Correcting Distortion of Image by Image Registration

T.Tamaki[1] T.Yamamura[3] N.Ohnishi[24]

[1]Dept. of Info. Eng., ***Niigata Univ.***, **JAPAN**

[2]Center for Info. Media Studies, ***Nagoya Univ.***, **JAPAN**

[3]Faculty of Info. Sci. Tec., ***Aichi Prefectual Univ.***, **JAPAN**

[4]Bio-Mimetic Control Research Center, ***RIKEN***, **JAPAN**

Background

Vision Applications
need accuracy

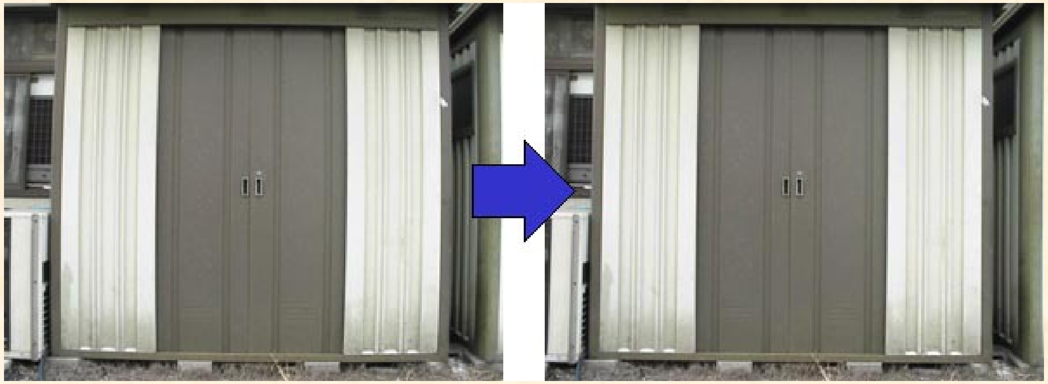
- **3D Reconstruction**
- **Structure from Motion**
- **Stereo Matching**

However...

Exact Estimation of Parameter

↑ ***affects***

Distortion* due to *Lens

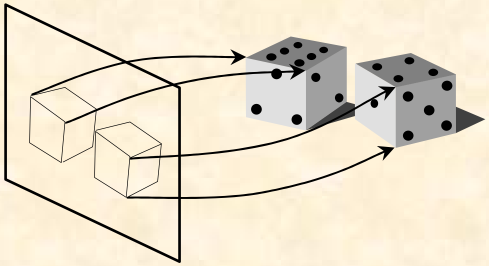


before after

Conventional Methods

Camera calibration:
Estimating internal camera parameters

N-to-N point correspondence



- ***Manual*** : too hard to do
- ***Detection*** : small number of correspondences

Image registration

- Establish the correspondence using all points in the image

Objective

Proposed a method for correcting distortion without point-to-point correspondences.

Based on image registration.

Requirements

- **Make a calibration pattern**

  Any kind of digital image
  (scanned photo,cg,etc.)

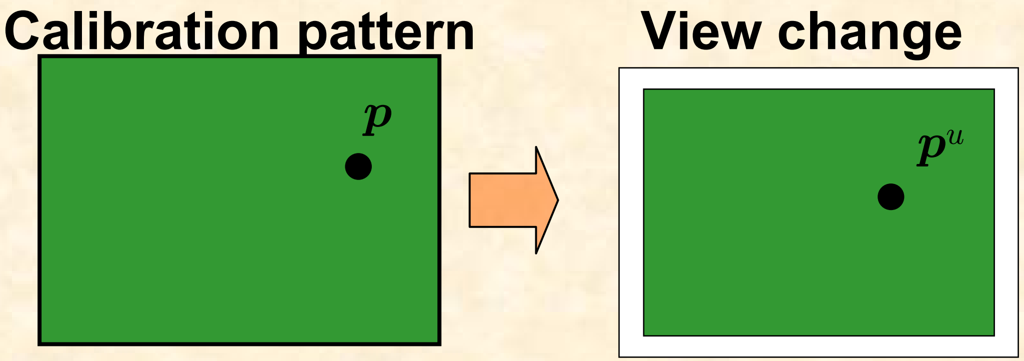
- **Print the pattern on a sheet of paper**

