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Motion of a Spherical Domain Wall and the Large-Scale Structure Formation

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The evolution of a wall-like structure in the universe is investigated by assuming a simplified model of a domain wall. The domain wall is approximated as a thin spherical shell with domain wall-like matter, which is assumed to interact with dust-like dark matter in an entirely inelastic manner, and its motion in an expanding universe is numerically studied in the general-relativistic treatment. We evaluate the lifetime of the wall, which is defined as the characteristic time for the wall to shrink due to its own tension. It is necessary that this time is not smaller than the cosmic age, in order that the walls avoid the collapse to the present time and play an important role in the structure formation of the universe. It is shown that, in spite of the above interaction, the strong restriction is imposed on the surface density of the domain walls and the allowed values are too small to have any influences on the background model.

§ 1. Introduction

Our understanding about the origin and evolution of the universe is changing under the influences of recent cosmological observations. They have revealed the existence of very large-scale structures in the universe. For example, one of them is the Great Wall, which is a large-scale structure extending beyond a few hundred Mpc.[1),2)] Broadhust et al. report a periodic distribution of galaxies with a characteristic scale of $128\,h^{-1}$Mpc in the direction near the Galactic poles.[3)] Moreover, Dressler and Faber found a very large-scale gravitational source from the peculiar velocities of galaxies, which is called "Great Attractor".[4)]

On the other hand it has been reported that an anisotropy of cosmic microwave background radiation (CBR) is very small, and only the upper limits have been observed.[5)] Because baryons are interacting with photons until the decoupling time, the present anisotropy of CBR represents the density perturbation of the baryons at the decoupling time roughly. In general it is considered that after the decoupling time the density perturbations evolved in proportion to the scale factor of the universe due to the gravitational instability and formed the structures in the universe. However it is difficult to explain the present structures in our universe, because the constraint on the anisotropy observations of CBR restricts strongly the density perturbation at the decoupling time.

Theoretically domain walls have recently attracted a great deal of attention, because they may be a possible candidate to be the origins of structures in the universe and they can also play a role of the positive cosmological constant. The existence of the cosmological constant has been argued recently from an astrophysical point of view. For instance, Fukugita et al. did a test of cosmological models using the

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relation of red-shift and number count of faint galaxies, and concluded that the observations favored universe models with the positive cosmological constant.[6] The study of ages of globular clusters in our Galaxy has also supported the necessity of the positive cosmological constant.[7),8)]

Domain walls are a kind of topological defects which are produced at the time of cosmological phase transition, as is predicted in grand unified theories of particle physics. Hill, Schramm and Fry showed a possibility that domain walls of very low energy scale might be formed after the decoupling time by a cosmological late time phase transition, and the structures in the universe might be formed by density perturbations due to these domain walls.[9] As these domain walls do not interact directly with photons but only through gravity, it may be possible to explain the structure formation consistent with the observations of the anisotropy of CBR.

Several numerical simulations have been performed to study dynamics of domain wall network and to clarify whether the domain walls are the possible origins of the structure formation as we expect.[10] In these simulations only the equations of the scalar field were solved in the given background space-time. As a result of their numerical simulations they have concluded that it is difficult for any simple models of domain walls to be the origins of the structure formation. According to their numerical simulations the domain walls are accelerated by the tension, their velocities soon reach a speed near the light velocity, the wall collapses and reconnections occur almost as fast as causality allows, and only the domain walls with the horizon size can survive the collapses. Such domain walls cannot be the origins of the structure formation. The rapid motion of domain walls is the most crucial difficulty of this scenario.

To avoid the rapid collapse of the domain walls or decelerate their motion, several wall models are considered. As one of them we have domain walls which are formed by a modified axion-like field. It has been, however, showed by numerical simulations that these domain walls are also unstable.[11]

There is another possibility suggested by Massarotti,[12] which we are paying attention to. He assumed domain walls interacting with dark matter, and discussed the possibility to decelerate the motion of the domain walls and the application to the large-scale structure formation. We also study such a possibility in the general-relativistic treatment. Though such an interaction has not been discussed so far, we assume the interaction between domain wall and dark matter phenomenologically from the viewpoint of structure formation, and discuss the applications to the large-scale structure formation.

We investigate a simplified model of a domain wall which interacts with dust-like dark matter in an entirely inelastic manner. Here the entire inelasticity is assumed as the most effective interaction in this possibility. We restrict ourselves to a spherically symmetrical case, in order to avoid unnecessary complexities, and we then approximate the domain wall by a thin wall which contains dust and domain wall-like matter. In order to treat a realistic thick domain wall, we must consider complicated situations associated with the non-sphericity and the interaction between dust-like matter and the scalar field forming the wall. Here we assume a spherical thin domain wall for simplicity. Moreover we assume that the space-time inside the wall is approximated by the Friedmann universe and that the space-time in the outside region is described by the Schwarzschild space-time, because the dust matter is trapped by the wall as the wall shrinks. Outside of these two regions, there is the background space-time described by the Friedmann space-time, which is not affected by the system inside. We pay attention to the two regions, that is, inside Friedmann space-time and Schwarzschild space-time, and study the motion of the domain wall using Israel's relativistic junction condition. We investigated the possibility of preventing the wall from shrinking and collapsing by the above mechanism.

We shall show that there is an upper bound for the surface energy density of domain walls which avoid shrinking and are slowly moving even at the present time. To generate the seeds of primordial density fluctuations from such walls, we need some modification in the scenario of the late time phase transition. The case when the spatial curvature of the Friedmann space-time is negative will also be studied and we get the results similar to those in the flat case. We shall also find that the full relativistic numerical results are consistent with the analysis based on the approximation in which the wall is moving slowly and that such domain walls cannot occupy so much of the total energy of the universe to play a role of the dominant positive cosmological constant.

In § 2 we review Israel's junction condition, apply it to our system, and derive the basic equations which determine the motion of a domain wall interacting with dust-like dark matter. In § 3 we solve these equations numerically and show the above results. Section 4 is assigned to a summary and discussion.

In this peper we adopt the convention in which a space-time metric has a signature, $-+++$, and $c=1$ unit, and G refer to the gravitational constant.

§ 2. A simplified model of domain wall interacting with dark matter

As described in the previous section we consider a spherical domain wall which interacts with dust-like dark matter in the entirely inelastic manner. We assume that the energy of dust-like dark matter dominates that of baryon matter, and that the domain wall is formed due to a late time phase transition at a certain time after the decoupling time. If at the initial time a closed domain wall is co-moving to the average matter motion, it begins to shrink thereafter due to its tension relative to the average motion. Because the wall traps the dust-like dark matter during the shrinking motion, the following three regions appear around the wall (see Fig. 1): 1. the inner Friedmann region (I), 2. the vacuum region and 3. the outer Friedmann region (II). The space-time in the vacuum region is expressed by the Schwarzschild metric. The domain wall is between the Friedmann region (I) and the vacuum region. If we neglect the width of the wall, we can regard the motion of the wall as that of the boundary surface, which contains the domain wall matter and the trapped dust-like matter. The motion of such a boundary surface can be investigated by using the junction condition which was formulated by Israel,[13] and extended by Maeda[14] to more general cases of bubbles either in the expanding universe or in the vacuum Schwarzschild-de Sitter space. On the basis of Israel's and Maeda's works we derive the basic equation in our system.

