Transverse stability of bunch trains

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The transverse stability of bunches in a bunch train is determined by solving the equations of betatron motion for macroparticles circulating in a high energy storage ring. We ignore multibunch modes that are more likely to be serious with equal bunch spacing, and find that a nonexponential beam breakup instability may develop, which would not be found by the usual instability analysis with an exponential ansatz. In the absence of radiation or other damping mechanisms, the amplitudes of the trailing bunches would grow with a power law and would soon be lost if the first bunches performed a betatron oscillation about the closed orbit. Experimental observations on a large electron-positron collider are also discussed.

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I. INTRODUCTION

In this paper, we explore the transverse stability of bunch trains circulating in a storage ring of the circumference $2\pi R$ [1]. The analysis is based on a macroparticle model [2, 3], i.e., internal bunch motion is neglected. The bunch train considered consists of N_b bunches traveling at the speed of light c. The nth bunch is assumed to be located at a distance $z_n (z_1 = 0)$ behind the first bunch. The distance between the mth and nth bunch is, accordingly, indicated by $d_{nm} = |z_n - z_m|$. Single bunch effects due to short-range wake fields are not treated here since they are the same as in a ring with equal spacing between bunches [4]. We also disregard the effects of long-range wake fields which may lead to the coupled-bunch instabilities which have been analyzed in the past [5]. In any case, they are known to be most critical when the spacing between bunches is equal [6], hence they are of less concern for operation with bunch trains. Here we concentrate our attention on specific effects of transverse wakes of intermediate range, corresponding to the distances between bunches in a train.

For simplicity, we assume that all wake fields are caused by localized impedances such as rf cavities. Wakes originating from other sources, in particular distributed ones, are not included. The cavity wakes are assumed to be damped away before arrival of the next train, since only effects of a single bunch train are of interest. This assumption should be roughly valid in large machines like the large electron-positron storage ring (LEP) where the time interval between neighboring trains is generally long enough for the wakes to have major resistive damping. The rf cavities are taken to be located at $s = s_1, s_2, \ldots, s_{N_c}$, where s is the distance along the design particle orbit, and N_c is the total number of cavities.

In Sec. II, we derive the equation for betatron motion of macroparticle bunches in a single transverse plane, while the closed orbit—in the same plane—is determined in Sec. III for each bunch, taking into account the effects of the localized wakes. In Sec. IV, we use the LEP parameters as an example to make an estimate for a typical wake strength. We then solve the equation of motion in Sec. V, concluding that a transverse displacement of a leading bunch from the closed orbit can be a source of nonexponential beam breakup in the trailing bunches. In Sec. VI, we investigate several methods to control the growth of the betatron amplitudes by introducing various additional forces. The effect of radiation damping is briefly explored in Sec. VII. After some discussion of the present results in Sec. VIII, recent experimental observations on bunch trains in LEP are described in Sec. IX. The results of the paper are then summarized in Sec. X.

II. EQUATION OF BETATRON MOTION

Since the leading bunch in a train does not receive a wake field kick under the assumptions made, the equation of its betatron motion about the design orbit is simply given by

$$\frac{d^2 x_1}{ds^2} + K(s) x_1 = F(s), \tag{2.1}$$

where $K(s)$ corresponds to the focusing strength of the quadrupoles, $F(s)$ is a periodic function, i.e., $F(s) = F(s + 2\pi R)$, characterizing the effect of field errors and misalignments of the magnetic components installed around the ring. However, for later bunches in the train, we need to consider not only the effect of $F(s)$ but also that of the wake fields generated by the preceding bunches.

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The second bunch is kicked by the wake of the first bunch, and leaves an additional wake behind. Consequently, the wake seen by the third bunch is the sum of the two wakes left by the preceding bunches. If we define the transverse displacement of the nth bunch from the design orbit to be $x_n(s)$, this displacement is governed by the following equation of motion:

$$\frac{d^2x_n}{ds^2}+K(s)x_n=F(s)+\sum_{\ell=1}^{N_c}F_n^{(\ell)}(s)\delta_p(s-s_\ell), \tag{2.2}$$

where $\delta_p(s)$ denotes the periodic delta function with the period $2\pi R$, and $F_n^{(\ell)}(s=s_\ell)$ is the kick force at the ℓth cavity experienced by the nth bunch, given by

$$F_n^{(\ell)}(s)=\sum_{m=1}^{n-1}W_{nm}^{(\ell)}[x_m(s)+\Delta^{(\ell)}]. \tag{2.3}$$

Here, $\Delta^{(\ell)}$ stands for the misalignment of the ℓth cavity, and the coefficient $W_{nm}^{(\ell)}$ is the wake field generated in the ℓth cavity by the mth bunch, acting on the nth one.

For a specific mode, $W_{nm}^{(\ell)}$ is given by

$$W_{nm}^{(\ell)}=\frac{eQ_mW}{E_0}\sin\left(\omega_\ell\frac{d_{nm}}{c}\right), \tag{2.4}$$

where ω_ℓ is the resonant frequency of the deflecting mode in the ℓth cavity, W corresponds to the wake strength (in units of V/C/m), Q_m is the charge of the mth bunch, and E_0 the energy of the design particle. Damping within the bunch is neglected.

We now introduce Courant-Snyder variables [7] to write

$$x_n(s)=\sqrt{\beta(s)}y_n(\theta)\ \text{with}\ d\theta=\frac{ds}{\nu\beta(s)}, \tag{2.5}$$

where θ goes from 0 to 2π in one turn, and $2\pi\nu$ is the phase advance of the betatron oscillation per turn. This allows us to rewrite Eq. (2.2) as

$$\frac{d^2y_n}{d\theta^2}+\nu^2y_n=f(\theta)+\nu\sum_{\ell=1}^{N_c}\beta_\ell\sum_{m=1}^{n-1}W_{nm}^{(\ell)}\left[y_m(\theta)+\frac{\Delta^{(\ell)}}{\sqrt{\beta_\ell}}\right]\delta_p(\theta-\theta_\ell), \tag{2.6}$$

where $f(\theta)=\nu^2\beta^{3/2}F(s)$, and $\beta_\ell=\beta(s=s_\ell)$.

III. CLOSED-ORBIT DISTORTIONS

We now proceed to calculate the closed orbit for each bunch. Even if all rf cavities are perfectly constructed and precisely aligned, the first bunch may still leave a transverse wake behind if it has an offset from the cavity axis due to a closed-orbit distortion caused by field errors or quadrupole misalignment. Then *the closed-orbit distortion of the nth bunch*, denoted by Δy_n, *satisfies an equation of exactly the same form as the equation for betatron motion*, Eq. (2.6). For the first bunch, the periodic solution to the equation of motion can be written as

$$\Delta y_1(\theta)=\int_0^{2\pi}d\theta' f(\theta')\left[\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\frac{e^{in(\theta-\theta')}}{\nu^2-n^2}\right], \tag{3.1}$$

where the Green function in brackets has been expressed as an explicit function of $|\theta-\theta'|$. In a similar fashion, Δy_n can be obtained from Eq. (2.6) as

$$\Delta y_n(\theta)=\Delta y_1(\theta)+\frac{\nu}{2\pi}\sum_{\ell=1}^{N_c}\beta_\ell\sum_{m=1}^{n-1}W_{nm}^{(\ell)}\left[\Delta y_m(\theta_\ell)+\frac{\Delta^{(\ell)}}{\sqrt{\beta_\ell}}\right]\sum_{k=-\infty}^{\infty}\frac{e^{ik(\theta-\theta_\ell)}}{\nu^2-k^2}. \tag{3.2}$$

