.
- Develop skills in problem-solving.
- Develop skills in social and commercial mathematics.
- Develop and foster order, speed, and accuracy in problem-solving.
- Apply mathematical concepts to their environment.
- Develop interest in mathematical skills for everyday use.
- Develop understanding of measurements and shapes.
- Apply mathematical operations in problem-solving.

Although the syllabus is in the early stages of implementation, it is clear that, once again, the content has largely remained the same except for the replacement of *objectives* with *outcomes*. The manner in which continuous assessment will be incorporated in the teaching process and the final grading of students is not clear yet. It is also unclear what assistance teachers will receive to "localize" the curriculum and incorporate the overlapping issues of HIV/AIDS, life-skills, gender, and environment.

- **Mathematics at the secondary school level**

Until approximately two years ago, Zambia was using the *University of Cambridge Local Examinations Syndicate* syllabuses, although the setting of examinations was localized much earlier. Zambia now has its own syllabus that states the following:

"This syllabus covers the topics of Arithmetic, Algebra, Geometry, Trigonometry, Probability, Statistics and Elementary Calculus. ... The aims and objectives of teaching mathematics at Senior Secondary level have been derived from three sources: the Educational Reform Document, the Structure of Mathematics as an academic discipline, and the needs of the child. The syllabus is structured in such a way that the pupil is encouraged to put emphasis on the mathematical concepts, principles and creative thinking processes.

When using this syllabus, it should be realized that Mathematics is a discipline with integrated and hierarchical concepts and skills. It is therefore recommended that an integrated and spiral approach be used (When the overall contents of the syllabus are presented in the traditional compartments)."

It is ironical that the new syllabus includes more challenging topics such as *Earth Geometry* and *Calculus* considering that the performance on preceding syllabuses was a source of grave concern.

- **Reflections**

(1) Language

In the colonial education system, initial literacy was imparted in the (seven major) local languages. After independence, it was believed that the use of a few local languages as the medium of instruction was politically incorrect as it would lead to certain tribes feeling superior and others feeling inferior. Thus, in the interest of fostering a sense of national unity, English was adopted as the official medium of instruction, including initial literacy, at all levels of education. This posed a problem for attainment of literacy (having to learn the language and to read and write in the foreign language), and caused a problem in the teaching of other subjects as well.

The country has reverted to initial literacy in local languages, but more importantly a distinction has been made between the imparting of literacy and teaching of language. In the few years of reverting to this policy on literacy, there are already indications that literacy levels have increased. However, it is not clear how the language issue will affect mathematics teaching and learning. While teachers may explain mathematics in the local language, there is no movement toward developing a mutually agreed-upon, formalized teaching system of maths in the local languages—all the books are still written in English. The most that has been done is to transfer the teaching methodology used in imparting literacy (some form of a child-centered approach) to the teaching of mathematics.

(2) Assessment

As indicated in the Basic School Syllabus of 2003, Continuous Assessment will now be a regular aspect of the mathematics curriculum. Until now, the final (Grade 7) examination has meant "everything." Thus, teaching and learning was directed mostly toward passing the examinations.

The examination system in Zambia is norm-referenced, i.e., passing is interpreted by progression from Grade 7 to Grade 8. Only recently has the progression rate gone beyond 30%. Progression is based on the number of seats available in Grade 8 in each of the 9 provinces of the country. An aggregate score based on English, Mathematics, and Special Papers 1 and 2 ("general intelligence" papers) is used to determine progression. Thus, the achievement levels of students in mathematics are not known to the general public; in particular, it is the progression figure that captures media headlines.

(3) Agents of curriculum change

Till date, changes in the mathematics curricula have been a result of changes in the political direction of the country. There has been a lack of influence from research or practitioners in the change processes. The net effect of this is that change has been cosmetic instead of systematic. In other words, change has been affected in official documents but has not percolated down to the classroom. Where it has been "mathematics for life," and "outcomes-based" (as opposed to objective-based), the syllabus and the teaching methodology have essentially remained the same.

- **Conclusion**

The National Assessment of Learning Achievement at the Middle-Basic School level (Grade 4—based on literacy and numeracy) concluded in the report (Kelly and Kanyika, 2001) that in terms of education, Zambia was a nation at risk—very little education of the type society expects is actually being imparted in Zambian schools. There is an urgent requirement for serious reflection on the state of mathematics education in Zambia and charting a more productive way forward. "Towards Endogenous Development of Mathematics Education" cannot be a more timely initiative.

2.6.2 Basic Information

(1) Teachers' Qualification

All lower- and middle-basic school teachers are expected to have been trained for 2 years at a primary teachers' college in order to obtain Primary Certification. However, there is a shortage of teachers in certain schools, particularly in rural areas. In such cases, schools have had to hire students who have completed Grade 12 and live near a school to assist in teaching certain classes.

(2) School Calendar and Examination

The new school year in Zambia begins in January and ends in December. The school year is divided into 3 terms, as shown in table 1, and each term is of a duration of 3 months. There is a one-month break between each term.

Table 1. School calendar

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
	1st Term				2nd Term				3rd Term		

At the end of Grade 7, students take a national examination and those who are able to acquire the cut-off marks proceed to Grade 8. All items in the Grade 7 examination are multiple-choice and presented in English. Similarly, at the end of Grade 9, students take a national examination and those who qualify proceed to Grade 10. It is planned that in the future, if there are sufficient seats available in Grade 8, Grade 7 students will be automatically promoted to Grade 8.

(3) Medium of Instruction

The official language in Zambia is English, although there are approximately 72 tribes that have their own language (dialect), which students usually speak in their homes. English is also the medium of instruction; therefore, teachers basically conduct lessons in every subject in English. However, students, particularly in the initial years of schooling, usually speak local languages outside classes and outside school. Thus, students, particularly in lower grades have difficulty in understanding and speaking English. Occasionally, certain teachers use a local language to explain certain things to students; however, the problem is that certain students may not understand the local language and therefore will not be able to communicate with the teachers. In certain cases, the teacher may not know how to speak the main local language.

2.6.3 Results from the First Year Field Survey

(1) Survey Schedule

A one-man delegation visited Zambia from January 17–27, 2005 in order to facilitate the field survey with his Zambian counterpart. This implies that data collection was conducted merely two weeks after the commencement of the first term. The detailed schedule for data collection is tabulated as follows.

Table 2. Schedule of data collection

Date	Activity
16^{th} / Jan / 2005	Arrival in Lusaka, Zambia
17^{th}- 19^{th} / Jan / 2005	Preparation of the survey Discussion with District Education Offices and Targeted Schools
20^{th} – 27^{th} / Jan / 2005	Data collection in two sample schools in Lusaka Province and their subsequent remedial work
28^{th} / Jan / 2005	Departure from Lusaka, Zambia

(2) Target Schools and Samples

1) Sampling procedures

From among the 9 provinces in Zambia, Lusaka—the capital province—was selected as a sample due to the time constraints for completing the data collection. According to the Central Statistic Office, the extra-departmental organization under the Finance Ministry, an urban area is defined by three criteria—population size, economic activity, and facilities available in the area[2]. Taking into consideration these criteria, from among the 4 districts in the province, Lusaka District was selected to represent an urban area and Chongwe District to represent a rural area.

For the selection of average schools in both urban and rural areas, the survey team requested each District Education Office (DEO) to recommend appropriate schools. After a discussion with the District Education Standards Officers (DESOs), we selected one average government school from both the urban and rural districts. The criteria taken into consideration were as follows:

a. Ranking of the sample schools according to the results of the National Final Exam

 and

b. Social and economical strata in the catchment area of the sample schools.

However, the survey team did not have an opportunity to go through the actual data related to

[2] In addition to this definition, an urban area must have a minimum population size of 5,000 people. The main economic activity of the population must be non-agricultural, such as wage employment. In addition, the area must have basic modern facilities such as piped water, tarred roads, post office, police station, health facility, etc.

the abovementioned criteria.

For the selection of a sample Grade 4 class from the average urban school, it was reported that classes are not created in the order of the students' results. Thus, all the classes are assumed to be uniform and one sample Grade 4 class was selected since the timing of its lesson could fit into the overall schedule for data collection. In the average rural school, there was only one Grade 4 class; thus, all Grade 4 students have been targeted in the survey.

2) Basic school information

Currently, Zambia is transforming its school system from the 7-year primary and 5-year secondary education to a 9-year basic and 3-year high school education. There remain certain basic schools, particularly in remote areas, that accommodate only 4 grades due to a number of constrains. On account of these situations, there are various sizes of basic schools depending on the level of progress in their transformation, and two of our sample schools were not exceptional.

The following table presents the details of the sample schools.

Table 3. Profile of the sample schools

	Location	No. of Pupils	No. of teachers	No. of classes per grade	Progression rate to G. 8 in 2004 (%)	Others
Urban Primary School	2 km from the capital city, Lusaka.	1657 (G.1-7) 2063 (G.1-9)	44	5	17.0	- Within the heart of the capital city/ - Beside tarred road. - Piped water & flush toilets.
Rural Primary school	58 km from the capital city, Lusaka 13 km from District centre	328 (G.1-7)	7	1	70.7	- Outside of district center - About 1 km from tarred road. - Water pump & Pit latrine.

Table 4. Details of teachers observed

		Urban	Rural
Set		Female	Male
Age		41-50	41-51
Academic/professional qualifications		- Primary teacher's certificate	- Primary teacher's certificate (ZPC) - B. Tech Education Management
Length of service		29 years (?)	18 years
Special responsibility in the school		Senior teacher	Head teacher
Teaching Experience	G. 1		1
	G. 2	3	1
	G. 3	3	2
	G. 4	2	2
	G. 5	2	1
	G. 6		2
	G. 7	2	9
INSET taken		NISTOCOL	NBTL SITE ROC

As mentioned earlier in the sample procedure, one class each from both urban and rural average primary schools has been selected for administering both the students' questionnaire and mathematics achievement tests. The same Grade 4 classes are used to observe a lesson in mathematics. According to the survey guidelines, the initial aim was to observe a topic in fractions. However, since the topic was supposed to be taught toward the end of the first term in Zambia, we agreed to tape and observe the lessons on Sets and attempted to tape the lesson on the target topic later in the same term. In this survey, we could manage to obtain a video-tape of the lesson on Fractions taught by the same teacher only from the urban school in time. Therefore, in this survey we examine the findings from lessons given on different topics.

(3) Results of Interview

The result of the interview of head teachers is presented in the following table.

Table 5. Responses from head teacher

	Interview items	Urban school	Rural school
[Problem]	1-1 What do you think is the biggest problem in teaching mathematics in your school?	Lack of teachers. Complicated process for procurement of books. Availability of text books and other T-L materials are satisfactory.	
	1-2 What kind of action do you take against that problem as an administrator		
	1-3 Do you observe lessons by teachers? Yes or No. If yes, how often do you observe them?	Not frequently, but instead, Deputy Head observes the lessons frequently.	
	1-4 What kind of advice do you give to young teachers at your school?		
[In-service training]	2-1 Do you see any impact of in-service course offered to teachers? If yes, is it negative or positive? Please describe the impact a little more	Latest system for In-service Training does not appear effective very much.	
	2-2 What kind of training do you think is necessary for teachers in your school, if a new training course is designed?	How to deliver the information such as knowledge, skills or attitudes to the pupils.	

The result of the interview of mathematics teachers is presented in the following table.

Table 6. Responses from mathematics teachers

	Interview items	Urban school	Rural school
[Problem]	1-1 What do you think is the biggest problem in teaching mathematics in your class?	Books, enough only for teachers but not sufficient for pupils. Because of that, teacher can give very few questions.	
	1-2 What kind of action do you take against that problem?	Sometimes encourage the pupils to buy books by themselves.	
[Today's lesson]	2-1 What was the purpose of today's lesson?	As shown in the lesson plan	
	2-2 How much do you think the purpose was attained?	Not available.	
	2-3 What do you think are the most important factors for successful lesson?	Teachers to do research about the topic with more than one book.	
	2-4 What kind of teaching would you like to do?		
[In-service training]	3-1 Have you ever had a teacher training after you become a teacher?	Yes, INSET at NISTICOL and the training with regards to literacy program.	
	3-2 Which kind of training, if you had before, do you think is useful for your teaching?	ZPC (previous PRESET in Zambia)	
	3-3 What kind of training do you think is necessary for improvement of your lesson, if a new training course is designed?	Workshops to learn how to teach difficult topic such as multiplication.	

(4) Results of Lesson Plan Analysis

[1] Lesson Plan of Urban School

Subject: Mathematics

Teaching Aids and References: Teachers' Book, pp. 18–20
Chart of Fractions
Students' Book, pp. 91–105
Fruits

Date: Thursday, March 10, 2005

Duration: 30 min.

Topic/sub-Topic: Fractions

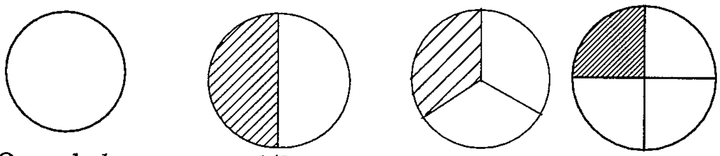
Objectives: By the end of the lesson, students will be able to

- know that fractions are equal parts of a whole,

- find the fraction shaded in each picture,
- find equal fractions from among other fractions shown,
- find the correct numeral in each box,
- put signs such as <, >, or =, and
- find the missing numbers in fractions.

Development:

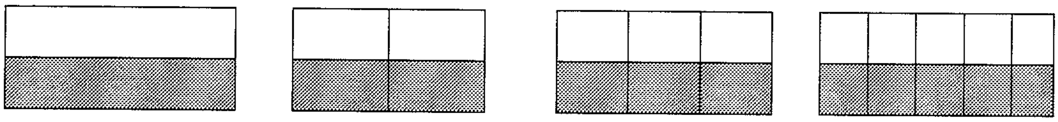
Example 1:



One whole 1/2 1/3 1/4

and 1/5 and 1/10

Example 2:



(a) How many quarters are there in 1/2?
(b) How many sixths are there in 1/2?
(c) How many tenths are there in 1/2?

1/2 is the same as 2/4, 3/6, and 5/10.

1/3 is the same as 2/6.

2/5 is the same as 4/10.

Example 3:

Put >, <, or =

One whole								
½				1/2				1/2 > 1/4
1/4		1/4		1/4		1/4		1/2 = 4/8
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/2 < 1/4

Equal $2/3 = 4/6$

Not equal $3/5 \neq 3/10$

Example 4:

$$\frac{1}{6}+\frac{3}{6}=\frac{1+3}{6}=\frac{4}{6}=\frac{2}{3}$$

Activity: Students are to draw the shapes from example 1 as an exercise, and cut fruits in fractions in groups.

[2] Lesson Plan of Rural School

Not available at the time.

Table 7. Responses of lesson observation checklists

		Urban school					Rural school				
Introduction		0	1	2	3	4	0	1	2	3	4
	The teacher starts the class on time	-	-	-	-	-	-	-	-	-	-
	The teacher made the objective clear								√		
	The objective suits to the level of children	-	-	-	-	-			√		
	Relationship with the previous lesson is clear	-	-	-	-	-				√	
Development	The teacher gives supports to pupils who seem to have little understanding		√								√
	The expresses appreciation for pupils' thinking attitudes			√				√			
	The teacher assesses the pupils' comprehension during teaching learning			√						√	
	The teacher uses easy language					√					√
	The teacher uses an appropriate and familiar example to illustrate main concept.				√					√	
	The teacher creates friendly atmosphere					√			√		
	The teacher accommodates discussion among pupils		√					√			
	The teacher gives hand-on activity				√		√				
	The teacher enjoys teaching				√				√		
	The teacher is impatient with wrong answer		√						√		
	The teacher involves children to say opinions freely.			√					√		
	The teacher encourages children to display diverse opinions	√					√				
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.			√					√		
	The teacher combines individual work and group work appropriately.			√				√			
Summary	At the end of the lesson, the teacher summarizes the lesson.	√					√				
	The teacher assigns homework at the end of lesson clearly.	√					√				
	The teacher explains about a connection between today's lesson and next lesson.	√					√				
General	The teacher prepares a lesson plan.					√	√				
	The teacher prepares a plan for taking note on the blackboard.	√					√				

Assess who dominate solving problems during the lesson observed.	Teacher dominates in solving problems.	Teacher dominates in class but students were solving problems in drill session.
Assess which of the followings is regarded as the most important in the lesson observed. Understanding concept/mastering the procedure/thinking mathematically/applying to the daily life/finding correct answer.	Understanding concept.	Understanding concept.

Rating scale: 0-never, 1-seldom/to a little extent, 2-sometimes/to some extent, 3-often/to a considerable extent, 4-very often/to a great extent

With limited information from the average urban school, it is noted that there exist certain differences in the opinions of teachers. For example, the Head Teacher felt that the availability of teaching/learning materials was not really a matter of concern, while it was taken more seriously by the class teacher. This is simply because more materials have been extended to a number of basic schools in major subjects since 2003 through the initiative of the ministry in the new educational development plan in Zambia. With regard to procurement, the voucher system has been introduced for certain grades this year; thus, each school has begun to enjoy a greater amount of freedom in selecting textbooks. These changes have probably caused the Head Teacher to become more optimistic towards the materials in school.

On the other hand, the initiatives have not been implemented fully in the schools; thus, in practice, teachers and students in certain grades have not been able to enjoy them as yet. This situation may have caused such a difference in the opinions of the Head Teacher and class teachers.

With regard to the issues of teaching in class, both the Head Teacher and the class teachers have a similar perception that delivery of information to students in class is the major problem as compared with the knowledge itself.

With regard to the issues of INSET, both parties appeared to be unsatisfied with the latest program. It can be said that this is because the new program requires a large amount of effort in terms of preparing for the lessons and assessment, which made it rather difficult for them to sustain the activities.

(5) Results of Video Analysis and Classroom Observation Checklists

The examination of the lesson plan on fractions has brought to light certain aspects, which are as follows. First, it appeared to be rather overloaded with contents, although the duration of a lesson was merely 30 min. This resulted in sparing much less time to teach the concept of fractions, although this appeared to be the first lesson in the topic. Second, there is an attempt to make the topic more student-friendly by using familiar materials like fruits; however, the relation to the essential concept was not very clear in the plan. Third, the plan did not describe the manner in which the level of understanding of the students would be assessed.

The observation of each lesson in keeping with the framework of the given checklist produced the following findings.

> There are relatively high scores in "The teacher uses easy language" and "The teacher uses an appropriate and familiar example to illustrate main concept." This implies that teachers were putting in more efforts for students to understand the topics they were teaching and follow instructions in class. Actually, teachers were occasionally using the local languages of the area, particularly when it came to the important portions of the lesson in order to supplement the explanation made in English. They also attempted to

create a friendly atmosphere in class. It was important for teachers to be considerate with regard to these aspects as students were also developing their English proficiency while learning the topic. Due to such an unavoidable situation, the lessons, to a certain extent, appeared like language lessons rather than those on mathematics.

On the other hand, there are relatively low scores in items related to teacher's interest in the response of students or using their own thinking in class. The actual lessons themselves appeared to be dominated mainly by teachers, even at the stage of problem–solving, except in drills. This implies that the focus of teachers may be to impart knowledge to the students or the mastery of mathematical operations such as computations, rather than induction of students toward developing mathematical concepts.

There were typical situations that confirmed the abovementioned findings in both lessons. In the urban school, there was an attempt to relate one of the basic concepts in fractions to daily life by making the students cut fruits (guavas) into pieces. This appeared to be a good approach. However, the manner of cutting was left to the students and the size of the pieces was unequal; therefore, missing the point that fractions are "equal parts of a whole." This did not appear to draw much attention in the class.

At the stage of drills in the rural school, a number of students were unable to copy the questions written in English on the board by the teacher. Students appeared to not recognize the words in sentences, so they merely wrote a series of alphabets without any spaces between the words. This reveals that the students did not understand the meaning and instructions in the questions. However, both the teacher and students did not appear to be concerned about this and the students continued to answer questions. The teacher was even marking the students' answers while moving around in the class.

As a next step, these situations can be reviewed and discussed with the concerned teachers in order to ascertain the explicit or implicit intentions and reasons behind them.

It was also a common observation that teachers neither summarize the topic nor give any assignments at the end of the lesson. They also did not relate the day's lesson to the next lesson. However, we cannot generalize these findings as a common feature of the teaching style in Zambia because we observed merely one lesson of each teacher.

As observed above, there were certain features of teaching mathematics in class. On the other hand, we did not sufficiently explore the teachers' perceptions of teaching mathematics. Thus, in the second year it is necessary to examine their perceptions with regard to the development of students, the weightage they assign to different contents, attitudes or teaching skills for mathematics, etc.

2.6.4 Results from the Second Year Field Survey

(1) Survey Schedule

The survey was conducted during the period October 17–27, 2004; the details of the schedule are presented in table 8.

Table 8. Schedule of data collection

Date	Activity
16th / Oct / 2005	Arrival to Lusaka, Zambia
17th / Oct / 2005	Discussion with Mr. Nkhata at University of Zambia
18th – 21st / Oct / 2005	Data collection in three sample schools
23rd / Oct / 2005	Departure from Lusaka, Zambia

(2) Target Schools and Samples

Urban and Rural School A were those that we targeted the last time. These were selected as the average schools in the urban and rural areas. This time, we would like to compare the result of tests that were conducted with an explanation to students in the local language and those that were conducted without such an explanation. Rural School A had only one class in each grade; thus, we needed to include an additional rural school in order to make the comparison. Therefore, Rural School B was selected as an average rural school in Mazabuka District in the Southern Province of Zambia.

(Explanation in local language implies the translation into local language of questions that are written in English, and explanation with regard to the manner in which the answers must be written, particularly in the case of multiple-choice questions.)

Table 9. Location of schools

	School location
Urban Primary School	2 km from the capital city, Lusaka
Rural Primary School A	58 km from the capital city, Lusaka 13 km from District Centre
Rural Primary School B	24 km from District Centre

Five teachers participated in this research—two of them were from the urban school and the other three from the rural schools. Table 10 provides the details.

Table 10. Brief profiles of teachers

School	Sex	Teaching experience	Subjects taught
School in urban area	(1) Female (2) Female (3) Female	(1) 29 years (2) 20 years 4 months (3) 25 years 5 months	(1) All subjects (2) All subjects (3) All subjects
School in rural area	(1) Male (2) Female	(1) 18 years 9 months (2) 22 years 4 months	(1) All subjects (2) All subjects

As is evident from the above table, a majority of the teachers who participated in the study had over 20 years of teaching experience.

(3) Results of Teachers' Questionnaire

Table 11 shows the responses of teachers with regard to their perception of the test given to students and on the teaching as well as learning of mathematics in general and fractions in particular. The following is the result of the Teachers' Questionnaire; the comments of the teachers are unedited.

Table 11. Teachers' perception on the test, teaching and learning of mathematics and Fractions

Questionnaire Items	Responses by teachers in urban school	Responses by teachers in rural schools
Teacher's forecast of s' average score	(1) forcasted 50% (2) gave no response (3) forcasted10%	(1) forcasted 20% (2) forcasted 75%
Pupils' familiarity to the given test	(1) Yes : It's one of the topics in G4 mathematics on Fractions. (2) No : They usually write multiple questions and the	(1) No : They are usually tested on a variety of topics and also materials for such testing not adequate as we usually used

	questioners are mixed not based on one topic. (3) No : They're not used to writing on such papers due to luck of paper. They're not used to reading small writings like the ones on this paper.	boards. (2) No : They are not accustomed to this kind of test because in most cases they used to have no teacher and as a result they find it very hard to answer this type of work. In short, they never break through.
Any questions which the pupils cannot solve.	(1) Yes : Pupils forget easily unless before testing them do the revision. (2) Yes : They usually mix fractions and work them as a whole number. (3) Yes : The way of question is not their level.	(1) Yes : Q7. They may not show the process. Q8. They may not do it because of language. Q10. They have problems in dealing with abstractions. Q2. They may not relate divisions to fractions. (2) Yes : 3,8,9 and 10 are hard for them because they have not yet tackled fraction. Further more, it now when they are starting G4 work. All along they have been drilled with G3 work.
Difficulties in teaching "Fractions".	(1) Easy : Teaching of Fractions is easy especially when you start with pupil using the concrete objects like orange by cutting 2 pieces, 4 pieces and even 3 pieces. (2) Difficult : When explaining they understand but when it comes to writing they write other things except a few pupils in class. (3) Easy : It is easy because when I teach, I have to use concrete objects like oranges, to show them the real parts of a whole.	(1) Easy : Easy but you need more time in order to get the concept of Fractions as they find it easier dealing whole numbers. (2) Easy : It is easy to teach Fractions depending the ability of pupils and the way you have introduced or presented the topic. In short, it is very easy to teach Fractions because in most case pupils are good at sharing things together. So Fractions involve sharing and parts of a whole.
The most difficult topic/s to teach in Grade 4.	(1) "Decimals" Pupils mostly misplace the points. (2) "Division, Fractions, Social Arithmetic" They don't complete the long division. Fractions they work out directly. (They do add denominators) They find the answer then they fail to add to get the total price. (3) "Division" It is difficult to teach division because to find the answer you have to use addition, subtraction and multiplication.	(1) "Division" Pupils fail in problem understanding the concept of division as this is not introduced in G1 and a bit of it is G2. (2) "Division and Length" Most pupils find it is difficult to solve division problems because they are lazy to master multiplication table which is a key to solve division cases. And Length also, there is a bit of problem to some pupils because they cannot differenciate centimeters and meters.
The easiest topic/s to teach in Grade4.	(1) "Fractions" Because Fractions are equal parts of a whole and you can use a lot of teaching aids/materials. (2) "Addition, Subtraction" Children find it very easy to add, subtract, make sets and identify sets. (3) "Addition and Subtraction"	(1) "Addition" Pupils normally deal with additions in their daily lives and therefore easy to relate examples to their lives. (2) Addition and Subtraction are the easiest because it is very rare for pupils to get nothing from these tow topics. Without giving them

	Because most of the pupils know on what to do through the games they play as children.	examples, they can still get them correct in each topic given.
Teacher's confidence in teaching "Fractions"	(1) confident (2) very confident (3) very confident	(1) very confident (2) very confident
Level of influence of the examination on teacher's teaching	(1) "Very little" It depends if you did not teach the topic in that G4 or may be you never explained well to pupils, it is the pupils can fail. (2) "Very much" As a teacher, you feel you have taught but when the exam comes pupils do not perform well. They do not know remember the formula used in finding certain topics. (3) "Very much" Because I will be able to learn and find whether what I taught was well understood by my pupils and be able to improve where I did not teach well.	(1) "Very much" The examinations really affect our teaching in a sense that when the examinations. Begin, no other class is allowed to come and the lessons and as a result, this makes us remain behind the syllabus. This happens because of lack of classrooms. You find all the classes are occupied by the examination classes. (2) "Very much" Once the concept are not clearly put forward to the pupils, the will not master the correct way of dealing with a problem and therefore they fail to answer correctly.
Degree of difficulty for the pupils to learn "Fractions"?	(1) easy (2) easy (3) difficult	(1) difficult (2) no response given
Points of difficulty for the pupils to learn the concept of "Fractions"?	(1) Teaching of Fractions can be easy if you use proper concrete objects. (2) They mix numbers : e.g. 1/2 of 100m instead of multiplying 1×100 divide by 2 they will just go straight to addition = 100/2 = 1+100 = 100. (3) The idea of finding the biggest fraction when there are no diagrams.	(1) They do not seem just understand the whole concepts. Difficult to pin-point exactly where and why. (2) The difficult part of it is that, when a child is not present at a time of presentation, it will be very difficult for that particular child to acquire the concept of "Fractions". At the time of presenting a topic in fractions, I very much put emphasis to pupils not to stay away from lessons.
Existence of pupils' difficulty with the medium of instruction in learning mathematics	(1) Yes : Same pupils are slow learner. (2) Yes : Language because some terms in mathematics have no names in venacular. (3) Yes : Because reading a problem to them, hence do not know what to do.	(1) Yes : Sometimes they fail to understand the question especially if it's in English. (2) No
Importance of learning "Fractions" with comparison to any other topics in mathematics.	(1) Yes : It is an important topic because Fractions can be used in our daily life. (2) Yes : Pupils do sharing every day in their activities so they should learn more on fractions to know more or sharing. (3) Yes : It involves sharing and	(1) Yes : They deal with our daily activities in life. (2) Yes : It is yes an important topic a sense that, at a lower grade like G4, where pupils are still learning sharing, it really helps a lot in that a child, can be, able to share something amongst his/her

	finding out a part of a whole.	life forends. e.g. 1/2 of an orange or 1/4 of it.
Teacher's main point/s of concern to the pupils in teaching "Fractions"?	(1) You teach them that Fractions are equal parts of a whole and it is being used in our lives. (2) The main point is to build, share and arrive at the correct answer. (3) Sharing or part of a whole.	(1) They must understand the four operations on Fractions and they should be able to apply practically. (2) The main point is that, pupils should know the names of both top and bottom numbers at a lower level, which are numerator and denominator.

The responses of teachers when asked to describe how they taught the following problem to students are presented in table 12:

"Which is longer 1/4m or 1/3m?"

Table 12. Teachers' Srategies in Taching "Fractions" (1)

Responses from teachers in urban school	Responses from teachers in rural school
(1) When it comes to meters you can use concrete objects like a ruler and then measure on the stick to get 4 quarters as well as 3 thirds. Then you can use semi- concrete objects like drawing diagrams on the board and divide them into 4parts and then 3 parts. (3) I would draw or write a meter on the board and divide it in section, then pupils will be able to find out 1/4 is less than 1/3 because 1 meter will be divided in 4 sections or parts where as 1/3 is bigger because a meter will be divided in 3 sections or parts.	(1) First of all, I will draw to different diagrams on the board for them to see and answer questions on the same as I do no explaining to them. The first one will be showing the whole thing divided into quarters. Equally the second one is longer than the other. Pupils will tell whether they are equally or one is longer than the other. By doing so, then I shall now know which method to use. (2) I would bring 2 strings of paper(same size), then divide one into quarters and the other into thirds using same scale.

When asked how they would deal with a student who when asked the question "What is half of 2m?"responds by saying "It is 1/2m," they responded as shown below.

Table 13. Teachers' Srategies in Taching "Fractions" (2)

Responses of teachers in an urban school	Responses of teachers in a rural school
(1) First of all you explain to the students or pupils that 1/2 of 2m is the same as $1/2 \times 2m$. Then you show how to get 1/2 of 2m using multiplication for example. $1/2 \times 2m = 1m$ (2) I need to explain like this. 2 meters. 200 centimeters when you are traveling on a distance of 2 meters, suddenly you are told to stop in the midway how many meters have you covered. The pupil will be given a ruler to see and give a correct answer. The correct answer will be 1m or 100cm.	(1) The answering itself will tell me that the pupil has an idea. As a teacher, what I will do is just to correct him or her, to say instead of saying 1/2m, I would advise that child to say 1m, unless if he/she changes the whole number into a fraction, then she should say it as a fraction. (2) I would tell him he is right it is one out of two but I want another way we can say the same thing. Until we arrive the answer 1m.

Half of 2m is 1m or 100cm. (3) I would ask the pupils to explain how she got the answer. If she fails I will get a string of 2m and fold it into 1/2 and ask which is half of a meter.	

2.6.5 Discussion

* Experience of teachers

All teachers in this research have sufficient teaching experience. However, their prediction regarding the performance of students does not match the actual score.

* Gap between teaching and learning

Four teachers believe that students are not familiar with the type of test and all of them believe that there are certain questions that the students will not be able to solve. Two teachers feel that it is a difficult topic for students and that their performance will be rather low. The reasons for the predicted poor performance were that students are not accustomed to the quality of the test paper and the type of tests. Moreover, the predictions varied greatly with every teacher.

Four teachers believe that teaching Fractions is easy because concrete objects are a great aid in this topic. Moreover, all teachers are rather confident with regard to teaching this topic. Thus, this implies that Fractions is a topic that is easy to teach, but learning it is difficult for students.

* Easy and difficult topics

The teachers feel that the easiest topic in Grade 4 is addition and subtraction, and the most difficult is division. The reason provided for the latter is that division involves other operations and has a longer process.

* Difficulty with Medium of Instruction

Four teachers identified that there is a problem with the medium of instruction. Several teachers identified the fluency problem faced by students. One teacher identified that there is a concept gap between English and the local language. There was no statement with regard to whether or not the teachers are conscious of the students' understanding of problems.

* Influence of Examination

All teachers, except one, believe that examinations affect their teaching to a great extent. Several teachers stated that examinations affect everyday teaching. One teacher stated that the results are indicative of the students' weaknesses, thereby helping him/her to improve teaching.

*Importance of Fractions

The teachers believe that Fractions is an important topic because of its close linkage with daily life. However, no teacher mentioned the importance of fractions as a mathematical concept. With regard to the main concern in teaching Fractions, one teacher answered that four operations of fractions are important, another answered that remembering which is the denominator and numerator is the main point, and the other three teachers, who are all from the urban school, believe that the main point is "sharing."