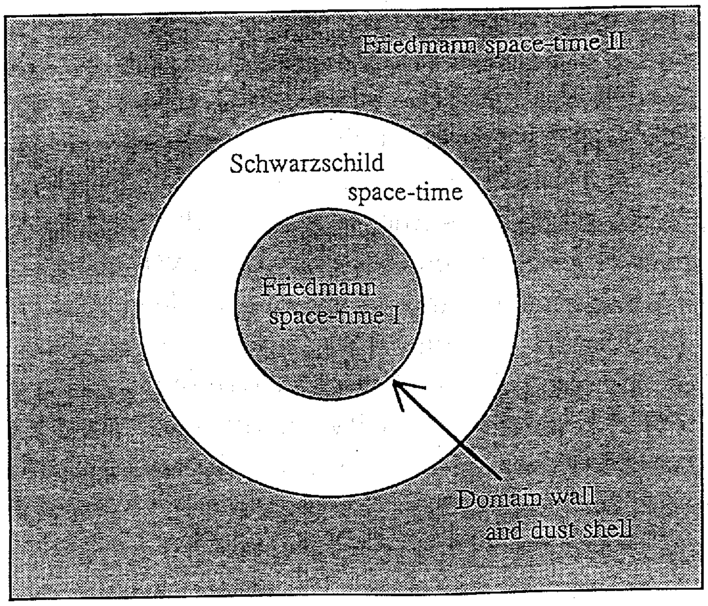

Fig. 1. A domain wall and three regions. The innermost region is represented by the Friedmann space-time I, and a domain wall surrounds this region. As the domain wall shrinks trapping the dust in the wall, the outer region of this wall is represented by the Schwarzschild space-time. In the outermost region there is the Friedmann space-time II, which is independent of the motion of the wall.

Let us consider a time-like hypersurface Σ which divides a space-time V into two regions V^+ and V^-. We define n^a to be a space-like unit vector orthonormal to this hypersurface. The intrinsic 3-dimensional metric h_{ab} on Σ is written as $h_{ab}=g_{ab}-n_a n_b$, where g_{ab} is a 4-dimensional metric of V. The extrinsic curvature K_{ab} on Σ is defined as follows:

$$K_{ab}=h_a{}^c h_b{}^d \nabla_c n_d\,, \tag{2·1}$$

where ∇_a is a covariant derivative operator associated with g_{ab}.

Israel's junction condition is composed of two parts. One of them is derived from the following Gauss-Codacci relations:

$$\begin{aligned} {}^{(3)}R+K_b{}^a K_a{}^b-(K_a{}^a)^2 &= -2G_{ab}n^a n^b\,, \\ D_b K_a{}^b - D_a K_b{}^b &= G_{cd} n^c h_a{}^d\,, \end{aligned} \tag{2·2}$$

where G_{ab} is the Einstein tensor, D_a is a covariant derivative operator associated with h_{ab} (that is, $D_a \equiv h_a{}^b \nabla_b$) and ${}^{(3)}R$ is a 3-dimensional scalar curvature on Σ. These relations must be satisfied on both sides of Σ. Another one comes from the evolution equation of the Einstein equation. Let the subscripts "±" refer to values associated with $V^{\pm}$, respectively. Israel showed that the following relation between K^+_{ab} and K^-_{ab} must be satisfied, when the stress-energy tensor T_{ab} on Σ has a δ-function like singularity,

$$K^+_{ab}-K^-_{ab}=-8\pi G\left(S_{ab}-\frac{1}{2}h_{ab}S_c{}^c\right), \tag{2·3}$$

where S_{ab} is an integral of T_{ab} on Σ with respect to the proper distance l, crossing Σ

in the direction of the normal vector n^a, that is, $S_{ab}=\int dl T_{ab}$. As was shown by Israel, S_{ab} is interpreted as the energy-momentum tensor of the matter on the singular surface.

We can combine these conditions (2·2) and (2·3) to get the following equations:

$$^{(3)}R+\tilde{K}_b{}^a\tilde{K}_a{}^b-\tilde{K}_a{}^{a2}=-16\pi^2G^2\Big(S_b{}^aS_a{}^b-\frac{1}{2}(S_a{}^a)^2\Big)-8\pi G\{T_{ab}n^an^b\}^{\pm}, \tag{2·4a}$$

$$\tilde{K}_a{}^bS_b{}^a=[T_{ab}n^an^b]^{\pm}, \tag{2·4b}$$

$$D_a\tilde{K}_b{}^a-D_b\tilde{K}=4\pi G\{T_{dc}n^dh_b{}^c\}^{\pm}, \tag{2·4c}$$

$$D_bS_a{}^b=-[T_{dc}n^dh_a{}^c]^{\pm}, \tag{2·4d}$$

where we use the notations $\{\Psi\}^{\pm}=\Psi^++\Psi^-$ and $[\Psi]^{\pm}=\Psi^+-\Psi^-$ for an arbitrary quantity Ψ on Σ, and $2\tilde{K}_{ab}=K_{ab}^++K_{ab}^-$.

We will write down the above equations in our system, where in the V^+ region there is the Schwarzschild space-time and in the V^- region there is the Friedmann space-time bounded by the spherical wall. Maeda has studied the junction condition of a spherical symmetric system, to formulate the dynamical equations for a spherical bubble in the expanding universe[14] in more general background space-time. In general the metric of a spherically symmetric space-time V can be written as follows:

$$ds^2=-e^{2\mu(x,t)}dt^2+e^{2\lambda(x,t)}dx^2+r(x,t)^2d\Omega^2. \tag{2·5}$$

$$(d\Omega^2=d\theta^2+\sin^2\theta d\phi^2)$$

A spherically symmetric hypersurface Σ is represented as $t=t_\Sigma(\tau)$, $x=x_\Sigma(\tau)$, where we define τ as the proper time on Σ. Then the metric on Σ can be expressed as

$$ds_\Sigma{}^2=-d\tau^2+R(\tau)^2d\Omega^2, \tag{2·6}$$

by the use of coordinates τ, θ and ϕ, and Eqs. (2·4a)~(2·4d) are rewritten in the following in terms of quantities on Σ.