  Paste it on a flat board

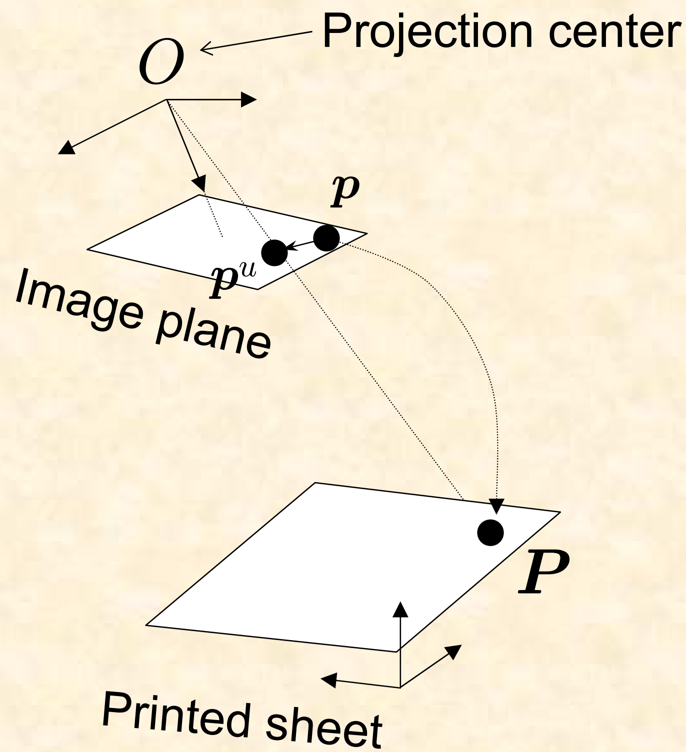
- **Take a distorted image of the printed pattern**