Next we derive an interesting expression for Δy_n which is valid under some simplifying assumptions. We use Eq. (2.4) for $W_{nm}^{(\ell)}$, and assume that the deflecting-mode frequency is the same in all rf cavities, i.e., $\omega_\ell\approx\omega_0$. We further assume equally spaced and equally populated bunches in the train, i.e., $d_{n+1,n}\equiv\Delta d$, and $Q_m=Q_0$. Neglecting possible cavity misalignments, we can find the closed orbit for the second bunch from Eq. (3.2) as

$$\Delta y_2(\theta)=\Delta y_1(\theta)+\frac{\nu\xi\sin\zeta}{\pi}\sum_{\ell=1}^{N_c}\beta_\ell\Delta y_1(\theta_\ell)\times\sum_{n=-\infty}^{\infty}\frac{e^{in(\theta-\theta_\ell)}}{\nu^2-n^2}, \tag{3.3}$$

where $\xi\equiv eQ_0W/2E_0$, and $\zeta\equiv\omega_0\Delta d/c$. Noting that

$$\sum_{n=-\infty}^{\infty}\frac{e^{in(\theta-\theta_\ell)}}{\nu^2-n^2}=\frac{\pi\cos\nu(|\theta-\theta_\ell|-\pi)}{\nu\sin\nu\pi}$$

for $-2\pi<\theta-\theta_\ell<2\pi$, we get

$$\Delta y_2(\theta)=\Delta y_1(\theta)+\xi\sin\zeta\sum_{\ell=1}^{N_c}\Delta y_1(\theta_\ell)p_\ell(\theta-\theta_\ell), \tag{3.4}$$

where we introduced the function

$$p_\ell(\theta)\equiv\beta_\ell\left(\sin\nu|\theta|+\frac{\cos\nu\theta}{\tan\nu\pi}\right).$$

Equation (3.4) indicates that the closed orbit for the second bunch differs from that for the first bunch by a factor proportional to $\beta_\ell\xi$, which is quite small in most cases. In fact, as shown later, this factor is less than 0.0003 in the case of LEP, even for rather high intensity beams.

Similarly, the closed orbit for the third bunch can be expressed as

$$\Delta y_3(\theta)=\Delta y_1(\theta)+\xi\sum_{\ell=1}^{N_c}[\Delta y_1(\theta_\ell)\sin 2\zeta+\Delta y_2(\theta_\ell)\sin\zeta]p_\ell(\theta-\theta_\ell). \tag{3.5}$$

If ξ is small, $\Delta y_2(\theta)$ can be approximated by $\Delta y_1(\theta)$ in Eq. (3.5). In this way, we find from Eq. (3.5), together with Eq. (3.4),

$$\Delta y_3(\theta)-\Delta y_2(\theta)\approx\xi\sin 2\zeta\sum_{\ell=1}^{N_c}\Delta y_1(\theta_\ell)p_\ell(\theta-\theta_\ell). \tag{3.6}$$

An analog analysis for the $(n+1)$st bunch results in

$$\Delta y_{n+1}(\theta) - \Delta y_n(\theta) \approx \xi \sin(n\zeta) \sum_{\ell=1}^{N_c} \Delta y_1(\theta_\ell) p_\ell(\theta - \theta_\ell). \tag{3.7}$$

We now see that successive bunches have closed orbits differing from each other only by a relative factor ξ, but have the *same azimuthal dependence.* Since the problem is linear, it is straightforward to generalize this formula to the case where $W_{nm}^{(\ell)}$ includes several deflecting modes.

IV. CAVITY WAKE STRENGTH

In this section, we will estimate the strength of the cavity wakes for later reference. We first consider a single deflecting mode, using the expression given in Eq. (2.4). It is worthwhile to note that, at the intermediate distances considered here, the strength of the wake potential is not well determined since it oscillates rapidly. This can be seen from Eq. (2.4), when the distance d_{nm} is large compared to the wavelength c/ω_ℓ. The wake can be very small, or even accidentally equal to zero, depending on the mode frequencies and the bunch spacing. In reality, the wake potentials will not be identical in all cavities because of dimensional differences due to fabrication tolerances, as well as due to possibly different operating temperatures, tuner settings (used to compensate frequency deviations of the accelerating mode), and so on. Therefore the frequencies ω_ℓ of the deflecting modes are no longer the same in all cavities. They will be represented as $\omega_\ell = \omega_0 + \Delta\omega_\ell$, where $\Delta\omega_\ell$ denotes the frequency deviation at the ℓth cavity from the average ω_0 over all cavities. Because of the indeterminacy of the cavity modes, it will be necessary to perform a statistical estimate for the most probable strength of the wake kicks, and *no definite predictions for stability can be made.*

As an example, let us consider the following expressions for the mth bunch acting on the nth one in the ℓth cavity:

$$S_{nm} \equiv \sum_{\ell=1}^{N_c} \beta_\ell W_{nm}^{(\ell)}. \tag{4.1}$$

As will become clear later, these expressions are key factors for the growth rates and the maximum amplitudes of the betatron oscillations. If a very large number of cavities are present, we can approximately replace the sum over the cavity number by an integral, introducing the normalized weight function $g(\omega)$ describing the distribution of the mode frequencies. Equation (4.1) may then be written as

$$S_{nm} \approx \frac{eQ_m W\bar{\beta}}{E_0} \int_{-\frac{\Delta\omega}{2}}^{\frac{\Delta\omega}{2}} g(\omega) \sin\left[(\omega_0 + \omega)\frac{d_{nm}}{c}\right] d\omega, \tag{4.2}$$

where $\Delta\omega$ is the maximum deviation of the deflecting-mode frequencies, and we have assumed for simplicity that the values of the betatron function at the cavity positions have only small differences from their average value $\bar{\beta}$. If the distribution in frequency is Gaussian, we can put

$$g(\omega) = \frac{N_c}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\omega^2}{2\sigma^2}\right), \tag{4.3}$$

where σ is the rms width of the distribution in ω.

To make a first estimate of this effect, we substitute Eq. (4.3) into Eq. (4.2), expanding the range of integration to infinity, to obtain

$$S_{nm} = \frac{N_c eQ_m W\bar{\beta}}{E_0} \sin\left(\frac{\omega_0 d_{nm}}{c}\right) \times \exp\left[-\frac{1}{2}\left(\frac{\sigma d_{nm}}{c}\right)^2\right]. \tag{4.4}$$

We thus find an extra factor which may considerably reduce the integrated effect of the wake kicks, depending quite sensitively on the product σd_{nm}.

The existence of many higher-order modes (HOM's) requires summation of the contributions over all modes for the evaluation of the total kick received by each bunch. The factor $\sin(\omega_0 d_{nm}/c)$ may as well be positive as negative, depending on the mode frequency ω_0, and accordingly we get another reduction factor in the kick strength. The effect of this factor may be more significant than that of the mode frequency spread considered above. If we write the wake strength of the kth mode as W_k, Eq. (4.4) should be modified to

$$S_{nm} = \frac{N_c eQ_m W\bar{\beta}}{E_0} \times \sum_k W_k \sin\left(\frac{\omega_{0k} d_{nm}}{c}\right) \exp\left[-\frac{1}{2}\left(\frac{\sigma_k d_{nm}}{c}\right)^2\right], \tag{4.5}$$

where ω_{0k} and σ_k are, respectively, the central frequency and a typical frequency spread of the kth mode.

We will take LEP parameters as an example to estimate the order of magnitude of S_{nm}. Then $R = 4.24$ km, $\bar{\beta} \sim 40$ m, N_c=120, $\nu = 90.27$ in the horizontal, and 76.24 in the vertical plane. The energy E_0 is evaluated at injection (20 GeV) since the instability is more severe at the lowest energy because of weaker radiation damping. As to the reduction factor, the estimated frequency spread of the HOM's due to fabrication tolerances in the LEP cavities is, unfortunately, rather small: it has been determined to be less than 0.1% [8]. However, possibly different positions of both fixed and variable tuners in the cavities might increase the actual spread of the HOM frequencies. Provided that the distances between bunches d_{nm} are sufficiently large, this could considerably reduce the magnitude of S_{nm}. However, as demonstrated later, the maximum betatron amplitude is always associated with the factor $S_{n+1,n}$, i.e., the distance between neighboring bunches Δd is of particular importance if the bunches in a train are equally spaced. Therefore, noting the fact that Δd adopted for bunch trains in LEP is presently about 74 m, the reduction factor is very near unity and can be ignored for a conservative estimate. Then we obtain $S_{nm} = 0.0213 I_m V(d_{nm})$, where I_m is the current in the mth bunch in units of mA, while $V(d_{nm})$ has the units of V/pC/m as the wake potential, defined by

$$V(z) = \sum_k W_k \sin\left(\omega_{0k}\frac{z}{c}\right). \tag{4.6}$$

Figure 1 shows the function $V(z)$, obtained from the wake potential for a five-cell LEP copper cavity for a bunch length $s = 2$ cm by a mesh-code analysis [9]. We observe that the peak transverse wake strength oscillates with an amplitude of about 5 V/pC/m between 70 and 80 m, which corresponds to the presently favored distance between two bunches in the trains for LEP. Later bunches, at multiple distances from the first ones, might feel somewhat smaller wakes because of resistive damping (which is not included in the mesh code), but this reduction is certainly very small for a short train. To be on the safe side, we shall assume $V(z)$ to be 5 V/pC/m over the whole range of a single bunch train in LEP. The value of S_{nm} derived from the above formula is then $S_{nm} = 0.1065 I_m$.