First we show the explicit expressions for the components of extrinsic curvature on Σ in the general spherical space-time (2·5). We define a unit time-like vector v^a and a unit space-like vector n^a orthonormal to Σ, so that a set of vectors (v^a, n^a, $r^{-1}(\partial/\partial\theta)^a$, $(r\sin\theta)^{-1}(\partial/\partial\phi)^a$) becomes an orthogonal tetrad on Σ. It is noted that the basis vector in the τ direction is v^a. We can write v^a and n_a using the coordinate in the metric (2·5) as follows:

$$v^a=\Big(\frac{dt_\Sigma}{d\tau},\frac{dx_\Sigma}{d\tau},0,0\Big), \tag{2·7a}$$

$$n_a=e^{\lambda_\Sigma+\mu_\Sigma}\Big(\frac{-dx_\Sigma}{d\tau},\frac{dt_\Sigma}{d\tau},0,0\Big), \tag{2·7b}$$

$$\frac{dt_\Sigma(\tau)}{d\tau}=e^{-\mu_\Sigma}\Big[1+e^{2\lambda_\Sigma}\Big(\frac{dx_\Sigma(\tau)}{d\tau}\Big)^2\Big]^{1/2}\equiv e^{-\mu_\Sigma}\gamma, \tag{2·7c}$$

where λ_Σ and μ_Σ are functions of $t_\Sigma(\tau)$ and $x_\Sigma(\tau)$. From these relations and the definition of the extrinsic curvature we can write down its components as

$$K_\tau{}^\tau = -K_{\tau\tau} = -v^a v^b \nabla_a n_b$$

$$= \frac{e^{\lambda_\Sigma}}{\gamma}\left[\frac{d^2 x_\Sigma}{d\tau^2} + 2\frac{\partial \lambda}{\partial t} e^{-\mu_\Sigma} \gamma \frac{dx_\Sigma}{d\tau} + \left(\frac{\partial \lambda_\Sigma}{\partial x} + \frac{\partial \mu_\Sigma}{\partial x}\right)\left(\frac{dx_\Sigma}{d\tau}\right)^2 + e^{-2\lambda_\Sigma}\frac{\partial \mu_\Sigma}{\partial x}\right], \tag{2·8}$$

$$K_\theta{}^\theta = K_\phi{}^\phi = \frac{1}{r^2}\nabla_\theta n_\theta = e^{\lambda_\Sigma - \mu_\Sigma}\partial_t(\log r)\frac{dx_\Sigma}{d\tau} + e^{-\lambda_\Sigma}\partial_x(\log r)\gamma\,, \tag{2·9}$$

and the other components vanish due to the symmetry of the space-time.

Using the above relations, explicit expressions of the extrinsic curvature on Σ are obtained, when the space-time is the Schwarzschild space-time or the Friedmann space-time. In the V^+ region the metric is written in terms of the Schwarzschild coordinate as follows:

$$ds^2 = -\left(1 - \frac{2GM_s}{x_+}\right)dt_+{}^2 + \left(1 - \frac{2GM_s}{x_+}\right)^{-1} dx_+{}^2 + x_+{}^2 d\Omega^2\,, \tag{2·10}$$

where M_s denotes the Schwarzschild mass. From Eqs. (2·8) and (2·9) the extrinsic curvature on Σ in the side of V^+ is reduced to

$$K_\tau{}^{\tau+} = \frac{1}{A_+}\left(\ddot{R} + \frac{GM_s}{R^2}\right), \quad K_\theta{}^{\theta+} = K_\phi{}^{\phi+} = \frac{A_+}{R}\,, \tag{2·11}$$

$$A_+ \equiv [1 + \dot{R}^2 - 2GM_s/R]^{1/2}\,, \tag{2·12}$$

where $\dot{R} \equiv dR/d\tau$. In the V^- region, on the other hand, the space-time is the Friedmann space-time. We first consider the case when the spatial curvature of the Friedmann space-time is flat. When the spatial curvature is negative, it is possible to give the similar formulation, as will be referred to later. We write the metric of the Friedmann space-time as

$$ds^2 = -dt_-{}^2 + a(t_-)^2(dx_-{}^2 + x_-{}^2 d\Omega^2)\,. \tag{2·13}$$

In the case of a dust universe, the Einstein equation gives the following relations between scale factor a and energy density ρ:

$$H^2 = \frac{8\pi G\rho}{3}\,, \quad \frac{2}{a}\frac{d^2 a}{dt^2} + H^2 = 0\,, \tag{2·14}$$

where we express $H = (da/dt)/a$. We can write the components of the extrinsic curvature by using Eqs. (2·8), (2·9) and the above relations. As in the V^+ region, we can again rewrite their expressions in terms of $R(\tau)$, which satisfy

$$R(\tau) = a(t(\tau)_-)x(\tau)_-\,. \tag{2·15}$$

Their final expressions are

$$K_\tau{}^{\tau-} = \frac{1}{A_-}\left(\ddot{R} + \frac{GM_-}{R^2}\right), \quad K_\theta{}^{\theta-} = K_\phi{}^{\phi-} = \frac{A_-}{R}\,, \tag{2·16}$$

where we used the following notations:

$$M_- \equiv m + 4\pi R^3 \rho (a\dot{x}_-)^2\,, \tag{2·17a}$$

$$A_- \equiv \left[1+\dot{R}^2-\frac{2Gm}{R}\right]^{1/2}, \tag{2·17b}$$

$$m \equiv \frac{4\pi}{3}R^3\rho, \tag{2·17c}$$

$$a\dot{x}_- = \frac{\dot{R}-HRA_-}{1-(HR)^2}. \tag{2·17d}$$

Next we consider the matter distribution on Σ, S_{ab}. The domain wall and the dust trapped by a motion of the domain wall are distributed on Σ. Therefore we can write S_{ab} as[15)]

$$S_{ab} = dv_a v_b - \sigma h_{ab}, \tag{2·18}$$

where σ and d are the surface energy densities of the domain wall and dust, respectively, and depend only on τ. Realistically there may be a dissipation term representing the interaction between the wall and dust, but we omitted it for simplicity.

We can now write down the explicit expressions for Eqs. (2·4a)~(2·4d). It is shown in Appendix A that only the following two equations obtained from Eq. (2·3) are independent:

$$\frac{1}{A_+}\left(\ddot{R}+\frac{GM_s}{R^2}\right)-\frac{1}{A_-}\left(\ddot{R}+\frac{GM_-}{R^2}\right)=4\pi G(d-\sigma), \tag{2·19}$$

$$\frac{A_+-A_-}{R} = -4\pi G(d+\sigma). \tag{2·20}$$

Equation (2·20) is an integral form of energy conservation equation, and Eq. (2·19) can be regarded as an equation of motion for the radius R. For three variables, R, d and σ, we have another equation which comes from the conservation law for the number of dust particles in addition to the above two equations. The number of particles constituting the dust matter is always conserved. It is shown in Appendix B that the surface density of the dust is constrained as

$$4\pi dR^2 + 4\pi\rho R^3/3 = m_i(\text{constant}). \tag{2·21}$$

On the other hand, we can easily verify that the basic equations (2·19) and (2·20) are invariant under the following scaling transformations:

$$R \to \alpha R, \quad d \to d/\alpha, \quad \sigma \to \sigma/\alpha,$$
$$\tau \to \alpha\tau, \quad M_s \to \alpha M_s, \quad t(\tau) \to \alpha t(\tau). \tag{2·22}$$

