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A GRAPH THEORETIC PROBLEM ON LOCAL AREA NETWORK DESIGN

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1. Introduction

In this paper we focus on a graph theoretic problem related to the design of a local area network(in short LAN) illustrated as in Fig.1, where the computing devices are separated into the user layer and the server layer. These two layers correspond to the work stations and the resources, respectively. The LAN plays a role of interconnection between two layers.

A virtual computer system for a user can be represented as a triangle of Fig.2, where the leftmost vertex means a work station. The triangle represents also a hierarchical structure of program modules, and then the LAN layer is inserted between two layers, i.e., two separated sets of program modules. If the data between two layers flows as fast as in a single computer, the problem does not occur. The realistic LAN, however, is not so fast, and then the problem to find the optimum cutset where the LAN is inserted becomes siginificant. We propose a theoretical problem arising from such background, and state an efficient algorithm to solve this.

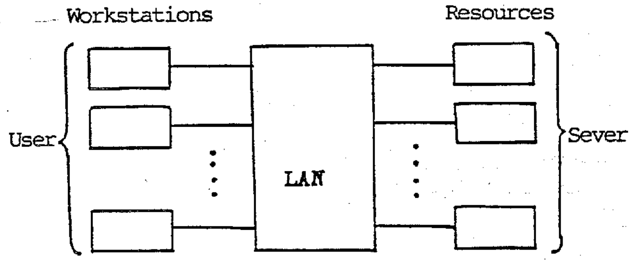

Fig.1 A conceptual LAN system

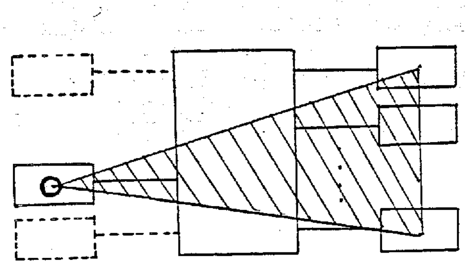

Fig.2 A virtual subsystem on LAN system

2. Minimum Cost Cutset Problem

Since the problem is to find the minimum cost LAN which divides the system into two layers, a graph model is proposed for this purpose.

First, we introduce a kind of data flow graph that is a data flow tree where each edge has the value of required data flow. The data flow tree differs from the conventional tree graph in variety of the type of nodes. In the case of LAN the tree must include both And-type node and OR-type node, which are needed to represent the resource locking and the conditional branch in programs, respectively. If the tree consists of only AND-type nodes, the problem becomes the same as in the conventional graph[1]. An example of AND/OR tree is shown as in Fig.3 as a data flow model. The root(a double circle) is the initial point of each behavior on interconnected program modules, and the leaves(squares) are the terminal modules. An AND/OR tree represents a dynamic behavior of logical computing systems. The minimum cost problem of LAN for computing system is reduced to the problem to find the minimum cost cutset for an AND/OR tree where the data flows from the root to leaves.

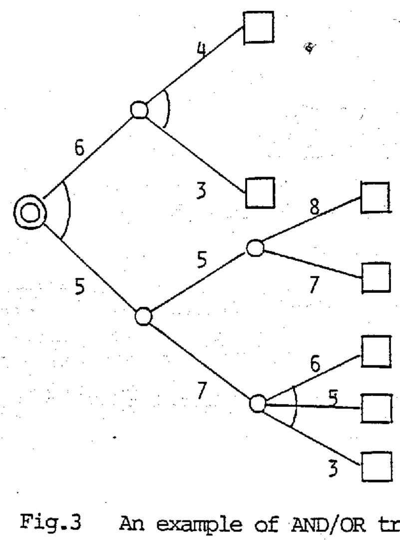

Fig.3 An example of AND/OR tree

3. Algorithm

Next, we show an algorithm to obtain the minimum cost cutset for an AND/OR tree.