V. BEAM BREAKUP IN BUNCH TRAINS

We are now in a position to look at the betatron motion of each bunch. From Eq. (2.6) together with Eq. (3.2), the bunch oscillation about the individual closed orbit is described by

$$\frac{d^2Y_n}{d\theta^2} + \nu^2 Y_n = \nu \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} Y_m(\theta_\ell)\delta_p(\theta - \theta_\ell), \tag{5.1}$$

where $Y_n = y_n - \Delta y_n$. Since the right-hand side of Eq. (5.1) depends only on the motion of the preceding bunches, we can easily get the general solution for $Y_n(\theta)$,

$$Y_n(\theta) = a_n \cos(\nu\theta) + b_n \sin(\nu\theta) + \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} \times \int_0^\theta d\theta' Y_m(\theta') \sin[\nu(\theta - \theta')]\delta_p(\theta' - \theta_\ell), \tag{5.2}$$

where $n \geq 2$, and the initial conditions have been introduced according to

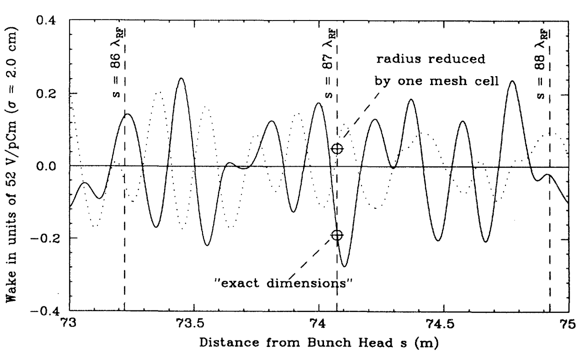

FIG. 1. Transverse wake potential of five-cell LEP cavity.

$$a_n \equiv Y_n(\theta = 0) \text{ and } b_n \equiv \frac{1}{\nu}\left(\frac{dY_n}{d\theta}\right)_{\theta=0}.$$

For the first bunch, the third term on the right side of Eq. (5.2) disappears, and the solution simply becomes

$$Y_1(\theta) = a_1 \cos(\nu\theta) + b_1 \sin(\nu\theta). \tag{5.3}$$

Equation (5.3) contains enough information to determine the second-bunch motion from Eq. (5.2), leading to the betatron amplitude after N turns,

$$Y_2(2\pi N) = a_2 \cos(N\mu) + b_2 \sin(N\mu) + \frac{N S_{21} a_1}{2} \sin(N\mu) + \frac{a_1}{2}\frac{\sin(N\mu)}{\sin\mu} \sum_{\ell=1}^{N_c} \beta_\ell W_{21}^{(\ell)} \sin(\mu - 2\nu\theta_\ell), \tag{5.4}$$

where $\mu = 2\pi\nu$, S_{nm} has been defined in Eq. (4.1), and we have assumed $b_1 = 0$ for simplicity. The third term in the right-hand side of Eq. (5.4) expresses a growing oscillation proportional to the number of turns N for the oscillation amplitude of the second bunch.

The solution for the third bunch can be derived from Y_1 and Y_2 in an analogous way. Obviously, the contribution from the first bunch results in the terms of the same form as in Eq. (5.4), yielding a linearly growing amplitude, while the wake parameter $W_{21}^{(\ell)}$ must be replaced by $W_{31}^{(\ell)}$. The stable part in the second-bunch solution also gives rise to terms growing linearly with the number of turns. On the other hand, the unstable part, i.e., the third term in Eq. (5.4), generates terms which grow with the square of the turn number. This N^2-dependent term reaches an amplitude

$$\frac{N^2 S_{32} S_{21} a_1}{8}. \tag{5.5}$$

It is easy to see that this term leads to an N^3-dependent amplitude in the solution for the fourth bunch. Thus we come to the conclusion that the oscillations of the nth bunch have amplitudes involving all powers of N up to the order of N^{n-1}, provided that the leading bunch executes betatron motion about the closed orbit. More correctly, the betatron motion of the mth bunch produces divergent terms proportional to $a_m N^{n-m}$ and $b_m N^{n-m}$ in the solution for the trailing nth bunch. Figure 2 shows the severe growth of the betatron amplitudes originating from this nonexponential instability. The ordinate of the figures represents the ratio of betatron displacement to the initial value, taken to be the same for all bunches. This figure has been obtained by tracking a bunch train with four bunches in LEP, with 16 rf cavities located at positions corresponding to the centers of groups of eight cavities. The wake strength at each cavity has, accordingly, been taken about eight times larger than the value estimated for a single cavity in the last section.

In order to evaluate the magnitude of the nth bunch amplitude after N turns approximately, we distribute the kicks over the whole circumference of the ring. Equation (5.1) then leads to

$$\frac{d^2Y_n}{d\theta^2} + \nu^2 Y_n = \frac{\nu}{2\pi}\sum_{m=1}^{n-1} S_{nm} Y_m. \tag{5.6}$$

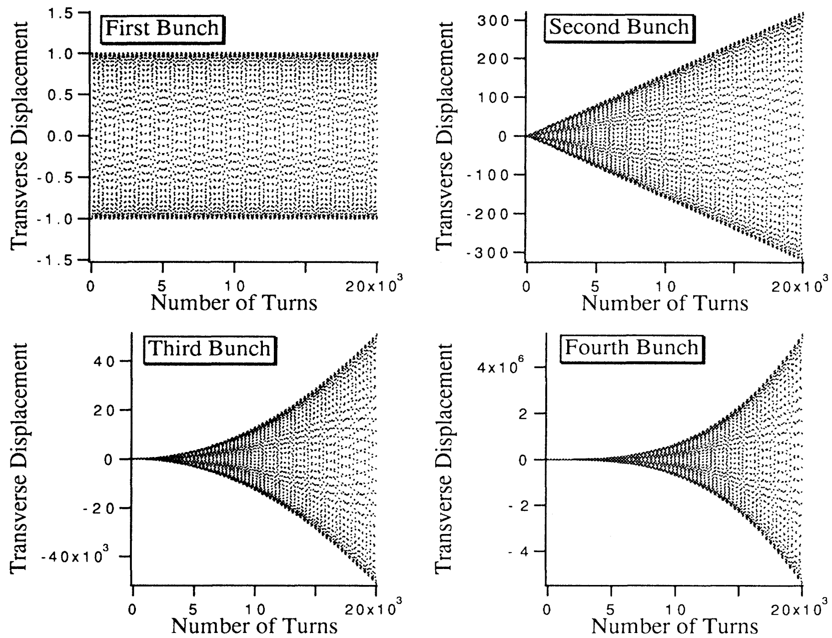

FIG. 2. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); no damping; bunch currents (mA): $I_1 = I_2 = I_3 = I_4 = 0.3$.

Furthermore, we assume that the amplitude and phase of each bunch are slowly varying quantities per turn. Writing $Y_n(\theta) = u_n(\theta)\exp(i\nu\theta)$, where u_n is a complex amplitude, we obtain from Eq. (5.6) the approximate equation

$$\frac{du_n}{d\theta} = \frac{1}{4\pi i}\sum_{m=1}^{n-1} S_{nm}u_m. \tag{5.7}$$

If we consider u_n on successive turns, we can rewrite Eq. (5.7) in terms of the turn number N as

$$\frac{du_n(N)}{dN} = \frac{1}{2i}\sum_{m=1}^{N-1} S_{nm}u_m(N). \tag{5.8}$$

We now proceed one bunch at a time, assuming that the first bunch has a constant amplitude u_1 since $du_1/dN = 0$. For the second bunch, Eq. (5.8) yields for $N \gg 1$

$$u_2 = \frac{NS_{21}u_1}{2i} \tag{5.9}$$

and for the third bunch

$$\frac{u_3(N)}{u_1} = \frac{1}{2}\left(\frac{N}{2i}\right)^2 S_{32}S_{21} + \frac{N}{2i}S_{31}. \tag{5.10}$$

Provided that NS_{nm} reaches a large value before damping of the betatron motion becomes important, only the highest power of NS_{nm} need be retained, and we find for the maximum amplitude

$$\frac{u_{n+1}(N)}{u_1} = \frac{1}{n!}\left(\frac{N}{2i}\right)^n \tilde{S}_n, \tag{5.11}$$

where

$$\tilde{S}_n \equiv \prod_{m=1}^{n} S_{m+1,m}. \tag{5.12}$$

Equation (5.11) agrees with the numerical results in Fig. 2 as well as with the solutions given in Eqs. (5.4) and (5.5). Thus—without radiation damping—successive bunches may have ever increasing amplitudes and will be shifted in phase from the preceding one by $-90°$.