In the case when the Friedmann space-time in the inner region has a negative spatial curvature, we can derive similar basic equations for the motion of the wall and the only modification is replacing (2·15) and (2·17d) by

$$R(\tau) = a(t(\tau)_-)\sinh x(\tau)_-, \tag{2·15$'$}$$

$$a\dot{x}_{-}=\frac{\dot{R}\left[1+\left(\frac{R}{a}\right)^{2}\right]^{1/2}-HRA_{-}}{1+\left(\frac{R}{a}\right)^{2}-(HR)^{2}}. \tag{2·17d)'$$

§ 3. Numerical calculation and the results

In the previous section we derived the basic equations for the motion of a domain wall which interacts inelastically with dust-like dark matter. These equations cannot be solved analytically and so we investigate them numerically under the following initial conditions:

(1) The domain wall expands at the same speed as that of a co-moving shell in the background universe, that is, $\dot{R}=RH$.
(2) The dust surface density d is equal to 0.

We write the radius of the domain wall at the initial time as R_i, where R_i can be chosen arbitrarily outside the Schwarzschild horizon. Initial value of σ is also an arbitrary parameter at the initial time. We determine M_s so that Eq. (2·20) is satisfied at the initial time.

The basic equations consist of Eqs. (2·19), (2·20) and (2·21). From Eq. (2·21) d is given in terms of R, and we determine σ on each time step by using Eq. (2·20). The integration of Eq. (2·19) is performed in the Runge-Kutta method.

Arbitrary parameters in our model are R_i, σ and the initial state of the Friedmann space-time, which is specified by the Hubble parameter and the density parameter. We define the following dimensionless parameters:

$$\beta\equiv\frac{R_i}{R_h}, \qquad \gamma\equiv\frac{\sigma_i}{\rho_i R_i}=\frac{1}{3}\frac{4\pi R_i^2\sigma_i}{4\pi R_i^3\rho_i/3}, \tag{3·1}$$

where R_h is the initial Hubble radius $1/H_i$, and σ_i and ρ_i are the domain wall surface energy density and the dust energy density of the Friedmann space-time at the initial time, respectively. Therefore β represents a ratio of the radius of the domain wall to the horizon radius at the initial time, and γ represents a ratio of the domain wall energy to the energy of dust inside the domain wall. When the spatial curvature of the Friedmann space-time is flat, there exists a scaling law, as was described in the previous section. As these parameters are invariant under the scaling transformation, motions of the domain wall are determined by only these two parameters. For simplicity we first consider the case when the Friedmann space-time is spatially flat.

We are interested in the region of β and γ in which the wall does not shrink so fast. In general, motions of the domain wall deviate with time from those of a co-moving shell, even if they are initially equal. This deviation is represented by using δ defined by

$$R=R_f(1-\delta),$$

where R_f represents the radius of the co-moving shell. We define the characteristic time for the domain wall to shrink, as the time when δ reaches 0.1. This time is

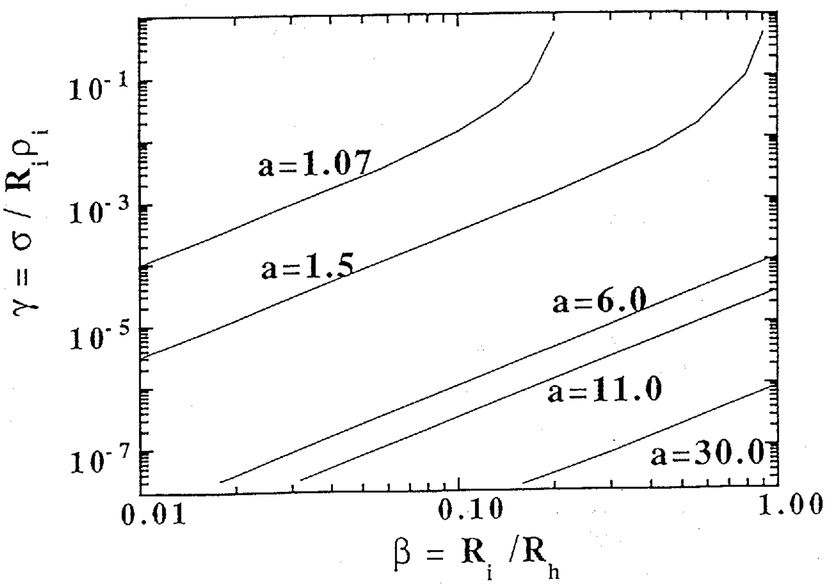

Fig. 2. The relation between the characteristic shrink time (a), β and γ. a is a scale factor of the Friedmann space-time normalized to unity at the initial time. The vertical axis represents γ and the horizontal axis represents β. Some lines of a=constant are drawn on this plane.

characteristic, because the spherical walls soon collapse after that time and the surface density of domain walls are almost constant during this period. We express this characteristic shrink time, in terms of the scale factor a of the Friedmann space-time which is normalized to unity at the initial time. Here we calculated numerically the characteristic shrink time and show the results in Fig. 2. The vertical axis represents γ and the horizontal axis does β. In this plane we draw the lines of a=constant corresponding to the shrinking time.

In a region where γ is less than 10^{-3}, the relation $a^3=0.025\beta^2/\gamma$ reproduces the numerical results well. In Appendix C we analyzed the behavior of the approximate solution representing the domain wall which is slowly moving relative to the background and has small δ. We obtained the same relation between a, β and γ as that in the numerical results. Figure 2 shows that it is necessary that the energy contribution of a domain wall to the total energy within the wall is very small, in order to prevent the domain wall from shrinking so fast.

We consider a role of such domain walls in the large-scale structure formation, assuming that the large-scale structures are formed by the slowly moving walls. According to the above calculations, it is confirmed that the walls can avoid the collapse and move slowly, only when the ratio of domain wall energy density to the total energy density of the universe is very small. We therefore neglect the gravitational influence of the domain walls on the evolution of the universe which is described by the Friedmann space-time. We adopted the condition $\delta=0.1$, as the criterion whether a domain wall is slowly moving or collapsing, and calculated the upper limits for the surface energy density of the domain wall, which was formed at the time with red-shift $z(=5\sim30)$ and is slowly moving still at the present time. This result is shown in Fig. 3. The vertical axis represents the surface density of the domain wall by units of MeV^3. The horizontal axis represents the present radius of the slowly