* Teaching Strategies for Fractions

With regard to the teachers' strategies for teaching Fractions, Q14, a majority of the teachers wrote that they draw two lines or objects and cut one into three equal parts and another into four equal parts. Then, they show the difference in length to the students. In Q15, certain teachers reveal their understanding of the state of thinking of the students, which may serve as a foundation for improving the quality of teaching.

Acknowledgment

We greatly appreciate the help rendered by Prof. Chistopher Haambokoma, the professor in the University of Zambia, in writing this report

References

Curriculum Development Centre (1983) Basic Education Mathematics Syllabus (Grades 1 - 9). Lusaka: Education Council of Zambia

Curriculum Development Centre (2000) The Basic School Curriculum Framework. Lusaka: Ministry of Education

Curriculum Development Centre (2003) Zambia Basic Education Syllabi: Grades 1 – 7. Lusaka: Ministry of Education

Ministry of African Education (1962) Approved Syllabus for Primary Schools. Lusaka: Northern Rhodesia Government

2.7 Japan

Chikara Kinone
Hiroshima University

2.7.1 Basic Information

(1) Teachers' Qualification

The present pre-service training system is such that a teaching license is awarded to those who acquire the credits specified in the Educational Personnel Certification Law by attendance at a university (for 4 years) or junior college (for 2 years) that has been accredited by the Minister of Education to conduct a pre-service training course.

In Japan, the standard access route into teaching is to give the teacher appointment examination organized by each prefectural board of education either while studying at university or junior college, or after graduation. If successful, they are to join the teaching profession, unless they fail to acquire the teaching license.

(2) School Calendar and Examination

Generally speaking, a school year in Japan begins in the second week of April and ends in the fourth week of March. It is divided into 3 terms and there are school holidays after every two terms, as shown in Table 1. (Recently, certain schools are introducing the two-term system.)

Table 1. School calendar in (year)

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
	Term 3			Term 1					Term 2		

At the elementary school level, the end-of-term test or national examination is not conducted in Japan as a legal requirement. However, a type of evaluation test is conducted in order to assess the achievement of students depending on each school or teacher.

(3) Medium of Instruction

In Japan, the official language and mother tongue for both children and teachers is Japanese in general. Therefore, the medium of instruction is also Japanese.

(4) Class Organization

In the elementary schools of Japan, each teacher is in-charge of a particular class and teaches all subjects (class-specific teacher). In general, each lesson has a duration of 45 minutes, and there are at least 23~27 lessons per week.

Table 2. School hours

	First shift									
Hour	8	9	10	11	12	13	14	15	16	17
Grade	All Grades									
Period	HR	1st	2nd	3rd	4th	Lunch Time Cleaning	5th	6th	HR	

(5) Transfer System

In Japan, the personnel administration of teachers is handled by prefectural boards of education in accordance with a procedure whereby the principal of each school is permitted to express an opinion regarding her/his wish to have a particular teacher transferred to her/his school; the local (city, town, or village) board of education forwards all opinions received to the prefectural board of education.

This systematic practice of transferring teachers employed in public sector schools at intervals of a number of years was developed on the basis of the following criteria: i) interchange between urban and rural areas; ii) interchange between isolated and populous areas (mountainous and flat areas); iii) maintaining an appropriate balance in terms of the age structure of teachers in a school; and iv) avoidance of a long term of service in one particular school.

In particular, newly-appointed teachers are generally dispatched to a school in an urban and populous area for a period of 3 years. Thereafter, they are transferred to a school in a rural and isolated area and serve there for 3~4 years.

2.7.2 Results from the Second Year Field Survey

(1) Survey Schedule

The survey in Japan was conducted in accordance with the following schedule:

Table 3. Schedule of data collection

Date	Activity
06/03/2006	Data collection in the school, Onomichi-city, Hiroshima

(2) Target Schools and Samples

Onomichi city is one of the cities in Hiroshima prefecture, located in the southeast of Hiroshima. The population of the city is approximately 150,000. The sample school was selected as an average school in Onomichi city based on an achievement test conducted by the education board of the city.

Table 4. Location of schools

	School location
Average Elementary School	On the top of a hill which is in Onomichi-city

Due to the time limitation, the survey in Japan included only 1 teacher of grade 4 in elementary school. The brief profile of the sample teacher is as follows:

Table 5. Brief profiles of teachers

	Sex	How long you have taught	Subjects you teach
Average Elementary School	(1) Male	(1) 10 months	(1) All subjects

(3) Results of Questionnaire

a) Test evaluation

The teacher responded on "Test evaluation" as follows.

First, he predicted that the average score of the students in the test would be 50% (this was in fact 60.7%).

Second, with regard to "students' familiarity with the given test," he indicated the frequency of classroom tests and students' high concentration when solving questions and believed that they were sufficiently familiar with the test. However, he was afraid that they were not used to tests with as many pages as this one.

Third, with regard to "questions that students were unable to solve," he indicated the existence of contents that were not previously taught, lack of time to deepen the understanding of students, and the difficulty in understanding fractions as numbers and the manner in which to decide denominators.

Table 6. Teacher's responses on "Test evaluation"

Questionnaire Items	Average school
(1) Teacher's forecast of pupils' average score	50%
(2) Pupils' familiarity to the given test	1. Yes Reason/s: ➢ Classroom tests are given frequently. ➢ The pupils concentrate very well when they solve questions in normal lessons. ➢ The pupils seem not to get used to such test with many pages as this.
(3) Any questions which the pupils cannot solve.	1. Yes Reason/s: ➢ There are some questions which the pupils don't learn yet. ➢ There was no time to make the pupils more deeply understand contents which they have already learnt. ➢ Some of the pupils don't understand the structure of fractions as "numbers". ➢ I haven't made clear that we can decide the denominators of fractions by the number of equal parts divided, on the basis of 1m or 1ℓ.

b) Self-evaluation

The teacher's responses on "Self-evaluation" are as follows.

First, he mentioned that the most difficult topic to teach in Grade 4 was "How to divide when the divisor is a two-digit number," and the easiest one was "Line graph."

Second, while he was confident in teaching fractions, he felt that it was difficult for students to picture fractions by reading statements written in words.

Third, he believed that examinations affected his teaching to a large extent because his students often expressed to him their impressions and questions with regard to his teaching either during or after lessons.

Table 7. Teacher's responses on "Self-evaluation"

Questionnaire Items	Average school
(4) Difficulties in teaching "Fractions".	3. Difficult Reason/s: ➢ When fractions are expressed as tape-diagrams visually, it seems easy for the pupils to understand them. ➢ However, when fractions are stated in written words, it seems difficult for the pupils to image them as numbers.
(5) The most difficult topic/s to teach in Grade 4.	Topic(s): How to divide when the divisor is a two-digit number Reason/s: ➢ I can't explain clearly for the pupils how to arrange the quotient in calculating divisions with large numbers in

	column form. ➢ I think that I make the pupils dislike learning divisions, because I needed to explain so many times in teaching how to calculate divisions with remainder. ➢ I have to learn more appropriate teaching methodology.
(6) The easiest topic/s to teach in Grade 4.	Topic(s): Line graph Reason/s: ➢ Pupils have some opportunities to see line graphs in their daily life, and don't start learning without any experiences on it. ➢ It is possible for pupils to draw line graphs, if they can interpret tables sufficiently.
(7) Teacher's confidence in teaching "Fractions"	3. Confident
(8) Level of influence of the examination on teacher's teaching	1. Very Much Example/s: ➢ Almost all of the pupils don't go to private cram schools, and they meet the learning contents at school for the first time. ➢ "Why?", "What for?" and so on, the pupils often tell me in the lessons. ➢ After the lessons, in addition, they also often tell me their impressions, such as "Today's lesson was very clear!" "I didn't understand that."

c) Evaluation of students

The teacher believed that it was difficult for his students to learn fractions. In particular, according to the teacher, it appears difficult for the student to understand the concept of fractions—considering a whole divided equally as 1m, 1ℓ, etc.

However, he mentioned that there was no existence of difficulty with the medium of instruction in learning mathematics.

Table 8. Teacher's responses on "Pupils-evaluation"

Questionnaire Items	Average school
(9) Degree of difficulty for the pupils to learn "Fractions"?	3. Difficult
(10) Points of difficulty for the pupils to learn the concept of "Fractions"?	➢ It seems difficult for the pupils to regard a whole thing divided equally as 1m, 1ℓ and so on. (It is necessary to attach units to fractions in dealing with them at Grade 4 in Japan.) ➢ At the beginning of teaching fractions, there were so many pupils who regarded four sixths of 2m as 4/6m.
(11) Existence of pupils' difficulty with the medium of instruction in learning mathematics	2. No

d) Evaluation of contents

The teacher thought that the topic "fractions" was important for learning mathematics, particularly for learning quantities such as "Area." Moreover, he mentioned that the main aspects of concern with regard to his students in teaching fractions were i) dividing a whole quantity such as 1m or 1ℓ into equal parts and ii) the implication of denominator as the number of the equal divided parts.

Table 9. Teacher's responses on "Contents-evaluation"

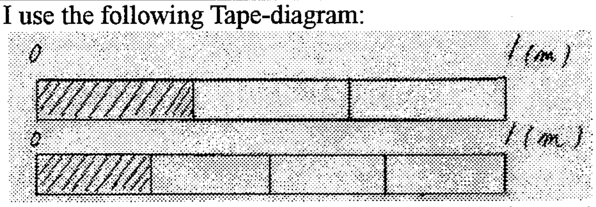
Questionnaire Items	Average school
(12) Importance of learning "Fractions" with comparison to any other topics in mathematics.	1. Yes Reason/s: ➢ It is very useful for pupils to understand quantities. ➢ Especially, it is important for the unit "Areas".
(13) Teacher's main point/s of concern to the pupils in teaching "Fractions"?	➢ Pupils need to know that there is need to divide 1m and 1ℓ equally at first. ➢ Then, pupils can regard the meaning of denominator as the number of the equal divided parts. (For example, it is important to regard 2/5 as 2 of 1/5.)

e) Teaching methodology

The teacher's response on Q14 was as follows: i) Visualization using tape diagram; ii) comparison by students; iii) summary by teacher; and iv) assigning similar exercises.

Regarding Q15, his idea was as follows: i) Questioning how many metres is 2/2m; ii) making students aware of the fact that 2/2m is equal to 1m; iii) asking which is longer—1/2m or 2/2m; iv) confirming that the total quantity is 1m, in this case; and v) respecting the student who provided an answer.

Table 10. Teacher's responses on "Teaching methodology"

Questionnaire Items	Average school
(14)Describe how to teach the following question to the pupils? "Which is longer 1/4m or 1/3m?"	➢ I use the following Tape-diagram:  ➢ Using Tape-diagram, I ask pupils to compare 1/4m or 1/3m by themselves. Then I explain which is longer visually. ➢ Giving similar exercises, I deepen their understanding.
(15)Suppose you posed the following question to the pupils in a lesson. "What is a half of 2m?" Then a student answered, "It is 1/2m." How do you deal with such a student in class?	➢ Through asking "how many metres 2/2m is", I make pupils notice that 2/2m is equal to 1m. ➢ Next, through asking pupils "which is longer, 1/2m or 2/2m", I make pupils notice that 1/2m is shorter, and that their answer was not correct. ➢ Then, I confirm that the whole quantity is 1m in the case. ➢ Finally, I pay respect to the pupil who answered, telling "We have deepened our understanding through her/his cooperation."

2.7.3 Discussion

Unfortunately, the survey in Japan included only 1 teacher, thereby making it difficult to generalize the all the characteristics of Japanese teachers through the survey. However, it is possible to observe certain connections between the teacher's teaching and the students' learning.

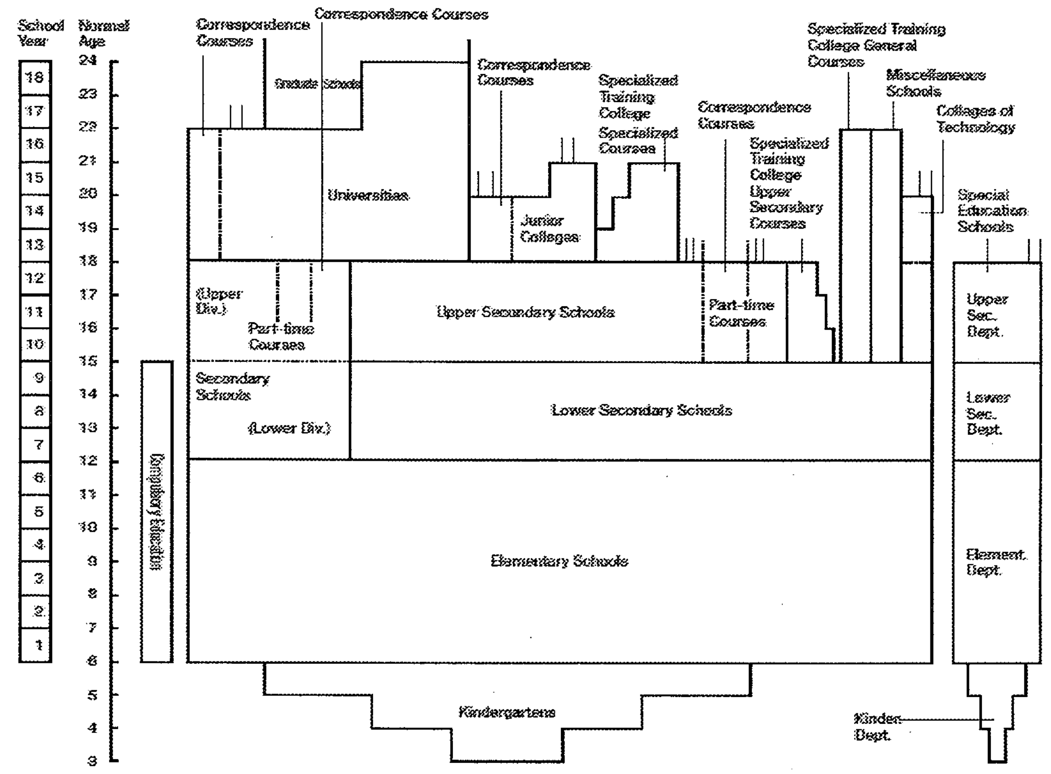
The teacher believed that while "Fractions" is important topic for learning of mathematics, it is difficult for students to understand the topic. Thus, he believed that it is necessary for his students to understand the basic concept of fractions. According to the teacher, he normally emphasized the following two aspects in teaching fractions: i) the concept of fractions based on the operation of dividing a whole quantity (such as 1m or 1ℓ) into equal parts; and ii) the meaning of denominator as the number of equally-divided parts. For this purpose, he attempted to introduce numerous visualizing techniques (using tape diagrams) and more concrete activities (comparing the lengths of real things) in his lessons.

As a result, his students appear to have obtained a deeper understanding with regard to fractions. For instance, they solved not only the questions that asked to represent fractions as tape diagrams, but also some of the questions that were not previously taught, thereby applying what they had already learnt before; for example, the questions comprising fractions with same denominators in the achievement test of the survey (Iwasaki, 2007).

Undoubtedly, there is a need to analyze more deeply and carefully the correlation between the teacher's teaching and the students' understanding. However, if the students did not have any ideas on the concept of fractions or ways of representing fractions as concrete models, it may have been impossible for them to solve such unknown questions.

Therefore, our challenges for the future are to analyze more deeply the correlation between the teaching of the teacher and the understanding of the students, and what type of teaching positively influences learning among students, so that we can seek ways of improving our mathematics education for teachers and students.

Annex 1: Organization of the School System in Japan



Source: Japan Ministry of Education, Culture, Sports, Science and Technology (2005)

Annex 2: Content development of fractions in the Course of Study for Japanese elementary school

Grade 4	1. Objectives To enable children to understand the meaning of decimals and fractions and how to represent them. 2. Contents A. Numbers and Calculations To enable children to understand the meaning of fractions and how to express them. a) To use fractions in order to represent the size of reminders, and the size of the proportion that is formed by division into equal parts. To know how to represent fractions. b) To know that fractions can be represented as a certain number of times unit fractions. [Terms and symbols] Denominator, Numerator, Mixed fraction, Proper fraction, Improper fraction
Grade 5	1. Objectives (1) To enable children to deepen their understanding of the meaning of decimals and fractions, and how to express them. ··· To enable children to understand the meaning of adding and subtracting fractions, to examine these calculations, and to make use of these calculations. 2. Contents A. Numbers and Calculations (4) To enable children to deepen their understanding of fractions. To enable children to understand the meaning of adding and subtracting fractions with the same denominators, and to make use of these calculations in appropriate way. a) In simple cases, to notice fractions of the same size. b) To convert whole numbers and decimals into fractions, and represent fractions in decimals. c) To understand that the result of dividing whole numbers can always be expressed as a

	single number using fractions. d) To examine adding and subtracting fractions with the same denominator, and do these calculations. 3. Points for consideration in dealing with contents (3) With regard to item (4) d) in "A. Numbers and Calculations", addition and subtraction as its opposite operation of two proper fractions should be included.
Grade 6	1. Objectives (1) To enable children to deepen their understanding of addition and subtraction of fractions, and to make appropriate use of these. To enable children to understand the meaning of multiplying and dividing fractions, to examine methods of calculating, and to make appropriate use of them. 2. Contents A. Numbers and Calculations (2) To enable children to deepen further their understanding of fractions, to understand the meaning of adding and subtracting fractions with different denominators, and to make appropriate use of these calculations. a) To understand that a fraction obtained by multiplying or by dividing the numerator and denominator of an existing fraction by the same number, has the same size as the existing fraction. b) To examine the equivalence and size of fractions, and organize the methods how to compare their size. c) To examine the methods of adding and subtracting fractions with different denominators, and do these calculations. (3) To enable children to understand the meaning of multiplying and dividing fractions, and to do these calculations appropriately. a) To understand the meaning of multiplication and division when the multipliers and divisors are whole numbers. b) To understand the meaning of multiplication and division when the multipliers and divisors are fractions, on the basis of an understanding of the calculations involved when the multipliers and divisors are whole numbers and decimals. c) To examine the methods of multiplication and division with fractions and decimals, and do these calculations. [Terms and symbols] Reduction to a common denominator 3. Points for consideration in dealing with contents (2) With regard to item (2) c) in "A. Numbers and Calculations", addition and subtraction of two proper fractions should be included. (3) With regard to item (3) in "A. Numbers and Calculations", calculations of mixed fractions should not be included. (4) With regard to item (3) b) in "A. Numbers and Calculations", simple cases should be handled, such as where the multipliers and divisors are fractions with a numerator of one.

References

Japan Ministry of Education, Culture, Sports, Science and Technology (2005) *Japan's Education at a Glance 2005.*

Institute for International Cooperation Japan International Cooperation Agency (2004) *The History of Japan's Educational Development -What implications can be drawn for developing countries today-*, Japan International Cooperation Agency.

Chapter 3

Discussions and Future Issues

Chapter 3 Discussions and Future Issues

3.1 Discussions of Present Status of Mathematics Education

Although in this research robust numerical comparisons were not always possible because of the limitations of our samples, making comparisons within each country and characterizing differences among the participating countries were possible and valuable. The study of syllabi and textbooks to support the analyses was done in some cases.

In the process of comparison, four categories were created: teacher-related context, teachers and teaching, language, and research method. Since "teaching" is a focus of this report, teachers and teaching are the natural one to address, while "teacher-related context" and "language" are somehow related to the preconditions of teaching, and "research method" focuses on the methodology to approach these aspects of mathematics education.

Here we didn't include the "children and learning" category, but this has been included in the second part of the joint study. In that part, we tried to uncover the status of children's learning in mathematics. These two reports jointly clarified the status of mathematics teaching and learning in each country.

3.1.1 Teacher related Context

The UNESCO Framework of Education Quality (UNESCO, 2004) emphasizes that context interacts with every part of the educational process. For teacher-related context we compiled the following list of five factors that can determine the present status of teachers.

(1) Curriculum: Syllabi, textbooks

All the participating countries have a national curriculum, but some introduced privatization in the production of textbooks, while in others the government produces the textbooks. In the syllabi and textbooks, the topics to be covered are more or less similar. In fact, Nebres (1988) noted the uniformity of mathematics curriculum despite cultural diversity.

As for the target topic of this research, fractions, some countries introduce it at very early stage such as grade 1, and others introduce it at a relatively later stage such as grade 4 when they judge that students have become mature enough. This means that there are two distinct approaches. One of them is to introduce the topic at an early stage and take time for the students to digest the concept. The other one is to wait for students' readiness and to deal with the topic in a concentrated manner.

(2) Working conditions: Teacher qualifications, salaries, employment, transfers

Some countries require a post-secondary diploma or an equivalent qualification while others require a bachelor degree. Teacher's salaries correspond with the economic size of each country and they are especially low for primary school teachers, whom we targeted in this research. In most cases, there is no systematic transfer system and so teachers tend to remain in the same school for many years. No formal scope for career development from classroom teacher to a higher position is available.

These low working conditions affect directly the motivation of not only teachers who are now in service, but also prospective teachers. The reasons to become a teacher can be an indicator of whether the job of teaching is attractive enough. They vary a little from country to country. This research has revealed that they are related to the job and the closeness of the place of living to the school besides economical reasons such as job security and a platform for better jobs.

(3) Teacher education: PRESET, INSET, inspections

Pre-service teacher education (PRESET) prepares student teachers to acquire a minimum level of competency of teaching in schools. Since the teaching job requires a lot of improvisation in the actual teaching, depending upon the students and the social conditions, it is important to improve this level of competency through in-service education (INSET). Cooney and Krainer (1996) conceptualize the in-service as a ground for integrating theory and practice. Besides, in this fast changing society, updating them continuously with educational innovations and new tasks is a must. That is why teachers are now called life-long learners. (UNESCO-UNDP, 1997)

In this society, images surrounding teaching also change. So, teachers' self images about the role of teachers and teaching, which were originally produced through their own educational experience and PRESET, may require renewal throughout the course of their teaching life. And Jaworski (1995) expressed reflection over practice is a key to the professional development. INSET is a major tool to bring about this change.

Besides teacher education, inspection is done by local education officers, head teachers or department heads to ensure the implementation of what is conveyed to teachers through teacher education. While images about teachers and teaching change, those concerning inspection also change. Inspection implies a more top-down and rigid form of monitoring, but now being a supervisor or an advisor is a preferred role for their job, because of its nature to give advice as a senior educator and to accommodate more self motivated development through a bottom-up approach.

(4) School management: Head teachers' initiatives, faculty and department meetings, monitoring

Together with teacher education, school and department management play crucial roles in ensuring the quality of education. How regularly faculty meetings and monitoring are being held, how principles of management are conveyed to faculty members, how teachers are motivated and provided with opportunity of in-service training inside and outside school, are all highly significant.

(5) Social conditions: status, strikes, examinations

Social status and strikes are related to working conditions such as employment and salary, although the direct casual relation between them is difficult to establish. In many countries, the social status of teaching is relatively high despite the low salary. So it attracts some people to work as teachers, but it may not be able to retain them so long, because they regard teaching as a stepping stone to a next stage.

This research has revealed that examinations are a very influential factor in each country both in relation to teaching and to learning. Teachers tend to teach only what will be tested, as this research has revealed, and they judge students' ability in terms of test results.

So further study should be designed to clarify what kinds of preconditions and practices teachers and students have in terms of examination, including extra tuition besides regular class teaching.

3.1.2 Teachers and teaching

Here we focused on the category of mathematics teachers and teaching. The teachers hold certain views, beliefs and values (Bishop, et al., 2003) that influence their teaching. By differentiating between teachers as people and the teaching methods they employ, we may be able to identify how teachers can change their values and teaching methods through in-service training and other support.

(1) Teachers' experience and perception about students

Gaps between teacher confidence and student performance suggest that teaching is quite different from students' learning. This may also explain that some teachers employ the teaching method of providing information to students in a unidirectional manner. If teachers think primarily teaching as to provide knowledge to students, then naturally they will do so.

The majority of teachers have sufficient teaching experience. However, in many cases their predictions for student performance did not match the actual scores. This mismatch cast a question over the assumption that teachers were aware of students' backgrounds and weaknesses.

(2) Content related beliefs

Teachers consider Fractions to be an important topic, for example, because of the close relationship with daily life, but fewer teachers mentioned the importance of Fractions in terms of mathematical concepts. When asked about the main point in teaching Fractions, they listed part-whole relation, four operations involving fractions, the names of "denominator" and "numerator", and sharing. The majority of the teachers expressed a strong degree of confidence in this topic. Some of them, however, judged that it is difficult for students. This suggests that according to their perception, Fractions is a topic that is easy for teachers to teach, but difficult for the students to learn.

(3) Textbook related belief and implicit examination orientation

Textbook related beliefs are prevalent in many countries, partly because the examination results influence students' futures and the styles are quite similar to the textbook styles. Therefore textbooks and examination related beliefs form a peculiar teaching style, and make it difficult for teachers to change their teaching style even after receiving INSET on new modes of teaching, unless the textbooks or the examinations are also changed.

Some teachers responded that they use examination results as an indicator of the success of their lessons. In the sense of self and formative evaluation for the improvement of the quality of lessons, it is helpful to use the examination results, but properly. It is, however, dangerous to judge whether the teaching is properly done, based solely on examination results.

Besides, some cases during the field survey were reported where students possibly cheated on the test questions and even where the teachers assisted the students in solving some questions. Although these were extreme cases, nevertheless they indicate the disproportionate importance of test scores.

(4) Teaching Strategies for Fractions

Regarding the teaching strategies of comparison of Fractions in Q14, most responded that they draw two lines or use objects and cut one into three equal parts and the other into four equal parts. Then they show the differences in length or size to their pupils. Responses to Q15 indicate how the teachers understand their pupils' state of thinking, which may serve as a foundation for improving the quality of teaching.

3.1.3 Language of Instruction

In some countries the language of instruction is the same as the language used in daily life, but in other countries this is not the case. In fact, language issues in mathematics education have been major discussion topics in the four-yearly ICMEs. Various conditions such as vocabulary, accessibility to latest information, availability of learning material, need to be considered. So, the selection of a language as a medium of instruction is a very important and political issue, and we cannot make a simple judgment about which language is better for instruction. It is, however, necessary to take some action to make meaningful learning happen.

Discussions on the medium of instruction cover the complexity of language conditions, difficulty in translation, and language fluency for both teachers and students, because teachers may not be able to speak fluently the language of the students, and vice versa. The language condition is a notable precondition for education in any country. In some countries it is further complicated because there are many local languages. The situation can be quite complex when there are various languages in use, such as the language of an ex-colonial government, a national language and local languages (Ex.: In Accra, Ghana they use at least the three languages of Ga, Asante and English).

(1) Medium of instruction and daily language

Many teachers pointed at fluency problems among their pupils, where the medium of instruction and the daily language are different. A few teachers also identified conceptual gaps between English and local languages. Regarding the difficulty of learning mathematics in a second language, the problems are classified into two such as A-type and B-type problems (Berry, 1985). The former is related to language fluency and the latter is related to conceptual gap.

In order to approach this problem, special measures are needed to shift from the local language to the official language. This situation makes it much harder for children to express their opinion and ideas freely. Besides, the official position regarding the medium of instruction such as using local language up to grade 3 as complementary language sometimes differs from reality, and even the teacher and students speak different languages in some cases.

There is a lot of literature on how difficult it is for students to attain a certain academic level and for teachers to manage the classrooms when the language used for teaching is different from the pupils' mother tongue (e.g. Fafunwa et al. 1989), in addition to the above-mentioned A-type and B-type problems.

(2) Cultural appropriateness

Multi-linguistic and multi-cultural situations in some participating countries demand which language/s are to be used. In some cases there is no word in the language that can express the concept, or the word, which can be translated, may have a different meaning. For example, some concepts such as "minus degree" during the first-year survey were not culturally appropriate if, for example, the students have not experienced minus degrees in their daily lives. The second-year survey touched on the concepts of "sharing", "fractions", "half", "more than and less than" and "one-third or one-over-three" which seemed to cause difficulties.

This is a main reason why we insist on endogenous development. The researchers of the participating countries gave us opinions on the relevance of the terms, and we had to consider many issues.

(3) Social implication of language

School education puts a stress on written language, but some local languages only have a spoken form. This situation implies that providing school education automatically means a bias towards the official language. In certain situations, this creates less self-esteem to the minority students about the value of their language and also their culture.

Besides this, the choice of which language to use in the classroom leads to economic implications. This is because the acquisition of an international language gives students greater access to international information and greater opportunities for employment abroad. In some countries, earning foreign currency cannot be ignored.

3.1.4 Research method

Fractions was decided as a focus of the main survey, and the length of the test was adjusted to a manageable level after reflecting on the results of the first-year survey. In fact, some countries finished the test earlier than the stipulated time. The following points such as selection of samples, linguistic influences and new types of problems were taken up as issues.

(1) Selecting samples and combining research methods

The members agreed that this research would deal with the selected school using certain criterion as an "average" school in terms of the local context, and then make a more in-depth analysis using more qualitative descriptions. Therefore, the first task was to select an "average" school in order to describe an "average" picture of mathematics education in each country. This was by no means an easy task because the criteria of determining an "average" school within each country were not uniform and some countries have clear guidelines and others don't. All the participating countries have unique socio-economic conditions and backgrounds. For example, an average school in Thailand has the minimum level of teaching materials and school infrastructure, but an average school in Zambia has fewer resources by comparison, although we have no intention to compare hardware.

The members recognized the importance of qualitative analysis after the first year surveys were made because each report included rich and interesting data suitable for further comparative studies between countries and within individual countries. In order to view the subjects from different angles, we attempted to combine different research methods. There are a variety of combinations such as pre / post -lesson interviews, second / third-party observations and ideal / actual lessons. By this combination, we tried to excavate the actual picture of mathematics education in each country.

(2) Linguistic influences on achievement

Various methods were employed in order to explore linguistic influences on achievement. These included the provision of additional explanations, the Newman method and the identification of difficult words. Since the medium of instruction is different in each country, translation was necessary and a back translation method was employed.

In order to understand the influence of language (fluency or conceptual gap) on performance we employed, when applicable, the Newman procedure during the second-year survey. While separating A type and B type problems, we realized that many students failed even at the reading level in Zambia, and some Bangladesh students proceed to further levels even if they were not able to explain the problems.

During the second-year survey, some countries added explanations in the local languages on an experimental basis due to the poor understanding of the intention of the questions. Even this did not have enough impact on students' understanding level as expected.

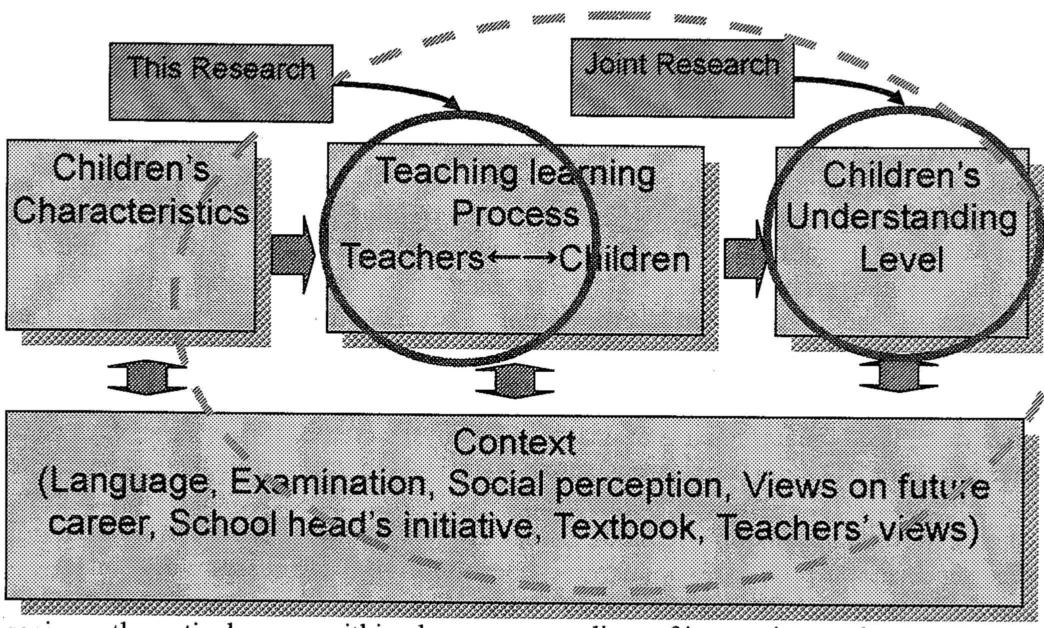
(3) New types of questions (problem posing)

Many countries now explore more student-centered lessons and the promotion of logical thinking skills at least in their policy, in order to improve the quality of education. So it is our intention to measure students' problem-posing ability in line with this direction. We realized that students' ability in this sense is very limited.

3.2 Future Issues

3.2.1 Revisiting endogenous development

Endogenous development of mathematics education is to start with its own culture and to modify it according to the necessity of the society and its era. Here it is important for local experts not to copy a model directly from some countries but to modify a model or models, based on their reality and necessity. Culture in this sense is dynamic, and it is subject to change with time. So, how can we contribute to change initiated by local experts? Culture here means things such as values attached to education, mathematical activities (ethnomathematics),



socio-mathematical norms within classrooms, medium of instruction, and etc.