VI. SUPPRESSION OF BEAM BREAKUP

A. Octupole nonlinearity

The nonexponential instability of the betatron oscillations of the later bunches is caused by a resonance with the driving force in Eq. (5.1) which has the same frequency. Therefore the growth can be limited by modifying this frequency. This occurs naturally if the betatron tune is amplitude dependent. For this purpose, we take into account the effect of an octupole nonlinearity. Starting from the averaged version of the equation, i.e., Eq. (5.6), and adding an octupole term to it, we get for the second bunch

$$\frac{d^2Y_2}{d\theta^2} + \nu^2 Y_2 = \frac{\epsilon\nu^2}{|u_1|^2}Y_2^3 + \frac{\nu}{2\pi}S_{21}Y_1, \tag{6.1}$$

where the last term must be extended to all turns. The parameter ϵ is related to the detuning of the betatron frequency of the first bunch. (In fact, $\delta\nu/\nu = 3\epsilon/8$ for the first bunch.) We now assume the solution to Eq. (6.1) to be $Y_n = A_n\sin(\nu\theta + \alpha_n)$, where A_n and α_n are slowly varying functions of θ. In terms of turn number, this can be rewritten as

$$Y_n(N) = A_n\sin(\mu N + \alpha_n), \tag{6.2}$$

and Eq. (6.1) becomes

$$\frac{d^2Y_2(N)}{dN^2} + \mu^2 Y_2(N) - \frac{\epsilon\mu^2}{A_1^2}[Y_2(N)]^3 = \mu S_{21} Y_1(N). \tag{6.3}$$

We choose

$$\frac{dA_2}{dN}\sin(\mu N + \alpha_2) + A_2\frac{d\alpha_2}{dN}\cos(\mu N + \alpha_2) = 0. \tag{6.4}$$

The use of Eq. (6.4) after substituting Eq. (6.2) into Eq. (6.3) results in

$$\frac{dA_2}{dN}\cos(\mu N + \alpha_2) - A_2\frac{d\alpha_2}{dN}\sin(\mu N + \alpha_2) = \frac{\epsilon\mu A_2^3}{A_1^2}\sin^3(\mu N + \alpha_2) + S_{21}A_1\sin(\mu N + \alpha_1). \tag{6.5}$$

Combining Eqs. (6.4) and (6.5), and averaging over the rapidly varying terms, we eventually obtain

$$\frac{dA_2}{dN} = \frac{S_{21}A_1}{2}\sin(\alpha_1 - \alpha_2), \tag{6.6}$$

$$A_2\frac{d\alpha_2}{dN} = -\frac{3\epsilon\mu A_2^3}{8A_1^2} - \frac{S_{21}A_1}{2}\cos(\alpha_1 - \alpha_2). \tag{6.7}$$

Since A_1 and α_1 are independent of N, Eqs. (6.6) and (6.7) lead to the constant of motion C satisfying the relation

$$\cos(\alpha_1 - \alpha_2) = -\frac{3\epsilon\mu}{16S_{21}}\left(\frac{A_2}{A_1}\right)^3 + C\frac{A_1}{A_2}, \tag{6.8}$$

which has been plotted in Fig. 3 in the form of $\cos(\alpha_2 - \alpha_1)$ vs A_2/A_1 for various values of C. If the initial amplitude A_2/A_1 is of the order of unity and $S_{21}/\epsilon\mu$ is large, we can neglect C in Eq. (6.8) and obtain, for the maximum value of A_2/A_1,

$$\left(\frac{A_2}{A_1}\right)_{\max} \approx \left(\frac{16S_{21}}{3\epsilon\mu}\right)^{1/3}. \tag{6.9}$$

Clearly, without the nonlinearity, A_2/A_1 will grow without limit (or until radiation damping takes over).

In order to extend this analysis to successive bunches, we assume, as in the case without nonlinearity, that only the preceding bunch is important. This is suggested, e.g., by Eq. (5.11) indicating that the fastest growing term in the nth bunch solution originates only from the largest term in the previous bunch solution. Then, the assumption enables us to write for the third bunch

$$\frac{dA_3}{dN} = \frac{S_{32}A_2}{2}\sin(\alpha_2 - \alpha_3), \tag{6.10}$$

$$A_3\frac{d\alpha_3}{dN} = -\frac{3\epsilon\mu A_3^3}{8A_2^2} - \frac{S_{32}A_2}{2}\cos(\alpha_2 - \alpha_3). \tag{6.11}$$

Unfortunately, A_2 and α_2 are now functions of N, as shown by Eqs. (6.6) and (6.7), and it is no longer possible to obtain an exact integral of motion. Instead we invoke an approximate model which we will then test on Eqs. (6.6) and (6.7). Specifically, we assume $\alpha_1 - \alpha_2$ starts at $\pi/2$, remaining there until α_2 grows rapidly near the end of the growth of A_2. In this way, we approximate A_2 as $A_2 \approx S_{21}A_1N/2$. The growth of $\alpha_1 - \alpha_2$ is governed by Eq. (6.7), where we drop the second term since $\cos(\alpha_1 - \alpha_2) \approx 0$ during most of the growth of A_2. Thus Eq. (6.7) can be approximately solved to give

$$\alpha_1 - \alpha_2 \approx \frac{\pi}{2} + \frac{\epsilon\mu S_{21}^2 N^3}{32}. \tag{6.12}$$

From Eq. (6.6), we see that the growth of A_2 stops when $\alpha_1 - \alpha_2 = \pi$. This occurs when $N \approx (16\pi/\epsilon\mu S_{21}^2)^{1/3}$ by which time the growth has reached its maximum value

$$\left(\frac{A_2}{A_1}\right)_{\max} \approx \left(\frac{2\pi S_{21}}{\epsilon\mu}\right)^{1/3}. \tag{6.13}$$

Since $(2\pi)^{1/3} \approx 1.845$ and $(16/3)^{1/3} \approx 1.747$, our model yields excellent agreement with Eq. (6.9).

This approximate model can easily be extended to the bunches which follow. Specifically, we find

$$\frac{A_{n+1}}{A_1} \approx \frac{1}{n!}\left(\frac{N}{2}\right)^n \tilde{S}_n, \tag{6.14}$$

$$\alpha_n - \alpha_{n+1} \approx \frac{\pi}{2} + \frac{3}{8}\frac{\epsilon\mu}{2n+1}\left(\frac{\tilde{S}_n}{2^n n!}\right)^2 N^{2n+1}. \tag{6.15}$$

The maximum amplitude of the $(n+1)$st bunch is reached near the turn number,

$$N \approx \left[\frac{4\pi(2n+1)}{3\epsilon\mu}\left(\frac{2^n n!}{\tilde{S}_n}\right)^2\right]^{\frac{1}{2n+1}} \tag{6.16}$$

and the approximate magnitude of the corresponding amplitude can be evaluated from

$$\left(\frac{A_{n+1}}{A_1}\right)_{\max} \approx \left[\left(\frac{2\pi(2n+1)}{3\epsilon\mu}\right)^n \frac{\tilde{S}_n}{n!}\right]^{\frac{1}{2n+1}}. \tag{6.17}$$

To obtain more reliable results, one must use the wake field of several bunches, as well as distribute the kicks over several cavities.