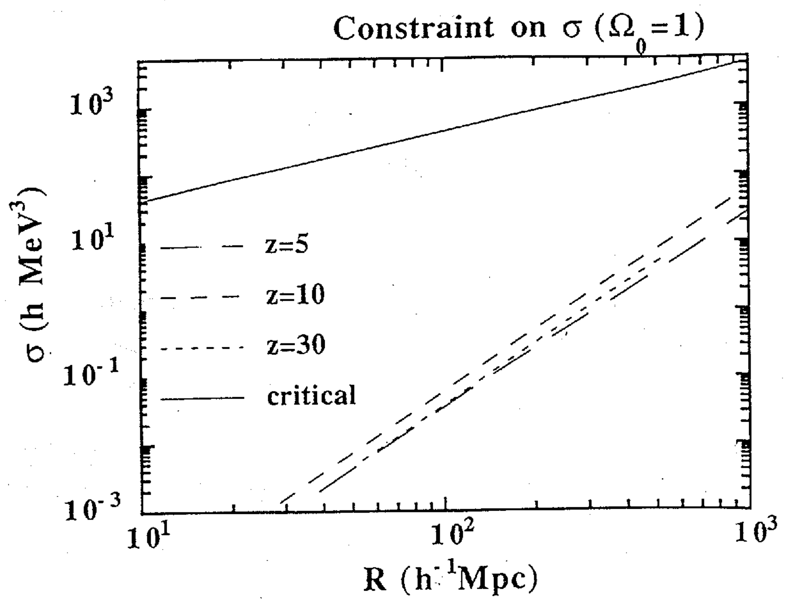

Fig. 3. The upper limits of the surface energy density σ for the domain wall with the characteristic shrink time $t_s \geq$ the present time t_0, where this is the case when the present density parameter, Ω_0, is equal to 1.0. The vertical axis represents the surface energy density of the domain wall in units of MeV^3, and the horizontal axis represents the present curvature radius of the domain wall which have expanded. The dotted lines are the upper limits for the walls which were formed at red-shift 5, 10 and 30. In the region under these lines the walls satisfy the condition $t_s \geq t_0$. The solid line shows the critical line where the energy of the domain wall becomes comparable to the dust energy in the universe.

moving domain wall, R_f. For example, if the surface density of the domain walls which were formed at time $z=5$ is less than the dotted line in Fig. 3, then the domain walls are slowly moving still at the present time and have the curvature radii shown at the horizontal axis. As was shown in Fig. 3, the lines representing the upper limits are similar and do not so depend on the formation times of the wall. This reason can be explained as follows: When the initial red-shift z becomes larger, the time intervals from the formation time to the present time become longer, while the initial energy ratios of the domain wall to the universe become smaller, and these effects to the upper limits of the wall surface density cancel each other.

The solid line in Fig. 3 shows the critical case where the domain wall energy in the universe is equal to the dust energy. Here we assumed that the domain walls are located in a lattice-like manner at the intervals R. This also shows that the energy contribution of the domain walls to the background universe must be very small to keep the walls to move slowly relative to the background. In the model of Hill, Schramm and Fry the surface density of their domain wall is $1\,MeV^3 \sim 10^4\,MeV^3$ corresponding to a neutrino mass, $0.1\,MeV \sim 10^{-3}\,MeV$.

When the spatial curvature of the Friedmann space-time is negative, the situation is not so simple, because there are not such scaling transformations. We have, however, investigated the upper limit of the surface energy density in the domain wall, in which the wall is moving slowly still at the present time, when the present density parameter Ω_0 is 0.1, and the Hubble parameter is equal to 100km/s/Mpc. The result is shown in Fig. 4. According to a decrease of the density of the Friedmann space-

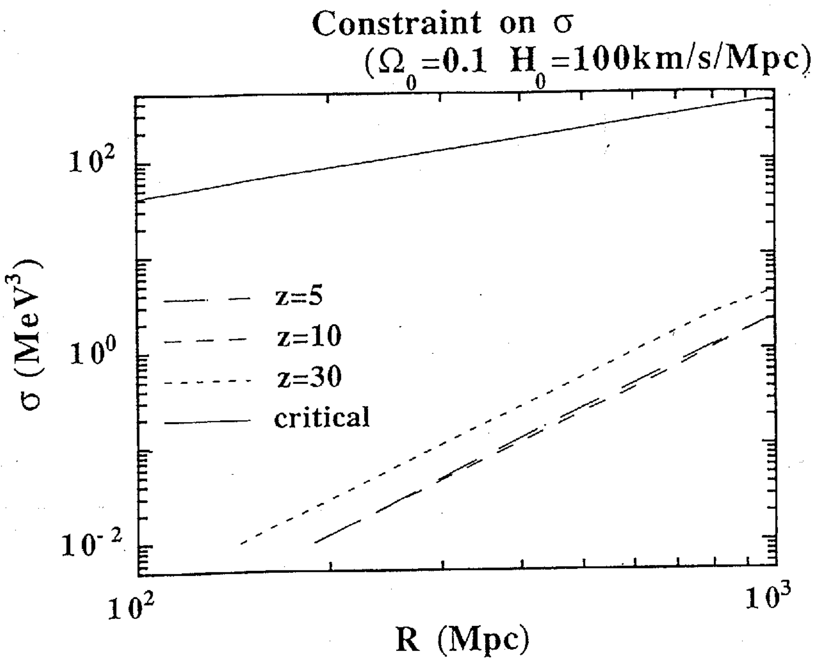

Fig. 4. The upper limits for the surface energy density σ of the domain wall in the case when Ω_0 is equal to 0.1.

time, the upper limit of the surface density decreases. The behaviors are similar to those in the case, $\Omega_0=1$. This implies that the curvature effect is negligible and the motions of the walls are determined by β and γ.

In addition we also investigated the case when the dark matter changes its equation of state, after it is trapped on the domain wall. In particular, we considered the case the dust-like dark matter changed into radiation-like matter. This case is represented replacing Eq. (2·18) by the following equation:

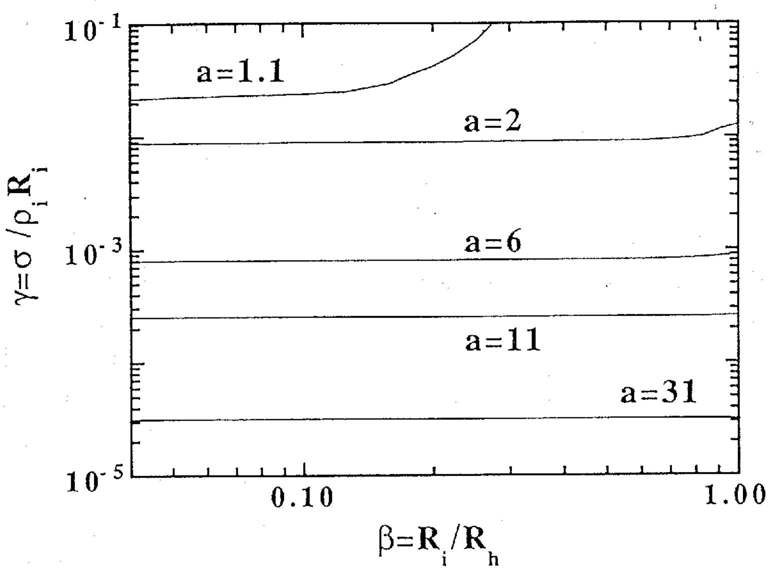

Fig. 5. The relation between the characteristic shrink time (a), β and γ, when the equation of state for the dark matter change into that of radiation-like matter. The axes are the same as in Fig. 2. In this case the characteristic shrink time is independent of β.