  Large enough in the image

View Change

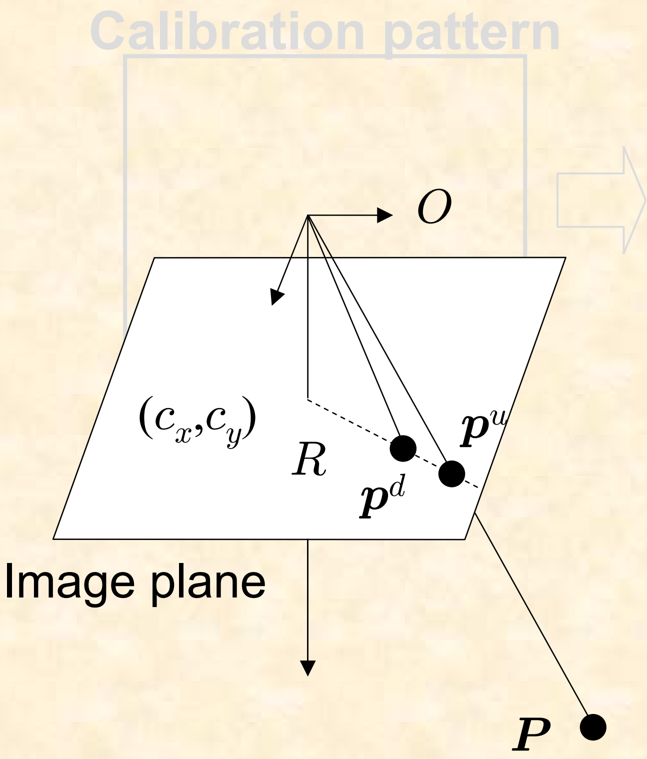


Calibration pattern
p
View change
p^u

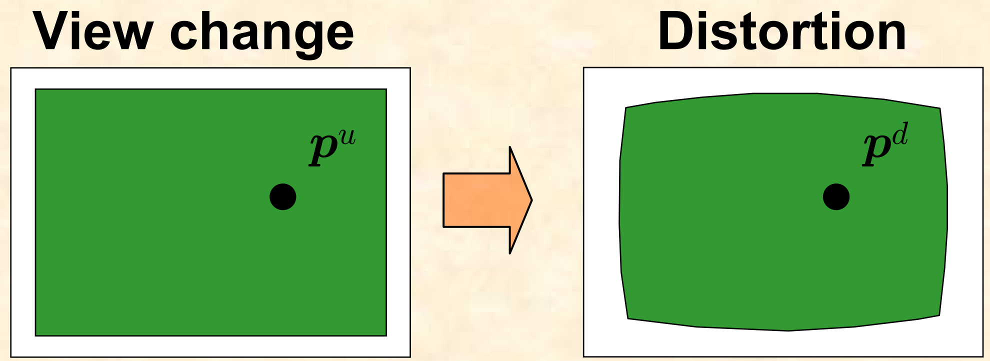


O
Projection center
p
p^u
Image plane
P
Printed sheet

Distortion



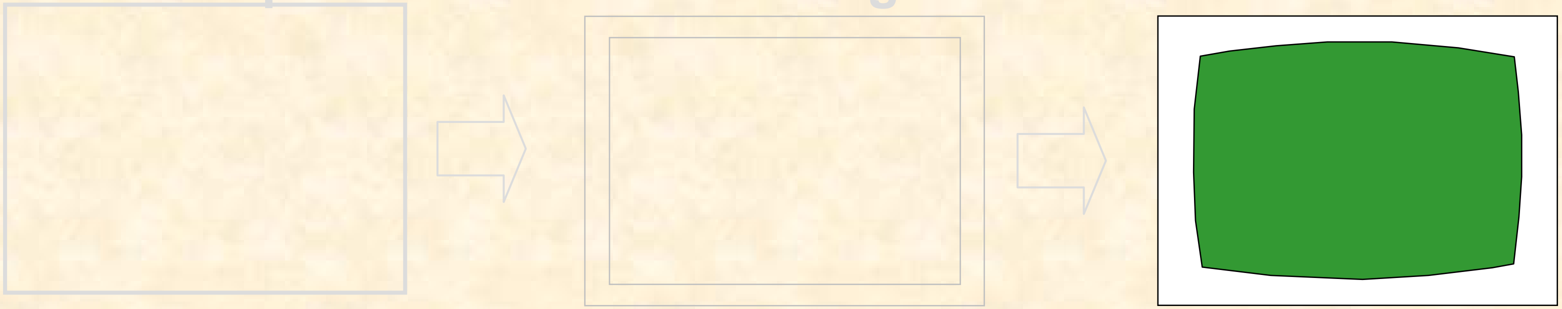
Calibration pattern
O
(c_x, c_y)
R
$\boldsymbol{p}^d$
$\boldsymbol{p}^u$
Image plane
P



View change
Distortion
$\boldsymbol{p}^u$
$\boldsymbol{p}^d$

Illumination Change

Calibration pattern View change Distortion

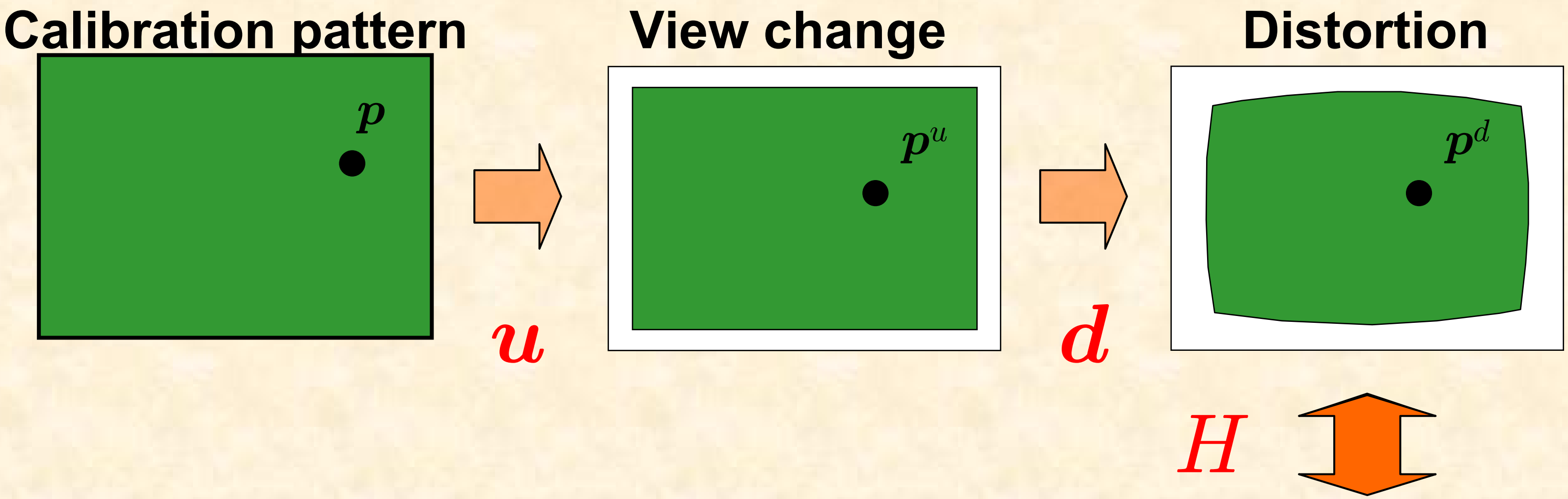


Actual image
taken by camera

Gradation of shade, shadow

Printer density (toner/ink)