The amplitude-dependent tune shift treated in this section could be a possibility to limit the growth in betatron amplitude if a sufficiently large value of ϵ can be provided. However, for the LEP case, currently only eight octupole

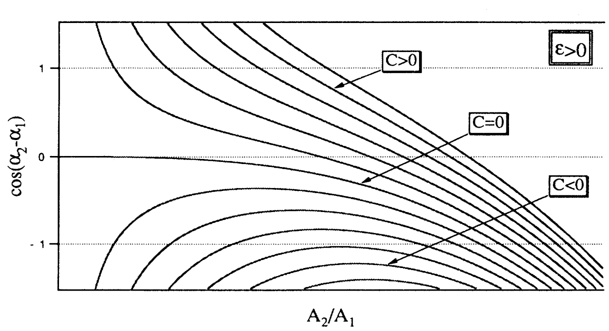

FIG. 3. Trajectories in the $\cos(\alpha_1 - \alpha_2)$ vs A_2/A_1 space for various values of the adiabatic invariant C.

magnets are installed and their field strength is too weak to provide sufficient suppression of the beam blowup. On the other hand, a large number of sextupoles, which also yield a tune shift, are installed in LEP. But, unfortunately, the effect of sextupole nonlinearity on betatron tune is much weaker than that of an octupole. Therefore, the nonlinear components on LEP will not help much to prevent the amplitude from reaching the critical size.

B. Current-dependent tune shift

Image charges and currents, induced by the beam in the surrounding vacuum enclosure, are also a source of tune shifts. Since these shifts are proportional to the beam intensity, it is possible to provide bunch-dependent tunes by intentionally storing different currents in each bunch. Keeping only the linear part of the image forces and making use of the transformation in Eq. (2.5), the starting equation reads

$$\frac{d^2Y_n}{d\theta^2} + K_n(\theta)Y_n = \nu \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} Y_m \delta_p(\theta - \theta_\ell), \tag{6.18}$$

where the closed orbits in Eq. (3.2) have been redefined to incorporate the image forces, and $K_n(\theta)$ is the periodic function including the image effect on the nth bunch. Smoothing the periodic force by writing the approximate tune of the nth bunch as ν_n, Eq. (6.18) becomes

$$\frac{d^2Y_n}{d\theta^2} + \nu_n^2 Y_n = \nu \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} Y_m \delta_p(\theta - \theta_\ell). \tag{6.19}$$

The tune ν_n can be decomposed into a zero-current tune ν and the bunch-dependent shift $\Delta\nu_n$ due to the image fields; namely, $\nu_n = \nu + \Delta\nu_n$. The shift $\Delta\nu_n$ can be represented as $\Delta\nu_n = (d\nu/dI)I_n$, where I_n is current of the nth bunch, and an explicit analytic form of the detuning factor $(d\nu/dI)$ is given, for example, by Laslett's formula [10]. In practice, the value of $(d\nu/dI)$ should be determined through experimental observations. In LEP, past experiments show that $(d\nu/dI) \approx -0.129$ (mA^{-1}) vertically and $(d\nu/dI) \approx -0.064$ (mA^{-1}) horizontally [11].

Needless to say, Eq. (6.19) yields a solution similar to Eq. (5.2). However, the first two terms on the right-hand side of Eq. (5.2) now have the frequency ν_n, and the factor $\sin[\nu(\theta - \theta')]$ in the third term must be modified to $\sin[\nu_n(\theta - \theta')]$. The first-bunch solution is again a harmonic oscillation with the frequency ν_1. The second-bunch amplitude, after N turns, is then found to be

$$Y_2(2\pi N) = a_2 \cos(2\pi N\nu_2) + \frac{\nu b_2}{\nu_2}\sin(2\pi N\nu_2) + \frac{\nu a_1}{2\nu_2}\sum_{\ell=1}^{N_c} \beta_\ell W_{21}^{(\ell)} \left\{ \frac{\sin(N\mu_+)}{\sin\mu_+} \sin\left[\left(1 - \frac{\theta_\ell}{\pi}\right)\mu_+ - N\mu_-\right] - \frac{\sin(N\mu_-)}{\sin\mu_-} \sin\left[\left(1 - \frac{\theta_\ell}{\pi}\right)\mu_- - N\mu_+\right]\right\}, \tag{6.20}$$

where we have put $b_1 = 0$ and $\mu_\pm = \pi(\nu_1 \pm \nu_2)$. It has been confirmed that the growing term no longer exists because $\nu_1 \neq \nu_2$. Equation (6.20) is substantially a superposition of two stable oscillations with the frequencies ν_1 and ν_2. In actual cases, these two frequencies are usually chosen close to each other, and the resulting effect is *beating*.

To evaluate the third bunch solution, Eq. (6.20) as well as the first-bunch solution are substituted into Eq. (6.19). The driving terms now involve two modes having the frequencies ν_1 and ν_2, but the free-oscillation tune of the third bunch ν_3 is different from both of them. Therefore the third-bunch motion is also stable and beating. Because of the linearity of the problem, it is obvious that the nth bunch solution $Y_n(\theta)$ is generally composed of the n stable modes oscillating at the frequencies ν_m ($m = 1, 2, \ldots, n$). If each frequency is different, the endless growth of the betatron amplitude can be totally suppressed. Thus the use of current-dependent tune shift is a simple and effective way to avoid beam breakup in bunch trains.

However, it is important to notice that the oscillation amplitude in Eq. (6.20) can still become very large owing to the factor $1/\sin\mu_-$ when the difference of the bunch currents is quite small. When $\nu_1 \approx \nu_2$, the last term on the right-hand side plays a dominant role, and the peak beating amplitude is of the order of

$$\frac{a_1\nu}{2\nu_2}\left|\frac{S_{21}}{\sin\mu_-}\right|. \tag{6.21}$$

For the third and later bunches, it may not be straightforward to obtain such a compact formula as Eq. (6.21) for quick evaluation of the maximum amplitudes. However, it is possible to derive a rough criterion when the current differences in adjacent bunches are sufficiently small and approximately the same, i.e., $\Delta\nu = \nu_n - \nu_{n-1} \ll 1$ regardless of the bunch number n. In this case, the maximum beating amplitude of the $(n+1)$st bunch can be estimated from the expression

$$\left(\frac{Y_{n+1}}{a_1}\right)_{\max} = \frac{1}{n!}\left(\frac{1}{2\pi\Delta\nu}\right)^n \tilde{S}_n \equiv h_{n+1}. \tag{6.22}$$

The tune shift $\Delta\nu$ should be chosen such that the amplitude increase h_n takes an acceptable value depending on various conditions: the minimum aperture size, the expected value of a_1, etc. If $h_n \approx 1$ is adopted for all bunches, beating could be completely eliminated, but this choice might be too conservative. With strong wake fields and/or a very small tune shift, h_n usually becomes larger for a later bunch.

The effect of the current-dependent tune shift is shown in Fig. 4 for the (smaller) horizontal LEP detuning parameter $(d\nu/dI) \approx -0.064$ (mA^{-1}). Although the total current of the train is even higher than for the situation

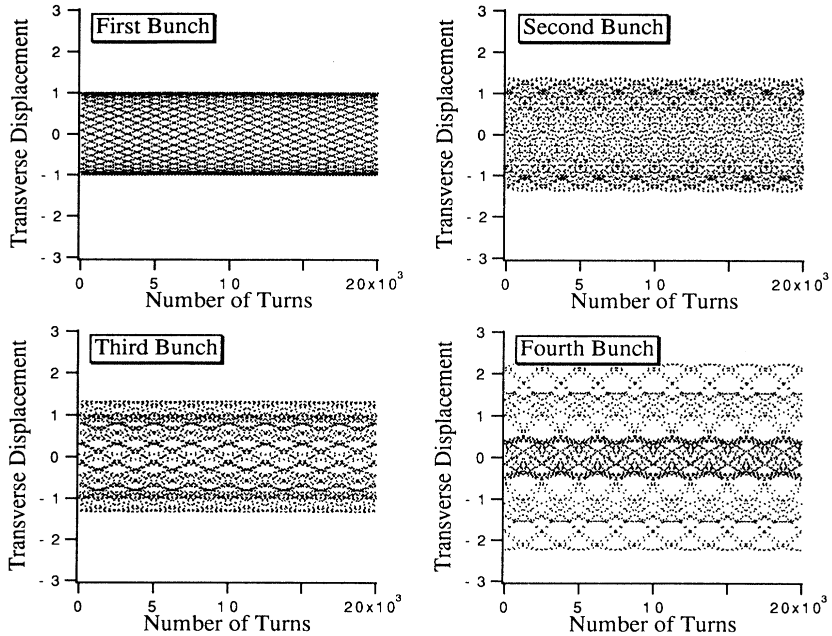

FIG. 4. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); no damping; bunch currents (mA): $I_1 = 0.45$, $I_2 = 0.40$, $I_3 = 0.35$, $I_4 = 0.3$.

in Fig. 2, we observe well-bounded oscillations in all four bunches. The value of h_n in this example is about two for each bunch, which roughly agrees with Fig. 4.