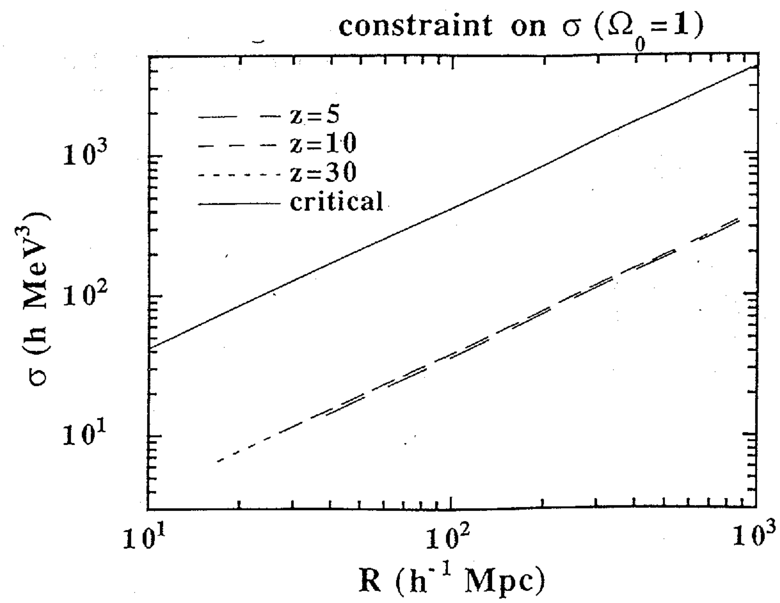

Fig. 6. The upper limits for the surface energy density σ of the domain wall corresponding to Fig. 5, where the present density parameter, Ω_0, is equal to 1.0. The axes are the same as in Fig. 3.

$$S_{ab}=dv_a v_b+p(h_{ab}+v_a v_b)-\sigma h_{ab}\ ,$$

where p is the pressure of the radiation-like dark matter on Σ, and $p=d/2$. As the negative pressure of the domain wall causes the shrinking of the wall, it is expected that the pressure of radiation-like dark matter set off that of the domain wall and prevent the wall from shrinking. The numerical results in the case with radiation-like matter are shown in Figs. 5 and 6 for $\Omega_0=1$. Figure 5 shows the characteristic shrink time in this case. Different from Fig. 2 the characteristic shrink time depends only on γ and the relation $a^2=0.04/\gamma$ reproduces the numerical results. Figure 6 shows the upper limit on the surface energy density of the domain wall. In this case the upper limit is not so severe as in the case of Fig. 3, but it is also difficult to play an important role as a cosmological constant.

§ 4. Summary and discussion

In this paper we investigated the motion of a spherical domain wall which interacts inelastically with dust-like dark matter by using Israel's general relativistic junction condition. In our model the motion of the slowly moving domain wall is characterized by the two parameters. One of them is the initial ratio of the wall radius to the horizon radius. Another one is the initial ratio of the energy of the domain wall to the energy of dust matter inside the wall. We defined the characteristic time for the wall to shrink and begin to collapse, and investigated the relation between this time and the above two parameters analytically and numerically.

We obtained the upper limit for the surface energy density of the domain wall such that the domain walls are slowly moving till now, relative to the background. To explain the large-scale structure extending to 100 Mpc using this wall model, it is necessary that the surface energy density of the domain wall is less than 0.1 MeV^3, when the present density parameter of dark matter is 1. If the dark matter changes its equation of state on the domain wall, it is possible to weaken the constraint. Figures 3 and 6, however, imply that such a wall also does not play an important role as a cosmological constant.

In the case when the interaction between the wall and dust is smaller and parts of dust pass through the wall, the role of dust preventing the wall from collapsing is smaller evidently. In this paper we assumed the vacuum region outside the inner Friedmann region (I). In the case when there is surrounding matter near the region (I), it falls at a slower speed that the wall, even if it freely falls. Accordingly it has no influence on the dynamics of the wall.

It is a remaining problem to make a model of interactions between domain wall and dark matter, to get the amplitude or spectrum of density perturbation which may be made by such a domain wall, and to discuss a possibility of large-scale structure formation due to such a scenario.

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Appendix A

In Appendix A we show that the equations obtained from Eqs. (2·4a)~(2·4d) are reduced to two independent equations (2·19) and (2·20) obtained from Eq. (2·3).

From Eq. (2·18) we write the following expressions by projecting S_{ab} onto the coordinate basis on the hypersurface Σ:

$$S_\tau{}^\tau = -d-\sigma\,, \quad S_\theta{}^\theta = S_\phi{}^\phi = -\sigma\,. \tag{A·1}$$

We easily get Eqs. (2·19) and (2·20) from Eq. (2·3) by using Eqs. (2·11), (2·16) and the above expressions.

Here we pay attention to Eqs. (2·4a)~(2·4d). First we show that the equation obtained from Eq. (2·4b) is satisfied, if Eqs. (2·19) and (2·20) are satisfied. Using (A·1), Eq. (2·4b) is written down as follows:

$$\tilde{K}_\tau{}^\tau(-d-\sigma)+2\tilde{K}_\theta{}^\theta(-\sigma) = -T_{\bar{a}b}n^a n^b\,. \tag{A·2}$$

Because we get the following relations from Eqs. (2·11) and (2·16):

$$\tilde{K}_\tau{}^\tau = \frac{1}{2A_+}\left(\ddot{R}+\frac{GM_s}{R^2}\right)+\frac{1}{2A_-}\left(\ddot{R}+\frac{GM_-}{R^2}\right),$$

$$\tilde{K}_\theta{}^\theta = \tilde{K}_\phi{}^\phi = \frac{1}{2}\left(\frac{A_+}{R}+\frac{A_-}{R}\right), \tag{A·3}$$

and we have $T_{\bar{a}b}n^a n^b = \rho(a\dot{x}_-)^2$, Eq. (2·4b) is rewritten down as follows:

$$-\frac{d+\sigma}{2A_+}\left(\ddot{R}+\frac{GM_s}{R^2}\right)-\frac{d+\sigma}{2A_-}\left(\ddot{R}+\frac{GM_-}{R^2}\right)-\frac{\sigma(A_++A_-)}{R}=-\rho(a\dot{x}_-)^2\,. \qquad (\text{A}\cdot 4)$$

$\ddot{R}$ is eliminated from this equation by using Eq. (2·19),

$$\frac{2G(M_--M_s)}{R^2}+4\pi G(d-\sigma)(A_++A_-)+\frac{2\sigma(A_++A_-)(A_--A_+)}{(d+\sigma)R}$$

$$=\frac{2}{d+\sigma}\rho(a\dot{x}_-)^2(A_--A_+)\,. \qquad (\text{A}\cdot 5)$$

From Eq. (2·20), the definitions of $A_\pm$, Eqs. (2·12) and (2·17b), we can write $A_\pm$ in terms of d and σ as

$$A_\pm=\frac{1}{2}\left[\frac{M_s-m}{2\pi R^2(d+\sigma)}\mp 4\pi GR(d+\sigma)\right], \qquad (\text{A}\cdot 6)$$

respectively. We insert the above $A_\pm$ and the definition of M_- into (A·5), then we can show that Eq. (A·5) is trivially satisfied.