We, local experts, are key persons in this community of mathematics education. We each relearn other what seems to be trivial by making comparison with other countries. For this purpose, it is the first step to form the research community among mathematics education researchers in the participating countries. Although oversimplification must be cautioned, reflection through comparison is an effective tool to start with.

In this research, we were able to identify some important points that need to be clarified further. In particular, "language problems" and "teacher-related context" have become keywords for pursuing a spirit of endogenous development. When we consider these keywords, it becomes clear that greater consideration of the research framework is needed. Therefore, for the short-term we compile in this report a list of the issues that we faced during the three years of this study. This, in the long term, forms the starting point for our future research and leads to a comprehensive framework for mathematics education research in developing countries.

3.2.2 Future research

For the endeavours of this research group, we have to have a target to improve teaching in each country although we should not be too short-sighted. For this purpose, we have to do analyses of students' common mistakes, and students' and teachers' home, educational and cultural backgrounds.

(1) Assumption

Endogenous development will continue to be a very basic assumption for future research. And this endogenous quality should be considered in curriculum, teachers, and students. The endogenous development of curriculum is based upon the society's needs and may involve considering the possibility of applying ethnomathematics into teaching and learning. The endogenous development of teaching may require the reflection of trivial values and socio-mathematical norms within the school culture in each country, and the creation of new educational practices. The endogenous development of learning requires the consideration of what students are to acquire by the end of their school education. All these endogenous qualities are related to each other.

(2) Which points to focus on?

The majority of teachers have sufficient teaching experience and expertise. However, some of their predictions for student performance did not match the actual scores. This mismatch cast a question over the assumption that teachers were aware of their students' backgrounds and weaknesses.

The context and language of instruction are points to focus on for future researches. Context was considered extensively in this research, and the identified ones are curriculum (Syllabi, textbooks), Working conditions (Teacher qualifications, salaries, employment, transfers), Teacher education (PRESET, INSET, inspections), School management (Head teachers' initiatives, faculty and department meetings, monitoring), Social conditions (status, strikes, examinations).

The language of instruction is one of the major issues to tackle with. Combining some research methods are necessary to clarify the important issues.

(3) Future research methods

Action research as a research method was proposed during the project. This is supported by the fact that this research involves many factors and their relation is always complicated. So having "thick" descriptions about present status and reorganizing conjectures is necessary. In any case, flexibility is required to adjust to the reality of each participating country.

References

Berry, J.W. (1985) 'Learning Mathematics in a Second Language : some cross—cultural issues', *For the Learning of Mathematics*, 5(2), 18-23.

Bishop, A.J., Seah, W.T., Chin, C. (2003), Values in Mathematics Teaching- The Hidden Persuaders?, in Bishop et al. eds, Second International handbook of Mathematics Education, Kluwer Academics, pp. 717-765.

Cooney, T. & Krainer, K. (1996) Inservice Mathematics Teacher Education: The Importance of Listening, in Bishop, A.J. eds. *International Handbook of Mathematics education*, Kluwer, Academics pp.1155-1185.

Fafunwa et al. (1989) *Education in Mother Tongue: The Ife Primary Education Research Project (1970-1978).*

Jaworski, B. (1994) *Investigating Mathematics Teaching: A Constructivist Enquiry*, The Falmer Press.

Nebres, B.F. (1988) 'School Mathematics in the 1990s': Recent Trends and the Challenge to the Developing Countries', *Proceeding of the Sixth International Congress on Mathematical Education*, 13-27.

UNESCO-UNDP (1997) *Teacher as Lifelong Learners: Case Studies of Innovative In-service Teacher training Programmes in the E-9 Countries*, Paris.

UNESCO (2004) EFA Global Monitoring Report 2005, Paris.

Part II

Chapter 4

Special Contribution

Chapter 4 Special Contribution

4.1 Numeracy and Mathematics – A Cultural Perspective on the Relationship

Alan J. Bishop
Monash University

4.1.1 Introduction - numeracy in a global cultural context

The concept of global citizenship has gained currency in recent years. The mass movement of people around the globe and the creation of regional entities such as the European Union and ASEAN have generated significant interest in the concept of global citizenship, as law-makers and curriculum policy advisors question the legitimacy of traditional notions of citizenship. These developments have tremendous implications for those of us who work in education, particularly in mathematics education.

In Australia for example, one must always be aware when considering educational issues, that it is predominantly a migrant country. A Report by the Public Affairs Section of the Department of Immigration and Multicultural Affairs, Canberra in 1996 states:

"Today nearly one in four of Australia's 18.5 million people was born overseas. In 1995–96 the number of settlers totalled 99 139. They came from more than 150 countries." In the particular cases of numeracy and lifelong education, I believe this recognition is essential for any educational initiatives to succeed. Even in Japan, which is rarely considered to be a multi-cultural society, the construct of 'culture' has strong meanings, and therefore important implications for numeracy education.

Whenever I am discussing educational issues I am always aware of three main constructs: curriculum, teaching, and learning. These are all of course embedded in social, cultural and political contexts which structure, support, and constrain them. When considering numeracy education, other people have tended to focus mainly on the learning aspects. So in this paper I wish to focus on the other two main areas which I feel are often neglected, and in particular on these ideas:

1. culturally, and socially - based numeracies, and
2. their relation to the mathematics curriculum.

There are clearly issues and challenges about defining and clarifying the nature and role of numeracy in today's society, and in particular its relationship with mathematics (e.g. FitzSimons et al., 2003) Is numeracy a part of mathematics? Is it a simple form of mathematics? Is it the same as mathematics? Is it radically different from mathematics? I will offer my own interpretation of the relationship, one which suggests some different ideas about a numeracy curriculum than those normally given, and one that recognises firstly that there are many numeracies in our societies, and secondly that numeracies are socially and culturally based.

4.1.2 The challenge of culturally-based mathematical knowledge.

In the last 20 years, two main strands of research have developed in mathematics education. The first was research on **ethnomathematics**, which has fundamentally changed many of our ideas and constructs in considering mathematics education (D'Ambrosio, 1985; Gerdes, 1995). The most significant influences have been in relation to:

Cultural roots. Ethnomathematics research has made us aware of the cultural starting points, and different histories, of mathematics. We now realise that Western Mathematics is but one form among many, and that Wasan was a special form of mathematics in ancient Japan.

Interactions between mathematics and languages. Ethnomathematics research has shown us that languages, and other symbol systems act as the principal carriers of mathematical ideas and values in different cultures and societies.

Human interactions. Ethnomathematics research focuses on mathematical activities and practices in society, and it thereby draws attention to the roles which people other than teachers and learners play in mathematics education.

Values and beliefs. Ethnomathematics research has made us realise that any mathematical activity involves values, beliefs and personal choices.

However the curriculum structures for mathematics teaching that we generally see in schools are typically culture and value free, and have evolved to suit the preparation of an elite minority of students who will study mathematics at university. When considering general education, and especially for those students who will never study mathematics at university, this elitist curriculum is highly inappropriate. It has contributed significantly to the widespread problems of alienation felt by many students, of all ages, towards mathematics.

But the problem is not just one of alienation, it is much more to do with the fact that a majority of the population has been debarred from any discussion or critique of the underlying mathematical constructs used in today's highly mathematically formatted and structured society (Skovsmose, 1994). Research therefore still needs to explore ways of making mathematics curricula more culturally and socially responsive, in order to encourage more societal participation at the higher levels particularly amongst cultural and social minority groups. I believe that a numeracy-based curriculum is one way to emphasise the societal and cultural aspects of mathematics.

The second trend in research which has developed in the past two decades, and which relates to this discussion, has demonstrated the significance for teaching, learning, and the curriculum, of the conceptual idea of **context**, that is the context within which the mathematical practice is situated. Indeed the construct of **"situated mathematical practice"** is now attracting a great deal of attention (see for example, Kirshner & Whitson, 1997; Zevenbergen, 2000). Much of this research work is based on the challenge coming from studies of mathematical activities occurring outside school (e.g. Abreu et al., 2002; Lave & Wenger, 1991) where the characteristics of the practice are markedly different from those in school.

These practices, many of which have been the subject of ethnomathematics research, are to my mind the roots of numeracy education. This idea demonstrates powerfully that the context of school is markedly different from the contexts outside school where numeracy practices occur. If we wish to develop numeracy curricula, how can we ensure that the school context doesn't ignore, or indeed over-ride, the outside-school numeracy contexts?

Bearing these two research trends in mind, the issue for researchers becomes: how can we develop a **numeracy-based mathematics curriculum** for schools, one which satisfies both the needs of a modern democratic society for having fully educated and politically contributing adults, and the needs of the mathematical research establishment which still wants competent, and in some cases brilliant, mathematicians?

4.1.3 Numeracies and mathematical theory

Numeracy is being defined in many different ways, particularly in its relation to mathematics, and particularly in the context of adult education (see for example, Bessot and Ridgway, 2000, and FitzSimons, 2002). There are those who define it as a part of mathematics, often as simple or basic arithmetic. Others see numeracy as far more than just arithmetic. Still others prefer to link numeracy to literacy rather than to mathematics (as UNESCO does), or to consider numeracy as mathematical literacy (Jablonka, 2003). A recent book from the USA focuses on Quantitative Literacy (Steen, 2001) although in the text the word 'numeracy' is often used in its place.

However I like to approach the issue differently. Firstly I believe we need a conceptualisation of numeracy which meets three criteria:

- it recognises the existence of many numeracies (Buckingham 1997, Tout, 2001)
- it clarifies the links between numeracies and mathematics, and
- it clarifies the educational task facing those responsible for numeracy education.

The first aspect of the relationship is to recognise the way that ethnomathematical ideas can be structured to enable them to be seen in relation to numeracy and to mathematics. In my original research (Bishop, 1988) I conceptualised a structure based on six fundamental and universal activities, which are found in all societies. These activities are the bases of the numeracies in the societies in which they appear. However they are also the bases of Western Mathematics, and this is the heart of the relationship we are seeking. Here are the six activities:

◆ **Counting**: This is the activity concerned with the question "How many?" in all its forms and variants eg. there are many ways of counting and of doing numerical calculations. The mathematical ideas derived from this activity are numbers, calculation methods, number systems, number patterns, numerical methods, statistics, etc.

◆ **Locating**: This activity concerns finding your way in the structured spatial world of today, with navigating, orienting oneself and other objects, and with describing where things are in relation to one another. We use various forms of description including maps, figures, charts, diagrams and coordinate systems. This area of activity is the 'geographical' aspect of mathematics. The mathematical topics derived from this activity are: dimensions, Cartesian and polar coordinates, axes, networks, loci, etc.

◆ **Measuring**: "How much?' is a question asked and answered in every society, whether the amounts which are valued are cloth, food, land, money or time. The techniques of measuring, with all the different units involved, become more complex as the society increases in complexity. Mathematical topics derived here are: order, size, units, measure systems, conversion of units, accuracy, continuous quantities, etc.

◆ **Designing**: Shapes are very important in the study of geometry and they appear to derive from designing objects to serve different purposes. Here we are particularly interested in how different shapes are constructed, in analysing their various properties, and in investigating the ways they relate to each other. The mathematical topics derived are: shapes, regularity, congruence, similarity, drawing constructions, geometrical properties, etc.

◆ **Playing**: Everyone enjoys playing and most people take playing very seriously. Not all play is important from a mathematical viewpoint, but puzzles, logical paradoxes, some games, and gambling all involve the mathematical nature of many activities in this category. Mathematical ideas derived here are: rules, procedures, plans, strategies, models, game theory, etc.

♦ **Explaining**: Trying to explain to oneself and to others why things happen the way they do is a universal human activity. In mathematics we are interested in, for example, why number calculations work, and in which situations, why certain geometric shapes do or do not fit together, why one algebraic result leads to another, and with different ways of symbolising these relationships. Mathematical topics derived here are: logic rules, proof, graphs, equations etc.

The next aspect of the relationship appears when we realise that the cultural history of Western Mathematics, which I shall continue to refer to by Mathematics with a capital 'M', (see Bishop, 1988) shows us that its essence is its **generality**. On the other hand the ethnomathematics literature indicates that numeracies are all about **particular practices**. To use the word 'practices' here is not just to refer to skills or algorithms, nor to minimise numeracy as just number work, but as with ethnomathematics, and the six activities above, numeracies include meanings, and conceptualisations also. No power relationship between Mathematics and numeracy is intended, as both are powerful forms of knowledge in their own contexts. It is just that the 'situated context' of Mathematics is the abstract world of the theorist, whereas the 'situated context' of numeracy is the pragmatic world of the ethnomathematical practitioner.

The relationship between them is therefore best understood to my mind as one between theory and practices, with **numeracies being the practices and Mathematics being the theory behind the practices**. Mathematics explains, theorises, and clarifies the rationales underlying those practices, and also gets applied through various developments of those practices. This conceptualisation of the relationship between the two forms of knowledge helps to construct a meaning for numeracy, which meets the three criteria above.

As we can see with the activity of 'explaining' above, the question "Why does any particular numeracy practice work?" is an important key to understanding the relationship, and in my view is the key to clarifying the educational task. Learners of all ages can bring many numeracy, and ethnomathematical, practices to their education, mainly from their families but also from their wider society contacts – friends, media, other adults etc. However the naive learner generally lacks any understanding of the underlying Mathematical theories that help to explain those practices. Without the Mathematical theories they lack the tools to understand, analyse and critique those practices. Without these tools they become trapped by those practices, as most adults currently are, without the understanding to question and develop alternatives.

There are many 'why' questions that can be asked of school students even within the current mathematics curriculum, such as these simple ones:

- Why do the many different practices for counting, addition, subtraction etc. all succeed? It is because of the underlying mathematical structures of those algorithms.

- Why do two negative numbers when multiplied together give a positive number? It is because of the wish to keep the rules of number theory consistent when dealing with positive and negative integers.

- Why can you divide one fraction by another by 'turning it upside down and multiplying'? Because multiplying the numerator and denominator of a fraction by the same number doesn't change the value of that fraction.

- Why does a three-legged stool always balance, while a four-legged one doesn't? Because 3 points determine a plane.

Of course it may seem to many that asking these 'why' questions is inappropriate when talking about the Mathematics curriculum, especially since the demise of proof from the school curriculum in many countries. Proofs are the ultimate mathematical explanation. But it is not necessary to learn proofs of theorems just for regurgitation at examination time. It is far more important to understand the role of the process of 'proving' in the development and education of the numerate and Mathematically literate student. 'Why' questions can be asked at any level of Mathematics and numeracy practice, and within Mathematics itself the answers to those questions will often be found by proving.

Thus my conceptualisation of the relationship between numeracies and Mathematics is that those numeracies are culturally, socially, and historically determined, and practically powerful. Mathematics on the other hand has developed culturally in a different way, and in a different context, as an extremely powerful way of theorising, explaining, and extending our knowledge of the world.

The implications for school mathematics curricula are many, but here are some of the main ones:

- These curricula need to recognise, and to be built around, the many numeracy practices which the learners have learnt at home and in the wider society, and which they bring to the education situation. Thus not all the curricular content can be determined centrally. Much must be determined locally within a national or regional framework. The six activities model can provide such a framework.

- These curricula need also to introduce learners to the many numeracy practices which powerful others in society practise and which can impinge on their lives in crucial ways. There are of course many of these in adult life, but the criterion should be whether, and how, they impinge on the lives of the learners.

- These numeracy practices need to be clarified, theorised, and critiqued in terms of the Mathematical theories behind them. This is to reverse the usual approach of teaching the Mathematical theory followed by its applications. The particular curricular challenge which this implication presents, is how to create sensible sequential designs which can help to structure that approach. This is a challenge of a high order.

- Generally, mathematical ideas should always be related to the various contexts and numeracy practices where they are used. The complaint is often voiced that mathematics is a meaningless and abstract subject, which reflects the need for this implication. If the relationship between the two were better understood by teachers, and then by the learners, perhaps we would not find so much alienation in our schools.

References

Abreu, G. de, Bishop, A.J. & Presmeg, N.C. (2002) *Transitions between contexts for mathematics learning*. Dordrecht, Holland: Kluwer.

D'Ambrosio, U. (1985) Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of mathematics*, 5(1), 44-48

Bessot, A. & Ridgway, J. (Eds.) (2000) *Education for mathematics in the workplace*. Dordrecht, Holland: Kluwer

Bishop, A.J. (1988) *Mathematical enculturation: a cultural perspective on mathematics education*. Dordrecht, Holland: Kluwer

Buckingham, E.A. (1997) Specific and generic numeracies of the workplace: How is numeracy learnt and used by workers in production industries, and what learning/working environments promote this? Burwood, Victoria, Australia:Centre for Studies in Mathematics, Science, and Environmental Education, Deakin University

FitzSimons, G. (2002) What counts as mathematics: Technologies of power in adult and vocational education. Dordrecht, Holland: Kluwer

FitzSimons, G., Coben, D. & O'Donoghue, J. (Eds.) (2003) Lifelong mathematics learning. In A.J.Bishop, M.A.Clements, C.Keitel, J.Kilpatrick, & F.K.S.Leung, (Eds) *Second international handbook of mathematics education (pp.103-142)* Dordrecht, Holland: Kluwer

Gerdes, P. (1995) *Ethnomathematics and education in Africa*, Sweden: Stockholm University.

Jablonka, E. (2003) Mathematical literacy. In A.J.Bishop, M.A.Clements, C.Keitel, J.Kilpatrick, & F.K.S.Leung, (Eds) *Second international handbook of mathematics education (pp.75-102)* Dordrecht, Holland: Kluwer

Kirshner, D. & Whitson, J.A.(1997) *Situated cognition: social, semiotic, and psychological perspectives.* New Jersey: Lawrence Erlbaum Associates

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge: Cambridge University Press.

Skovsmose, O. (1994) Towards a philosophy of critical mathematics education. Dordrecht, Holland: Kluwer

Steen, L.A. (ed.) (2001) Mathematics and democracy: the case for quantitative literacy. Washington: National Council on Education and the Disciplines.

Tout, D. (2001) What is numeracy? What is mathematics? In G.E.FitzSimons, J.O'Donaghue & D.Coben (Eds.) *Adult and lifelong education in mathematics.* Melbourne, Australia: Language Australia

Zevenbergen, R. (2000) Ethnography and the situatedness of workplace numeracy. In A. Bessot & J. Ridgway (Eds.) *Education for mathematics in the workplace.(pp. 209-224)* Dordrecht, Holland: Kluwer

4.2 The World Role of Culture in Mathematics Education[3]

Norma Presmeg
Illinois State University

Mathematics education has experienced a major revolution in perceptions (cf. Kuhn, 1970) comparable to the Copernican revolution that no longer placed the earth at the center of the universe. This change has implicated beliefs about *the role of culture* in the historical development of mathematics (Eves, 1990), in the practices of mathematicians (Civil, 2002; Sfard, 1997), in its political aspects (Powell & Frankenstein, 1997), and hence necessarily in its teaching and learning (Bishop, 1988a; Bishop & Abreu, 1991; Bishop & Pompeu, 1991; Nickson, 1992). The change has also influenced methodologies that are used in mathematics education research (Pinxten, 1994). Researchers now increasingly concede that mathematics, long considered value- and culture-free, is indeed a cultural product, and hence that the role of culture—with all its complexities and contestations—is an important aspect of mathematics education.

Topics that are central in addressing this role of culture are those arising from and extending the notions of ethnomathematics and everyday cognition (Nunes, 1992, 1993). Various broad theoretical fields are relevant. Some of the theoretical notions that are apposite are rooted in—but not confined to—situated cognition (Kirshner & Whitson, 1997; Lave & Wenger, 1991; Watson, 1998), cultural models (Holland & Quinn, 1987), notions of cultural capital (Bourdieu, 1995), didactical phenomenology (Freudenthal, 1973, 1983), and Peircean semiotics (Peirce, 1992, 1998). From this partial list, the breadth of this developing field may be recognized. This paper does not attempt to treat the general theories in detail: The interested reader is referred to the original authors. Instead, some key notions that have explanatory power or usefulness are the central focus, which is *the role of culture in learning and teaching mathematics all over the world*. The seminal work of Ascher (1991, 2002), Bishop (1988a, 1988b), D'Ambrosio (1985, 1990) and Gerdes (1986, 1988a, 1988b, 1998) on ethnomathematics is still centrally relevant and thus is treated in some detail.

In the last decade, the field of research into the role of culture in mathematics education has evolved from "ethnomathematics and everyday cognition" (Nunes, 1992), although both ethnomathematics and everyday cognition are still important topics of investigation. The developments have rather consisted in a broadening of the field, clarification and evolution of definitions, recognition of the complexity of the constructs and issues, and inclusion of social, critical, and political dimensions as well as those from cultural psychology involving valorization, identity, and agency (Abreu, Bishop, & Presmeg, 2002). This paper in its scope cannot do full justice to political and critical views of mathematics education (see Mellin-Olsen, 1987; Skovsmose, 1994; Vithal, 2003, for a full treatment), but some of the *landscapes* from these fields—as Vithal calls them—are used to deepen and problematize aspects of the treatment of culture in mathematics education.

This paper has four sections. The first addresses an introduction to issues and definitions of key notions involving culture in mathematics education. The organization of the second and third sections uses the research framework of Brown and Dowling (1998). In this framework, resonating theoretical and empirical fields surround and enclose the central research topic, and the description involves layers of increasing specificity as it zooms in on details of the

[3] This paper is an abridged version of Presmeg's Chapter 11, "The role of culture in teaching and learning mathematics", in F. K Lester (Ed.), *Second handbook of research on mathematics teaching and learning*, pp. 435-458. Charlotte, North Carolina: Information Age Publishing, 2007.

problematic and problems of the research issues, the empirical settings, and the results of the studies, only to zoom out again at the end in order to survey the issues in a broader field, informed by the results of the research studies examined, in order to see where further research on culture might be headed. Thus section 2 addresses theoretical fields that incorporate culture specifically in mathematics education; section 3 addresses salient empirical fields, settings, and some details of results of research on culture in mathematics education, and their implications for the teaching and learning of mathematics. Using a broader perspective, the fourth (final) section collects and elaborates suggestions for possible directions for future research on culture in mathematics education.

4.2.1 Definitions and Significance of Culture in Mathematics Education

Why is it important to address definitions carefully in considering the role of culture in teaching and learning mathematics? As in other areas of reported research in mathematics education, different authors use terminology in different ways. Particularly notions such as *culture, ethnomathematics,* and *everyday mathematics* have been controversial; they have been contested and given varied and sometimes competing interpretations in the literature. Thus these constructs must be problematized, not only for the sake of clarification, but more importantly for the roles they have played, and for their further potential to be major focal points in mathematics education research and practices. Further, especially in attempts to bridge the gap between the formal mathematics taught in classrooms and that used out-of-school in various cultural practices, *what counts as mathematics* assumes central importance (Civil, 1995, 2002; Presmeg, 1998a). Thus ontological aspects of the nature of mathematics itself must also be addressed.

(1) Culture

In 1988, when Bishop published his book, *Mathematical Enculturation: A Cultural Perspective on Mathematics Education* (1988a) and an article that summarized these ideas in *Educational Studies in Mathematics* (1988b, now reprinted as a classic in Carpenter, Dossey, & Koehler, 2004), the prevailing view of mathematics was that it was the one subject in the school curriculum that was value- and culture-free, notwithstanding a few research studies that suggested the contrary (e.g., Gay & Cole, 1967; Zaslavsky, 1973). Along with those of a few other authors (notably, D'Ambrosio), Bishop's ideas have been seminal in the recognition that culture plays a pivotal role in the teaching and learning of mathematics, and his insights are introduced repeatedly in this paper.

Whole books have been written about definitions of *culture* (Kroeber & Kluckhohn, 1952). Grappling with the ubiquity yet elusiveness of culture, Lerman (1994) confronted the need for a definition but could not find one that was entirely satisfactory. Yet, as he pointed out, culture is "ordinary. It is something that we all possess and that possesses us" (p. 1). Bishop (1988a) favored the following definition of culture in analyzing its role in mathematics education:

> Culture consists of a complex of shared understandings which serves as a medium through which individual human minds interact in communication with one another. (p. 5, citing Stenhouse, 1967, p. 16)

This definition highlights the communicative function of culture that is particularly relevant in teaching and learning. However, it does not focus on the continuous renewal of culture, the dynamic aspect that results in cultural change over time. This dynamic aspect caused Taylor (1996) to choose the "potentially transformative view" (p. 151) of cultural anthropologist Clifford Geertz (1973), for whom culture consists of "webs of significance" (p. 5) that we ourselves have spun.

This potentially transformative view assumes particular importance in the light of the necessity for negotiating social norms and sociomathematical norms in mathematics classrooms

(Cobb & Yackel, 1996). The culture of the mathematics classroom, which was brought to our attention as being significant (Nickson, 1994), is not monolithic or static but continuously evolving, and different in different classrooms as these norms become negotiated. The mathematics classroom itself is one arena in which culture is contested, negotiated, and manifested (Vithal, 2003), but there are various levels of scale. Bishop's (1988a) view of mathematics education as a social process resonates with a transformative, dynamic notion of culture. He suggested that five significant levels of scale are involved in the social aspects of mathematics education. These are the cultural, the societal, the institutional, the pedagogical, and the individual aspects (1988a, p. 14). Culture here is viewed as an all-encompassing umbrella construct that enters into all the activities of humans in their communicative and social enterprises. In addition to a view of culture in this macroscopic aspect, as in these levels of scale, culture as webs of significance may be central also in the societal, institutional, and pedagogical aspects of mathematics education considered as a social process. Thus researchers may speak of the culture of a society, of a school, or of an actual classroom. Culture in all of these levels of scale impinges on the mathematical learning of individual students.

Notwithstanding these general definitions of culture, the word with its various characterizations does not have meaning in itself. Vithal and Skovsmose (1997) illustrated this point starkly by pointing out that interpretations of *culture* (and by implication also anthropology) were used in South Africa to justify the practices of apartheid. They extended the negative connotations still attached to this word in that context to suggest that ethnomathematics is also suspect (as suggested in the title of their article: "The End of Innocence: A Critique of 'Ethnomathematics'"). Some aspects of their critique are taken up in later sections.

How are *culture* and *society* interrelated? The words are not interchangeable (Lerman, 1994), although connections exist between these constructs:

> One would perhaps think of gender stereotypes as cultural, but of 'gender' as socially constructed. One would talk of the culture of the community of mathematicians, treating it as monolithic for a moment, but one would also talk, for example, of the social outcomes of being a member of that group. (p. 2)

Alan Bishop stated succinctly, on several occasions (personal communication, e.g., July, 1985), that *society involves various groups of people, and culture is the glue that binds them together.* This informal characterization resonates with Stenhouse's definition of *culture* as a complex of shared understandings, and also with that of Geertz, as webs of significance that we ourselves have spun. Considering the cultures of mathematics classrooms, Nickson (1994) wrote of the "invisible and apparently shared meanings that teachers and pupils bring to the mathematics classroom and that govern their interaction in it" (p. 8). Values, beliefs, and meanings are implicated in these "shared invisibles" (p. 18) in the classroom. Nickson saw socialization as a universal process, and culture as the *content* of the socialization process, which differs from one society to another, and indeed, from one classroom to another.

Part of culture as webs of significance, taken at various levels, are the prevailing notions of what counts as mathematics.

(2) Mathematics

As Nickson (1994) pointed out, "one of the major shifts in thinking in relation to the teaching and learning of mathematics in recent years has been with respect to the adoption of differing views of the nature of mathematics as a discipline" (p. 10). Nickson characterized this cultural shift as moving from a *formalist* tradition in which mathematics is absolute—consisting of "immutable truths and unquestionable certainty" (p. 11)—without a human face, to one of *growth and change*, under persuasive influences such as Lakatos's (1976) argument that "objective knowledge" is subject to proofs and refutations and thus that mathematical knowledge has a strong social component. That this shift is complex and that both views of mathematics are held simultaneously by many mathematicians was argued by Davis and Hersh (1981). The formalist and socially mediated views of mathematics resonate with the two categories of absolutist and fallibilist conceptions discussed by Ernest (1991). Contributing the notion of mathematics as problem solving, the Platonist, the problem-solving, and the fallibilist conceptions are categories reminiscent of the teachers' conceptions of mathematics that Alba Thompson, already in 1984, gave evidence were related to instructional practices in mathematics classrooms.

Both Civil (1990, 1995, 2002; Civil & Andrade, 2002) in her "Funds of Knowledge" project and in her later research with colleagues into ways of linking home and school mathematical practices, and Presmeg (1998a, 1998b, 2002b) in her use of ethnomathematics in teacher education and research into semiotic chaining as a means of building bridges between cultural practices and the teaching and learning of mathematics in school, described the necessity of broadening conceptions of the nature of mathematics in these endeavors. Without such broadened definitions, high-school and university students alike are naturally inclined to characterize mathematics according to what they have experienced in learning *institutional* mathematics—more often than not as "a bunch of numbers" (Presmeg, 2002b). On the one hand, such limited views of the nature of mathematics inhibit the recognition of mathematical ideas in out-of-school practices. On the other hand, if definitions of what counts as mathematics are too broad, then the "everything is mathematics" notion may trivialize mathematics itself, rendering the definition useless. In examining the mathematical practices of a group of carpenters in Cape Town, South Africa, Millroy (1992) expressed this tension well, as follows:

> [It] became clear to me that in order to proceed with the exploration of the mathematics of an unfamiliar culture, I would have to navigate a passage between two dangerous areas. The foundering point on the left represents the overwhelming notion that 'everything is mathematics' (like being swept away by a tidal wave!) while the foundering point on the right represents the constricting notion that 'formal academic mathematics is the only valid representation of people's mathematical ideas' (like being stranded on a desert island!). Part of the way in which to ensure a safe passage seemed to be to openly acknowledge that when I examined the mathematizing engaged in by the carpenters there would be examples of mathematical ideas and practices that I would recognize and that I would be able to describe in terms of the vocabulary of conventional Western mathematics. However, it was likely that there would also be mathematics that I could not recognize and for which I would have no familiar descriptive words. (pp. 11–13)

Some definitions of mathematics that achieve a balance between these two extremes are as follows. *Mathematics* is "the language and science of patterns" (Steen, 1990, p. iii). Steen's definition has been taken up widely in reform literature in the USA (National Council of Teachers of Mathematics [NCTM], 1989, 2000). Opening the gate to recognizing a human origin of mathematics, Saunders MacLane called mathematics "the study of formal abstract structures arising from human experience" (as cited in Lakoff, 1987, p. 361). According to Ada Lovelace, mathematics is the systematization of relationships (as described by Noss, 1997). All of these definitions strike some sort of balance between the human face of mathematics and its formal aspects. Going beyond Steen's well-known pattern definition in the direction of stressing abstraction, in a critique of ethnomathematics, Thomas (1996) defined mathematics as "the science of detachable relational insights" (p. 17). He suggested a useful distinction between *real mathematics* (as characterized in his definition), and *proto-mathematics* (the category in which he placed ethnomathematics). In the next section, Barton's (1996) characterization of ethnomathematics, which resolves many of these issues and clarifies this dualism, is presented along with some evolving definitions of ethnomathematics. (For details, the reader should consult Barton's original article.)

(3) Ethnomathematics

What is *ethnomathematics*? In his illuminating article, Barton (1996) wrote as follows:

> In the last decade, there has been a growing literature dealing with the relationship between culture and mathematics, and describing examples of mathematics in cultural contexts. What is not so well-recognised is the level at which contradictions exist within this literature: contradictions about the meaning of the term 'ethnomathematics' in particular, and also about its relationship to mathematics as an international discipline. (p. 201)

Barton pointed out that difficulties in defining ethnomathematics relate to three categories: epistemological confusion, "problems with the meanings of words used to explain ideas about culture and mathematics" (p. 201); philosophical confusion, the extent to which mathematics is regarded as universal; and confusion about the nature of mathematics. The nature of mathematics is part of its ontology, and because both ontology and epistemology are branches of philosophy, all of these categories may be regarded as philosophical difficulties. The strength of Barton's resolution of the difficulties lies in his creation of a preliminary framework (he admitted that it might need revision) whereby the differing views can be seen in relation to each other. His triadic framework is an "Intentional Map" (p. 204) with the three broad headings of *mathematics*, *mathematics education*, and *society* (cf. the whole day of sessions dedicated to these broad areas at the 6th International Congress on Mathematical Education held in Budapest, Hungary, in 1988). The seminal writers whose definitions of ethnomathematics he considered in detail and placed in relation to this framework were Ubiratan D'Ambrosio in Brazil, Paulus Gerdes in Mozambique, and Marcia Ascher in the USA.