C. rf focusing

In order to make the tune bunch-dependent, we briefly explore here the application of a time-dependent linear kick on a bunch train, modifying Eq. (2.2) to

$$\frac{d^2 x_n}{ds^2} + K(s) x_n = F(s) + \sum_{\ell=1}^{N_c} F_n^{(\ell)} \delta_p(s - s_\ell) + \kappa(t) x_n \delta_p(s). \tag{6.23}$$

The additional kick has been assumed to occur at position $s = 0$; $\kappa(t)$ is a periodic function representing the time-varying kick strength.

This effect could be provided, for example, by installing an rf focusing device in the storage ring. The function $\kappa(t)$ then has the form $\kappa(t) = q\cos(\Omega t)$, where q is a constant related to the rf voltage, and Ω denotes the angular rf frequency which should be an integer multiple of angular revolution frequency of the design particle. The magnitude of the kick strength depends on the time when a bunch traverses the focusing element. The resulting tune shift can be made bunch-dependent by making a proper choice of the rf frequency and initial phase.

With the closed-orbit distortions redefined again by including the additional terms, Eq. (6.23) turns out to be of the same form as Eq. (6.18), but with the coefficient of the linear force term changed to

$$K_n(\theta) = \nu^2 - \nu\beta_0\kappa_n\delta_p(\theta), \tag{6.24}$$

where $\beta_0 \equiv \beta(s = 0)$, and κ_n is the kick strength experienced by the nth bunch. The second term yields a tune shift, and an approximate relation between the original tune ν and the shifted tune ν_n can be given by

$$\nu_n - \nu = -\frac{\kappa_n \beta_0}{4\pi}. \tag{6.25}$$

The smoothed version of Eq. (6.23) is identical to Eq. (6.19) except that we now need to employ Eq. (6.25) instead of the Laslett tune shift. The nth bunch solution is made stable anyway if $\nu_m \neq \nu_n$ for $m \neq n$. Although in principle such an rf kicker is a simple means to eliminate the resonant growth of betatron amplitude, it may be rather difficult to obtain a sufficiently large tune shift when the beam energy is high.

VII. RADIATION DAMPING

The effect of synchrotron radiation damping will now be considered. Although the natural damping force is quite weak at injection energy in LEP, its exponential nature is always strong enough to eventually limit the power-law beam blowup. By adding a frictional term to Eq. (5.1), the starting equation can be written as

$$\frac{d^2 Y_n}{d\theta^2} + 2\lambda\frac{dY_n}{d\theta} + \nu^2 Y_n = \nu \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} Y_m \delta_p(\theta - \theta_\ell), \tag{7.1}$$

where we have assumed the damping constant λ to be small, and the frictional force has been averaged over one turn. The redefined closed orbit here is the periodic solution of Eq. (2.2) together with the frictional term on the left-hand side. For $\nu \gg \lambda$, the general solution to Eq. (7.1) can be obtained, to a good approximation, as

$$Y_n(\theta) = e^{-\lambda\theta}[a_n \cos(\nu\theta) + b_n \sin(\nu\theta)] + e^{-\lambda\theta} \sum_{\ell=1}^{N_c} \beta_\ell \sum_{m=1}^{n-1} W_{nm}^{(\ell)} \times \int_0^\theta d\theta' Y_m(\theta') e^{\lambda\theta'} \sin[\nu(\theta-\theta')]\delta_p(\theta'-\theta_\ell). \quad (7.2)$$

The natural damping time is about 0.5 sec at injection energy in LEP, corresponding to about 6000 turns. The parameter λ is then about 2.7×10^{-5}. Although this value seems very small, an initial betatron oscillation will be damped to negligible amplitude in only a few seconds, removing the driving term for the nonexponential instability. It is, therefore, easy to make the instability mechanism ineffective by injecting subsequent bunches at larger time intervals, unless the injection of a later bunch excites oscillations of the preceding ones. All one needs to do is simply to wait for a few damping times before a new bunch is added to the train. Then, at the nth bunch injection, the functions $Y_m(\theta)$ $(m = 1, 2, \ldots, n-1)$ have already been damped away, and we only need to consider the first two terms in Eq. (7.2) which represent damped oscillations.

In practice, when the distance between bunches in a train is not very large, it will be difficult to inject a new bunch without any influence on the already stored bunches due to residual fields of the kicker magnet. All bunches will suffer weak kicks by these fields during accumulation of additional bunches. The driving terms will then reappear, and are clearly important if their oscillation frequency coincides with the resonant value, leading to the nonexponential instability. Although these oscillatory components are eventually again damped away and all bunches will get back to the closed orbits, the betatron amplitudes will first grow and may reach values which exceed the radius of the beam pipe. Later bunches will then be limited in current, or even be lost, and reaching the same levels as that of the preceding bunches might be difficult.

In order to evaluate the peak amplitude of the second bunch, we simply use the function $a_1 e^{-\lambda\theta}\cos(\nu\theta)$ as the first-bunch solution. Then only the last term on the right-hand side of Eq. (7.2) is of concern, which yields the amplitude after N turns,

$$Y_2(2\pi N) = \frac{N S_{21} a_1 e^{-2\pi\lambda N}}{2} \sin(N\mu) + \frac{a_1 e^{-2\pi\lambda N}}{2} \frac{\sin(N\mu)}{\sin\mu} \sum_{\ell=1}^{N_c} \beta_\ell W_{21}^{(\ell)} \sin(\mu - 2\nu\theta_\ell), \quad (7.3)$$

identical to the last two terms in Eq. (5.4), except for the damping factor $e^{-2\pi\lambda N}$. In particular, the first term on the right-hand side makes the dominant contribution to the maximum amplitude. The time when Y_2 reaches its maximum is clearly $\theta = 1/\lambda$, and the corresponding amplitude can be evaluated from

$$\frac{S_{21} a_1}{4\pi e\lambda}. \quad (7.4)$$

It is straightforward to show that the peak amplitude of the $(n+1)$st bunch occurs around $\theta = n/\lambda$, and the value is roughly of the order of

$$a_1 \left(\frac{n}{2^{(n+3)/2}\pi e\lambda}\right)^n \tilde{S}_n. \quad (7.5)$$

This expression is valid under the assumption that the first bunch has initially a finite betatron amplitude a_1. Needless to say, bunch stability is much better if $a_1 = 0$. Provided that the peak amplitude of the last bunch in a train is well below the minimum aperture, no bunches will be lost due to this nonexponential instability.

Figure 5 demonstrates the effect of radiation damping. The damping time is 3000 turns, corresponding to LEP at injection energy with all wigglers excited. Other parameters are identical to those used for Fig. 2. We recognize that the peak amplitude for the nth bunch actually occurs near turn number $(n-1) \times 3000$, which agrees with the values discussed above. Furthermore, the values of the maximum amplitudes are also in good agreement with Eq. (7.5).

VIII. DISCUSSION

From the present results, it follows that radiation damping in LEP works rather effectively in limiting the growth of bunch oscillations at modest intensity, even at injection energy. However, the maximum amplitudes of the third and fourth bunch shown in Fig. 5 are beyond the permissible range. This could be a possible explanation of recent experimental observations in LEP, where accumulation in the third and fourth bunch failed to reach the first-bunch intensity, about 0.45 mA, while reaching the same level in the second bunch was no problem [12].

To simulate the situation of the experiment, we increase the second-bunch current in Fig. 4 to 0.45 mA, keeping other parameters unchanged. Figure 6 illustrates the result with a damping time of 3000 turns. While the maximum amplitude in Fig. 4 was only twice the initial value, even without damping, we now observe a large growth of the amplitudes in all trailing bunches. This drastic change strongly suggests that one should avoid equal currents for the two leading bunches. In fact, it is interesting to note that the peak amplitudes of all three trailing bunches occur at 3000 turns, indicating excitation of the linear-growth mechanism, though we observe no such behavior in Fig. 4.