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<OUTLINE OF  ALGORITHM>
INPUT : AND/OR tree G=( V,E ) with label
OUTPUT : Set of Minimum Cost Cutset C ;

Procedure PAND ;
 Input:Subtree C0 whose node v has relation AND
       C0 = { ei ,.....,ei+k};
 Output: Edge e*i  whose label is the sum of all
       labels of ei+p ∈ C  (p=0,...,k);
Procedure POR ;
 Input:Subtree C0 whose node v has relation OR
       C0 = { ei,.....,ei+k};
 Output: Edge e*i  whose label is the maximum label
        of ei+p ∈ C0 (p=0,...,k);
Procedure PPATH ;
 Input: Subtree C0 whose node v has two edges
       C0 = { ei, ej (or e*j )}:
 Output: Edge e*i  whose label is smaller one of
        two in C0 ;

 begin                       /* main */
  C <-- E;                   /* initialization */
  for level := k downto 1 do
   while (set of nodes belong to current level)
       ≠ φ do
    begin
    if (the objective node v  has two or more
       edges) then
      begin
      if (node v  is AND-type node) then
       begin
       PAND ;                 /*call procedure AND*/
       C <-- C - C0+ { e*i  }
       end
      else
       begin
       POR ;                  /*call procedure OR*/
       C <-- C - C0+ { e*i  }
       end
      end;
    PPATH ;                  /*call procedure PATH*/
    C <-- C  - C0+ { e*i }   /* edge ei or ej is
                                deleted */
    end
 end.
```

The level means the distance from the root. C_0 is the objective set of edges, and $\{ e^*_i \}$ is an edge reduced from C_0 by each procedure. PAND, POR or PPATH which is called by the algorithm is a lower procedure for each type node(AND, OR or UNARY), where the star-type subtree is reduced to an edge. We demonstrate the algorithm for an AND/OR tree as in Fig.4.

4. Conclusion

As a result we obtain the following;

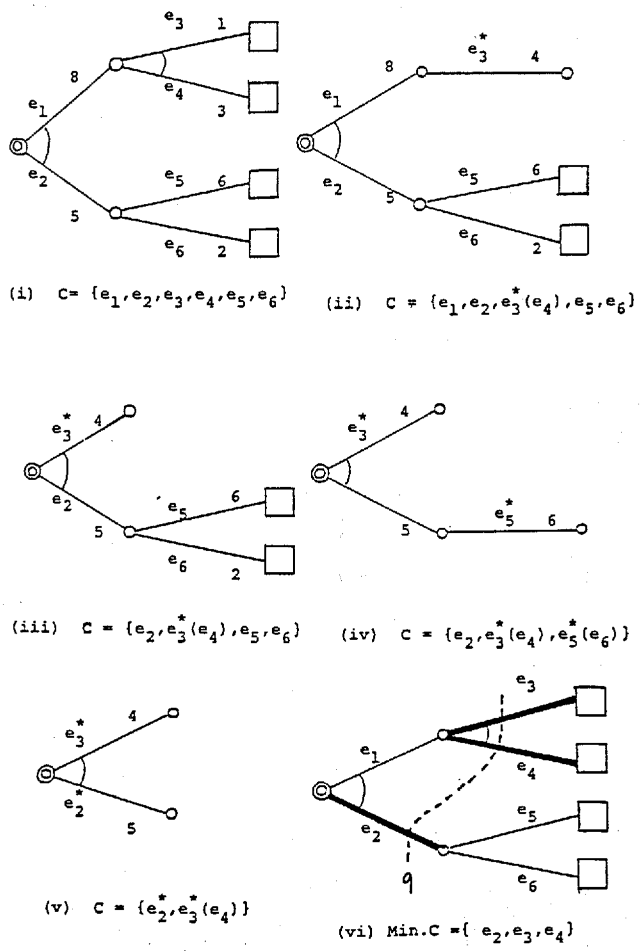

Fig.4 An application of ALGORITHM described in the paper

[Theorem]

There exists an O(n) algorithm to find the minimum cost cutset for an AND/OR tree. If the number of minimum cost cutsets are fixed, then the set of minimum cost cutsets is also obtained within the linear time.

An extended case of non-tree graph has been discussed. There still remain, however, some difficulties to utilize this result for the realistic LAN design, but the discussion of this paper reminds us of importance of data flow analysis in the distributed system [2].

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References

[1] W.Mayeda,"Graph Theory",John Wiely & Sons,1972

[2] H.S.Stone,"Multiprocessor Scheduling with Aid of Network Flow Algorithms",IEEE Tran.SE-3,1, pp.85-93,1977