As Barton (1996) pointed out, D'Ambrosio's prolific writings on the subject of ethnomathematics have influenced the majority of writers in this area. Thus on the Intentional Map, although D'Ambrosio's work (starting with his 1984 publication) falls predominantly in the socio-anthropological dimension between *society* and *mathematics*, some aspects of his concerns can be found in all of the dimensions. In his later work, he increasingly used his model to analyze "the way in which mathematical knowledge is colonized and how it rationalizes social divisions within societies and between societies" (Barton, 1996, p. 205). In his early writing, D'Ambrosio (1984) defined ethnomathematics as the way different cultural groups mathematize—count, measure, relate, classify, and infer. His definition evolved over the years, to include a changing form of knowledge manifest in practices that change over time. In 1985, he defined ethnomathematics as "the mathematics which is practiced among identifiable cultural groups" (p. 18). Later, in 1987, his definition of ethnomathematics was "the codification which allows a cultural group to describe, manage, and understand reality" (Barton, 1996, p. 207).

D'Ambrosio's (1991) well-known etymological definition of ethnomathematics is given in full in the following passage.

> The main ideas focus on the concept of *ethnomathematics* in the sense that follows. Let me clarify at the beginning that this term comes from an etymological abuse. I use *mathema(ta)* as the action of explaining and understanding in order to transcend and of managing and coping with reality in order to survive. Man has developed throughout each one's own life history and throughout the history of mankind *techné*'s (or *tics*) of *mathema* in very different and diversified cultural environments, i.e., in the diverse *ethno*'s. So, in order to satisfy the drive towards survival and transcendence in diverse cultural environments, man has developed and continuously develops, in every new experience, **ethno-mathema-tics.** These are *communicated* vertically and horizontally in time, respectively throughout history and through conviviality and education, relying on memory and on sharing experiences and knowledges. For the reasons of being more or less effective, more or less powerful and

> sometimes even for political reasons, some of these different *tics* have lasted and spread (ex.: counting, measuring) while others have disappeared or been confined to restricted groups. This synthesizes my approach to the history of ideas. (p. 3)

As in some of his other writings (1985, 1987, 1990), D'Ambrosio is in this definition characterizing ethnomathematics as a dynamic, evolving system of knowledge-the "process of knowledge-making" (Barton, 1996, p. 208), as well as a research program that encompasses the history of mathematics.

Returning to Barton's Intentional Map, the work of Paulus Gerdes is "practical, and politically explicit," concentrated in the *mathematics education* area of the Map (Barton, 1996, p. 205). Gerdes's definition of ethnomathematics evolved from the mathematics implicit or "frozen" in the cultural practices of Southern Africa (1986), to that of a mathematical movement that involves research and anthropological reconstruction (1994). The work of mathematician Marcia Ascher (1991, 1995, 2002), while overlapping with that of Gerdes to some extent, falls closer to the *mathematics* area on the Map, concerned as it is with cultural mathematics. Her definition is that ethnomathematics is "the study and presentation of the mathematical ideas of traditional peoples" (1991, p. 188). When Ascher (1991) worked out the kinship relations of the Warlpiri, say, in mathematical terms, she acknowledged that she was using her familiar "Western" mathematics. In that sense her ethnomathematics is subjective: The Warlpiri would be unlikely to view their kinship system through her lenses. Referring to mathematics and ethnomathematics, she stated, "They are both important, but they are different. And they are linked" (Ascher & D'Ambrosio, 1994, p. 38). In this view, there is no need to view ethnomathematics as "proto-mathematics" (Thomas, 1996), because it exists in its own right.

Finally, Barton (1996) found a useful metaphor to sum up the similarities and differences between the views of ethnomathematics held by these three proponents: "For D'Ambrosio it is a window on knowledge itself; for Gerdes it is a cultural window on mathematics; and for Ascher it is the mathematical window on other cultures" (p. 213). These three windows are distinguished by the standpoint of the viewer, and by what is being viewed, in each case. Although not eliminating the duality of ethnomathematics as opposed to mathematics (of mathematicians), these three distinct windows represent approaches each of which has something to offer. Taken together, they contribute a broadened lens on the role of culture in teaching and learning mathematics.

Several other writers in the field of ethnomathematics have acknowledged the need and attempted to define ethnomathematics. Scott (1985) regarded ethnomathematics as lying at the confluence of mathematics and cultural anthropology, "mathematics in the environment or community," or "the way that specific cultural groups go about the tasks of classifying, ordering, counting, and measuring" (p. 2). Several definitions of ethnomathematics highlight some of the "environmental activities" that Bishop (1988a) viewed as universal, and also "necessary and sufficient for the development of mathematical knowledge" (p. 182), namely counting, locating, measuring, designing, playing, and explaining. One further definition brings back the problem, hinted at in the foregoing account, of *ownership* of ethnomathematics. Whose mathematics is it?

> Ethnomathematics refers to any form of cultural knowledge or social activity characteristic of a social and/or cultural group, that can be recognized by other groups such as 'Western' anthropologists, but not necessarily by the group of origin, as mathematical knowledge or mathematical activity. (Pompeu, 1994, p. 3)

This definition resonates with Ascher's, without fully solving the problem of ownership. The same problem appears in definitions of everyday mathematics, considered next.

(4) Everyday Mathematics

Following on from the description of *everyday cognition* (Nunes, 1992, 1993) and important early studies that examined the use of mathematics in various practices, such as mathematical cognition of candy sellers in Brazil (Carraher, Carraher, & Schliemann, 1985; Saxe, 1991), constructs and issues are still being questioned. In this area, too, clarification of definitions is being sought, along with deeper consideration of the scope of the issues and their potential and significance for the classroom learning of mathematics.

Brenner and Moschkovich (2002) raised the following questions.

> What do we mean by *everyday mathematics*? How is everyday mathematics related to *academic mathematics*? What particular everyday practices are being brought into mathematics classrooms? What impact do different everyday practices actually have in classroom practices?" (p. v)

In a similar vein, and with the benefit of 2 decades of research experience in this area, Carraher and Schliemann (2002) examined how their perceptions had evolved, as they explored the topic of their chapter, "Is Everyday Mathematics Truly Relevant to Mathematics Education?"

All the authors of chapters in the monograph edited by Brenner and Moschkovich (2002) in one way or another set out to explore these and related questions. Several of these authors pointed out that it is problematic to oppose everyday and academic mathematics, for several reasons. For one thing, for mathematicians academic mathematics *is* an everyday practice (Civil, 2002; Moschkovich, 2002a). For another, studies of everyday mathematical practices in workplaces reveal a complex interplay with sociocultural and technological issues (FitzSimons, 2002). In the automobile production industry, variations in the mathematical cognition required of workers have less to do with the job itself than with the decisions of management concerning production procedures and organization of the workplace. Highly skilled machinists display spatial and geometric knowledge that goes beyond what is commonly taught in school: In contrast, assembly-line workers and some machine operators find few if any mathematical demands in their work, which is deliberately stripped of the need for decisions involving knowledge of mathematics beyond elementary counting (Smith, 2002). The complex relationship between use of technology and the demand for mathematical thinking in the workplace is a theme that is explored in a later section.

Another aspect that is again apparent in all the chapters of Brenner and Moschkovich's monograph is the importance of perceptions and beliefs about the nature of mathematics, both in the microculture of classroom practices (Brenner, 2002; Masingila, 2002) and in the broader endeavor to bridge the gap between mathematical thinking in and out of school (Arcavi, 2002; Civil, 2002; Moschkovich, 2002a). An essential element in all of these studies is the concern to connect knowledge of mathematics in and out of school. (This issue is revisited later.) Because of the difficulties surrounding the construct *everyday mathematics* the terminology that will be adopted in this chapter follows Masingila (2002), who referred to in-school and out-of-school mathematics practices (p. 38).

The developments described in this section parallel the genesis of the movement away from purely psychological cognitive and behavioral frameworks for research in mathematics education, towards cultural frameworks that embrace sociology, anthropology, and related fields, including political and critical perspectives. The following section introduces some relevant theoretical issues and lenses that have been used to examine some of these developments.

4.2.2 Theoretical Fields That Incorporate Culture in Mathematics Education

The notion of theoretical and empirical fields is drawn from Brown and Dowling (1998) and provides a useful framework for characterizing components of research. This section addresses some theoretical fields pertinent to culture in the teaching and learning of mathematics. Their instantiation in empirical studies is described in the next section. The reader is reminded again to consult the original authors for a full treatment of theoretical fields that are introduced in this section, which has as its purpose a wide but by no means exhaustive view of the scope of theories that are available for work in this area.

As suggested in Barton's (1996) sense-making article introduced in the previous section, in the last 2 decades there has already been considerable movement in theoretical fields regarding the interplay of culture and mathematics. One such movement is discernible in the definitions of ethnomathematics given by D'Ambrosio, Gerdes, and Ascher, as their theoretical formulations moved from more static definitions of ethnomathematics as the mathematics of different cultural groups, to characterizations of this field as an anthropological research program that embraces not only the history of mathematical ideas of marginalized populations, but the history of mathematical knowledge itself (see previous section). D'Ambrosio (2000, p. 83) called this enterprise *historiography*.

(1) Historiography

Moving beyond earlier theoretical formulations of ethnomathematics, its importance as a catalyst for further theoretical developments has been noted (Barton, 1996). D'Ambrosio played a large role and served as advisory editor in the enterprise that resulted in Helaine Selin's (2000) edited book, Mathematics Across Cultures: *The History of Non-Western Mathematics*. The chapters in this book are global in scope and record the mathematical thinking of cultures ranging from those of Iraq, Egypt, and other predominantly Islamic countries; through the Hebrew mathematical tradition; to that of the Incas, the Sioux of North America, Pacific cultures, Australian Aborigines, mathematical traditions of Central and Southern Africa; and those of Asia as represented by India, China, Japan, and Korea. As can be gleaned from the scope of this work, D'Ambrosio's original concern to valorize the mathematics of colonized and marginalized people (cf. Paolo Freire's *Pedagogy of the Oppressed* in 1970/1997) has broadened to encompass a movement that is both archeological and historical in nature, based on the theoretical field of "historiography" and visions of world knowledge through the "sociology of mathematics" (D'Ambrosio, 2000, pp. 85–87).

Although these antedated D'Ambrosio's program, earlier studies such as Claudia Zaslavsky's (1973) report on the counting systems of Africa and Glendon Lean's (1986) categorization of those of Papua New Guinea (see also Lancy, 1983), could also be thought of as historiography, as could anthropological research such as that of Pinxten, van Dooren, and Harvey (1983) who documented Navajo conceptions of space. Also in the cultural anthropology tradition, Crump's (1994) research on the anthropology of numbers is another fascinating example of historiography. More recent studies such as some of those collected as *Ethnomathematics* in the book by Powell and Frankenstein (1997) and the work of Marcia Ascher (1991, 1995, 2002) also fall into this category. Many of the cultural anthropological studies of various mathematical aspects of African practices, such as work on African fractals (Eglash, 1999); *lusona* of Africa (Gerdes, 1997); and women, art, and geometry of Africa (Gerdes, 1998), may also be regarded as historiography. As part of ethnomathematics conceived as a research program, this ambitious undertaking of historiography is designed to address lacunae in the literature on the history of mathematical thought through the ages. Much of the work of members of the International Study Group on Ethnomathematics (founded in 1985), and of the North American Study Group on Ethnomathematics (founded in 2003)—including research by Lawrence Shirley, Daniel Orey, and many others—could be placed in the category of historiography (see the list of some of the available web sites following the references).

Because historiography addresses some of the world's mathematical systems that have been ignored or undervalued in mathematics classrooms, it reminds us that there is *cultural capital* involved in the power relations associated with access in school mathematics. Some relevant theories are outlined in the following paragraphs.

(2) Cultural Capital and Habitus

The usefulness of the theoretical field outlined by Bourdieu (1995) for issues of culture in the learning of mathematics has been indicated in research on learner's transitions between mathematical contexts (Presmeg, 2002a). Resonating with D'Ambrosio's original issues of concern (although D'Ambrosio did not use this framework), Bourdieu's work belongs in the field of sociology. The relevance of this work consists in "the innumerable and subtle strategies by which words can be used as instruments of coercion and constraint, as tools of intimidation and abuse, as signs of politeness, condescension, and contempt" (Bourdieu, 1995, p. 1, editor's introduction). This theoretical field serves as a useful lens in examining empirical issues related to the social inequalities and dilemmas faced by mathematics learners as they move between different cultural contexts, for example in the transitions experienced by immigrant children learning mathematics in new cultural settings (Presmeg, 2002a). This field embraces Bourdieu's notions of *cultural capital, linguistic capital,* and *habitus*. Bourdieu (1995) used the ancient Aristotelian term *habitus* to refer to "a set of dispositions which incline agents to act and react in certain ways" (p.12). Such dispositions are part of culture viewed as a set of shared understandings. Various forms of capital are *economic capital* (material wealth), *cultural capital* (knowledge, skills, and other cultural acquisitions, as exemplified by educational or technical qualifications), and *symbolic capital* (accumulated prestige or honor) (p. 14). *Linguistic capital* is not only the capacity to produce expressions that are appropriate in a certain social context, but it is also the expression of the "correct" accent, grammar, and vocabulary. The *symbolic power* associated with possession of cultural, symbolic, and linguistic capital has a counterpart in the *symbolic violence* experienced by individuals whose cultural capital is devalued. Symbolic violence is a sociological construct. In that capacity it is a powerful lens with which to examine actions of a group and ways in which certain types of knowledge are included or excluded in what the group counts as knowledge (for examples embracing the learning of mathematics, see the chapters in Abreu, Bishop, & Presmeg, 2002).

(3) Borderland Discourses

The notion of symbolic violence leaves a possible theoretical gap relating to the ways in which individuals choose to construct, or choose *not* to construct, particular knowledge—mathematical or otherwise. Bishop (2002a) gave examples both of immigrant learners of mathematics in Australia who chose not to accept the view of themselves as constructed by their peers or their teacher—and of others who chose to accept these constructions. One student "shouted back" when peers in the mathematics class shouted derogatory names.

Discourse is a construct that is wider than mere use of language in conversation (Philips, 1993; Wood, 1998): It embraces all the aspects of social interaction that come into play when human beings interact with one another (Dörfler, 2000; Sfard, 2000). The notion of *Discourses* formulated by Gee (1992, his capitalization), and in particular his extension of the construct to *borderland Discourses*, those "community-based secondary Discourses" situated in the "borderland" between home and school knowledge (p. 146), are in line with Bourdieu's ideas of habitus and symbolic violence. Borderland Discourses take place in the borderlands between primary (e.g., home) and secondary (e.g., school) cultures of the diverse participants in social interactions. In situations where the secondary culture (e.g., that of the school) is conceived as threatening because of the possibility of symbolic violence there, the borderland may be a place of solidarity with others who may share a certain habitus. These ideas go some way towards closing the theoretical gap in Bourdieu's characterization of symbolic violence by raising some issues of individuals' choices, because individuals choose the extent to which they will participate in various forms of these Discourses (see also Bishop's, 2002b, use of Gee's constructs).

Gee's work was in the context of second language learning but is also useful in the analysis of meanings given to various experiences by mathematics learners in cultural transition situations. Bishop (2002a) used this theoretical field in moving from the notion of cultural conflict to that of cultural mediation, in analyzing these experiences of learners of mathematics. If one considers the primary Discourse of school mathematics learners to be the home-based practices and conversations that contributed to their socialization and enculturation (forming their habitus) in their early years, and continuing to a greater or lesser extent in their present home experiences, then the secondary Discourse, for the purpose of learning mathematics, could be designated as the formal mathematical Discourse of the established discipline of mathematics. The teacher is more familiar with this Discourse than are the students and thus has the responsibility of introducing students to this secondary Discourse. As students become familiar with this field, their language and practices may approximate more closely those of the teacher. But in this transition the borderland Discourse of interactive classroom practices provides an important mediating space.

The enculturating role of the mathematics teacher was suggested in the foregoing account. However, as Bishop (2002b) pointed out, the learning of school mathematics is frequently more of an acculturation experience than an enculturation. The difference between these anthropological terms is as follows. *Enculturation* is the induction, by the cultural group, of young people into their own culture. In contrast, *acculturation* is "the modification of one's culture through continuous contact with another" (Wolcutt, 1974, p. 136, as quoted in Bishop, 2002b, p. 193). The degree to which the culture of mathematics, as portrayed in the mathematics classroom, is viewed as their own or as a foreign culture by learners would determine whether their experiences there would be of enculturation or acculturation.

(4) Cultural Models

Allied with Gee's (1992) notion of different Discourses is his construct of *cultural models*. This construct, defined as "'first thoughts' or taken for granted assumptions about what is 'typical' or 'normal'" (Gee, 1999, p. 60, quoted in Setati, 2003, p. 153), was used by Setati (2003) as an illuminating theoretical lens in her research on language use in South African multilingual mathematics classrooms. Already in 1987, D'Andrade defined a cultural model—which he also called a *folk model*—as "a cognitive schema that is intersubjectively shared by a social group," and he elaborated, "One result of intersubjective sharing is that interpretations made about the world on the basis of the folk model are treated as if they were obvious facts about the world" (p. 112). The transparency of cultural models may help to explain why mathematics was for so long considered to be value- and culture-free. The well-known creativity principle of making the familiar strange and the strange familiar (e.g., De Bono, 1975) is necessary for participants to become aware of their implicit cultural beliefs and values, which is why the anthropologist is in a position to identify the beliefs that are invisible to many who are within the culture.

In the context of mathematics learning in multilingual classrooms, Adler (1998, 2001) pointed out aspects of the use of language as a cultural resource that relate to the transparency of cultural models. Particularly in classrooms where the language of instruction is an *additional language*—not their first language—for many of the learners (Adler, 2001), teachers must at times focus on the language itself, in which case the artifact of language no longer serves as a "window" of transparent glass through which to view the mathematical ideas (Lave & Wenger, 1991). In this case the language used is no longer *invisible*, and the focus on the language itself may detract from the conceptual learning of the mathematical content (Adler, 2001).

(5) Valorization in Mathematical Practices

If transparency of culture necessitates making the familiar strange before those sharing that culture become aware of the lenses through which they are viewing their world, then this principle points to a reason for the neglect of issues of valorization in the mathematics education research community until Abreu's (1993, 1995) research brought the topic to the fore. The *value* of formal mathematics as an academic subject was for so long taken for granted that it became a *given* notion that was not culturally questioned. Especially in its role as a gatekeeper to higher education, this status in education is likely to continue. But if ethnomathematics as a research program is to have a legitimate place in broadening notions both of what counts as mathematics and of which people have originated these forms of knowledge, then issues of valorization assume paramount importance.

Working from the theoretical fields of cultural psychology and sociocultural theory, Abreu and colleagues investigated the effects of valorization of various mathematical practices on Portuguese children (Abreu, Bishop, & Pompeu, 1997), Brazilian children (Abreu, 1993, 1995), and British children from Anglo and Asian backgrounds (Abreu, Cline, & Shamsi, 2002). As confirmed also in the research of Gorgorio, Planas, and Vilella (2002), many of these children denied the existence of, or devalued, mathematics as used in practices that they associated with their home- or out-of-school settings.

Valorization, the social process of assigning more value to certain practices than to others, is closely allied to Wertsch's (1998) notion of *privileging*, defined as "the fact that one mediational means, such as social language, is viewed as being more appropriate or efficacious than another in a particular social setting" (Wertsch, 1998, p. 64, in Abreu, 2002, p. 183). Abreu (2002) elaborated as follows.

> From this perspective, cultural practices become associated with particular social groups, which occupy certain positions in the structure of society. Groups can be seen as mainstream or as marginalised. In a similar vein individuals who participate in the practices will be given, or come to construct, identities associated with certain positions in these groups. The social representation enables the individual and social group to have access to a 'social code' that establishes relations between practices and social identities. (p. 184)

Thus Abreu argued strongly that in the learning of mathematics, valorization operates not only on the societal plane but also on the personal plane, because it impacts the construction of social identities. At this psychological level, the construction of mathematical knowledge may be subordinated to the construction of social identity by the individual learner in cases of cultural conflict, as suggested by Presmeg (2002a).

Abreu's ideas are embedded in the field of cultural psychology. Also taking the individual and society into account, another field that has grown in influence in mathematics education research in recent decades is that of situated cognition.

(6) Situated Cognition

The theoretical field of situated cognition explores related aspects of the interplay of knowledge on the societal and psychological planes. Hence it has been a useful lens in research that takes culture into account in mathematical thinking and learning (Watson, 1998). As Ubiratan D'Ambrosio by his writings founded and influenced the field of ethnomathematics, so Jean Lave in analyzing and reporting her anthropological research has influenced the theoretical field of situated cognition. From her early theorizing following ethnography with tailor's apprentices in Liberia (Lave, 1988) to her more recent writings, following research with grocery shoppers and weight-watchers (Lave, 1997), the notion of transfer of knowledge through abstraction in one context, and subsequent use in a new context, was questioned and problematized. In collaboration with Etienne Wenger, her theorizing led to the notion of cognitive apprenticeship and *legitimate peripheral participation* (Lave & Wenger, 1991; Wenger, 1998). In this view, learning consists in a centripetal movement of the apprentice from the periphery to the center of a practice, under the guidance of those who are already masters of the practice. This theory was not originally developed in the context of or for the purpose of informing mathematics education. However, in its challenge to the cognitive position that abstract learning of mathematics facilitates transfer and that this knowledge may be readily applied in other situations than the one in which it was learned (not born out by empirical research), the theory has been powerful and influential. The research studies inspired by Lave and reported by Watson (1998) bear witness to this strong influence.

In her later writings, Lave attempted to bring the theory of socially situated knowledge to bear on the classroom teaching and learning of mathematics. But issues of intentionality and recontextualization separate apprenticeship and classroom situations, although enough commonality exists in the two situations for both legitimately to be called *practices* (Lerman, 1998). Mathematics learners in school are not necessarily aiming to become either mathematicians or mathematics teachers. As Lerman pointed out in connection with the issue of voluntary and nonvoluntary participation, students' presence in the classroom may be nonvoluntary, creating a very different situation from that of apprenticeship learning, and calling into question the assumption of a goal of movement from the periphery to the center. At the same time, the teacher has the intention to teach her students mathematics, notwithstanding Lave's claim that teaching is not a precondition of learning. The learning of mathematics is the goal of the enterprise, and teaching is the teacher's job. In contrast, in the apprenticeship situation the learning is not a goal in itself, for example, in the case of the tailors the goal is to make garments efficiently. Thus, as Adler (1998) suggests, "It is in the understanding of the aims of school education that Lave and Wenger's seamless web of practices entailed in moving from peripheral to full participation in a community of practice is problematic" (p. 174).

Although the theory of legitimate peripheral participation may not translate easily into the classroom teaching and learning of mathematics, the view of learning as a social practice has powerful implications for this learning and has been an influence in changes that have taken place in the practice of teaching mathematics, such as an increased emphasis on communication (NCTM, 1989, 2000) in the mathematics classroom. Even more, situated cognition as a theory has given a warrant to attempts to bridge the gap between in- and out-of-school mathematical practices.

(7) Use of Semiotics in Linking Out-Of-School and In-School Mathematics

In the USA, the *Principles and Standards for School Mathematics* (NCTM, 2000) continued the earlier call (NCTM, 1989) for teachers to make connections, in particular between the everyday practices of their students and the mathematical concepts that are taught in the classroom. But various theoretical lenses of situated cognition (Lave & Wenger, 1991; Kirshner & Whitson, 1997) remind us that these connections can be problematic. There are at least three ways in which the activities of out-of-school practices differ from the mathematical activities of school classrooms (Walkerdine, 1988), as follows.

- The goals of activities in the two settings differ radically.
- Discourse patterns of the classroom do not mirror those of everyday practices.
- Mathematical terminology and symbolism have a specificity that differs markedly from the useful qualities of ambiguity and indexicality (interpretation according to context) of terms in everyday conversation.

A semiotic framework that uses chains of signification (Kirshner & Whitson, 1997) has the potential to bridge this apparent gap through a process of chaining of signifiers in which each sign "slides under" the subsequent signifier. In this process, goals, discourse patterns, and use of terms and symbols all move towards that of classroom mathematical practices in a way that has the potential to preserve essential structure and some of the meanings of the original activity.

This theoretical framework resonates with that of *Realistic Mathematics Education* (RME) developed by Freudenthal, Streefland, and colleagues at the Freudenthal Institute (Treffers, 1993). *Realistic* in this sense does not necessarily mean out-of-school in the real world: The term refers to problem situations that learners can imagine (van den Heuvel-Panhuizen, 2003). A theory of semiotic chaining in mathematics education resonates with RME in that the starting points for the learning are realistic in this sense. But more specifically the chaining model is a useful tool for linking out-of-school mathematical practices with the formal mathematics of school classrooms (an example of such use is presented in the next paragraph).

In brief, the theory of semiotic chaining used in mathematics education research, as it was initially presented by Whitson (1997), Cobb, Gravemeijer, Yackel, McClain, & Whitenack (1997), and Presmeg (1998b), followed the usage of Walkerdine (1988). Although she was working in a poststructural paradigm, Walkerdine found the Swiss structural approach of Saussure, as modified by Lacan, useful in building chains of signifiers to link a home practice, such as that of a daughter and her mother pouring drinks for guests, with more formal learning of mathematics, in this case the system of whole numbers. Walkerdine's chain had the following structure.

Signified 1: the actual people coming to visit;
Signifier 1 (*standing for* signified 1): the names of the people.

} **Sign 1**

The daughter is asked to raise a finger for each name.
Signified 2: the names of the guests;
Signifier 2: the raised fingers.

} **Sign 2**

The daughter is asked to count her raised fingers.
Signified 3: the raised fingers;
Signifier 3: the numerals, one two, three, four, five.

} **Sign 3**

In this process, signifier 3 actually comes to stand for all of the preceding links in the chain: Five guests are coming to tea. Note that each signifier in turn becomes the next signified, and that at any time any of the links in the chain may be revisited conceptually. As mother and daughter move through the three signs, the discourse shifts successively from actual people to their names, to the fingers of one hand, and finally to the more abstract discourse of the numerals of the whole number counting system.

In her distinctive style, Adler (1998) characterized the associated recontextualizing process as that of crossing a discursive bridge:

> That there is a bridge to cross between everyday and educated discourses is at the heart of Walkerdine's (1988) argument for 'good teaching' entailing chains of signification in the classroom where everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice. (p. 174)

Using this process, a teacher can use chains of signification as an instructional model that develops a mathematical concept starting with an out-of-school situation and linking it in a number of steps with formal school mathematics (Hall, 2000). Building on the work of Presmeg (1998b) in his dissertation research, Hall taught three 4th-grade teachers to build semiotic chains appropriate to the practices of their students and the instructional needs of their individual classrooms. Using a similar semiotic chaining model, Cobb et al. (1997) reported on the emergence (using their *emergent* theoretical perspective) of chains of signification in one 1st-grade classroom. In addition to these sources, examples of mathematical chaining at elementary school, high school, and college levels may be found in Presmeg's (1998b, 2006) writings. In later research on building bridges between out-of-school and in-school mathematics, Presmeg (2006) preferred to use a nested triadic model based on the writings of Charles Sanders Peirce (1992, 1998). In addition to an *object* (which could be an abstract concept) and a *representamen* that stands for the object in some way, each nested sign has a third component that Peirce called the *interpretant*. This triadic model explicitly allows for learners' individual construction of meaning (through the interpretant) in a way that linear chains of signification can do only implicitly.

Attempts to facilitate teachers' use of children's out-of-school mathematical practices in their classrooms provide examples of practical uses of theoretical fields. This theme is continued in the next section.

4.2.3 Empirical Fields That Incorporate Culture in Mathematics Education

The last section dealt with some theoretical fields that have been useful, or have the potential to be useful, in research on the role of culture in mathematics education. In the current section, some associated empirical fields are discussed. Empirical fields entail broad methodologies of empirical research, but in addition to these, in this section specific methods of research and also the participants, data, and results of selected studies are introduced. These details of participants, time, and place in the research are termed the *empirical settings* (Brown & Dowling, 1998). Thus appropriate empirical fields and settings are the focus of this section, along with some results of research on the role of culture in mathematics education.

Because ethnography is the special province of holistic cultural anthropological research (Eisenhart, 1988), which has as a broad goal the understanding of various cultures, it is natural to expect that ethnography (sometimes in modified form) would be used by researchers who are interested in the role of culture in teaching and learning mathematics. This is indeed the case. However, researchers in mathematics education in general sometimes call their studies ethnography when they have not spent long enough in the field to warrant that term for their research (Eisenhart, 1988). For instance, valuable as studies such as that of Millroy (1992) are for mathematics educators to learn more about the use of mathematical ideas in out-of-school settings such as that of Millroy's carpenters in Cape Town, South Africa, the $5\frac{1}{2}$ month duration of her fieldwork might be considered brief for an ethnography, and most ethnographic studies in mathematics education have a shorter duration than Millroy's. For this reason, the field of such research is often more appropriately called *case studies* (Merriam, 1998), because they satisfy the criterion of being bounded in particular ways. If such studies are investigating a particular topic, such as ways of bringing out-of-school mathematics into the classroom, and using several cases to do so, they could be called *instrumental case studies* (Stake, 2000). Where the individual cases themselves are the focus, case studies are *intrinsic* (ibid.).

Most of the studies mentioned in this section may be regarded as instrumental case studies in the sense explained in the foregoing. Research that concerns in-school and out-of-school mathematical practices falls into two main categories. In the first category are studies that attempt to build bridges between informal or nonformal mathematical practices (Bishop, Mellin-Olsen, & van Dormolen, 1991) and those of formal school mathematics. In the second category are studies that link with formal mathematics education less directly but with the potential to deepen understanding of this education, by exploring the culture of mathematics in various workplaces. The studies in both of these categories are too numerous for a summary chapter such as this one to do full justice to the aims, research questions, methodologies and methods for data collection, and the subtleties unearthed in the results of such research. (Again, the reader is reminded to read the original literature for depth of coverage.) What follows is an introduction to the range of research in each of these two categories, with discussion of some of the empirical settings, a sampling of issues addressed, and some of the results.

4.2.4 Linking Mathematics Learning Out-Of-School and In-School

In the first category, studies that specifically investigated linking out-of-school and in-school mathematics are of several types. Some researchers interviewed parents of minority learners and their teachers to investigate relationships between home and school learning of mathematics. In England, Abreu and colleagues conducted such interviews for the purpose of investigating how the parents of immigrant learners are able to support their children in their transitions to learning mathematics in a new school culture (Abreu, Cline & Shamsi, 2002). Civil and colleagues in the USA interviewed such parents for the purpose of finding out which home practices of immigrant families might be suitable for pedagogical purposes in mathematics classrooms (Civil, 1990, 1995, 2002; Civil & Andrade, 2002). Related empirical studies in which researchers investigated the transitions experienced by immigrant children in their learning of mathematics have been reported by Bishop (2002a) in Australia and by Gorgorio, Planas, and Vilella (2002) in Catalonia, Spain.

In the next type of study in this category, research was conducted in mathematics classrooms to investigate the effects of bringing out-of-school practices into that arena. Using video recording as a data collection method, Brenner (2002) investigated how four teachers and their junior high school students went about an activity in which the students were required to decide cooperatively which of two fictitious pizza companies should be given the contract to supply pizza to the school cafeteria. In a different classroom video study, Moschkovich's (2002b) research addressed the mathematical activities of seventh-grade students during an architectural design project: Students working collaboratively tried to design working and living quarters for a team of scientists in Antarctica in such a way that space would be maximized while still taking into account the heating costs of various designs. These studies emphasized the point that through their pedagogy and instructional decisions in the classroom, teachers are an important component of the success (or lack of success) of attempts to use out-of-school practices effectively in mathematics classrooms. Beliefs of teachers about the nature of mathematics, what culture is, and its role in the classroom learning of mathematics are strong factors in teachers' decisions in this regard (Civil, 2002; Presmeg, 2002b).

Like the researchers in the preceding paragraphs, Presmeg (1998b, 2002b, 2006) and Hall (2000) recognized the lack of congruence of out-of-school cultural practices and those "same" practices when brought into the school mathematics classroom (Walkerdine, 1988). Their research used semiotic chaining as a theoretical tool (see previous section) in an attempt to bridge the gaps, not only between these practices in- and out-of-school, but also between mathematical ideas implicit in these activities and the formal mathematics of the syllabi that teachers are expected to use in their practices. The fourth-grade teachers with whom Hall worked built chains of signifiers that they implemented with their classes, starting with a cultural practice of at least some of their students and aimed at linking mathematical ideas in this practice in a number of steps with a formal school mathematics topic. Chains constructed by the teachers were designated as either *intercultural*-bridging two or more cultures, or *intracultural*-having a chain that remained within a single culture. Examples of the first type involved number of children in students' families, pizzas, coins, and measurement of students' hands, linking in a series of steps with classroom mathematical concepts. These are intercultural because the cultures and discourse of students' homes or activities are linked with the different discourse and culture of classroom mathematics, for instance, the making of bar graphs. Manipulatives were frequently used as intermediate links in these chains. The intracultural type, involving chains that were developed within the culture of a single activity, was evidenced in a chain involving baseball team statistics. The movement along the chain could be summarized as follows:

Baseball Game → Hits vs. At Bats → Success Fraction → Batting Average.

It was not the activity that was preserved throughout the chain, but merely the culture of baseball (at least as it was imported into the classroom practice) within which the chain was developed. The need for interpretation at each link in the chain led to further theoretical developments using a triadic nested model that was a better lens for interpreting the results (Presmeg, 2006).