Next we test the case where the currents in all four bunches of a train are almost equal, but any two adjacent bunches have slightly different intensities. As expected, the nonexponential growth is no longer dominant in this case, but beating can still lead to large amplitudes as seen in Fig. 7. With parameters for motion in the vertical plane of LEP, Fig. 7 is altered to Fig. 8. Comparison of these two figures leads to the conclusion that horizontal beam blowup in LEP is more severe than vertical growth because the detuning factor is only half of the vertical

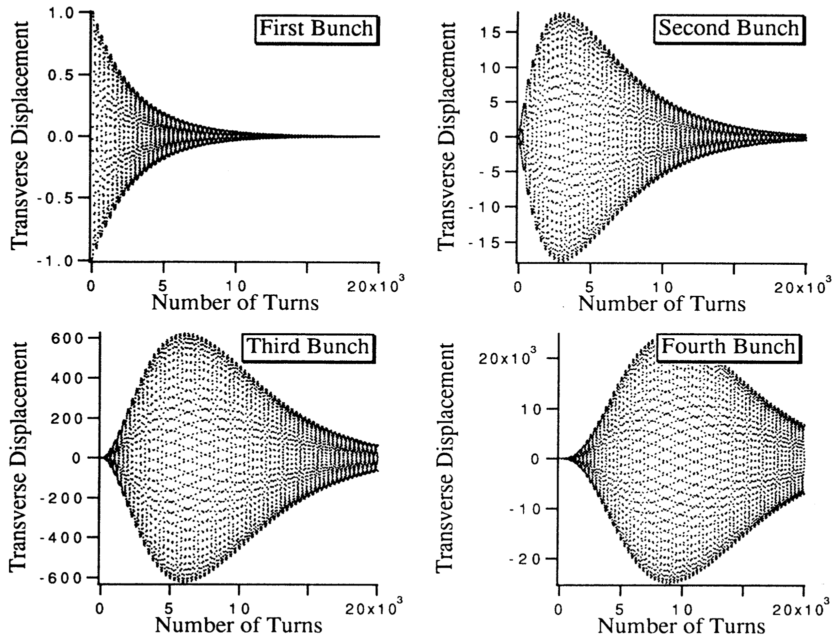

FIG. 5. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); damping time 3000 (turns); bunch currents (mA): $I_1 = I_2 = I_3 = I_4 = 0.3$.

value. However, the LEP beam pipe cross section is an ellipse whose horizontal semiaxis, $a \approx 70$ mm, is twice its vertical size, $b \approx 35$ mm. Therefore, even a slower vertical growth could be just as dangerous for beam loss as the horizontal one.

We can use Eq. (6.22) to evaluate the required tune difference for keeping the betatron amplitudes small. With the same total current of the trains in Figs. 7 and 8, and adopting the condition $h_4 \approx 15$, we find that the desirable tune split is $\Delta\nu \approx 0.0013$ corresponding to a bunch current difference of 0.02 mA in the horizontal plane. We therefore set the bunch currents at $I_1 = 0.355$ mA, $I_2 = 0.335$ mA, $I_3 = 0.315$ mA, and $I_4 = 0.295$ mA, resulting in Fig. 9. While the first-bunch intensity is even higher than in Fig. 7, the stability of the bunch train has improved remarkably. Also the peak amplitudes are in reasonable agreement with Eq. (6.22); i.e., $h_2 = 4.6$, $h_3 = 10.4$, and $h_4 = 14.5$.

It should be noticed that, except for the nonlinear situation discussed in Sec. VI A, the maximum amplitude always depends linearly on the initial offset a_1 of the first bunch, measured from its closed orbit. Therefore, if an error at injection doubles the value of a_1, this will immediately result in doubling the amplitudes of all fol-

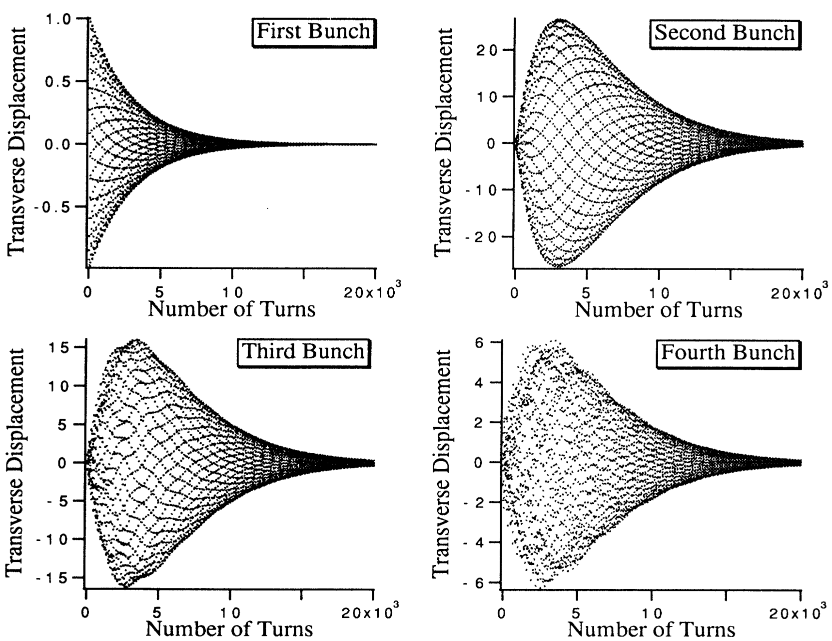

FIG. 6. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); damping time 3000 (turns); bunch currents (mA): $I_1 = 0.45$, $I_2 = 0.45$, $I_3 = 0.35$, $I_4 = 0.3$.

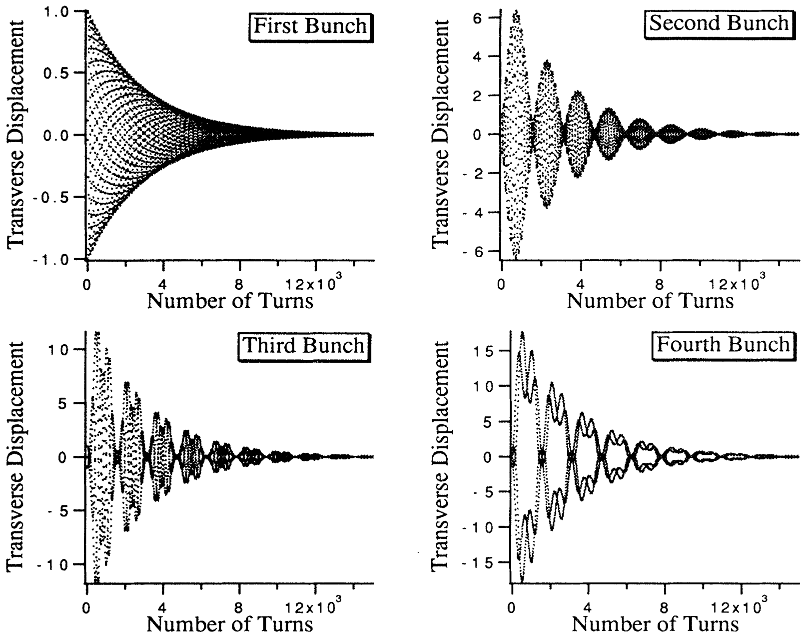

FIG. 7. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); damping time 3000 (turns); bunch currents (mA): $I_1 = 0.345$, $I_2 = 0.335$, $I_3 = 0.315$, $I_4 = 0.305$.

lowing bunches. Inversely, if the size of a_1 is minimized by performing better injection of the trailing bunches, it will considerably improve stability of all bunches. In fact, the leading two bunches would be completely stable if $a_1 = 0$ and, furthermore, the third-bunch amplitude would only grow linearly, which is much less dangerous than the quadratic increase. Thus, it is crucial to avoid accidental kicks to preceding bunches.

As easily seen from the above formulas, e.g., Eq. (5.11), the maximum amplitude is usually determined by the factor $S_{n+1,n}$. The strongest term in the motion of the nth bunch originates from the strongest motion of the $(n-1)$st bunch. For a bunch train filled with equally spaced bunches, this fact implies that the magnitude of the wake function at the distance $\Delta d \equiv d_{n+1,n}$ is of particular importance. Minimization of the sum of $S_{n+1,n}$ over all HOM's by optimizing Δd is thus essential to achieving better stability of a bunch train.