Second let us consider 3-dimensional scalar curvature on Σ, ${}^{(3)}R$. It is calculated as

$${}^{(3)}R=4\frac{\ddot{R}}{R}+\frac{2}{R^2}(\dot{R}^2+1)\,, \qquad (\text{A}\cdot 7)$$

and Eq. (2·4a) is written down as follows:

$$4\frac{\ddot{R}}{R}+\frac{2}{R^2}(\dot{R}^2+1)-2\tilde{K}_\theta{}^\theta(\tilde{K}_\theta{}^\theta+2\tilde{K}_\tau{}^\tau)=-8\pi^2G^2(d+\sigma)(d-3\sigma)-8\pi G\rho(a\dot{x}_-)^2\,. \qquad (\text{A}\cdot 8)$$

We insert $\tilde{K}_\tau{}^\tau=(\rho(a\dot{x}_-)^2-2\sigma\tilde{K}_\theta{}^\theta)/(d+\sigma)$ obtained from Eq. (A·2), and the expression of $\tilde{K}_\theta{}^\theta$ into Eq. (A·8), then we get the following equation:

$$4\frac{\ddot{R}}{R}+\frac{2}{R^2}(\dot{R}^2+1)-\frac{A_++A_-}{R}\left(\frac{(d-3\sigma)(A_++A_-)}{2R(d+\sigma)}+2\frac{\rho(a\dot{x}_-)^2}{d+\sigma}\right)$$

$$=-8\pi^2G^2(d+\sigma)(d-3\sigma)-8\pi G\rho(a\dot{x}_-)^2\,.$$

Eliminating $\ddot{R}$ and $\dot{R}$ by the use of Eq. (2·19) and the definition of A_-, we get

$$\frac{4}{R}\left[\frac{G(A_-M_s-A_+M_-)}{R^2(A_+-A_-)}-\frac{4\pi G(d-\sigma)A_+A_-}{A_+-A_-}\right]+\frac{2}{R^2}\left(\frac{2Gm}{R}+A_-{}^2\right)$$

$$-\frac{(d-3\sigma)(A_++A_-)^2}{2R^2(d+\sigma)}-\frac{2\rho(a\dot{x}_-)^2(A_++A_-)}{R(d+\sigma)}$$

$$+8\pi^2G^2(d+\sigma)(d-3\sigma)+8\pi G\rho(a\dot{x}_-)^2$$

$$=0\,.$$

If we again insert $A_\pm$ of Eq. (A·6) into this equation, then we can show that the left-hand side of this equation is equal to 0 after a little troublesome calculations.

Next we write down the τ component of Eq. (2·4c) and obtain

$$-2\frac{d}{d\tau}(\tilde{K}_\theta{}^\theta)-2\frac{\dot{R}}{R}(\tilde{K}_\tau{}^\tau-\tilde{K}_\theta{}^\theta)=4\pi G\rho(a\dot{x}_-)^2[1+(a\dot{x}_-)^2]^{1/2}. \qquad (\text{A}\cdot 9)$$

If we eliminate $\tilde{K}_\tau{}^\tau$ using the relation $\tilde{K}_\tau{}^\tau=(\rho(a\dot{x}_-)^2-2\sigma\tilde{K}_\theta{}^\theta)/(d+\sigma)$, then the left-hand side of (A·9) is reduced to

$$\text{r.h.s. of (A}\cdot 9)=-2\frac{d}{d\tau}(\tilde{K}_\theta{}^\theta)-2\frac{\dot{R}(d+3\sigma)}{R(d+\sigma)}\tilde{K}_\theta{}^\theta+\frac{\dot{R}\rho(a\dot{x}_-)^2}{R(d+\sigma)}.$$

Inserting the expression of $\tilde{K}_\theta{}^\theta$ into the above expression, we get

$$\begin{aligned}\text{r.h.s. of (A}\cdot 9)&=-2\frac{d}{d\tau}\left(\frac{A_++A_-}{2R}\right)-\frac{2(d+3\sigma)\dot{R}}{(d+\sigma)R}\left(\frac{A_++A_-}{2R}\right)+\frac{2\dot{R}\rho(a\dot{x}_-)^2}{R(d+\sigma)}\\
&=\frac{(A_++A_-)\dot{R}}{R^2}-\frac{\dot{R}\ddot{R}}{R}\left(\frac{1}{A_+}+\frac{1}{A_-}\right)+\frac{1}{R}\left(\frac{GM_s\dot{R}}{A_+R^2}+\frac{Gm\dot{R}}{A_-R^2}-\frac{G\dot{m}}{RA_-}\right)\\
&\quad-\frac{(d+3\sigma)(A_++A_-)\dot{R}}{(d+\sigma)R^2}+\frac{2\dot{R}\rho(a\dot{x}_-)^2}{R(d+\sigma)}\\
&=\frac{G}{R^2A_-}(\dot{m}+4\pi R^2\dot{R}\rho(a\dot{x}_-)^2),\end{aligned}$$

where we used Eq. (A·4) to eliminate $\ddot{R}$. On the other hand we can show the following relation by using (2·17a)~(2·17d):

$$\dot{m}+4\pi R^2\dot{R}\rho(a\dot{x}_-)^2=4\pi R^2\rho a\dot{x}_-[1+(a\dot{x}_-)^2]^{1/2}A_-, \qquad (\text{A}\cdot 10)$$

so that the left-hand side of Eq. (A·9) is equal to $4\pi G\rho a\dot{x}_-[1+(a\dot{x}_-)^2]^{1/2}$, which is equal to the right-hand side of Eq. (A·9). The equations obtained from θ and ϕ components of Eq. (2·4c) are trivially satisfied.

Last we differentiate Eq. (2·20) with respect to τ,

$$4\pi\frac{d}{d\tau}(R^2(d+\sigma))=-\frac{d}{d\tau}\left(\frac{R}{G}(A_+-A_-)\right).$$

If we eliminate $\ddot{R}$ using Eq. (2·19) and adjust it using (A·10), then we get the following equation after a little troublesome calculations:

$$4\pi\frac{d}{d\tau}(R^2(d+\sigma))-4\pi\sigma\frac{d}{d\tau}R^2=-4\pi R^2\rho a\dot{x}_-[1+(a\dot{x}_-)^2]^{1/2}. \qquad (\text{A}\cdot 11)$$

This is nothing but the equation obtained from the τ component of Eq. (2·4d).