One study that is ethnographic in its scope and methodology has less explicit claims to link out-of-school and in-school mathematics, although implicit ties between the two contexts are present. This study is a thorough investigation of mathematical elements in learning the practice of selling newspapers in the streets by young boys (called in Portuguese *ardinas*) on the island of Cabo Verde in the Atlantic ocean (Santos & Matos, 2002). Using Lave and Wenger's (1991) theoretical framework of legitimate peripheral participation, Santos and Matos described the goal of their research as follows.

> Our goal was to look into the ways (mathematics) learning relates to forms of participation in social practice in an environment where mathematics is present but that escapes the characteristics of the school environment. Because we believe that culture is an unavoidable fact that shapes our way of seeing and analyzing things, we decided to look at a culturally distinct practice and that constituted a really strange domain for us: the practice of the *ardinas* at Cabo Verde islands in Africa. (p. 81)

In addition to interesting explicit and implicit mathematical aspects of the changing practice of the *ardinas* as they moved from being newcomers to full participants, methodological difficulties in this kind of research were foregrounded by Santos and Matos (2002):

> The fact that the research is studying a phenomena [*sic*] which was almost totally strange to us in most of its aspects, led us to realize that we had to go through a process which should involve, to a certain extent, our participation in the (*ardinas'*) practice with the explicit (for us and for them) goal of learning it but not in order to be a full member of that community of practice. This starting point (more in terms of knowing that there are more things that we don't know than that we know) opened that community of practice to us but also gave us consciousness that methodological issues were central in this research. (p. 120)

Many of the difficulties that Santos and Matos described with sensitivity and vividness are common to most anthropological research and thus also impact investigations of cultural practices in the field of mathematics education. They were required to become part of the culture sufficiently to be able to interpret it to others who are not participants, but at the same time not to become so immersed in the culture that it would be transparent-a lens that is not the focus of attention because one looks *through* it. They faced the difficulty that entering the group as an outsider might in fact change the culture of that group to a greater or lesser extent. On several occasions there was evidence that the mathematical reporting by key informants among the *ardinas* was influenced by the fact that the fieldworker was a Portuguese-speaking woman, whom these boys might have associated with their schoolteachers. Finally, the researchers recognized the not inconsiderable difficulties associated with practical matters concerned with collecting data in the street.

The investigation of the *ardinas'* mathematical thinking could be classified as an ethnographic workplace study. Thus this short account of this research provides a transition to the second aspect of this empirical section of the chapter, mathematics in the workplace.

(1) Culture of Mathematics in the Workplace

In the second category of studies in this section, the culture of mathematics in various workplaces was investigated intensively by FitzSimons (2002), by Noss and colleagues (Noss & Hoyles, 1996a; Noss, Hoyles & Pozzi, 2000; Noss, Pozzi & Hoyles, 1999; Pozzi, Noss, and

Hoyles, 1998), and by several other researchers (see later). FitzSimons investigated the culture and epistemology of mathematics in Technical and Further Education in Australia, and how this education related or failed to relate to the cultures of mathematics as used in workplaces. Noss, Hoyles, and Pozzi examined workplace mathematics in more specific detail: *Inter alia* their studies addressed mathematical aspects of banking and nursing practices. A common thread in these studies is the *demathematization* of the workplace: "As the seminal work of researchers Richard Noss and Celia Hoyles among others indicates, mathematics actually used in the workplace is contingent and rarely utilizes 'school mathematics' algorithms in their entirety, if at all, or necessarily correctly" (FitzSimons, 2002, p. 147).

The same point was brought out strongly, but contingently, in an investigation of mathematical activity in automobile production work in the USA (Smith, 2002). One result of Smith's study was that "the organizational structure and management of automobile production workplaces directly influenced the level of mathematics expected of production workers" (p. 112). This level varied from a minimal expectation on assembly lines-designed to be "worker-proof"-to "a surprisingly high level of spatial and geometric competence, which outstripped the preparation that most K–12 curricula provide" (p.112), e.g., as manifested by skilled machinists who translated between two- and three-dimensional space with sophistication and accuracy in creating products that were sometimes one-of-a-kind. Organization that used "lean manufacturing principles" (p. 124) to move away from assembly lines and give more autonomy to workers was more likely to enhance and require these highly developed forms of mathematical thinking. Workers are not usually allowed to decide what level of mathematics will be required in their work:

> The fact that the organization of production systems and work practices mediates and in some cases limits workers' mathematical expectations and their access to workplace mathematics is a reminder that the everyday mathematics of work is inseparable from issues of power and authority. In large measure, someone other than the production workers themselves decides when, how often, and how deeply they will be called on to think mathematically. (Smith, 2002, p. 130)

Other sources of insights into the mathematical ideas involved in practices of various workplaces are detailed in FitzSimons's (2002) book. These include the following:

> operators in the light metals industry (Buckingham, 1997); front-desk motel and airline staff (Kanes, 1997a, 1997b); landless peasants in Brazil ...(Knijnik, 1996, 1997, 1998); carpet layers (Masingila, 1993); commercial pilots (Noss, Hoyles, & Pozzi, 2000); ... draughtspersons (Strässer, 1998); semi-skilled operators (Wedege, 1998b, 2000a, 2000b) and swimming pool construction workers (Zevenbergen, 1996). Collectively, these reports highlight not only the breadth and depth of mathematical concepts encountered in the workplace, but underline the complex levels of interactions in the broad range of professional competencies as outlined above, where mathematical knowledge can come into play – when permitted by (or in spite of) management. (p. 72)
>
> In addition to this broad range of reported research, in an early study Mary Harris (1987) investigated "women's work," challenging her readers to "derive a general expression for [knitting] the heel of a sock" (p. 28). The more recent mathematics education conferences have included presentations on the topic of the mathematics of the workplace (e.g., Strässer, 1998).

From the foregoing, the breadth of this field, the variety of practices that have been investigated from a mathematical point of view, and the scope of methodologies employed are apparent. Many of these studies may be characterized as investigations in ethnomathematics, for example, those of Masingila (1993) with the art of carpet laying, and Harris (1987). This overlap in classification also applies to research on mathematics in the world of work that is concerned with its artifacts and tools, and with its technology. The confluence of technology and culture in mathematics education is the topic of the last part of this section.

(2) Influences of Technology

A study that stands at the intersection of workplace mathematics studies and those that are concerned with the role and influence of technology in mathematics education is that by Magajna and Monaghan (2003). In their research they investigated both the mathematical elements and the use of computer aided design (CAD) and manufacture (CAM) in the work of six skilled technicians who designed and produced moulds for glass factories. The technicians specialized in "moulds for containers of intricate shapes" (p. 102), such as bottles in the form of twisted pyramids, stars, or guitars. The study is interesting because the mathematics used by the CAD/CAM technicians in various stages of their work (for instance, calculating the interior volume of a mould) was not elementary, and in theory school-learned mathematics could have been used. However, the researchers reported as follows.

> Although the technicians did not consider their activity was related to school mathematics there is evidence that in making sense of their practice they resorted to (a form of) school mathematics. The role of technology in technicians' mathematical activity was crucial: not only were the mathematical procedures they used shaped by the technology they used but the mathematics was a means to achieve technological results. Further to this the mathematics employed by the technicians must be interpreted within the goal-oriented behaviour of workers who 'live' the imperatives and constraints of the factory's production cycle. (p. 101)

An implication of results such as these, resonating with the conclusions of other workplace studies, is that there is no direct path from school mathematics to the mathematics used, sometimes indirectly, in various occupations, where the specific practices are learned on the job. It may not be possible then to gear a mathematics curriculum, even in vocational education (FitzSimons, 2002), to the specific requirements of a number of vocations simultaneously. However, as technology has developed in the last 6 decades, its influence has been felt in mathematics curricula of various time periods, as illustrated by Kelly (2003).

The research in this growing field is beyond the scope of this paper, but an example highlights the potential of graphing calculators and computers to change the culture of learning mathematics. The research of Ricardo Nemirovsky (2002) and his colleagues (Nemirovsky, Tierney, & Wright, 1998) demonstrates how the use of motion detectors and associated technology changes the culture of learning from one of fostering *formal* generalizations in mathematics (e.g., "all x are y"), to one of constructing *situated* generalizations. The latter kind of generalization is "embedded in how people relate to and participate in tasks, events, and conversations" (Nemirovsky, 2002, p. 250)—and it is "loaded" with the values of "grasping the circumstances and transforming aspects that appear to be just ordinary or incidental into objects of reflection and significance" (p. 251). Nemirovsky illustrated this kind of generalization in the informal mathematical constructions of Clio, an 11-year-old girl working with the problem of trying to predict, based on the graph created on a computer screen by a motion detector, where a toy train is situated in a tunnel.

In summary, then, technology, by entering all avenues of life, influences not only the mathematics of the curriculum and the ontology of mathematics itself, but also the culture of the future, through its children.

This section has examined in broad detail some of the empirical research relating to the role of culture in teaching and learning mathematics. Aspects that were considered included the linking of out-of-school and in-school mathematics learning, both from the point of view of comparing these different cultural practices and from the point of view of building bridges between cultural practices and the classroom learning of mathematics-of "bringing in the world" (Zaslavsky, 1996). A related strand that was outlined was the culture of mathematics in the workplace, and finally, some aspects of the cultural influence of technology on the learning of mathematics were suggested. Clearly in all of these strands ongoing research is needed and in progress. However, perhaps just as significantly, research is needed that will increase understanding of and highlight how these strands are related amongst themselves. For instance, as the use of technology both in the workplace and in the mathematics classroom changes the culture of learning mathematics in these broad arenas, hints of avenues to be explored in future work appear, for example in whether and how technology has the potential to bridge or decrease the gap between formal and informal mathematical knowledge. The theme of technology and culture in future research is elaborated, along with other significant areas, in the final section.

4.2.5 Future Directions for Research on Culture in Mathematics Education

The final section uses a wider lens to zoom out and consider areas of research on culture in mathematics education that require development or are likely to assume increasing importance in the years ahead. By its nature, this section is speculative, but trends are already apparent that are likely to continue. The influence of technology is one such trend (Morgan, 1994; Noss, 1994).

(1) Technology

Already in 1988, Noss pointed out the cultural entailments of the computer in mathematics education, as follows: "Making sense of the advent of the computer into the mathematics classroom entails a cultural perspective, not least because of the ways in which children are developing the computer culture by appropriating the technology for their own ends" (p. 251). He elaborated, "The key point is that children see computer screens as 'theirs', as a part of a predominantly adult culture which they can appropriate and use for their own ends" (p. 257). As these children become adults with a facility with technology beyond that of their parents (Margaret Mead's *prefigurative enculturation* comes into play as children *teach* their parents), the culture of mathematics education in all its aspects is likely to change in fundamental ways. Noss (1988) and Noss and Hoyles (1996b) put forward a vision for the role that computer microworlds have played and might play in the future of mathematics education. With accelerating changes in platforms, designs, and software, research must keep pace and inform resulting changes in the cultures of teaching and learning of mathematics in schools. An aspect of potential use for the computer stems from its ability to mimic reality in a world of virtual reality. Noting the complex relationship between formal and informal mathematics (ethnomathematics), Noss (1988) suggested, "I propose that the technology itself—specifically the computer—can be the instrument for bridging the gap between the two" (p. 252). This area remains one in which research is needed, both in its own right and in ways that the change of context of bringing the virtual reality of computer images into mathematics classrooms changes the culture of teaching and learning in those classrooms.

(2) Bridging Mathematics In-School and Out-Of-School

Resonating with the Realistic Mathematics Education viewpoint that *real* contexts are not confined to those concrete situations with which learners are familiar (van den Heuvel-Panhuizen, 2003), Carraher and Schliemann (2002) made a useful distinction between *realism* and *meaningfulness*:

> What makes everyday mathematics powerful is not the concreteness of the objects or the everyday realism of the situations, but the meaning attached to the problems under consideration (Schliemann, 1995). In addition meaningfulness must be distinguished from realism (D. W. Carraher & Schliemann, 1991). It is true that engaging in everyday activities such as buying and selling, sharing, or betting may help students establish links between

> their experience and intuitions already acquired and topics to be learned in school. However, we believe it would be a fundamental mistake that schools attempt to emulate out-of-school institutions. After all, the goals and purposes of schools are not the same as those of other institutions. (p. 137)

The dilemma, then, is *how* to incorporate out-of-school practices in school mathematics classrooms in ways that are meaningful to students and that do not trivialize the mathematical ideas inherent in those practices. This issue remains a significant one for mathematics education research.

Reported research (Civil, 2002; Masingila, 2002; Presmeg, 2002b) has shown that learners' beliefs about the nature of mathematics affect what mathematics they identify *as* mathematics in out-of-school settings. Students' and teachers' conceptions of mathematics as decontextualized and abstract may limit what can be accomplished in terms of meaningfulness derived from out-of-school practices (Presmeg, 2002b). And yet, abstract thinking is not necessarily antagonistic to the idea of reasoning in particular contexts, as Carraher and Schliemann (2002) pointed out, which led them to propose the construct of *situated generalization*. They summarized these issues as follows.

> Research sorely needs to find theoretical room for contexts that are not reducible to physical settings or social structures to which the student is passively subjected. Contexts can be imagined, alluded to, insinuated, explicitly created on the fly, or carefully constructed over long periods of time by teachers and students. Much of the work in developing flexible mathematical knowledge depends on our ability to recontextualize problems – to see them from diverse and fresh points of view and to draw upon our former experience, including formal mathematical learning. Mathematization is not to be opposed to contextualization, since it always involves thinking in contexts. Even the apparently context-free activity of applying syntax transformation rules to algebraic expressions can involve meaningful contexts, particularly for experienced mathematicians. (p. 147)

They mentioned the irony that the mechanical following of algorithms characterizes the approaches of both highly unsuccessful and highly successful mathematical thinkers.

Taking into account both the need for meaningful contexts in the learning of mathematics and the necessity of developing mathematical ideas in the direction of abstraction and generalization (in the flexible sense, not to be confused with decontextualization), at least two extant fields of research have the potential to address these issues in significant ways. The notions of *horizontal* and *vertical* mathematizing that have informed Realistic Mathematics Education research for several decades (Treffers, 1993) could resolve a seeming conflict between abstraction and context. In harmony with these ideas, recent attempts to use semiotic theories in linking out-of-school and in-school mathematics also have the potential for further development (Hall, 2000; Presmeg, 2006).

As Moschkovich (1995) pointed out, a tension exists between educators' attempts to engage learners in "real world" mathematics in classrooms and movements to make mathematics classrooms reflect the practices of mathematicians. Multicultural mathematics materials for use in classrooms have been available for some time (Krause, 1983; Zaslavsky, 1991, 1996). However, the tensions, the contradictions, and the complexity of trying to incorporate practices for which "making change" serves as a metonymy at the same time that students are "making mathematics" (Moschkovich, 1995) will engage researchers in mathematics education for some time to come. Bibliographies such as that compiled by Wilson and Mosquera (1991) will continue to be necessary, to inform both researchers and practitioners what has already been accomplished as the field of culture in mathematics education continues to grow.

(3) Teacher Education

In all of the foregoing areas of potential cultural research in mathematics education, the role of the teacher remains important. Noting that concrete and abstract domains in mathematical thinking are not necessarily disparate, Noss (1988) suggested,

> The key idea is that of focusing attention on the important relationships involved, a role in which—as Weir (1987) points out—the computer is rather well cast; but not without the conscious intervention of educators, and the careful development of an ambient learning culture. (p. 263)

Bishop (1988a, 1998b) was also intensely aware of the role of the *mathematical enculturators* in personifying the values that are inherent in the teaching and learning of mathematics. In the final chapter of his seminal book (1988a) he suggested requirements for the education-rather than the more restricted notion of "training"-of those who will be mathematics teachers, at both the elementary and secondary levels. He did not distinguish between these levels, for teachers at both elementary school and secondary school have important mathematical enculturation roles. Bishop (1988a) summarized the necessary criteria as follows.

I propose, then, these four criteria for the selection of suitable Mathematical enculturators:

- ability to 'personify' the mathematical culture
- commitment to the Mathematics enculturation process
- ability to communicate Mathematical ideas and values
- acceptance of accountability to the Mathematical cultural group. (p. 168)

These ideas still seem timely; in fact the literature on discourse and communication has broadened in the decades since these words were written, to suggest that communication amongst all involved in the negotiation of the cultures of mathematics classrooms (teacher and learners) plays a significant role in the learning of mathematics in those arenas (Cobb et al., 1997; Dörfler, 2000; Sfard, 2000). *Language and Communication in Mathematics Education* was the title of a Topic Study Group at the 10th International Congress on Mathematical Education (Copenhagen, 2004), and the literature in this field is already extensive. But *how* to educate future teachers of mathematics to satisfy Bishop's four criteria is still an open field of research.

As Arcavi (2002) acknowledged, much has already been accomplished in curriculum development, research on teachers' beliefs and practices, and "the development of a classroom culture that functions in ways inspired by everyday practices of academic mathematics" (p. 27). However, open questions still exist concerning ways of using the recognition that the transition from out-of-school mathematical practices to those within school is sometimes not straightforward, in order to inform the practices of mathematics teaching. Arcavi (2002) gave examples of such questions.

> However, much remains to be researched. For example, is it always possible to smooth the transition between familiar and everyday contexts, in which students use ad hoc strategies to solve problems, and academic contexts in which more general, formal, and decontextualized mathematics is to be learned? Are there breaking points? If so, what is their nature? Studies in everyday mathematics and in ethnomathematics are very important contributions, not only because of their inherent value but also because of the reflection they provoke in the mathematics education community at large. There is much to be gained from those contributions. (p. 28)

Clearly ethnomathematics conceived as a research program (D'Ambrosio, 2000) has already permeated the cultures of mathematics education research and practice in various ways. The influence, and the need for research that addresses the complexities of the issues involved, are ongoing.

Acknowledgements

The author wishes to thank Ubiratan D'Ambrosio and Joanna Masingila for helpful comments on an early draft of the *Handbook* chapter on which this paper is based, and Ubiratan D'Ambrosio for supplying the list of useful web sites, including those of relevant dissertations.

References

Abreu, G. de (1993). *The relationship between home and school mathematics in a farming community in rural Brazil.* Unpublished doctoral dissertation, University of Cambridge.

Abreu, G. de (1995). Understanding how children experience the relationship between home and school mathematics. *Mind, Culture and Activity, 2*(2), 119-142.

Abreu, G de (2002). Towards a cultural psychology perspective on transitions between contexts of mathematical practices. In G de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 173-192). Dordrecht, The Netherlands: Kluwer Academic.

Abreu, G. de, Bishop, A. J., & Pompeu, G. (1997). What children and teachers count as mathematics. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 233-264). Hove, East Sussex, England: Psychology Press.

Abreu, G. de, Bishop, A. J., & Presmeg, N. C. (Eds.) (2002). *Transitions between contexts of mathematical practices.* Dordrecht, The Netherlands: Kluwer Academic.

Abreu, G de, Cline, T., & Shamsi, T. (2002). Exploring ways parents participate in their children's school mathematical learning: Case studies in multiethnic primary schools. In G de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 123-147). Dordrecht, The Netherlands: Kluwer Academic.

Adler, J. (1998). Lights and limits: Recontextualising Lave and Wenger to theorise knowledge of teaching and of learning school mathematics. In A. Watson (Ed.), *Situated cognition and the learning of mathematics* (pp. 162-177). Oxford, England: University of Oxford Department of Educational Studies.

Adler, J. (2001). *Teaching mathematics in multilingual classrooms.* Dordrecht, The Netherlands: Kluwer Academic.

Arcavi, A. (2002). The everyday and the academic in mathematics. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 12-29). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Ascher, M. (1991). *Ethnomathematics: A multicultural view of mathematical ideas.* New York: Chapman and Hall.

Ascher, M. (1995). Models and maps from the Marshall Islands: A case in ethnomathematics. *Historia Mathematica, 22,* 347-370.

Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures.* Princeton, NJ: Princeton University Press.

Ascher, M. & D'Ambrosio, U. (1994). Ethnomathematics: A dialogue. *For the Learning of Mathematics, 14*(2), 36-43.

Barton, B. (1996). Making sense of ethnomathematics: Ethnomathematics is making sense. *Educational Studies in Mathematics, 31,* 201-233.

Bishop, A. J. (1988a). *Mathematical enculturation: A cultural perspective on mathematics education.* Dordrecht, The Netherlands: Kluwer Academic.

Bishop, A. J. (1988b). Mathematics education in its cultural context, *Educational Studies in Mathematics, 19*(2), 179-191.

Bishop, A. J. (2002a). The transition experience of immigrant secondary school students: dilemmas and decisions. In G. de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 53-79). Dordrecht, The Netherlands: Kluwer Academic.

Bishop, A. J. (2002b). Mathematical acculturation, cultural conflicts, and transition. In G. de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 193-212). Dordrecht, The Netherlands: Kluwer Academic.

Bishop, A. J. & Abreu, G. de (1991). Children's use of outside school knowledge to solve mathematical problems at school. In F. Furinghetti (Ed.) *Proceedings of the 15th International Conference for the Psychology of Mathematics Education, Assisi, Italy, 1*, 128-135.

Bishop, A. J., Mellin-Olsen, S., & van Dormolen, J. (Eds.) (1991). *Mathematical knowledge: Its growth through teaching.* Dordrecht, The Netherlands: Kluwer Academic.

Bishop, A. J. & Pompeu, G. (1991).Influences of an ethnomathematical approach on teacher attitudes to mathematics education. In F. Furinghetti (Ed.) *Proceedings of the 15th International Conference for the Psychology of Mathematics Education, Assisi, Italy, 1*, 136-143.

Bourdieu, P. (1995). *Language and symbolic power.* Cambridge, MA: Harvard University Press.

Brenner, M. E. (2002). Everyday problem solving and curriculum implementation: An invitation to try pizza. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 63-92). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Brenner, M. E. & Moschkovich, J. N. (Eds.) (2002). *Everyday and academic mathematics in the classroom. Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Brown, A. & Dowling, P. (1998). *Doing research/reading research: A mode of interrogation for education.* New York: Falmer Press.

Carpenter, T. P., Dossey, J. A., & Koehler, J. L. (Eds.) (2004). *Classics in mathematics education research.* Reston, VA: National Council of Teachers of Mathematics.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology, 3*, 21-29.

Carraher, D. W. & Schliemann, A. D. (2002). Is everyday mathematics truly relevant to mathematics education? In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 131-153). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Civil, M. (1990). "You only do math in math": A look at four prospective teachers' views about mathematics. *For the Learning of Mathematics, 10*(1), 7-9.

Civil, M. (1995). Connecting home and school: Funds of knowledge for mathematics teaching. In B. Denys & P. Laridon (Eds.), *Working group on cultural aspects in the learning of mathematics: Some current developments* (pp. 18-25). Monograph following the 19th Annual Meeting of the International Group for the Psychology of Mathematics Education, Recife, Brazil. Paris, France.

Civil, M. (2002). Everyday mathematics, mathematicians' mathematics, and school mathematics: Can we bring them together? In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 40-62). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Civil, M. & Andrade, R. (2002). Transitions between home and school mathematics: Rays of hope amidst the passing clouds. In G. de Abreu, A. J. Bishop, & N. C. Presmeg, (Eds.), *Transitions between* *contexts of mathematical practices* (pp. 149-169). Dordrecht, The Netherlands: Kluwer Academic.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first grade classroom. In. D. Kirshner & J.A. Whitson (Eds.), *Situated Cognition: Social Semiotic, and Psychological Perspectives* (pp. 151-233). Hillsdale, NJ: Lawrence Erlbaum Associates.

Cobb, P. & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175-190.

Crump, T. (1994). *The anthropology of numbers.* Cambridge, England: Cambridge University Press.

D'Ambrosio, U. (1984). Socio-cultural bases for mathematics education. Plenary presentation, *Proceedings of the 5th International Congress on Mathematical Education, Adelaide, Australia.*

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

D'Ambrosio, U. (1987). Ethnomathematics, what it might be. *International Study Group on Ethnomathematics Newsletter, 3*(1).

D'Ambrosio, U. (1990). The role of mathematics education in building a democratic and just society. *For the Learning of Mathematics, 10*(3), 20-23.

D'Ambrosio, U. (1991). *On ethnoscience.* Campinas, Brazil: Interdisciplinary Center for the Improvement of Science Education.

D'Ambrosio, U. (2000). A historiographical proposal for non-Western mathematics. In H. Selin (Ed.), *Mathematics across cultures: The history of non-Western mathematics* (pp. 79-92). Dordrecht, The Netherlands: Kluwer Academic.

D'Andrade, R. (1987). A folk model of the mind. In D. Holland & N. Quinn (Eds.), *Cultural models in language and thought.* Cambridge, England: Cambridge University Press.

Davis, P. J. & Hersh, R. (1981). *The mathematical experience.* Boston: Houghton Mifflin.

De Bono, E. (1975). *Mechanisms of mind.* Harmondsworth, England: Penguin.

Dörfler, W. (2000). Means for meaning. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 99-131). Mahwah, NJ: Lawrence Erlbaum Associates.

Eglash, R. (1999). *African fractals: Modern computing and indigenous design.* New Brunswick, NJ: Rutgers University Press.

Eisenhart, M. A. (1988). The ethnographic research tradition and mathematics education research. *Journal for Research in Mathematics Education, 19*(2), 99-114.

Ernest, P. (1991). *The philosophy of mathematics education.* London: The Falmer Press.

Eves, H., (1990). *An introduction to the history of mathematics* (6th ed.). Philadelphia, PA: Saunders College Publishing.

FitzSimons, G. E. (2002). *What counts as mathematics? Technologies of power in adult and vocational education.* Dordrecht, The Netherlands: Kluwer Academic.

Freire, P. (1970/1997). *Pedagogy of the oppressed.* New York: Continuum.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, The Netherlands: Kluwer Academic.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures.* Dordrecht, The Netherlands: Kluwer Academic.

Gay, J. & Cole, M. (1967). *The new mathematics in an old culture.* New York: Holt, Rinehart and Winston.

Gee, J. P (1992). *The social mind: Language, ideology and social practice.* New York: Bergin & Garvey.

Geertz, C. (1973). *The interpretation of culture.* New York: Basic Books.

Gerdes, P. (1986). How to recognise hidden geometrical thinking: A contribution to the development of anthropological mathematics. *For the Learning of Mathematics, 6*(2), 10-17.

Gerdes, P. (1988a). On possible uses of traditional Angolan sand drawings in the mathematics classroom. *Educational Studies in Mathematics, 19*(1), 3-22.

Gerdes, P. (1988b). On culture, geometrical thinking and mathematics education. *Educational Studies in Mathematics, 19*(2), 137-162.

Gerdes, P. (1994). Reflections on ethnomathematics. *For the Learning of Mathematics, 14*(2), 19-22.

Gerdes, P. (1997). *Lusona – Geometrical recreations of Africa.* Montréal, Canada: L'Harmattan.

Gerdes, P. (1998). *Women, art, and geometry in Southern Africa.* Trenton, NJ: Africa World Press.

Gorgorio, N., Planas, N. & Vilella, X. (2002). Immigrant children learning mathematics in mainstream schools. In G de Abreu, J. Bishop, & N. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 23-52). Dordrecht, The Netherlands: Kluwer Academic.

Hall, M. (2000). Bridging the gap between everyday and classroom mathematics: An investigation of two teachers' intentional use of semiotic chains. Unpublished doctoral dissertation, The Florida State University.

Harris, M. (1987). An example of traditional women's work as a mathematics resource. *For the Learning of Mathematics, 7*(3), 26-28.

Holland, D. & Quinn, N. (Eds.) (1987). *Cultural models in language and thought.* Cambridge, England: Cambridge University Press.

Kelly, B. (2003). The emergence of technology in mathematics education. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Kirshner, D. & Whitson, J. A. (Eds.) (1997). *Situated cognition: Social, semiotic, and psychological perspectives.* Mahwah, NJ: Lawrence Erlbaum.

Krause, M. (1983). *Multicultural mathematics materials.* Reston, VA: National Council of Teachers of Mathematics.

Kroeber, A. L. & Kluckhohn, C. (1952). *Culture: A critical review of concepts and definitions.* New York: Vintage Books.

Kuhn, T. S. (1970). *The structure of scientific revolutions.* Chicago: University of Chicago Press.

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, England: Cambridge University Press.

Lakoff, G. (1987). *Women, fire and dangerous things: What categories reveal about the mind.* Chicago: University of Chicago Press.

Lancy, D. (1983). *Cross cultural studies in cognition and mathematics.* New York: Academic Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge, England: Cambridge University Press.

Lave, J. (1997). The culture of acquisition and the practice of understanding. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 17-35). Mahwah, NJ: Lawrence Erlbaum Associates.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge, England: Cambridge University Press.

Lean, G. A. (1986). *Counting systems of Papua New Guinea.* Lae, Papua New Guinea: Research Bibliography, Department of Mathematics, Papua New Guinea University of Technology.

Lerman, S. (Ed.) (1994). *Cultural perspectives on the mathematics classroom*. Dordrecht, The Netherlands: Kluwer Academic.

Lerman, S. (1998). Learning as social practice: An appreciative critique. In A. Watson (Ed.), *Situated cognition and the learning of mathematics* (pp. 34-42). Oxford, England: University of Oxford Department of Educational Studies.

Magajna, Z. & Monaghan, J. (2003). Advanced mathematical thinking in a technological workplace. *Educational Studies in Mathematics, 52*(2), 101-122.

Masingila, J. O. (1993). Learning from mathematics practice in out-of-school situations. *For the Learning of Mathematics, 13*(2), 18-22.

Masingila, J. O. (2002). Examining students' perceptions of their everyday mathematical practice. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 30-39). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht, The Netherlands: D. Reidel.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.

Millroy, W. L. (1992). *An ethnographic study of the mathematics of a group of carpenters. Journal for Research in Mathematics Education* Monograph No. 5. Reston, VA: National Council of Teachers of Mathematics.

Morgan, C. (1994). The computer as a catalyst in the mathematics classroom? In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 115-131). Dordrecht, The Netherlands: Kluwer Academic.

Moschkovich, J. (1995, April). *"Making change" and "making mathematics" as proposals for mathematics classroom practices.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco.

Moschkovich, J. N. (2002a). An introduction to examining everyday and academic mathematical practices. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 1-11). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Moschkovich, J. N. (2002b). Bringing together workplace and academic mathematical practices during classroom assessments. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 93-110). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Nemirovsky, R. (2002). On guessing the essential thing. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 233-256). Dordrecht, The Netherlands: Kluwer Academic.

Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction, 16*(2), 119-172.

Nickson, M. (1994). The culture of the mathematics classroom: An unknown quantity? In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 7-35). Dordrecht, The Netherlands: Kluwer Academic.

Noss, R. (1988). The computer as a cultural influence in mathematics learning. *Educational Studies in Mathematics, 19*(2), 251-268.

Noss, R. (1994). Sets, lies and stereotypes. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 37-49). Dordrecht, The Netherlands: Kluwer Academic.

Noss, R. (1997). Meaning mathematically with computers. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 289-314). Hove, East Sussex, England: Psychology Press.

Noss, R. & Hoyles, C. (1996a). The visibility of meanings: Modelling the mathematics of banking. *International Journal of Computers for Mathematical Learning, 1*(1), 3-31.

Noss R. & Hoyles, C. (1996b). *Windows on mathematical meanings: Learning cultures and computers.* Dordrecht, The Netherlands: Kluwer Academic.

Noss, R., Hoyles, C,. & Pozzi, S. (2000). Working knowledge: Mathematics in use. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 17-35). Dordrecht, The Netherlands: Kluwer Academic.

Noss, R., Pozzi, S., & Hoyles, C. (1999). Teaching epistemologies: Meanings of average and variation in nursing practice. *Educational Studies in Mathematics, 40*(1), 25-51.

Nunes, T. (1992). Ethnomathematics and everyday cognition. In D. A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 557-574). New York: Macmillan.

Nunes, T. (1993). The socio-cultural context of mathematical thinking: research findings and educational implications. In A. J. Bishop, K. Hart, S. Lerman & T. Nunes, *Significant influences on children's learning of mathematics.* (UNESCO Publcation No. 44, pp. 27-42). Paris.

Peirce, C. S. (1992). *The essential Peirce: Selected philosophical writings.* Volume 1 (1967-1893). (Edited by N. Houser & C. Kloesel). Bloomington, IN: Indiana University Press.

Peirce, C. S. (1998). *The essential Peirce: Selected philosophical writings.* Volume 2 (1893-1913). (Edited by the Peirce Edition Project. Bloomington, IN: Indiana University Press.

Philips, S. U. (1993). *The invisible culture: Communication in classroom and community on the Warm Springs Indian Reservation.* Prospect Heights, IL: Waveland Press.

Pinxten, R. (1994). Anthropology in the mathematics classroom? In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 85-97). Dordrecht, The Netherlands: Kluwer Academic..

Pinxten, R., van Dooren, I., & Harvey, F. (1983). *Anthropology of space: Explorations into the natural philosophy and semantics of Navajo Indians.* Philadelphia: University of Pennsylvania Press.