IX. EXPERIMENTAL OBSERVATIONS IN LEP

Recently, a number of MD (machine development) shifts have been devoted to the study of bunch trains in preparation for the possible operation of LEP with

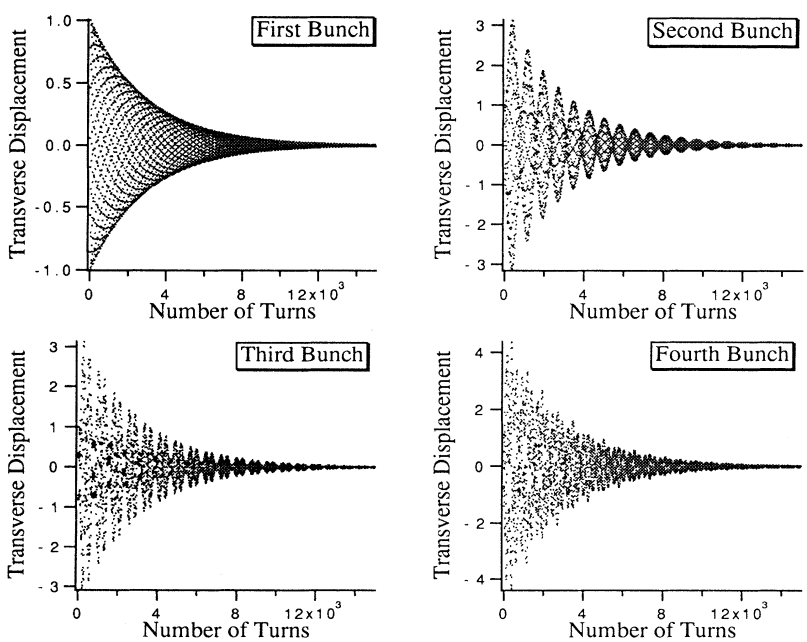

FIG. 8. Tune $\nu = 76.24$; detuning factor $d\nu/dI = -0.129$ (mA^{-1}); damping time 3000 (turns); bunch currents (mA): $I_1 = 0.345$, $I_2 = 0.335$, $I_3 = 0.315$, $I_4 = 0.305$.

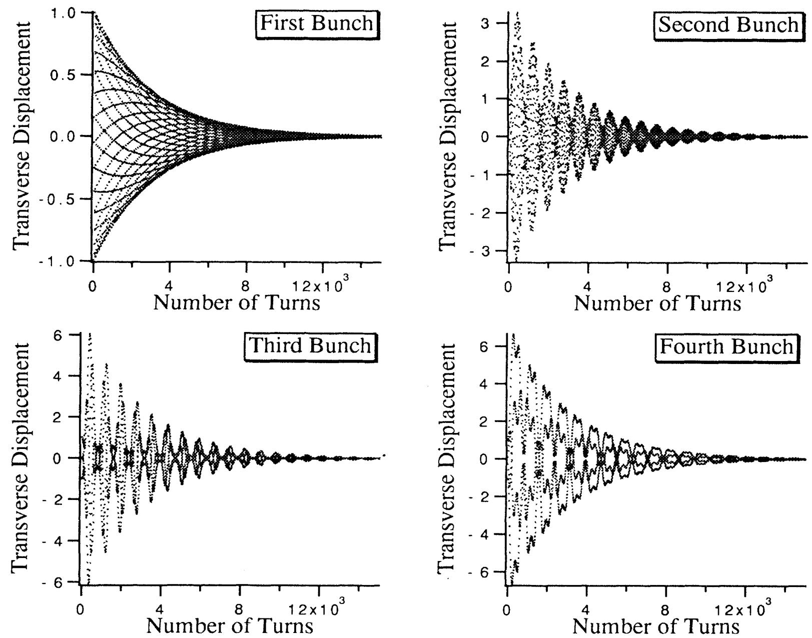

FIG. 9. Tune $\nu = 90.27$; detuning factor $d\nu/dI = -0.064$ (mA^{-1}); damping time 3000 (turns); bunch currents (mA): $I_1 = 0.355$, $I_2 = 0.335$, $I_3 = 0.315$, $I_4 = 0.295$.

such trains in the near future. In the absence of additional vertical electrostatic separators, which will only be installed during the end-of-year shutdown, only a single beam could be stored for most of the year. Usually it consisted of four equally spaced trains with up to four bunches each, distanced by 87 λ_{rf} (about 74 m). Near the end of the year, existing horizontal separators in intersection regions No. 4 and No. 8 were rotated into the vertical plane, and two beams with two trains could be tested.

In the earlier runs, the bunch currents in a train were rather limited and quite unequal, as instrumentation to measure the individual bunches was not yet fully available. However, in the course of the year, diagnostics were developed and large improvements could be made: with sixteen bunches per beam, record intensities for the total current were achieved.

In particular, currents of the first two bunches often reached the transverse mode-coupling threshold for single bunches, typically about 0.6 mA for standard synchrotron tunes and bunch lengths with all wigglers excited. However, it was not possible to reach the same high values in the third and fourth bunch, although no coherent transverse oscillations could be detected on the streak camera. The ultraviolet beam emittance radiation monitor (BEUV) showed consistently larger beam sizes, which could be due either to emittance growth or to oscillation, which cannot be distinguished as the device integrates over many turns.

The results of some experiments are summarized in Fig. 10, where the increase of current in all four bunches, of a train is shown during injection. A small overshoot at injection, and some loss of current in the earlier bunches during injection of the later bunches, can be seen, but the individual bunch currents could be made more equal by refilling the weakest bunch without loss of current in the others. Since the injection kick is long enough to influence all four bunches, this indicates that the wakes were not strong enough at these current levels to cause excessive amplitude growth of the later bunches.

X. CONCLUSIONS

Beam breakup was never clearly observed with the streak camera during single beam experiments with up to four bunches in a train in LEP, but possible amplitude or emittance growth was observed on the UV beam monitors. We hope to be able to carry out more experiments during the coming year with better instrumentation.

The transverse stability of the bunches in a train was better than feared from predictions, which had to be based on pessimistic worst-case values for the strength of the kicks by transverse wake potentials. Due to the large number of oscillations of these potentials over the

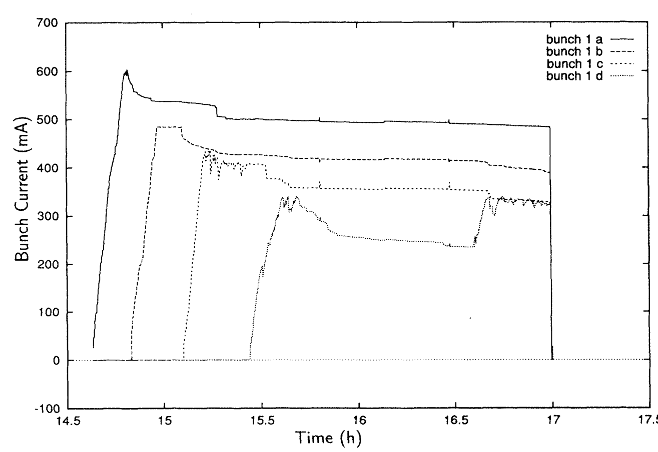

FIG. 10. Bunch current (mA) vs time (h) in four bunches of a train.

rather large distance between the bunches in a train, and also due to the large number of rf cavity cells with possibly slightly different HOM frequencies, no exact values for these kicks can be given, and only an upper limit can be estimated.

Unequal bunch currents were found to be most useful to reduce possible beam breakup, but were actually not required. On the other hand, they are dangerous for synchrobetatron resonances, as they are much harder to avoid due to their different working points in both planes for each bunch caused by different detuning with current.

Another problem, which has not been studied yet, is the effect of bunches crossing very close to the rf cavities. This will occur when the additional separators are installed around interaction regions No. 2 and No. 6, i.e., when LEP gets back into operation. Depending on the phase of the oscillating field left behind by the first bunch, the next bunch from the opposing beam could be much more strongly kicked than for operation with equal bunch spacing, where the bunches cross at the interaction region which is at a much larger distances from the rf cavities. Current limitations might result, and compensation by adjustment of the phases of all dangerous HOM's may be difficult. The effect is expected to be much weaker in the superconducting cavities since they have a smaller transverse impedance.

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