Thus Eqs. (2·19) and (2·20) are the only independent equations, which satisfy junction conditions (2·4a)~(2·4d).

Appendix B

In Appendix B we describe the number conservation law of dust in our system. Introducing a new radial coordinate $\bar{r}$, we can set up the following metric around hypersurface Σ:

$$ds^2=-d\tau^2+d\bar{r}^2+R^2d\Omega^2. \qquad (\text{B}\cdot 1)$$

This coordinate is interpreted as the local Lorentz frame on the hypersurface Σ. In this coordinate, Σ is located at $\bar{r}=\bar{r}_0$ (constant). The number conservation law is written as follows:

$$\partial_\tau(\sqrt{-g}\,\rho u^\tau)+\partial_{\bar{r}}(\sqrt{-g}\,\rho u^{\bar{r}})=0\,. \tag{B·2}$$

If we integrate this equation crossing Σ,

$$\lim_{\epsilon\to 0}\int_{\bar{r}_0-\epsilon}^{\bar{r}_0+\epsilon} d\bar{r}\,[\partial_\tau(\sqrt{-g}\,\rho u^\tau)+\partial_{\bar{r}}(\sqrt{-g}\,\rho u^{\bar{r}})]=0\,. \tag{B·3}$$

In this coordinate we have $\sqrt{-g}=R^2\sin\theta$, and the surface energy density of dust on Σ is written by definition as follows:

$$d=\lim_{\epsilon\to 0}\int_{\bar{r}_0-\epsilon}^{\bar{r}_0+\epsilon} d\bar{r}\rho\,,$$

so that Eq. (B·3) becomes

$$\frac{d}{d\tau}(R^2 d)-\lim_{\epsilon\to 0}[R^2\rho u^{\bar{r}}]_{\bar{r}=\bar{r}_0-\epsilon}=0\,. \tag{B·4}$$

As we can obtain the relation $u^{\bar{r}}|_{\bar{r}=\bar{r}_0-\epsilon}=-a\dot{x}_-$ by the coordinate transformation, the number conservation law of dust is described as

$$\frac{d}{d\tau}(R^2 d)=-R^2\rho\left(a\frac{dx_-}{d\tau}\right), \tag{B·5}$$

and this equation can be integrated to give

$$4\pi dR^2+\frac{4\pi}{3}\rho R^3=\text{constant}\,. \tag{B·6}$$

Appendix C

When the wall is moving at a speed near that of a co-moving shell in the background universe, we can treat a deviation of the wall from the motion of the co-moving shell as a small perturbation. Here we consider the case when the Friedmann space-time in the inside region is spatially flat.

Let us express the radius R of the wall as

$$R=R_f(1-\delta)\,, \tag{C·1}$$

where R_f represents the expansion of the background universe, that is, a co-moving shell in the expanding universe, and we assume $\delta\ll 1$. R_f is written as $R_f=R_i a$, where R_i is an initial radius of the domain wall and a is a scale factor of the Friedmann space-time which is normalized to unity at the initial time. As $a\dot{x}_-$ is of the first order with respect to δ, we get $t_-=\tau$ within the linear approximation because $\partial t_-/\partial\tau=[1+(a\dot{x}_-)^2]^{1/2}$ and the following expression differentiating Eq. (C·1):

$$\dot{R}=R_f H\left(1-\delta-\frac{\dot{\delta}}{H}\right), \tag{C·2}$$

where we used $H=\dot{a}/a$. Moreover let us express ρ and m as

$$\rho=\frac{\rho_i}{a^3}, \quad m=m_i(1-3\delta), \tag{C·3}$$

where ρ_i and m_i denote the initial values of ρ and m. Then we can expand A_-, M_s, A_+, $a\dot{x}_-$ and M_- in terms of δ using Eqs. (C·1)~(C·3) as follows:

$$A_-=1-(R_fH)^2\frac{\dot{\delta}}{H}, \tag{C·4}$$

$$M_s=m_i+4\pi G\sigma R_i^2, \tag{C·5}$$

$$A_+=1+(R_fH)^2\left(-\frac{3}{2}\delta-\dot{\delta}H\right)-\frac{4\pi G\sigma R_i^2(1+\delta)}{R_f}, \tag{C·6}$$

$$a\dot{x}_-=-R_ia\dot{\delta}, \tag{C·7}$$

$$M_-=m. \tag{C·8}$$

Using the above equations we can expand Eq. (A·11) in terms of δ and get the following equation:

$$\frac{d}{dt}(4\pi dR^2)=4\pi\rho_iR_i^3\dot{\delta},$$

and this is easily integrated to give

$$4\pi R_f^2d=3m_i\delta. \tag{C·9}$$

This equation represents the number conservation law for dust. This expression is realized in spite of whether σ=constant or not, so that the number conservation law does not conflict with the condition σ=constant in the linear approximation.

As we have solved with respect to d, we will next derive δ. We rewrite Eq. (A·4) as

$$\ddot{R}=\frac{1}{1/A_++1/A_-}\left[\frac{-G}{R^2}\left(\frac{M_s}{A_+}+\frac{M_-}{A_-}\right)-\frac{2\sigma(A_++A_-)}{R(d+\sigma)}+\frac{2\rho(a\dot{x}_-)^2}{d+\sigma}\right]. \tag{C·10}$$

This equation can be interpreted as the equation of motion for radius R. The first term represents a deceleration rate of the Friedmann space-time, the second term is interpreted as a deceleration rate due to the tension within the domain wall, and the third term is neglected, because it is the second order of δ. If d dominates σ in the second term and if the second term is kept smaller than the first one, then the domain wall would remain to be slowly moving, relative to the background. We expand this equation in terms of δ using Eqs. (C·1)~(C·9), and we get

$$\ddot{\delta}+2H\dot{\delta}-\frac{3}{4}H^2\delta-2\left(\frac{3m_i\delta}{4\pi\sigma}+R_f^2\right)^{-1}=0. \tag{C·11}$$

If the domain wall is slowly moving, $\ddot{\delta}$ must be negligible and the inequality, $\sigma \ll d$ is satisfied as described above. Then we approximate Eq. (C·11) as

$$2H\dot{\delta}-\frac{3}{4}H^2\delta=2\left(\frac{3m_i\delta}{4\pi\sigma}\right)^{-1}. \quad (C\cdot 12)$$

This differential equation is solved to give

$$a^3=\frac{9}{8}\delta^2\frac{\beta^2}{\gamma}. \quad (C\cdot 13)$$

If we set $\delta=0.1$ to define the characteristic lifetime for the domain wall to shrink, then we get $a^3=0.011\beta^2/\gamma$. This is roughly the same as our numerical condition for the characteristic shrink time.

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