Pompeu, G. Jr. (1994). Another definition of ethnomathematics? *Newsletter of the International Study Group on Ethnomathematics, 9*(2), 3.

Powell, A. B. & Frankenstein, M. (Eds.) (1997). *Ethnomathematics: Challenging Eurocentrism in mathematics education.* Albany: State University of New York Press.

Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics, 36*(2), 105-122.

Presmeg, N. C. (1998a). Ethnomathematics in teacher education. *Journal of Mathematics Teacher Education, 1*(3), 317-339.

Presmeg, N. C. (1998b). A semiotic analysis of students' own cultural mathematics. Research Forum Report, in A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education,* Vol.1 (pp. 136-151). Stellenbosch, South Africa: University of Stellenbosch.

Presmeg, N. C. (2002a). Shifts in meaning during transitions. In G de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 213-228). Dordrecht, The Netherlands: Kluwer Academic.

Presmeg, N. C. (2002b). Beliefs about the nature of mathematics in the bridging of everyday and school mathematical practices. In G. Leder, E. Pehkonen, & G. Törner (Eds.) *Beliefs: A hidden variable in mathematics education?* (pp. 293-312). Dordrecht, The Netherlands: Kluwer Academic.

Presmeg, N. C. (2006). Semiotics and the "connections" standard: Significance of semiotics for teachers of mathematics. *Educational Studies in Mathematics, 61*, 163-182.

Santos, M. & Matos, J. F. (2002). Thinking about mathematical learning with Cabo Verde *ardinas*. In G de Abreu, A. J. Bishop, & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 81-122). Dordrecht, The The Netherlands: Kluwer Academic.

Saxe, G (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Scott, P. R. (1985). Ethnomathematics: What might it be? *Newsletter of the International Study Group on Ethnomathematics, 1*(1), 2.

Selin H. (Ed.) (2000). *Mathematics across cultures: The history of non-Western mathematics*. Dordrecht, The Netherlands: Kluwer Academic.

Setati, M. (2003). Language use in a multilingual mathematics classroom in South Africa: A different perspective. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 151-158). Honolulu, HI: University of Hawai'i Press.

Sfard, A. (1997). The many faces of mathematics: Do mathematicians and researchers in mathematics education speak about the same thing? In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 491-511). Dordrecht, The Netherlands: Kluwer Academic.

Sfard, A. (2000). Symbolizing mathematical reality into being – or how mathematical discourse and mathematical objects create each other. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 37-98). Mahwah, NJ: Lawrence Erlbaum Associates.

Skovsmose, O. (1994). *Towards a critical philosophy of mathematics education*. Dordrecht, The Netherlands: Kluwer Academic.

Smith, J. P. III (2002). Everyday mathematical activity in automobile production work. In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 111-130). *Journal for Research in Mathematics Education* Monograph No. 11. Reston, VA: National Council of Teachers of Mathematics.

Stake, R. E. (2000). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 435-454). Thousand Oaks, CA: Sage.

Steen, L. A. (1990). *On the shoulders of giants: New approaches to numeracy*. Washington, D. C.: National Academy Press

Stenhouse, L. (1967). *Culture and education*. New York: Weybright & Talley.

Strässer, R. (1998). Mathematics for work: A didactical perspective. In C. Alsina, J. M. Alvarez, B. Hodgsen, C. Laborde, & A. Pérez (Eds.), *8th International Congress on Mathematical Education: Selected lectures* (pp. 427-441). Seville, Spain: S. A. E. M 'THALES'.

Taylor, P. C. (1996). Mythmaking and mythbreakiing in the mathematics classroom. *Educational Studies in Mathematics, 31*(1-2), 151-173.

Thomas, R. S. D. (1996). Proto-mathematics and/or real mathematics. *For the Learning of Mathematics, 16*(2), 11-18.

Thompson, A. G (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Treffers, A. (1993). Wiskobas and Freudenthal realistic mathematics education. *Educational Studies in*

Mathematics, 25, 89-108.

Van den Heuvel-Panhuizen (2003). The didactical use of models in Realistic Mathematics Education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9-35.

Vithal, R. (2003). *Towards a pedagogy of conflict and dialogue for mathematics education.* Dordrecht, The Netherlands: Kluwer Academic.

Vithal, R. & Skovsmose, O. (1997). The end of innocence: A critique of 'ethnomathematics.' *Educational Studies in Mathematics, 34*(2), 131-157.

Walkerdine, V. (1988). *The mastery of reason: Cognitive developments and the production of rationality.* New York: Routledge.

Watson, A. (Ed.) (1998). *Situated cognition and the learning of mathematics.* Oxford, England: University of Oxford Department of Educational Studies.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge, England: Cambridge University Press.

Wertsch, J. V. (1998). *Mind as action.* Oxford: Oxford University Press.

Whitson, J. A. (1997). Cognition as a semiosic process: From situated mediation to critical reflective transcendance. In D. Kirshner & J. A. Whitson (Eds), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 97-149). Mahwah, NJ: Lawrence Erlbaum Associates.

Wood, L. N. (1998). Communicating mathematics across culture and time. In H. Selin (Ed.), *Mathematics across cultures: The history of non-Western mathematics* (pp. 1-12). Dordrecht, The Netherlands: Kluwer Academic.

Zaslavsky, C. (1973). *Africa counts: Number and pattern in African cultures.* Chicago: Lawrence Hill Books.

Zaslavsky, C. (1991). World cultures in the mathematics class. *For the Learning of Mathematics, 11*(2), 32-36.

Zaslavsky, C. (1996). *The multicultural mathematics classroom: Bringing in the world.* Portsmouth, NH: Heinemann.

Useful web sites on culture and the learning and teaching of mathematics:

http://www.csus.edu/indiv/o/oreyd/once/once.htm
http://www.geometry.net/pure_and_applied_math/ethnomathematics.html
http://www.dm.unipi.it/~jama/ethno/
http://phoenix.sce.fct.unl.pt/GEPEm/
http://www.fe.unb.br/etnomatematica/
http://www2.fe.usp.br/~etnomat
http://web.nmsu.edu/~pscott/spanish.htm
http://etnomatematica.univalle.edu.co
http://www.rpi.edu/~eglash/isgem.htm
http://chronicle.com/colloquy/2000/ethnomath/ethnomath.htm
http://chronicle.com/free/v47/i06/06a01601.htm
http://chronicle.com/colloquy/2000/ethnomath/re.htm
http://www.ecsu.ctstateu.edu/depts/edu/projects/ethnomath.html

4.3 Teaching for numeracy: Re-examining the role of cultural aspects of mathematics

Wee Tiong Seah
Monash University

4.3.1 Numeracy in the school mathematics curriculum

Developments and advancement – scientific / technological or otherwise – in the world we live in have meant that over the years, we acquire or adapt new knowledge, new skills, new ways of doing the same tasks, new outlooks, new attitudes, and new perspectives. Likewise, the purpose of formal school education in general, and also institutional aims for the teaching and learning of mathematics in schools in particular, have evolved in new forms in response to new sociocultural demands. For school mathematics, one of the significant moves in the early 2000s in countries such as Australia, New Zealand, Sweden and United Kingdom is towards the teaching of mathematics for numeracy. This move acknowledges that general education for all implies that not all school students need to be trained to be mathematicians. Rather, general education for the masses has provided nations with the opportunity to educate their respective citizens so that they acquire the capacity to cope with the evolving changes in related demands in daily life. This is generally observed in most countries, although we are still reminded today that what topics constitute the study of mathematics in schools remain to be socio-politically regulated by different government systems. For example, the study of statistics had been removed from the secondary mathematics syllabi of schools in some countries.

It is in this climate that numeracy assumes a significant role in countries such as Australia, New Zealand, Sweden and United Kingdom. This term is also expressed differently in different cultures, such as 'numerical literacy', 'functional mathematics', and, in the United States, 'quantitative literacy'. In Australia, the notion of numeracy in schools was thrown in the spotlight in the 1999 *Adelaide Declaration on National Goals for Schooling in the Twenty-First Century*, in which Goal 2.2 states that students should have "attained the skills of numeracy and English literacy; such that every student should be numerate, able to read, write, spell and communicate at an appropriate level" with mathematics as one of eight key learning areas.

Numerous definitions of numeracy have been made, reflecting different approaches (Perso, 2006), and presenting "an enormous set of conceptual and pedagogical issues" (Coco, Kostogriz, Goos, & Jolly, 2006). In this chapter, the approach which appears to have been embraced by the intended curricula in many education systems (such as those in the UK and the different states within Australia) will be the reference for discussion. In this 'mathematical literacy' approach (Perso, 2006), the description made by the Australian Association of Mathematics Teachers (1997) is apt, in which "to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (p. 15). Likewise, Willis (1998) associates numeracy skills with the capability of one to conduct 'intelligent practical mathematical action in context'.

4.3.2 Numeracy in Victorian primary schools

In the state of Victoria, Australia, much of the primary school mathematics curricular activities has been influenced by the findings and recommendations of two state-level research projects in the early 2000s, namely, the Early Numeracy Research Project (Victoria Department of Education Employment and Training et al, 2002) and the Middle Years Numeracy Research Project (Victoria Department of Education Employment and Training et al, 2001). For example, students entering the first formal school year (i.e. Grade Prep) will have their individual numeracy standards assessed (and subsequently reported) through the School Entry Assessment, which takes the form of individualised interview sessions. While the school subject is formally called 'mathematics', the introduction of initiatives such as 'numeracy blocks' in lesson planning has meant that many primary school teachers in Victoria use the terms 'mathematics' and 'numeracy' interchangeably, a phenomenon which is also frequently observed in the UK. This is despite the fact that at the intended curriculum level, there is only one 'unforced' use of the term 'numeracy' in the chapter of the Victorian Essential Learning Standards for Mathematics (VCAA, 2005), which spells out the context of the discipline in the school system before the outcomes for each learning level are specified in subsequent chapters. Interestingly, that one inclusion may be interpreted to make an explicit distinction between mathematics and numeracy, where one aim of the curriculum is for students to "demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society" (p. 4). The other occurrences of the term 'numeracy' are in the last section (p. 9), which describes how student achievement is reported against the National Numeracy Benchmarks.

While numeracy in the early years of schooling is mostly (but not exclusively) concerned with equipping students with the necessary understandings (such as number sense and data sense) to facilitate more efficient learning in the middle years and beyond (such as how basic number sense aids in the development of algebraic thinking as described in Willis, 2000), the focus of this chapter is more on the numeracy lessons planned for the middle years of schooling (typically, the last two years of primary school education and the first two years of secondary school education) and beyond, where mathematical tasks in context allow students to apply what Willis (1998) calls the 'intelligent practical mathematical actions'. In this context, numeracy appears to be promoted in Victorian primary school classrooms in one of two ways. In the first way, mathematical concepts and/or processes are introduced to students, before opportunities for student applications and/or investigations are provided. In the second way, numeracy is delivered interdisciplinarily and contextualised in sessions where students work on investigations / projects to solve identified or pre-specified problems, often in groups. The necessary mathematical skills are acquired by students in context, and teachers are expected to direct student learnings of relevant, emerging mathematical concepts.

4.3.3 Does numeracy mirror daily life?

Both these teaching approaches are likely to not foster students' affective development in mathematics / numeracy, however. Given the reality of school lessons and the accompanying constraints, there will always be a limit to the authenticity and realism of mathematical problems in numeracy lessons. This may be especially so in the first of the teaching approaches, where the initial classroom learning of mathematical concepts and processes may stimulate students to ask for the reasons why they need to be learnt, or when they will ever be used in life. However, this does not mean that students experiencing the second, more holistic education approach would feel that what they learn in numeracy lessons are relevant to numeracy demands in daily life. The validity of this point is supported by observations that quite often, authentic numeracy activities are not really authentic, though it must be reminded at the same time that it is not attributed to teachers alone.

A few examples may be illustrative. In Victoria (as in most other educational systems in the world), rules for rounding decimal numbers ending with the numeral 5 are not the same as those used in similar operations in the Australian optical fibre industry (Fielding, 2003). Similarly, Victorian primary school students may learn to execute rounding to the nearest five cents in schools, yet children know that the cash registers of different retail outlets conduct the same operation for cash transactions in different ways (i.e. some round off, some round down, and others round up to the nearest five cents). The use of modals (such as 'might' and 'should') in numeracy questions might also stimulate some children to think that the price for several items of a particular commercial product needs not be proportional to their unit price (as is the case in real life), a situation which is likely to be assessed as their teachers to represent wrong computations!

This gap between what constitute school mathematics and daily/workplace mathematics remains not only because of the nature of schooling with its constraints and conditions, but also because very often the invisibility of mathematics in daily/workplace tasks has been 'black-boxed' paradoxically by the discipline itself. Latour (1999) refers 'black-boxing' to

> the way scientific and technical work is made invisible by its own success. When a machine runs efficiently, when a matter of fact is settled, one need focus only on its inputs and outputs and not on its internal complexity. (p. 304)

4.3.4 Emphasising the understanding of others' mathematics

Thus, introduction of numeracy in mathematics classrooms (at both the primary and secondary school levels) needs to be complemented by an explicit acknowledgement by schools and teachers that the very nature of lesson timetabling and also often of the intended curriculum mean that school mathematics will neither be academic mathematics, nor will it be daily/workplace mathematics. In education systems such as those found in Australia where numeracy is the aim for mathematics teaching in schools, teacher explicitness in highlighting that not all real-life situations can be reasonably modelled within the constraints of lesson planning is important in sustaining student interest and faith in the subject, and in emphasising instead the value in one's capacity to apply mathematical knowledge in daily life or in the workplace.

How might this capacity be developed? Williams and Wake (2007) advocate that

> to make sense of workplace mathematics, outsiders need to develop flexible attitudes to the way mathematics looks, to the way it is 'black-boxed' .... Students had little awareness that the College mathematics they had learnt was itself idiosyncratically 'conventional', and that mathematical conventions might vary in time and place. (p. 338)

The intended curriculum can play a crucial contributing role here while still promoting the satisfaction of numeracy outcomes. In fact, it potentially adds depth to what it means to be functionally numerate in the society when students acquire the skill of "making sense of the mathematics of others" (Williams & Wake, 2007, p. 339). These 'others' would refer to members of a diversity of cultures – national, ethnic, workplace, religious, gender, for examples.

In this regard, Bishop's (1996) conception that cultures universally engage in six types of activities from which mathematical ideas evolve is a particularly useful one. At one point or another in time and space, all cultures find the need to count, locate, measure, explain, design, or play, and, Bishop (1996) argued, all cultures perform all these six mathematical activities. From the pedagogical point of view, this is more than enabling students (and teachers) to appreciate that different cultures perform mathematics tasks differently, that, for example, the Anglo-Australian parents emphasise the skill of counting in their young, whilst their Aboriginal-Australian counterparts traditionally emphasise the importance of their children to perceive their respective locations within the three-dimensional space. Without disputing that this in itself has rich mathematics pedagogical implications, the focus here is on the potential for students to understand mathematical situations.

Thus, making sense of how cashiers often provide the change for a cash transaction through 'adding up' rather than subtracting has the potential of enabling students to go beyond making sense and understanding. It also provides an opportunity for students to relate this change-giving method to the 'textbook' method (of using the decimal subtraction algorithm), to relate and understand more of the operations of addition and subtraction, to evaluate relative strengths and constraints, to explain how formal, school and workplace mathematics support one another.

Clearly, there is also the bonus advantage of fostering students' higher-order cognitive outcomes.

If a numeracy program seeks to expose students to real-life mathematics in the daily life or workplace, then one which identifies and discusses how mathematical processes are executed by 'others' can bring the mathematics out of the 'black-box', thereby making the underlying concepts and ideas more visible. It also acknowledges the limitation of the school mathematics framework in possibly discussing and examining selected mathematical practices of selected cultures, without appearing to advocate that the numeracy developed through school mathematics is sufficient in equipping the individual student to apply related concepts and skills to the world beyond the school gates. In fact, by highlighting the cultural aspects of mathematical practices in numeracy classes, student attainment and mastery of numeracy is situated within a cultural community of practice, which reflects the very essence of numeracy, that is, for effective sense-making and negotiation of demands arising from family, workplace, community and civis lives.

A numeracy approach which acknowledges the inevitable gaps amongst academic, school and daily/workplace mathematics can highlight to students that if mathematics is to be regarded as a form of language for communication, then different genres of this language evolve from the different contexts within which this language is used. Thus, school mathematics is one such genre, and the extent to which policy makers desire to inculcate numeracy in the young generation through school mathematics will be reflected in its closeness with the genres of daily/workplace mathematics. Most importantly, such an acknowledgement can address student sceptism abut the value of school mathematics, and the explicitness can foster student appreciation of the genre of school mathematics.

Students' cognitive 'movements' within and across the different genres of mathematics is also educationally valuable in that a meaningful appreciation and understanding of how 'others' do mathematics can promote positive development of their individual cultural intelligence [CQ]. First conceptualised by Earley and Mosakowski (2004), CQ extends the notion of emotional intelligence and is a measure of the extent to which an individual who is

> an outsider [to a particular culture] has a seemingly natural ability to interpret someone's unfamiliar and ambiguous gestures in just the way that person's compatriots and colleagues would, even to mirror them. (p. 139)

A numeracy curriculum which provides students opportunities to relate mathematical concepts and ideas to its various genres in different occupational, ethnic, gender or other categories of cultures is an exercise in interpreting these other cultures' unfamiliar ways of doing mathematics. This hidden curriculum of cultivating students' individual cultural intelligence extends beyond mathematics/numeracy learning, to the learning of other educationally desirable skills of social learning, such as interpersonal skills.

4.3.5 Closing remarks

The discussion above re-examines the role of the cultural aspects of mathematics in the teaching of numeracy in schools. It is expected to be relevant to educational systems in both developed nations and developing ones that seek to benefit from being part of the knowledge-production economies. Participation in these economies (as compared to agricultural, manufacturing, service and information economies) require more than ever before a workforce whose members are functionally, if not critically, literate and numerate. The argument made here is that achieving meaningful and sustained numeracy rates amongst a country's students may be supported by intended and implemented curricula which acknowledge the differences amongst academic, school, daily-life and workplace mathematical practices. By promoting student understanding of culturally-different ways of doing mathematics, the black-boxing of mathematical concepts and ideas in daily life and in the workplace can be reduced. This should optimise student progression through the school years with an increasing understanding and confidence in relating academic and school mathematics to the demands of tasks they encounter or will encounter in the respective personal and working lives they lead, with the additional potential of cultivating their individual cultural intelligence in the process.

Reference

Australian Association of Mathematics Teachers. (1997). *Numeracy = Everyone's business*. Adelaide, Australia: Australia Department of Employment, Education, Training and Youth Affairs.

Bishop, A. J. (1996, June 3-7). *How should mathematics teaching in modern societies relate to cultural values --- some preliminary questions.* Paper presented at the Seventh Southeast Asian Conference on Mathematics Education, Hanoi, Vietnam.

Coco, A., Kostogriz, A., Goos, M., & Jolly, L. (2006). *Dangerous liaisons: Home-business-school numeracy networks.* Paper presented at the Australian Association for Research in Education 2005 Conference, Parramatta, Australia.

Earley, C. P., & Mosakowski, E. (2004). Cultural intelligence. *Harvard Business Review, 82*(10), 139-146.

Fielding, P. (2003). Are we teaching appropriate rounding rules? In B. Clarke, A. Bishop, R. Cameron, H. Forgasz & W. T. Seah (Eds.), *Making mathematicians* (pp. 395-401). Victoria, Australia: The Mathematical Association of Victoria.

Latour, B. (1999). *Pandora's hope: Essays on the reality of science studies*. Cambridge, MA; London: Harvard Univeristy Press.

Perso, T. (2006). Teachers of mathematics or numeracy? *Australian Mathematics Teacher, 62*(2), 36-40.

Victorian Curriculum and Assessment Authority. (2005). *Discipline-based learning strand: Mathematics*. East Melbourne, Victoria: Victorian Curriculum and Assessment Authority.

Victoria Department of Education Employment and Training, Catholic Education Commission of Victoria, & Association of Independent Schools of Victoria. (2001). *Successful interventions: Middle years numeracy research project 5-9 (stage 2)* (Final report). Melbourne.

Victoria Department of Education Employment and Training, Catholic Education Commission of Victoria, & Association of Independent Schools of Victoria. (2002). *Early Numeracy Research Project (1999-2001): Summary of the final report* (Report). Melbourne: Victoria Department of Education and Training.

Williams, J., & Wake, G. (2007). Black boxes in workplace mathematics. *Educational Studies in Mathematics, 64*, 317-343.

Willis, S. (1998, September 27-30). *Numeracy for the(ir) future: Rite or right?* Paper presented at the National Conference of the Australian College of Education, Canberra, Australia.

Willis, S. (2000). Strengthening numeracy: Reducing risk. In *Improving numeracy learning* (pp. 31-33). Melbourne, Australia: Australian Council for Educational Research.

4.4 Minority Students and Teacher's Support: Reviewing International Assessment Results and Cultural Approach

Hideki Maruyama
National Institute for Educational Policy Research (NIER)

4.4.1 Introduction

One of the most influential international comparative studies on education is the Programme for International Student Assessment (PISA) for recent years. The program has been started as a response to the need for cross-nationally comparable evidence on student performance by the Organisation for Economic Co-operation and Development (OECD) in 1997, and the first assessment was conducted in 2000. The results of PISA revealed wide differences in the extent to which countries succeed in enabling young adults to access, manage, integrate, evaluate and reflect on written information in order to develop their potential. At the same time, the results showed the common difficulties across countries and unique issues within them. Some countries recognize the significant variation of the performance level among their children and concerns about equity in the distribution of learning opportunities, although they have the equivalent of several years of schooling and sometimes despite high investments in education as the other better performed countries.

Some European countries found reasons why the disparity exists between student's high and low performances. Immigrant students have a lot of different backgrounds from natives as both personal concerns like a language and social issues such as economic conditions. The results of the international assessments introduce the clear difference, as quantitative data, and the governments have started to focus on students' socio-economic backgrounds for better education system. And meanwhile, teachers in a host country for the immigrant students are natives in many countries.

Our IDEC research project started to describe phenomena of mathematics education and classroom interaction in the wide range of countries. We are prone to imagine that instructions in classroom cause lower performance of the students in the developing countries because of its poor pedagogical approach such as chalk and talk. The results of the first and second year survey can be interpreted as a finding that the students who do not use a test language as their first language achieve lower academic performance. The author believes the language problem is an explicit aspect of cultures which students generally have in their classrooms because by and large they are always in multi-cultural environment and the interaction between a teacher and students must be complicated like those in European classrooms. Minority students need more supports especially from teachers because the teacher's role in classroom is rather a controller in the project countries and their outer views are limitedly sensitive to the backgrounds of the minority students.

In this review, the author summarizes the PISA results of immigrant students in mainly Europe with literature and analyzes the backgrounds of the immigrants from the cultural point of view. The author also tries to discuss the context of the classroom in developing countries in which local teachers might miss the cultural backgrounds of the minority students.

4.4.2 Findings from the Assessments Concerning the Immigrant Students

Many types of education assessments are available when we try to analyze student's performance today. Here we look at PISA and its related research because it is one of the most influential to education in the world today. Following the brief description of the world wide assessment, let us see the performance of immigrants and their language problem.

(1) A Large-Scale International Assessment

PISA seeks to measure how the 15-year-old children, as the age of ending compulsory schooling in many countries, are prepared to use their knowledge and skills to meet real life challenges of today's knowledge society, rather than merely on the extent to which they have mastered a specific school curriculum. PISA is the most comprehensive and rigorous international program today for the participating countries/regions to assess their student performance and to collect data on student, family, and institutional factors that can help to explain differences in performance.

PISA has three cycles of its implementation procedure and three domains of student's knowledge and skills called "literacy,"(1) namely, reading, mathematical, and scientific ones. Each cycle covers all the domains but its focus is mainly on one of the three domains. The first PISA assessment was conducted, focusing on reading literacy, in 2000 and on mathematical literacy in 2003. More than a quarter of a million students and their school managers participated in the each. There will be the first final cycle for about 60 countries, including 30 OECD countries, on the scientific literacy in 2006. Besides the literacy, the participants also responded their family and economic backgrounds, their motivation to learn, their beliefs about themselves, and their learning strategies.

(2) Lower Performance of Immigrant(2) Students in the Assessments

The results of the series of PISA provide us with the data the students whose parents are not native show lower performance than native students in many countries. The greatest gap, of 93 points in mathematics scores, is in Germany, and students themselves born outside the country tend to lag even further behind, in Belgium by 109 points.(3) While circumstances of different immigrant groups vary greatly, and some are disadvantaged by linguistic or socio-economic background as well as their migrant status itself, two particular findings are worrying for some countries. One is the relatively poor performance even among students who have grown up in the country and gone to school there. The other is that after controlling for the socio-economic background and language spoken at home, a substantial performance gap between immigrant students and others remains in many countries: it is above half a proficiency level in Belgium, Germany, the Netherlands, Sweden and Switzerland.(4)

Schnepf (2004) used the results of ten countries(5) which had participated in IEA's TIMSS and PIRLS in addition to PISA because all the three assessments collected information in the same format regarding immigration variables of their parents origin and language used at home.(6) She explained the immigrants achieved significantly lower test scores than natives in almost all countries and surveys. She concludes the promoting language education of immigrant students and decreasing school segregation are likely to have a positive outcome on students' performance, based on the three findings from her statistical analysis: 1) the immigrants' socio-economic background might be lower than natives and be immigrants' educational disadvantages in some countries; 2) immigrants' educational disadvantage might derive from their problems of integration into the host country; and 3) the process of selection/acceptation of immigrants to the host country is likely to impact upon their achievement results.

Ammermüller (2005) used the data from PISA extension study(7) in Germany and tried to find determinants that the results of the immigrant students were lower than native students. He found that the use of different language at home from German in classroom is typical characteristics for the immigrant students, in addition to the higher grade level of German students and more home resources as measured by the amount of books at home. These factors explained the test score gap would decrease by 40 to 49 %, if the immigrant students had had the same returns to student background as native students. However, he also found that differences in parental education and family situation were less influential.

Rangvid (2005) made an analysis of the immigrant students' performance in Denmark by using the results of PISA-Copenhagen which is based on international PISA study design. The PISA-Copenhagen results suggest similar performance between the native students and the students who have one immigrant parent, but the mean scores of the immigrant students who have both non-native parents are much lower. She also pointed out the immigrant students experienced lower teacher expectations and the peer composition at schools attended by immigrant students was potentially less conducive to academic achievement.

The results of the international assessments show that immigrant students achieve lower level of knowledge and skills than natives in many European countries. We can assume that the language proficiency of the immigrant students is one of the main reasons why their performance is lower than natives.

(3) Teaching Limited English Proficient Students

Secada and Carey (1990) argue school practices might forge the links that the level of English language proficiency affects mathematics achievement. They believe that children enter school with a broad range of understandings about mathematical concepts. Instruction should build upon and develop the children's knowledge so that it would provide children's learning in mathematics and contribute to their confidence about their abilities. They describe one effort to accomplish this approach called Cognitively Guided Instruction (CGI).[(8)]

Although the CGI assessment focuses on the processes by which students get an answer and seems to be quite learner-centered approach to some countries in the IDEC project, the unique procedure relates with intercultural communication. To start assessment of student's thinking, a teacher should provide students with counters, pose a problem, and if the student responds, follow up by asking how he/she figured it out. The teacher then uses additional questions to help him/her clarify what it meant. Alternatively, if a student does not solve the problem correctly, the teacher has at least five options.[(9)] The point is to begin a conversation about the problem. Many limited English proficient students receive little encouragement to speak about their ideas and many girls are socialized to defer to boys. If teachers tend to call on students who answer first, the rest will often be left out of the conversation. Therefore, teachers need to reach out to these children and to be sure that they are included in the classroom's processes.

They recommend that students should be able to discuss how they are thinking about problems during mathematics. As a result, students learn mathematics from one another as well as from the teacher. This validates their thinking and students begin to recognize that both teacher and students are a source of knowledge in the classroom. They might share the backgrounds more.

4.4.3 Education for the Cultural Minority: *integration as policy and personal level*

We have seen the lower performance of the immigrant students. In addition to the language issue, minority student's cultural backgrounds call for our attention to more influence in classroom interaction and their academic performance than language. European countries such as Germany, Denmark, and the Netherlands have received immigrants in past decades have emphasized the efforts for integration policy of the immigrants into the native societies as a solution. But the policymakers found the results of the assessments show the integration policy still faces challenges. At school level, many teachers understand immigrant students' poor achievement stems from their language problem and different cultural backgrounds.

Immigrants also have difficulties, however sometimes impossibility, to cope with different environments. Immigrant students live with host culture at school and original culture at home.

Having focused on language, when their language ability is not enough to understand instruction in class, they not only feel less confident in learning lessons and participating classroom activities but also hesitate to ask what they do not understand and develop little concept of mathematics at last.

This section deals with cultural experience of the immigrant students, their cultural learning process, and possible supports by teachers. Descriptions can apply to some examples of IDEC project because this paper takes the problems of the immigrant students in mostly European countries as similar phenomena to the "cultural minority" groups.(10) Before proceeding, we should also remind ourselves of the risk of overgeneralization.

(1) Cultural Experience of Immigrants

Immigrant students live in two or more different cultures(11) and are also in co-culture groups.(12) Parents' views and values affect students, especially in childhood prior to schooling so strongly that children have bias and feel uncomfortable when they first face different views from other students in school. For example, they experience the different manners of expression in low-context(13) culture in Europe and feel the large gap between their own in-group and other out-group cultures.(14) And meanwhile, the native students of dominant culture, or majority students, also feel difference of paralanguage such as gestures and facial expression when interacting with "strangers." As mental defense mechanism that they try to avoid the unfamiliarity and reduce uncertainty, they tend to stay with those who have similar backgrounds for their comfort through the interaction among students.

This grouping process is different from the psychological stage as gang age but rather based on mental state called ethnocentrism. Ethnocentrism is a universal tendency of the belief that one's own group or culture is superior to other groups or cultures. People generally interpret other cultures according to the values of their own. When the students consciously separate in- and out-groups, they attribute the other group's mistake, for instance, to the characteristics of the whole group but own group's to personal. A German female student sees a Turkish student wearing headscarf in physical education class, she tend to thinks all the Turkish girls wear it. But she does not think that all the German girls go to Sunday school because it depends on family's attitude to daughter. This is not a serious problem very much until students start to develop prejudice from the bias after cultural encounter because they are barrier to understanding others. The students need a mentor to follow up their observation and understanding the difference properly.

Both minority and majority groups generally have bias and possibilities of conflicts between the two. The host society is basically more affirmative and supportive to the majority, and the minority groups have to follow many of its social norms. The society enforces the minority integration or assimilation to be the citizen so that a strong reaction such as marginalization sometimes comes up from the minority group. It is understandable that a group of immigrant students seems culturally homogeneous to the majority students and native teachers because their appearance and attitude resemble each other. And their communication can be categorized as intra-cultural communication if we compare communication between explicitly different cultures. The large scale survey and analysis also categorize they are the same group; otherwise there are too many cultural-specific cases. When we easily categorize and judge them based on ethnocentric view, prejudice overcasts our eyes.

Here, we need to shift our views. What we think as intra-cultural communication can be actually intercultural when we see their experiences by *emic* approach. This view is originally from the standpoints of Pike (1967) who has generalized from phonetics and phonemics to what he has called the *"etics"* and *"emics"* of all socially meaningful human behavior. Whatever the names one may choose to call them, the concepts of *etic* and *emic* are indispensable for understanding the problems of description and comparison. *Etic* is pertaining to a concept that is culture-general and is therefore easy to describe when we examine cultures from outside. *Emic* is pertaining to a concept that is culture-specific and is therefore difficult to translate from one language to another when we examine a culture from the inside. In short, *etic* is external and *emic* is internal view.(15)

It might be much easier for us to perceive the various backgrounds of immigrant students or cultural minority if we try to observe from the inside.(16) Religious consciousness may be one of the biggest and typical issues which majority/minority students and teachers concern in classroom. Parker-Jenkins (1995) illustrates an example:

> "Muslim children are taught to respect and not question elders or those in authority and accordingly they may appear to be stereotypically passive and accepting. If a child does not question teachers it does not automatically denote lack of interest or intelligence but rather respect for their authority and position. Furthermore, the value placed on deference to authority, and *haya* or modesty within the Islamic consciousness, helps to explain the difficulties and contradictions some Muslim children may experience in participating in [British] state school activities which call for assertiveness and extrovert behaviour."(17)

(2) Acculturation: Integration or Assimilation of the Immigrants

The immigrant students sometimes have no choice but follow the dominant group's norms. The students learn appropriate behaviors by living in a certain cultural context. This is called acculturation which could contain four types of strategies.

Herskovits and other American anthropologists (1936) proposed the following definition of acculturation. "Acculturation comprehends those phenomena which result when groups of individuals having different cultures come into continuous first-hand contact, with subsequent changes in the original cultural patterns of either or both groups."(18) Acculturation differs from cultural change in which the source of change is internal within the culture and either from enculturation.(19)

Berry (1992) categorizes acculturation strategies in which individual or group wishes to relate to the dominant society into four patterns: integration, assimilation, separation, and marginalization. They are conceptually the result of an interaction between ideas deriving from the culture change and the intergroup relations. In the former the central issue is the degree to which one wishes to remain culturally as one has been as opposed to giving it all up to become part of a larger society; in the latter it is the extent to which one wishes to have day-to-day interactions with members of other groups in the larger society as opposed to turning away from other groups and relating only to those of one's own group.

> When these two central issues are posed simultaneously, a conceptual framework [Fig. 1] is generated that posits four varieties of acculturation. When an acculturating individual does not wish to maintain culture and identify and seeks daily interaction with the dominant society, then the assimilation path or strategy is defined. In contrast, when there is a value placed on holding onto one's original culture and a wish to avoid interaction with others, then the separation alternative is defined. When there is an interest in both maintaining one's original culture and in daily interactions with others, integration is the option; here there is some degree of cultural integrity maintained, while moving to participate as an integral part of the larger social network. Integration is the strategy that attempts to "make the best of both worlds." Finally, when there is little possibility or interest in cultural maintenance, and then marginalization is defined.(20)

Although the officials try to promote integration of immigrants into the society, the immigrants sense its uncertainty and meanwhile the host local people are not interested in the promotion very much. Parker-Jenkins (1995) draws multiculturalism in Britain and details a phase of assimilation took place along with great restriction of immigration to the country in 1950s.

The author also interviewed German scholars about immigrants' integration in Germany and recognized some cases could be categorized as assimilation which possibly leads marginalization.[21] The immigrants or minority group of people follow the norms of the majority society with mental conflict.

Issue 1

Is it considered to be of value to maintain cultural identify and characteristics?

Issue 2		"YES" ↓	"NO" ↓
Is it considered to be of value to maintain relationships with other groups?	"YES" →	Integration	Assimilation
	"NO" →	Separation	Marginalization

Fig. 1 Four varieties of acculturation (Berry, 1992)

(3) Support from Teacher in Classroom

The minority students need supports from teachers and other students. Friends are the strongest supporter in and out of classroom but if no one wants to be a friend, then the immigrant students have only teachers to ask for help. Teachers become more important being today not only for the academic achievement of minority students but also for their coping strategy with the dominant culture. OECD recently operated a project of policy research on teachers in its member countries,[22] and its report introduces the importance of teachers:

> The demands on schools and teachers are becoming more complex. Society now expects schools to deal effectively with different languages and student backgrounds, to be sensitive to culture and gender issues, to promote tolerance and social cohesion, to respond effectively to disadvantaged students and students with learning or behavioural problems, to use new technologies, and to keep pace with rapidly developing fields of knowledge and approaches to student assessment. Teachers need to be capable of preparing students for a society and an economy in which they will be expected to be self-directed learners, able and motivated to keep learning over a lifetime.[23]

Teachers are not allowed to give lessons only for subjects anymore but need to keep learning in today's knowledge-based society. Communication is clearly important in classroom among students, and teachers actively receive feedbacks from them. In addition to receiving a message from them, teachers may have to imagine students' thoughts based on understanding on their backgrounds. It requires more efforts than before so that teachers might lose what to do first.

To imagine the student's needs, the first and explicit aspect of their backgrounds is their language. Their language proficiency and teacher's understanding of their language at home help both students and the teacher. The students' explanations may be tentative not only because of content mastery but also because of the language used in classroom.[24] Moreover, even when using their native languages, many students have difficulty in expressing their thoughts in as sophisticated a manner as teachers might like. Teachers must recognize students judge people who sound like they know what they are talking about have knowledge, while those who express themselves poorly do not have such knowledge. Teachers need to begin with what students understand and with how they can express their understandings. Therefore, mathematics teachers are required the knowledge of both subject and cultures today.[25]

To know more about cultures, Hofstede's framework can be helpful. Geert Hofstede (1984) categorizes four dimensions of values by surveying across 50 countries[26]: 1) power distance, 2) uncertainty avoidance, 3) individualism and collectivism, and 4) masculinity and femininity. Power distance is a value regarding the degree to which a society accepts the fact that power in institutions is distributed unequally. If it is small, equality is highly sensed. Uncertainty avoidance is the attempt to understand the other person in a communication setting by increasing predictability; brought about by the high need to understand oneself and others in interpersonal situations. If a society has strong uncertainty reduction, it is less tolerance to deviant behaviors of others. Individualism gives more importance to individual's will than to group, and collectivism is vice versa. Masculinity refers to emphasis on heroism, argument, achievement and etc., while femininity prioritizes compassion to the weak, human relation, quality of life and etc. Hofstede compares working staff at country level, but teachers in classroom can identify themselves with the framework and locate the background of student's behaviors within culture and between co-cultures. This shift of viewpoint can be *emic* approach from the teacher's *etic* views.

When teaching side shifts its view to the inside, teachers would be more able to understand minority student's backgrounds besides language. As an example of Muslim students in Britain, Parker-Jenkins (1995) categorizes student needs into four:(27)

1) As religious/cultural needs: Time of school assemblies should be coordinated for daily prayers. School diet should be considered and understand the need of fasting. Dress code should be flexible, especially in physical education for girls, etc.
2) As curriculum needs: Sex education should be paid more attention, especially need for HIV issue. Language instruction for minor languages should be encouraged. Islamic dimensions should be promoted, etc.
3) As linguistic needs: Support for English language competence/acquisition, etc.
4) As general needs: Home-school links should be emphasized because enlisting parental support is vital to all attempts to accommodate Muslim needs within the school system. Muslim organizations are encouraged to become more proactive in attempting to bring about change within the educational system for school governance.

She conducted the survey on the above needs and practices of British schools and found the similarities and differences between the perceptions of head teachers in state schools and private Muslim schools over Muslim children's needs. The similarities were in the issue of religion and religious observance, and the differences were the "conceptualization of religion" in the children's lives, English language acquisition, effective home-school links, and balanced curriculum. She points out that English language acquisition is considered the major area of concern by state school administrators because it is the medium of instruction in British schools but opportunity for various groups to learn their own language need to be made for its cultural importance and the matter of parity of languages. She also pointed that curriculum as a "transmission of the culture" transmits minority cultures and the right balance between the indigenous and minority cultures is an area of concern. She suggested understanding the processes of assimilation and acculturation within multicultural education is pertinent through learning from each other and emphasized that teachers cannot be expected to translate educational theory into practice without adequate training and the selection and recruitment of teachers from minority backgrounds are needed.

4.4.4 Conclusion: Cultural Sensitivities of Teachers

In this review, we have seen the immigrants' low performance shown by the international assessments and cultural backgrounds, their cultural experiences and personal coping strategies, and teacher's room to learn to shift the view to the insider. We may summarize them in the following three.

(1) *Importance of teacher's role*

The largest source of variation in student learning is attributable to differences of students' personal and cultural backgrounds such as their individual abilities, attitudes, family, and community. These factors are difficult for the outsiders to influence. On the other hand, factors concerning teachers and teaching are the most important influences on student learning. Teacher's education and experience influence on student achievement. But as OECD reports "a point of agreement among the various studies is that there are many important aspects of teacher quality that are not captured by the commonly used indicators such as qualifications, experience and tests of academic ability,"(28) we can focus more on teacher's potential to recognize the student backgrounds for their learning. In the IDEC project, we shall also remember more cultural-specific aspects, for example, the traditional African education(29) is based on oral communication as Reagan (2005) examines. The traditional education and training have its own contexts, and local teachers must be tolerance and sensitive to them. This is because "formal schooling played important roles in many non-Western traditions as one of fundamental distinctions."(30)

(2) *Cultural sensitivity*

OECD (2005) introduces the five broad areas for the development of professional knowledge and expertise in teaching, as Shulman (1992) identified(31): teacher's behavior, cognition, content, character, and knowledge of and sensitivity to cultural, social, and political contexts and environments of the students. The fifth area treats teacher's sensitivity to cultural backgrounds of the minority students and it is what this paper finds the importance. Not only OECD but all teachers who have minority students already know classrooms are becoming increasingly diverse with students of different cultural and religious backgrounds. "Teachers are expected to work for social cohesion and integration by using appropriate classroom management techniques and applying cultural knowledge about different groups of students."(32)

Teachers are symbolized powerful authority in some cultural contexts. For this reason, teachers themselves are required for tolerance and sensitivity. Children with limited English proficiency generally have very difficult time learning in regular classrooms and easily lag behind leading to lack of confidence in their academic performance. Less confidence in language and communication negatively affect student's motivation to learn, and a teacher sees student's attitude and has low expectation so that the vicious cycle of impotence appears within the minority students in classroom. In the similar context, special attention should be paid to girls because they are not encouraged to express their thoughts more than boys and their performance is lower than male as shown by the international assessments. This relates with the concepts of empowerment of women and equal opportunity to access the basic education in the field of international development cooperation. Teachers have to be the all sensor in classroom in any cultures.

(3) *Inclusive atmosphere in classroom*

The role of teachers becomes more important today, and they can create the cooperative atmosphere in classroom to promote more interaction among students. This is not only the cases in Europe but should also be African cases. Dei (2005) introduces the case of education policy in Ghana and emphasizes the inclusive education, even though Ghana has the goal of national integration, because postcolonial education in Ghana denies heterogeneity in local populations. He explains no attempts have been made to explore majority-minority relations in schooling or to draw comparisons with approaches to minority education in other pluralistic contexts. This situation missed the needs of students from minority backgrounds, and people take the difference negatively. For example, "girls are denied access to schooling precisely because of this marker of 'difference' that it they are different from boys, or more accurately (in terms of power, privilege and dominance), they are not boys."(33) He also introduces an interview result as reaction to the minority student: "For minority students it is very important to see that everybody can equally perform regardless of background. Specially here some people tend to look down on them, and you know, it hurts them psychologically. So, if in all places some of their community members serve as role models, it would really help them a lot."(34) When we observe a classroom in Ghana, we are hardly able, from the outside views, to understand the difference which the local people recognize.

Notes:

(1) The definitions of the each literacy are as follows (OECD, 2003):

- *Mathematical literacy*: an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.
- *Reading literacy*: understanding, using and reflecting written texts, in order to achieve one's goals, to develop one's knowledge and potential and to participate in society.
- *Scientific literacy*: the capacity to use scientific knowledge, to identify questions and to draw evidence-based conclusions in order to understand and help make decisions about the natural world and the changes made to it through human activity.

(2) There are various definitions of immigrants have been employed in the literature. Some take students born to one immigrant and one native parent as immigrants (Ammermüller 2005), other label only students with two immigrant parents as immigrants (OECD, 2001). As we are seeing the immigrant students' performance of PISA here, the immigrant students should be school children whose parents are not natives of the host country.

(3) OECD, 2003, p. 172

(4) See OECD, 2003, Table 4.2h, for more detail.

(5) Australia, Canada, France, Germany, the Netherlands, New Zealand, Sweden, Switzerland, the UK, and the USA.

(6) Her explanation shows the difference between PISA and TIMSS and the language influence to mathematics achievement: "We might expect that immigrants' educational disadvantage is smaller in subjects where language skills are generally of a lower importance like in math. However, achievement in math in PISA is measured by applying the 'life-skill' approach related to open-ended questions on wordy descriptions of 'real life' situations. Hence, it is not necessarily surprising, that immigrants do not fare better in math in PISA. This result stands in contrast to TIMSS: for all countries immigrants' educational achievement gap is significantly lower in TIMSS math than in TIMSS science. Hence, there seems to be the tendency that immigrants' educational disadvantage is smaller in technical subjects as long as achievement is assessed in a more curriculum based approach by using predominantly multiple-choice questions (Schnepf, 2004: 13)."

(7) It is so-called PISA-E in Germany which includes over 34,000 students compared to 5,500 students in the German OECD-PISA dataset. PISA-E data are detailed data on student performance for Germany and show 81% of the students have both German parents and 19% have one or both non-German parent. The data also show 40% of the 19% students speak another language than German at home.

(8) CGI is based on the following four interlocking assumptions: 1) Teachers should know how specific mathematical content is organized in children's minds; 2) Teachers should make mathematical problem solving the focus of their instruction of that content; 3) Teachers should find out what their students are thinking about the content in question; and 4)

Teachers should make instructional decisions based on their own knowledge of their students' thinking.

(9) First, pose a similar problem. If the student needs more help, secondly, the teacher might simplify the language of the problem even further. The third option is to add localized context to a similar problem. The contexts added to story problem is to refer to the children's home cultures and backgrounds. Forth, the teacher translates the problem into the students' native languages. If all else fails, the teacher might try an easier problem which the students will be surely able to solve.

(10) The minority groups here should be those who have different cultural backgrounds in classroom. This includes the students who are not fluent in a test language and who can hardly understand local contexts in classroom. As the results of the first year IDEC survey in Ghana show English is a key competency for the students because the teachers reported the difficulty for the students were sometimes a very language problem. Even the Chinese case gives us a variety of the students' background for languages. As the author explained above, the language is one of explicit aspects of culture as a symbol of power and personal ability.

(11) Culture is defined in many ways in literature. Here we rather take it generally as "the deposit of knowledge, experience, beliefs, values, attitudes, meanings, hierarchies, religion, timing, roles, spatial relations, concepts of the universe, and material objects and possessions acquired by a group of people in the course of generations through individual and group striving (Samovar, L. and Porter, R., 1991)."

(12) co-culture: a group of people living within a dominant culture, yet having dual membership in another culture.

(13) A form of communication in which the explicit coded message contains almost all of the information to be shared. In contrast, little information is spoken out in a high-context culture. The four differences between high- and low-context are: 1) verbal message is importance and shared in low-context so that people need to observe the situation to gain the information; 2) people in high-context trust oral expression less than those in low-context; 3) people in high-context are more sensitive to non-verbal communication and are more reading the situation; and 4) people in high-context think anyone can achieve communication without words so that do not use words as much as those in low-context.

(14) in-group: the ground that one belongs to (relatives, clans, organizations); out-group: people who are not members of an individual's group.

(15) Pike wrote more about *etic/emic*: creation / discovery of a system; external / internal view; external / internal plan; absolute / relative criteria; non-integration / integration; sameness / difference as measured vs. systemic; partial / total data; preliminary / final presentation; and etc. In addition, *etic* approach is different from structurism which compares cultures and leads universality between structures in which events cause and effects. Researchers study cultures with the imposed *etic* but need to make it from *emic*. In other words, we need to pause our judgment as observing a different culture with *emic* approach and switch to derived *etic* as describing the culture in our language.

(16) However, the author admits his criteria for taking anthropological or psychological approach are not constant in this paper for the moment.

(17) Parker-Jenkins, 1995, p. 28

(18) Redfield, Linton, and Herskovits, 1936, p. 149

(19) Enculturation refers to the process by which the individual learns the culture of the

belonging group. Socialization is also used for the same meaning. However, Herskovits (1948) defined enculturation as "the aspects of the learning experience which mark off man from other creatures, and by means of which, initially, and in later life, he achieves competence in his culture" and socialization as "the process by means of which an individual is integrated into his society (p.38-39)."

(20) Berry, 1992, p. 278-279

(21) Interviews were conducted as semi-structured and informal way at the different time and venues as the author had opportunities to communicate with German scholars. The interviewees were: Ms. Inge Steinstraβer, Deputy Director, VHS in Bonn (Dec. 2, 2004); Dr. Heribert Hinzen and Dr. Werner Hutterer, IIZ/DVV (Dec. 19, 2005); and Dr. Raphaera Henze, Ministry of Science and Health of Hamburg (Jan. 11, 2006).

(22) "Attracting, Development and Retaining Effective Teachers" OECD Project conducted over the 2002 to 2004 period.

(23) OECD, 2005, p. 97

(24) In the studies we reviewed above, the results explain the language as "one aspect of the integration issue is the pupils' capacity to communicate in the language of the host country. Regression analysis showed consistently across surveys that speaking a foreign language at home decreases pupils' achievement greatly in all countries compared. (Schnepf, 2004, p.36)"

(25) Secada & Carey say "the instruction for students from culturally diverse backgrounds often does not take account of the everyday sources of their informal knowledge, i.e., the knowledge that they bring from home. ... teachers of LEP [limited English proficient] students not only need to focus on their students' informal understandings, but they also need to seek those understandings in ways and in settings that are other than commonly supposed."

(26) Later, more than 80 countries, including Arabic and African countries (Hofstede and Hofstede, 2005).

(27) Parker-Jenkins, *op. cit.* p. 118-120.

(28) OECD, *op. cit.*, p. 27

(29) Because scholars have tended to equate "education" with "schooling," and because they have consistently focused on the role of literacy and a literary tradition, many important and interesting - indeed, fascinating - traditions have been seen as falling outside of the parameters of "legitimate" study in the history and philosophy of education (Reagan, 2005, p.6).

(30) Reagan, 2005, p. 248

(31) "Shulman, L., 1991, Ways of seeing, ways of knowing, ways of teaching, ways of learning about teaching, Journal of Curriculum Studies, vol. 23, pp. 393-395." The author was unable to find descriptions of the five areas in the article.

(32) OECD, *op. cit.*, p. 98

(33) Dei, 2005, p. 283

(34) *Ibid.*, p. 279

Reference

Ammermüller, A., 2005, Poor Background or Low Returns? Why immigrant students in Germany perform so poorly in PISA, ZEW DP 05-18.

Berry, J. W., Poortinga, Y. H., Segall, M. H. and Dasen, P. R., 1992, Cross-Cultural Psychology: Research and Applications, Cambridge University Press.

Brown, G. and Micklewright, J., 2003, Using TIMSS, PISA and IALS: A View from the Outside, UNESCO Institute for Statistics, Montreal.

Dei, G. J. S., 2005, The challenge of inclusive schooling in Africa: a Ghanaian case study, Comparative Education, Vol.41, No.3, p.267-289.

Herskovits, M. J., 1948, Man and His Works: the Science of Cultural Anthropology, Alfred A. Knopf, NY.

Hofstede, G. and Hofstede G.J., 2005, Cultures and Organizations: Software of the Mind, McGraw-Hill, NY.

OECD, 2001, Knowledge and Skills for Life - First Results from PISA 2000, OECD, Paris.

OECD, 2004a, Learning for Tomorrow's World - First Results from PISA 2003, OECD, Paris.

OECD, 2004b, Reviews of National Policies for Education - Denmark: Lessons from PISA 2000, OECD, Paris.

OECD, 2005, Teachers Matter: Attracting, Developing and Retaining Effective Teachers, OECD, Paris.

Parker-Jenkins, M., 1995, Children of Islam: A Teacher's Guide to Meeting the Needs of Muslim Pupils, Trentham Books, Staffordshire.

Pike, K. 1967, Language in Relation to a Unified Theory of the Structure of Human Behavior, Janua Linguarum, Series Maior 24, The Hague: Mouton and Co..

Rangvid, B. S., 2005, Sources of Immigrants' Underachievement: Results from PISA-Copenhagen, akf Working Paper, akf. Copenhagen.

Reagan, T. Non-Western Educational Traditions: Indigenous Approaches to Educational Thought and Practice, 2005, LEA, London.

Redfield, R., Linton, R., and Herskovits, M., 1936, Memorandum for the Study of Acculturation, American Anthropologist, vol. 38, pp149-152.

Samovar, L. and Porter, R., 1991, Communication between cultures, Wadsworth Publishing

Secada, W.G. and Carey, D. A., 1990, Teaching Mathematics with Understanding to Limited English Proficient Students, Urban Diversity Series No.101, ERIC Clearinghouse on Urban Education, NY.

Schnepf, S. V., 2004, How Different Are Immigrants? A Cross-Country and Cross-Survey Analysis of Educational Achievement, IZA Discussion Paper, Institute for the Study of Labor, Bonn.

Chapter 5

Literature Review

Chapter 5 Literature Review

5.1 Literature Review on the Professional Development of Teachers

Norimi Osaka
Hiroshima University

The life-course of teachers has been examined by studies in Japan as well as in other countries. These studies commonly demonstrate that teachers' life-course can be classified into six stages. Each stage has a characteristic of expertise development.

I describe several studies on expertise development in the following.

5.1.1 Studies in Japan

Kiyomi Akita (1997) provided several insights into and analyzed several past studies (Huberman, 1992; Yoshizaki, 1996; Shimahara, & Sasaki, 1995; Shulman, 1987; Berliner, 1988; Borko, & Putnam, 1996; Akita, 1994). She then divided teachers' expertise development into the following four stages: (1) Novice (first year of teaching) (2) Advance beginner (second to third year of teaching) (3) Middle (from the end of 20 years to the beginning of 40 years) (4) Expert.

Novice:

1. Confused about the gap between their ideals and the reality of their class.
2. Unable to see the students.
3. Make it a point to establish a bond with students.
4. Make it a point to teach students motivation and the correct attitude.

Advance beginner:

1. The behavior during lessons change, and self-image and self-esteem are formed.

Middle:

1. From their experience of success and failure, there is a change in the sense of class and the sense of role.
2. Become stagnant, conduct non-interactive lessons because of the demands of the job, and undergo a decline in the sense of sympathy for students.
3. Creation and staggering through their lessons.

Expert

1. Intuitive, act independently in uncertain situations, think along multiple lines of thought, and depending on the context.
2. Find new solutions through crisis situations.
3. Able to communicate about their personal experiences.

Figure 1: (Akita, 1997; this figure was created in Osaka, based on Akita, 1997)

Fujisawa (2004) studied the formation of the instructional competency of teachers.

He divided the process of formation of instructional competency into three stages: (1) the first stage (2) the middle stage (3) the final stage.

The first stage

The basic skill of creating lessons is achieved in this stage. They acquire the ability to conduct the job smoothly and efficiently due to the skills they gain. Their understanding of students and the curriculum is unsatisfactory at this point.

The middle stage

They gain an understanding of the students and curriculum in this stage. Their skill and understanding is gained through experience, and thus, they are able to accomplish the educational target. In this stage, it is possible for them to respond flexibly to students' reaction and to create lessons constructively.

The final stage

Teachers who not only have teaching experience but also self-study and vocation can qualify in this stage. Their aim is each student's intellectual development. They establish their own style of teaching in this stage.

Figure 2: The process of formation of instructional competency

Yamazaki (1992) pursues the development and growth process of teachers, using the "life-course study" method.

The development stages of teachers are then classified as follows: (Yamazaki, Matudaira, 2002)

First period: From reality recheck to establishment of identity (one ~ ten years experience)

In this period, daily informal educational activity is common. Although they may be experience shock upon realizing the gap between reality and their idealistic impressions, they are absorbed in their duty. Through their exercise, they establish their identity as to how to live as teachers.

Middle period: Branch and crossing of life-course (approximately 30 ~ 40 years)

Life course branches and crosses again.

Managerial-post period: separation from being practitioners to creation as managerial staff.

The life-course of male and female teachers cross again, and then, they are promoted to principal or vice-principal and put in charge of school administration and school management. Through their experiences, they establish their careers.

5.1.2 Berliner's research in the US

Berliner (1988) demonstrated the general theory of expertise development in pedagogy as consisting of the following five stages of skill development.

Stage 1: Novice
Students and those beginning the first year of their teaching careers

In this stage, the commonplace must be discerned. The element of the tasks to be performed need to be labeled and learned, and the novice learns a set of context-free rules to guide behavior. In learning to teach, the novice is taught the implication of terms such as higher-order question, reinforcement, and learning disabled. The behavior of the novice is rational, relatively inflexible, and tends to conform to whatever rules and procedures are to be followed. It is a stage where the novice gains experience. Further, it is the stage at which real-world experience appears to be far more important than verbal information.

Stage 2: Advanced beginner
The second and third years of their teaching careers

At this stage, experience can become melded with verbal knowledge, similarities across contexts are recognized, and episodic knowledge is developed. Strategic knowledge—when to ignore or break rules and when to follow them—is developed as context begins to guide behavior.

Experience affects behavior, but the advanced beginner still has no sense of what is important.

The novice and the advanced beginner, though intensely involved in the learning process, may also lack a certain responsibility for their actions. This occurs because they label and describe events, follow rules, recognize, and classify contexts, but they do not actively determine actions what is happening through their personal.

Stage 3: Competent
The third or fourth year

Competent performers of a skill have two distinguishing characteristics. First, they make conscious choices about what they are going to do. They set priorities and decide on plans. They have rational goals and choose sensible means to reach them. In addition, while enacting their skill, they can determine what is and what is not important. From their experience they know what to attend to and what to ignore.

In teaching, this is the stage where one learns not to make errors with regard to timing and targeting because one learns when teachers develop curriculums and instruction decisions, such as when to stay with a topic and when to move on.

Because they are more personally in control of the events around them, following their own plans, and responding only to the information that they choose to respond to, teachers at this stage tend to feel more responsibility for what happens. They are not detached. Thus, they often feel emotional about success and failure in a different and more intense sense than novices or advanced beginners do. However, competent performers are not yet fast, fluid, or flexible in their behavior.

Stage 4: Proficient
Around the fifth year: A modest number of teachers

This is the stage where intuition or know-how becomes prominent. These terms are not mysterious to the teachers. From the wealth of experience the proficient individual has accumulated comes a holistic recognition of similarities.

This holistic similarity recognition allows proficient individuals to predict events more precisely because they perceive more things as alike and, therefore, as having been experienced before.

Stage 5: Expert
Reached by few teachers

If novices, advanced beginners, and competent performers are rational and if proficient performers are intuitive, experts are categorized as "arational." They have an intuitive grasp of a situation and seem to sense in nonanalytic, nondeliberative ways the appropriate response to make. They exhibit fluid performances, as people do when they no longer have to choose their words when speaking or think about where to place their feet when walking.

Experts do things that usually work, and thus, when things are proceeding without a hitch, experts are not solving problems or making decisions in the usual sense of the terms. When anomalies occur, when things do not work out as planned or something atypical is noted, deliberate analytic processes are used in the situation. However, when things are going smoothly, experts rarely appear to be reflective about their performance.

(Berliner, 1988)

5.1.3 Humberman's research in Switzerland

Humberman demonstrated the general trends in the professional cycle of teachers. For the study, 160 secondary teachers were interviewed, each interview lasting five to six hours.

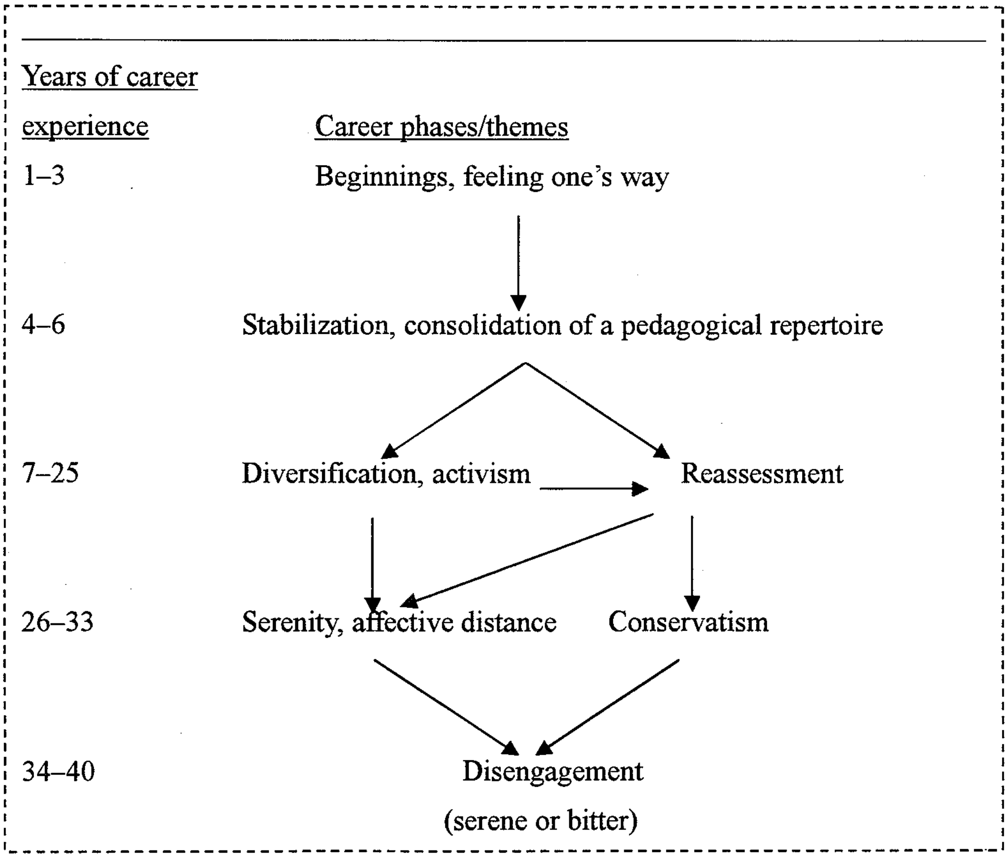


Figure 3: The human life cycle—a thematic model

Humberman (1993) stated that based on the interviews, they laid out a fragmentary, embryonic and, above all, highly speculative and normative sequence of the professional life cycle of the secondary teacher. There appear to be distinct phases, transitions and "crises" affecting the cohorts of subsets of practitioners. They have evoked central tendencies at general junctures, notably with respect to the leitmotivs of different phases and the ordering of these phases. The figure is a schematic and speculative model. It depicts a unique thread leading to stabilization, followed by multiple branches at mid-career, concluding at a single phase. Depending on the preceding course of events, the last phase may be experienced with tranquility or rancor. The most harmonious trajectory would be the following.

Diversification ⟶ Serenity ⟶ Serene Disengagement

The most problematic trajectories would be the following two.

Reassessment ⟶ (bitter) Disengagement

A) Reassessment ⟶ Conservatism ⟶ (bitter) Disengagement

Reference

Berliner,D.C. 1988 *The development of expertise in pedagogy*. AACTE Washington, DC 20036.

Humberman,M. 1993 *The lives of teachers*. Teacher College Press.

Shinsuke Hujisawa. 2004 *The teachers' process of development of competency as a reflectioner*. Kazama Syobo.

Junji Yamazaki. 1990 *A research on Professional Development of Teachers in Primary and Secondary Schools*. Research Report of Sizuoka University Education Faculty, 41.

Junji Yamazaki. 1992 *Life-Course-Research on Professional Development of Teachers in Primary and Secondary Schools*. Research Report of Sizuoka University Education Faculty, 43.

Junji Yamazaki. 1994 *Monograph; Life-Course-Research on Professional Development of a School Mistress*. Research Report of Sizuoka University Education Faculty, 46.

Junji Yamazaki. 2002 *Teacher's Life-Course-Research*. Sohusya.

Junji Yamazaki & Nobuhisa Matudaira. 2002 *Reconstruction of the image of teacher -Teachers' Life-History-*. Iwanamisyoten.

Junji Yamazaki. 2005 *A research on Professional Development of Teachers in Primary and Secondary School*. Research Report of Sizuoka University Education Faculty, 56.

Kiyomi Akita. 1997a *The teachers' Developmental challenges and confusion of the novice*. Jidoshinri, 51(5).

Kiyomi Akita. 1997b *The middle standing teachers' development and stagnation*. Jidoshinri. 51(7).

Kiyomi Akita. 1997c *Learn from expertise, The factor of development*, Jidoshinri, 51(8).

Tadahiko Inagaki & Masao Terasaki & Nobuhisa Matudaira. 2000 *Teacher's Life-Course-Research-Live as a Teacher in Syowa period-*. Tokyo University Syuppankai.

ANNEX

ANNEX 1

First year research Tool

Interview Items

Interview Items for Mathematics Teachers

Please answer freely the following items. In case you need some clarification, you can ask interviewer any time.

[Problem]

1-1) What do you think is the biggest problem in teaching mathematics in you class?

1-2) What kind of action do you take against that problem?

[Today's lesson]

2-1) What was the purpose of today's lesson?

2-2) How much do you think the purpose was attained?

2-3) What do you think are the most important factors for successful lesson?

2-4) What kind of teaching would you like to do?

[In-service training]

3-1) Have you ever had a teacher training after you become a teacher?

3-2) Which kind of training, if you had before, do you think is useful for your teaching?

3-3) What kind of training do you think is necessary for improvement of your lesson, if a new training course is designed?

Interview Items for Head-Teachers

Please answer freely the following items. In case you need clarification you can ask interviewer any time.

[Problem]

1-1) What do you think is the biggest problem in teaching mathematics in your school?

1-2) What kind of action do you take against that problem as an administrator?

1-3) Do you observe lessons by teachers? YES or NO

If YES, how often do you observe them?

1-4) What kind of advice do you give to young teachers at your school?

[In-service training]

2-1) Do you see any impact of in-service course offered to teachers? If yes, is it negative or positive? Please describe the impact a little more.

2-2) What kind of training do you think is necessary for teachers in your school, if a new training course is designed?

Lesson Observation Checklist

Lesson Observation Checklist

NAME of Observer:

SCHOOL GRADE: 4 DATE;

SUBJECT Mathematics TOPIC Fraction

No. of PUPILS: MALE FEMALE

During the lesson, please take a record of the lesson in the video, and after the lesson, indicate your assessment of the following aspects of the lesson by placing a tick in the appropriate box on the rating scale.

(Rating scale: 0 – never, 1 –seldom/ to a little extent, 2 – sometimes/ to some extent, 3 – often/ to a considerable extent, 4 - very often/ to a great extent)

		0	1	2	3	4
Introduction	The teacher starts the class on time.					
	The teacher made the objective clear.					
	The objective suits to the level of children.					
	Relationship with the previous lesson is clear.					
Development	The teacher gives supports to pupils who seem to have little understanding.					
	The teacher expresses appreciation for pupils' thinking attitudes.					
	The teacher assesses the pupils' comprehension during teaching learning.					
	The teacher uses easy language.					
	The teacher uses an appropriate and familiar example to illustrate main concept.					
	The teacher creates friendly atmosphere.					
	The teacher accommodates discussion among pupils.					
	The teacher gives hands-on activity.					
	The teacher enjoys teaching.					
	The teacher is impatient with wrong answer.					
	The teacher involves children to say opinions freely.					
	The teacher encourages children to display diverse opinions.					
	The children are actively engaged in learning, such as telling opinions, asking questions, solving problems etc.					
	The teacher combines individual work and group work appropriately.					
Summary	At the end of the lesson, the teacher summarizes the lesson.					
	The teacher assigns home work at the end of lesson clearly.					
	The teacher explains about a connection between today's lesson and next lesson.					
General	The teacher prepares a lesson plan.					
	The teacher prepares a plan for taking note on the blackboard.					

Describe objective of today' lesson.
Describe problems/ activities. (No. of problems, relation among them, their difficulty level etc.)
Describe children's opinions
Assess who dominate solving problems during the lesson observed.
Assess which of the followings is regarded as the most important in the lesson observed. Understanding concept/ mastering the procedure/ thinking mathematically/ applying to the daily life/ finding correct answer

ANNEX 2

Second year research Tool

Interview Items

Teachers' Questionnaire

School name:

Sex: Male or Female How long have you taught: ____yr/____mo

Subjects you teach: All subjects or Certain subjects

(Specify the subject(s))

Test-evaluation

(1) What do you perceive to be the average score of pupils on the given test?

(Expected average score in percentages: %)

(2) Are the pupils accustomed to this kind of test? Why do you think so?

1. Yes 2. No

Reason(s):

(3) Are there some questions which you think the pupils cannot solve? And if yes, what is the reason?

1. Yes 2. No

Question no. and Reason(s):

Self-evaluation

(4) To what extent do you feel it is easy or difficult to teach "Fractions"? And why do you feel so?

1.very easy 2.easy 3.difficult 4.very difficult

Reason(s):

(5) What is the most difficult topic for you to teach in Grade 4? And why do you think so?

You can answer more than one topic(s) in this question.

(Topic(s):)

Reason(s):

(6) What is the easiest topic for you to teach in Grade4? And why do you think so?

You can answer more than one topic in this question.

(Topic(s):)

Reason(s):

(7) How confident are you when you teach "Fractions"?

1. not confident 2. little confident 3. confident 4. very confident

(8) How do you think the examinations affect your teaching? Please describe an example how it

affects?

1.very much 2.much 3.little 4. very little

Description:

Pupils-evaluation

(9) How easy or difficult do you think it is for the pupils to learn "Fractions"?

1.very easy 2.easy 3.difficult 4.very difficult

(10) What is difficult for the pupils to learn the concept of "Fractions"?

Difficulty:

(11) Do the pupils have any difficulty with the medium of instruction in mathematics learning? And if yes, what is the difficulty?

1. Yes 2. No

Difficulty

Contents-evaluation

(12) Do you think "Fractions" is an important topic among any other topics? And why do you think so?

1. Yes 2. No

Reason(s):

(13) What is the main point for the pupils when you teach "Fractions" ?

Teaching Methodology

(14) Describe how to teach the following question to the pupils?

"Which is longer 1/4m or 1/3m?"

(15)Suppose you posed the following question to the pupils in a lesson.

"What is a half of 2m?"

Then a student answered, "It is 1/2m."

How do you deal with such a student in class?

ANNEX 3

Curriculum China

Grade 1 (4 classes per week)

Contents

(A) Number and Calculations

(1) Children should understand the meaning of number up to 20, and the meaning of addition and subtraction.

a) To count the number, to understand the composition, the order and size of the number, how to read and write, and the meaning of addition and subtraction.

b)

(2) Children should understand the meaning of the number up to 100, and the meaning of addition and subtraction.

a) To count the number, to understand the meaning of one-digit and two-digit numbers, the order and size of the number, how to read and write.

b) To add the multiple of ten to the two-digit numbers, to subtract the multiple of ten from the two-digit numbers, and to add the one-digit numbers to the two digit numbers, to subtract the one-digit numbers from two digit numbers by mental arithmetic. To solve two steps problem in addition and subtraction.

(B) Quantities and Measurements

(1)Children should be able to read clock

(2)To make out Yuan(unit of Chinese money),to calculate simple problem about a money

(C) Geometric Figures

(1) To recognize rectangular parallelepiped,cubes,circular cylinder,sphere viscerally

(2) To recognize rectangle, square, trianle, circle viscerally

(D) Applied problem

To solve relatively simple addition and subtraction in one step verbal problems

(E) Practical exercises

To chouse concret materials for daily life. For example, the number of students in a class, how it could make problem on the basis of distribution of the number in the graph

Tuitional requirement

(1) Through the counting the number of different thing, children should be able to make the number abstraction progressivily, classify the cardinal number and the ordinal number, understand the composition of the numbers up to ten, write the numbers accurately and finely.

(2) Children should know one or ten as a unit, understand the meaning of ones position and tens position, should count, read, write numbers up to 100 exactly. And children should understand composition of numbers up to 100 as a sum of multiple of ten and multiple of one, compare order or size of numbers.

(3) Children should know the term of each part of expression expressed by addition and

subtraction,and relationaship between addition and subtraction. Children should understand additon of two one-digit numbers, and inverse subtraction in mental arithmetic. Children should understand how to add two-digit number to multiple of ten, and how to subtract one-digit number from two-digit number in mental arithmetic.

And, children should solve two steps problems in addition and subtraction.

(4) Children should know the terms lile =, <, >, and by using this terms, children should express size of numbers

(5) Children should read time, understand hour. Children should know that 1 yuan = ten jiao, 1 jiao = ten fen. Children should think good deal of yuan.

(6) Children should solve relatively simple one step verbal probrems in addition and subtraction on the basis of contents of addition and subtraction. Children should know conditions and questions in problems, and make expression. And children should write the name of units in answer, and answer orally

(7) Children should be brought up good habitude, solving probrem with a serious mind, calculating exactry, writing finely

(8) Through the practice, children should have a good experience to know the close relationship between mathematics and daily life.

Grade 2 (5 classes par week)

<u>Contents</u>

(A) Number and Caluculations

(1) Addition and subtraction two two-digit numbers.

To formularise addition and subtraction. To solve two step probrems in addition and subtraction.

(2) Multiplication and division

(a) To know multiplication in entry-level. How to read multiplication table. To formularise multipication.

(b) To know division in ently-level. To find quotient in muntiplication table. To formularise division. To understand the meaning of remainder. To solve two steps probrem.

(3) How to read and write number up to 1000.

To count number. Hundreds position, thousands position, ten thousands position. To know how to read and write. To compare size of numbers.

(4) Addition and Subtraction

Addition and subtraction. successive addition. To check sums, or difference by using the idea of addition.

(5) Calculation including four operations

To know the order of operation, first multiplication and division,next addition and subtraction. To solve two steps problems. To know parenthesis.

(B) Quantities and Measurements

(1) To know the meaning of hours, minutes, seconds

(2) To know the meaning of meters, decimeters, centimeters, and simple calculation.

(3) To know the meaning of kilogram

(C) Geometric Figures

(1) To know the meaning of line and segment at ently-level.

(2) To know the meaning of angle and right angle at ently-level.

(D) Applied problem

(1) To solve the one step verbal problems in addition and subtraction.

(2) To solve the one step verbal problems in multiplication and divition.

(3) To solve conparatively simple two step verbal problems

(E) Practical exercises

To chouse concret materials for daily life. For examples, how it could make problem on the basis of condition of payment for a week in account book.

Tuitional requirement

(1) Children should know the meaning of hundreds, thousands, ten thousands as a unit of cardinal number, and relationship between adjacent numbers in decadal system. Children should know the order of the digit up to 10000, could write and read numbers, and conpare the size of numbers.

(2) Children should know how to add and subtract on a piece of papers. Children should be able to solve relatively simple problems in addititon by making expression. Children should be able to do addition and subtraction with two two-digit numbers, whose sum is less than one hundred, exactory by using mental arithmetic. And children should be able to do addition and subtraction with multiple of hundreds and thousands, do addition and subtraction with numbers, which is a composition of hundreds and tens, and multiple of hundreds or tens in mental arithmetic , and check sums by using commutative law in addition, and differences by using addition. Children should be trained to check answers progressively.

(3) Children should know the meaning of multiplication and division, the names of each part in expressions, the relationship of multiplication and division. Children should know the origin of how to read multiplication table, memorise exactly, be able to find products and quotients by using multiplication table. Children should be able to do divisition, whose divisor and quotient is a one-digit number, with remainder

(4) Children should know the order of operations,should be able to solve two steps problems, know parenthesis.

(5) Children should know the unit such as meter, decimeter, centimeter. Children should know the actual length of one meter, one centimeter. Children should know that one meter is equivalent to ten decimeter, one decimeter is equivalent to ten centimeter. Children should be able to calculate the length of mesurements.

(6) Children should know kilogram as the unit of weight, should build the concept of weight of one kilogram.

(7) Children should know the units of time such as a hour, minute, second. Children should know that one hour is equivalent to sixty minutes, one minute is equivalent to sixty seconds. Children should built the concept of time such as a hour, minute, second. Children should get the habit of being punctual.

(8) Children should know lines and segments at ently-level, should be able to mesure the length of segment and be able to draw segments whose lenth is within the integers, whose unit is only centmeter.

(9) Children should know the meaning of angle and right angle, should know the name of angle. Children should be able to determine whether right angle or not by using triangle ruler.

(10) Children should be able to solve one step problems in addition, subtraction, multiplication, and division. Children should solve the simple two step verbal problems dvided into some steps to make equation.

(11) Through activity, children should be trained to get a good sence of mathematics.

Grade 3

Contents

(A) Number and Caluculations

(1)Multiplication and Division with one-digit numbers

To understand multiplication whose multiplier is one-digit number(Multiplicand is less than three-digit number), and in the case of multiplier is zero. To understand division whose dividend is one-digit number. To comfirm quotient by using multiplication.

(2) Multiplication and Division with two-digit numbers

(a) To understand multiplication whose multiplier is two-digit number(Multiplicand is less than three-digit number),To understand how to calculate simple multiplication in the case of last place of multipliar is zero. To comfirm multiplication. To understand division whose dividend is two-digit number.

(b)

(3) Mixed Operation including addition, subtraction, multiplication, division.

To solve two steps problems. To understand how to use parenthesis

(4) To understand fraction at ently-level

To understand meaning of fraction, how to read and wright fractions. To compare size of frantion in the figures. To understand simple reduction to common denominators in addition and subtraction.

(B) Quantities and Measurements

(1) To understand meaning of kilometer and milimeter, and simple calculation.

(2) To understand meaning of ton and gram, and simple calculation.

(3) To know the unit of areas.

(C) Geometric Figures

(1) To understand the caractaristic of rectangle and square. To understand length of perimeter of rectangle and square.

(2) To recognize parallelogram.

(3) To understand meaning of area area of rectangles and squares.

(D) Applied problem

To understand relationship of numerical quantity which are often used. To solve two steps verbal problems.

(E) Practical exercises

To perform activity with concreat material around daily life. For example, children chould analyze, classify and arrange, record the weather forecast for ten days.

Tuitional requirement

(1) Children should know how to multiply numbers less than 1000 by one-digit numbers, and should comparatively be able to calculate skilfully. Children should be able to confirm quotient by using product(including remainder).

(2) Children should know how to multiply and divid with two-digit numbers on paper, should be able to do multiplication and division skillfully. Children should be able to confirm product by commuting position of multiplication.Children should be able to do multiplication and division with one-digits and two-digit numbers by mental arithmetic, whose product is less than 1000. In the case of multiplier and divisor are multiple of tens, Children should do mental arithmetic. Children should be able to solve simple problems.

(3) Children should know the order of operations including addition, subtraction, multiplication, division. Children should be able to solve three steps problem, should know how to use parenthesis.

(4) Children should know simple fraction, should be able to read and write. Children should be able to compare size of fraction with same denominators. Children should be able to do simple addition and subtraction of fractions with same denominators.

(5) Children should know the units of length kilometer and millimeter. Children should know that one kilometer is equivalent to one thousand meter, one centimeter is equivalent to ten millimeter. Children should know the units of weight ton and gram, should know that one ton is equivalent to one thousand gram, one kilogram is equivalent to one thousand gram. Children should be able to do simple calculation concerning length and weight.

(6) Children should understand the characteristics of rectangle and square. Children should be able to draw rectangles and squares on plotting paper. Children should understand the meaning of perimeter, should find the perimeter of rectangle and square by calculation.

(7) Children should understand the meaning of area, should know the units of area square meter, square decimeter and square centimeter. Children should understand the meaning of square meter, square decimeter and square centimeter. Children should understand the formulas to find area of rectangles and squares.

(8) Children should understand the popular mathematical relations. Children should be able to solve two step verbal problems.

(9) Through activity, children should be trained to get a good sence of mathematics.

Grade 4 (5 classes per week)

Contents

(A) Number and Caluculations

(1) How to read and write up to 100 million

To know the one hundred thousand, one million and ten million as units of cardinal numbers. To know relationship between adjacent numbers in decadal system. To know how to read and write up to 100 million. To know approximate number based on ten thousand.

(2) Addition and Subtraction

(a) To know how to add and subtract round numbers such as ten, one hundred.

(b) To know relationship between each terms of expressions in addition and subtraction. To find the value of unknown X

(3) Multiplication and Division

(a)*******.************. To know how to calculate if the final number of dividend ana divisor is zero.

(b) To know how to calculate if multipliers is close to multiple of ten, or hundred.

(c) To know relationship between each terms of expression in multiplication and division. To find the value of unknown X

(4) Estimating large numbers. Abacuse and Calculator.

(5) Calculation using four operations

To know the braces. To solve three steps problems

(6) Relationship between four operations with integers and rule of calculation

(a) To know natural numbers and integers, how to read and write of decimal notation system.

(b) To know the meaning of four operations. To know interactions of addition and subtraction, multiplication and division. To know dividable division by integers and division contain remainder

(c) To know the rules of calculation

(7) Meanings, characteristics, addition and subtraction of decimal fraction.

(a) To know the meanings and characteristics of decimal fraction. To compare the size of decimal fraction. To know the size of decimal fraction is changed in connection with the changing of the decimal point. To know the approximate value of the decimal fractions.

(b) To know addition and subtraction of the decimal fractions by applying the rule of calculation of integers.

(B) Quantities and Measurements

(1) How to record Year, month, day, normal year, intercalary year, century and 24 hours.

(2) Angle.

(3) Units of area.

(C) Geometric Figures

(1) The measurement of straight lines. Measurement of distance by using tools, feet and visual measurement.

(2) To draw segment, right angle, acute angle, obtuse angle, straight angle, perigon, straight line and parallel line.

(3) Characteristics of triangle. Sum of the interior angles of triangle.

(D) Statistics at entry-level

To organize simple date. To understand meaning of statical tables, average. To find average.

(E) Applied problem

To solve two step verbal problems and three step verbal problems.

(F) Practical exercises

To organize activity concerning something around us. For example, children set up an research group , calculate research budget for one person in vacation.

Tuitional requirement

(1) Children should know the units hundred thousand, one million, ten million, should understand rule of decadal system. Children shoud understand the meaning of natural numbers and integers. Children should be able to find approxicimate value by using round off based on requirement of problems.

(2) Children should be able to do addition and subtraction with multiple of ten thousand. Children should be able to do addition and subtraction with two numbers which conbined hundred and ten by mental arithmetic. Children should be able to to addition and subtraction with numbers which close to ten and hundred according to circumstances. Children should understand the relationship between each terms of the expressions in addition and subtraction, should be able to find the value of unknown X based on this relationship.

(3) Children should be able to confirm produnt by commuting position of multiplication. **************. If multiplier and divisor is multiple of one hundred, children should be able to multily and divide by mental arithmetic. Children should understand the relationship between each terms of the expressions in multiplication and division, should be able to find the value of unknown X based on this relationship.

(4) Children should understand meaning of four operations, interrelationship between addition and subtraction. Children should receive educational campain about dialectic materialism. Children should be able to solve some problems by through the use of calculus. Children should be able to do calculation associated with four operations including brace.

(5) Children should understand the meaning and characteristic of decimal fractions. Children should be able to do addition and subtraction on paper, and should be able to do it by mental arithmetic skilfully.

(6) Children should understand the meaning of units hour, month and day, should know how many days there are in a year, intercalary year, each month. Children should be able to inscribe times by using 24-hour system.

(7) Children should understand the meaning of segment and angle, should know the size of anlge, should be able to draw angle by using protractor. Children should understand the meaning of perpendicular lines and parallel lines, should be able to draw perpendicular lines, parallel lines, rectangle and square. Children should understand the caracteristics of triangle, should know the sum of the interior angles of triangle.

(8) Children should know the units of area such as hectare, square kilometer. Children should be able to measure length of straight lines on ground by using instruments.

(9) Children should understand the meaning of simple statistical table. Children should be able to collect and organize date. Children should understand the meaning of average, should be able to find value of average. Trough statistical materials, children should be convinced achievements of socialist system.

(10) Children should be able to solve two step and three step verbal problems.

(11) Trough answers and calculation of problems, children should be trained to confirm results of calculations, to get behavior to be responsible.

(12) Through activitys, children should be trained ability to discaver problems in daily life by using mathematical knowledge, and should be trained mathematical literacy.

Grade 5 (5 classes per week)

Contents

(A) Number and Caluculations

(1) Numbers which are dividable by integers

To know characteristics of numbers which are divisable by 2,5,3. To know odd and even numbers, prime numbers and numbers without prime numbers. To know the table of prime numbers up to 100. To be able to do prime factorization. To be able to find divisors and multiple, common divisor and common multiple, greatest common divisor and least common multiple

(2) Multiplication and division of decimal fractions

(a) To know multiplication and division, approximate value of product and quotient, nonterminating decimals. To extend the rule of multiplication to decimal fraction.

(b) Four operations of the decimal fractions within three step problems.

(3) To calculate large numbers by calculators. To search for pertinent rules.

(4) Meanings and characteristics of fractions.

To know meanings and units of fraction. To compare size of fractions. To know relationship between fraction and division. To know proper fractions, improper fraction and mixed fraction. To know characteristics of fractions. To know reduction and reduction to common denominator. To know conversion between fraction and decimal fraction.

(5) Addition and subtraction of decimal fractions

To know meanings of addition and subtraction of the decimal fractions. To know how to calculate addition and subtraction without improper fraction. To extend the rule of addition to fraction. To add and subtraction include fraction and decimal fraction.

(B) Algebla

To express numbers by using letters for variables. To solve verbal problems by writing equations.

(C) Quantities and Measurements

(1) The units of area

(2) **********************

(D) Geometric Figures

(1) Charactaristics of parallelogram and trapezoid. Area of triangle and trapezoid. Combined figure

(2) Charactaristics of rectangular solid and cube. Surface area of rectangular solid and cube. Meaning of volume. Volume of rectangular solid and cube.

(E) Statistics

Collecting, grouping and arrangement of date. Simple statistical table. To find value of average based on collecting date.

(E) Applied problem

*****************. To solve three step verbal problems.

(F) Practical exercises

To organize activity according to actual conditions in society. For example, children research payment of city water, electricity and gas for ten families in a month. Children make mathematical problems concerning about amount of production for ten farm families.

Tuitional requirement

(1) Children should know meanings of numbers which are dividable by integers, divisors and multiple, prime numbers and numbers without prime numbers, should understand relationship between them. Children should understand meanings of characteristics of numbers which are divisable by 2,5,3. Children should be able to do prime factorization within two-digit numbers. Children should be able to find divisors and multiple, common divisor and common multiple, greatest common divisor and least common multiple

(2) Children should be able to do multiplication and division of decimal fractions on paper,should be able to do it by mental arithmetic. Children should be able to find approximate value of product and quotient by using rouding off. Children should be able to calculate four operations of the decimal fractions within three step problems.

(3) Children should understand meanings and charactaristics of fractions. Children should be able to compare size of fractions, should be able to do reduction and reduction to common denominator. Children should be able to do conversion between fraction and decimal fraction. Children should understand meanings of addition and subtraction of fraction , and be able to calculate it skilfully. Children should be able to do four operations certainly, should be able to do addition and subtraction of fraction by mental arithmetic.

(4) Children should be able to express common relationship, rule of calculations and formulas by using letters for variable. Children should understand meanings of equations, should be able to write equations.

(5) Children should understand meaning of common units and convert of units. *******************

(6) Children should understand characteristics of parallelogram and trapezoid. Children should understand formula to find area of parallelogram and triangle.

(7) Children should understand charactaristics of rectangular solid and cube, should be able to find surface area of rectangular solid and cube. Children should understand meaning of volume, should know units of volume such as cubic meter, cubic decimeter, cubic centimeter, liter and milliliter. Children should understand fomulas to find volume of rectangular solid and cube.

(8) Children should be able to do collecting, grouping and arrangement of date, should be able to make simple statistical table. Children should be able to find value of average based on collecting date. Through realistic data, or collecting , grouping and arrangement of date, children should be trained to love one's country and socialism.

(9) Children should be able to solve three step verbal problems. Children should be able to solve verbal problems by making equation. Children should be able to solve concrete

problems in daily life through the use of mathematical knowledge.

(10) Through activitys, children should be trained ability to discaver problems in daily life by using mathematical knowledge, and should be trained mathematical literacy.

Grade 6 (5 classes per week)

Contents

(A) Number and Caluculations

(1) Multiplication and division of fraction

(a) To know the meaning of mumliplication of fractions. To extend rule of multiplication to fractions. Inverse number. To know the meaning of division of the fraction

(b) To know the meaning of division of fractions.

(2) Four operations of fractions

To be able to calculate four operations of fractions

(3) Percentage

To know the meaning of percentage, and how to write percentage. To know the conversion of percentage, fraction and decimal fraction.

(B) Ratio and Proportional

To know the meanings and characteristics of ratio. To be able to find ratio by calculation. To know the quantity of proportion and inverse proportion.

(C) Geometric Figures

(1) To know the meaning of circles and circle ratio. To be able to draw circles. To find the circumference and area of a circle. To know the meaning of quarter sector. To know the meanig of axisymmetric figures.

(2) To know the meanig of cylinders. To find surface area and volume of cylinders. To know the meaning of cones. To find volume of cones. To know the meaning of spheres. To know the meaning of radius and diameter of a spheres.

(D) Statistics

(1) Statistical table.

(2) Ber graphs, line graphs and circular graphs

(E) Applied Problems

To solve verbal problems including construction promblems by using four operations of fraction. Actual use of percentage, for example germination rate, examination pass rate, interest, tax charge. ******. Distribution according to proportion

(F) Practical exercises

To organize activity according to actual conditions in society, for example, To draw plane view of bed room.

(G) Arrangement and Review

Tuitional requirement

(1) Children should understand the meaning of multiplication and division of fractions, should be able to do addition and subtraction of fractions, and should be able to do it by mental

arithmetic. Children should be able to solve problems including four operations of fractions within three step problems

(2) Children should understand the meaning of percentage, should know the application of average, and should be able to do calculations concerning about average.

(3) Children should understand the meanings and characteristics of ratio. Children should be able to find ratio by calcuration and should be able to simplify ratio. Children should understand meaning of proportion and inverse proportion. Children should be able to determine whether proportion or inverse proportion. Through the teaching of proportion, Children should receive educational campain about dialectic materialism.

(4) Children should understand the meaning of circles, should be able to draw circles. Children should understand the meaning of formula to find circumference and area of a circle. Through the using of histrical material about circle ratio, children should receive patriotic education.

(5) Children should know the meanig of cylinders and cones, should be able to find surface area and volume of cylinders and cones.

(6) Children should be able to make statistical table accurately. Through the analysing statistical tables, children should study about nation.

(7) Children should be able to solve verbal problems about fraction and percentage within two step problems. Children should be able to solve verbal problems by using knowledge of ratio. Children should understand scale size on maps.

(8) Through activity, children should understand relationship between mathematics and society, and application of mathematics.

(9) Through systematization, arrangement, reviewal, children should understand mathematics which have been learned in elementary school more strongly. Children should select a simple way of solution according to situation reasonably, should be able to solve actula problems in daily life by using knowledge which have been learnd.

Curriculum Bangladesh

Mathematics Syllabus For Primary School (Grade-1 to Grade-5)

Objectives of Mathematics Learning for Primary School:

On the one hand, mathematics is the way to enter into the world of science and it enables to spread human creativity on the other. Moreover, to solve real problems of everyday life we need mathematics. So one of the major purposes of mathematics education would be to stimulate children's thoughtfulness so that they can fulfill the responsibilities of dutiful and innovative citizens. Again, another goal of mathematics education is to feel the contribution of mathematics in the race of modern science and to increase the ability to render proper responsibility as a citizen.

Mathematics Syllabus of Grade 1:

Numbers and Calculations:

Children should be able

To acquire the idea of number counting

To count concrete materials from 1 to 50

To express the group idea of materials into numbers from 1 to 50

To count number from 1 to 50

To count 10-based numbers from 10 to 50

To know the symbol of numbers from 1 to 9

To acquire the idea of zero

To read and write number from 1 to 50.

To write from 1 to 20 in Bangla

To identify odd and even numbers from 1 to 50

To acquire the idea of number order and to arrange ascending and descending order of numbers from 1 to 10

To understand the meaning of addition and subtraction.

To add and subtract numbers up to 50

To solve addition and subtraction related simple problems

To know Bangladeshi currency

***Quantities and Measurements*:**

Children should be able

To compare the length, height, weight and volume of objects directly.

To identify small number and big number comparing from 1 to 10

C) Geometrical figures:

To understand shape of objects.

Symbols:

+, −, =

Mathematics Syllabus of Grade 2:

A) Numbers and Calculations:

Children should be able

1) To count concrete materials from 51 to 100 numbers
2) To express the group idea of materials into numbers from 51 to 100
3) To count numbers up to 100
4) To count 10-based numbers from 50 to 100
5) To read and write from 50 to 100
6) To wrote from 21 to 100 in Bangla
7) To understand what is the place value of digits used in numbers up to 100 and to write the place value of digits.
8) To identify odd and even numbers from 51-100
9) To acquire the idea of number order and to arrange ascending and descending order of numbers from 11 to 100

10) To add and subtract using concrete materials up to sum 100
11) To add and subtract of two digits numbers
12) To understand how to add and subtract zero from one or two digit numbers.
13) To solve addition and subtraction related problems
14) To acquire the idea of multiplication and division using materials and to learn method of multiplication and division
15) To know about the multiplication table from 1 to 10, and to multiply and divide by zero accurately.
16) To perform division accurately when the dividend is maximum two digits and divisor is maximum one digit number except zero
17) To solve multiplication and division related problems.
18) To acquire the idea of fraction and to utilize it.
19) To use Bangladeshi currency in daily life.

B) ***Quantities and Measurements*:**

Children should be able

1) To identify small number and big number comparing from 11 to 100
2) To understand the meaning of measurement and the units of measurement for length.

2) To understand (know the unit of length) millimeter (mm), centimeter (cm), and meter (m).

3) To know about weight and volume.

C) ***Geometrical figures***:

Children should be able

1) To draw and construct the triangle, square, quadrilateral and so forth.
2) To construct and analyze the various shapes: square, cylinder, solid etc.

Terms and symbols:

Unit ,×

Mathematics Syllabus of Grade3:

A) ***Numbers and calculations:***

Children should be able

1) To count numbers up to ten thousands.
2) To read and write numbers from 101 to 10000
3) To write from 101 to 10000 in Bangla
4) To find out the place value of digit up to ten thousands
5) To identify small number and big number comparing from 101 to 10000.
6) To find out odd and even number from 101 to 10,000.
7) To arrange ascending and descending order of numbers from 11 to 100

8) To add and subtract up to 4 digits numbers, and to understand that these calculations can be done using the same basic methods as for two digits numbers. To understand how to do these calculations in column form.
9) To solve addition and subtraction related problems of two steps
10) To know the multiplication table from 11 to 20.
11) To multiply maximum three digits numbers with maximum two-digits number and to understand that these calculations can be done based on the multiplication tables.
12) To say what is multiplicand, multiplication and multiplier
13) To say what is dividend, divisor, quotient and remainder.
14) To divide accurately when the dividend is 3 digits and divisor is one digit.
15) To solve multiplication and division related problems
16) To know the mathematical symbols: > (greater than), < (less than).
17) To know about the fraction and to utilize in daily life.
18) To identify small and big fraction comparing between same denominator fraction
19) To add and subtract of same denominator fraction

B) Quantities and Measurements

Children should be able.

1) To know the units of length: the kilometer (km), millimeter (mm).
2) To understand the meaning of the units of volume, weight and their measurement.
3) To know the units of volume for liquid: the liter (l)
4) To know the units of weights: the gram (g).
5) To know the relationship between kilometer, meter and centimeter etc.
6) To know days, hours, minutes and seconds, and to understand the relationship *between them.*

C) *Geometrical figures:*

Children should be able

1) To know about the squares, rectangles, quadrilateral and right angled triangles, and to draw and construct these.
2) To know about the circle.

Terms and symbols:

Right angle, ÷

Mathematics Syllabus of Grade 4:

Numbers and Calculations:

Children should be able

To count number up to 100million.

To read and write number up to 100million.

To write any number up to 100 million in Bangla

To read and write the Roman number from 1 to 12.

To find out the place value of digit up to 100million.

To make greatest number and lowest number with four digits.

To subtract from five digits number to five digits number

To solve addition and subtraction related problems of three steps

To multiply when the multiplicand is 4 digits number and multiplier is 3 digits number

To divide when the divisor is 2 digits number, and the dividend is 4 digits number, and to understand how to do these calculations in column form.

To solve multiplication and division related problems (maximum three digits number)

To know the multiple and factor.

To know the prime number and composite number.

To know the Highest Common Factor (H.C.F), and Least Common Multiple (L.C.M), and to find out the HCF and LCM.

To know mathematical symbol (<, >) and to utilize it.

To know denominator and numerator of a fraction.

To know different types of fraction: mixed fraction, proper fraction, improper fraction and decimal fraction and to know how to express fraction.

To add, subtract, multiply and divide fraction related simple problems (maximum two digits number)

To understand the system of units M.K.S. (meter, kilometer, second) and F.P.S (feet, pound, second)

Quantities and Measurements

Children should be able

To find out the area of square, rectangle.

To know unit of area: square centimeter.

To know the unit of angles: degree (°) and to find out the angle of triangle using instruments.

Geometrical figures:

Children should be able

To draw bar graph of simple data including population data.

To draw parallelogram, rhombus, rectangle, square, circle etc.

To draw the point, plane and line.

Terms and symbols:

Factor, Fraction, Denominator, Numerator, Point, Plane, Line.

Mathematics Syllabus of Grade-5

A) Numbers and Calculations:

Children should be able

1) To make greatest number and lowest number with maximum six digits number
2) To add and subtract up to 6 digits number.
3) To solve addition, subtraction, multiplication and division related problems of three steps
4) To multiply 4 digits number by 3 digits number, and to divide 5 digits number by 3 digits number.
5) To solve daily life problems using unitary method
6) To know parenthesis and to solve parenthesis, addition, subtraction, multiplication and division related problems.
7) To acquire the idea of average and to find out average
8) To solve average related problems
9) To find out the highest common factor (H C F) and least common multiple (LCM) using various methods and to solve H C F & L C M related problems.
10) To add and subtract of fractions with same denominator and different denominator, and to compare various fraction.
11) To know the idea of proper fraction, mixed fraction and improper fraction
12) To find out small and big fraction comparing various fractions and to arrange ascending and descending order of fraction using mathematical symbols.
13) To add and subtract proper fraction, improper fraction and mixed fraction related problems.
14) To multiply mixed fraction and proper fraction by integer, and to multiply fraction by fraction, and to divide fraction by fraction & integer, and to divide integer by fraction.
15) To solve addition, subtraction, multiplication and division related problems of decimal fraction of three steps.
16) To acquire the idea of percentage and to solve percentage related problems

B) Quantities and Measurements:

Children should be able

1) To know the characteristics of square, rectangular, rhombus, parallelogram, and to find out the area of triangle and parallelogram.
2) To draw different types of triangles and to find out angle of the triangle.

C) Geometrical figures:

Children should be able

1) To know that circles' have a center, diameter, and radius, and to draw it
2) To understand relationship between parallel lines and perpendicular lines.
3) To draw bar graph including population data and to collect the information from bar graph.

Terms and symbols:

Parallel, perpendicular, %