Total transformation



Estimate the parameters of functions $\boldsymbol{u}$, $\boldsymbol{d}$ and H by nonlinear optimization

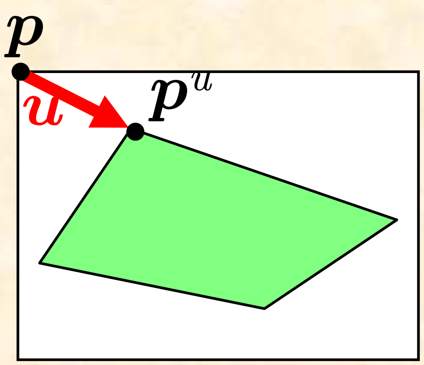
Actual image taken by camera

Modeling View Change

Same planar object from different view point

Planar perspective motion model

with eight parameters $\boldsymbol{\theta}^u$



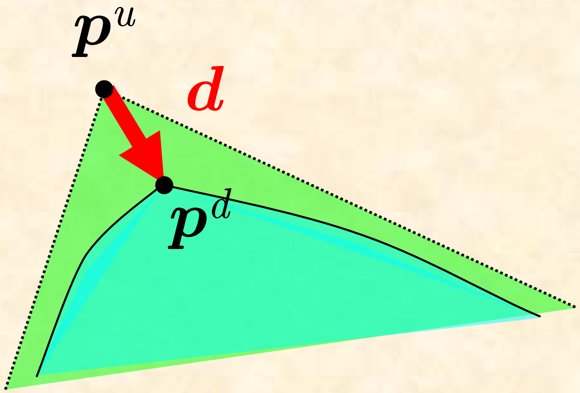
$$\boldsymbol{p}^u = \boldsymbol{u}(\boldsymbol{p}; \boldsymbol{\theta}^u)$$

$$= \frac{1}{\theta_1^u x + \theta_2^u y + 1} \begin{pmatrix} \theta_3^u x + \theta_4^u y + \theta_5^u \\ \theta_6^u x + \theta_7^u y + \theta_8^u \end{pmatrix}$$

$$\frac{\partial \boldsymbol{p}^u}{\partial \theta^u} = \begin{pmatrix} -x^2 & -xy & x & y & 1 & 0 & 0 & 0 \\ -xy & -y^2 & 0 & 0 & 0 & x & y & 1 \end{pmatrix}$$

Modeling Distortion

Projected point $\boldsymbol{p}^u$ is moved to $\boldsymbol{p}^d$ due to distortion.



$$\boldsymbol{p}^d = \boldsymbol{d}(\boldsymbol{p}^u ; \boldsymbol{\theta}^d)$$

$$\boldsymbol{p}^d = \boldsymbol{d}(\boldsymbol{u}(\boldsymbol{p} ; \boldsymbol{\theta}^u) ; \boldsymbol{\theta}^d)$$

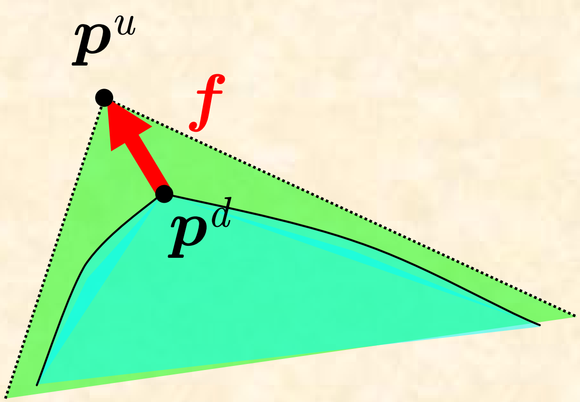
$\boldsymbol{p}^u$: undistorted point

$\boldsymbol{p}^d$: distorted point

$\boldsymbol{\theta}^d$: distortion parameters

Model of Distortion

Projected point $\boldsymbol{p}^u$ is moved to $\boldsymbol{p}^d$ due to distortion.



$$\boldsymbol{p}^u = \boldsymbol{f}(\boldsymbol{p}^d; \boldsymbol{\theta}^d)$$

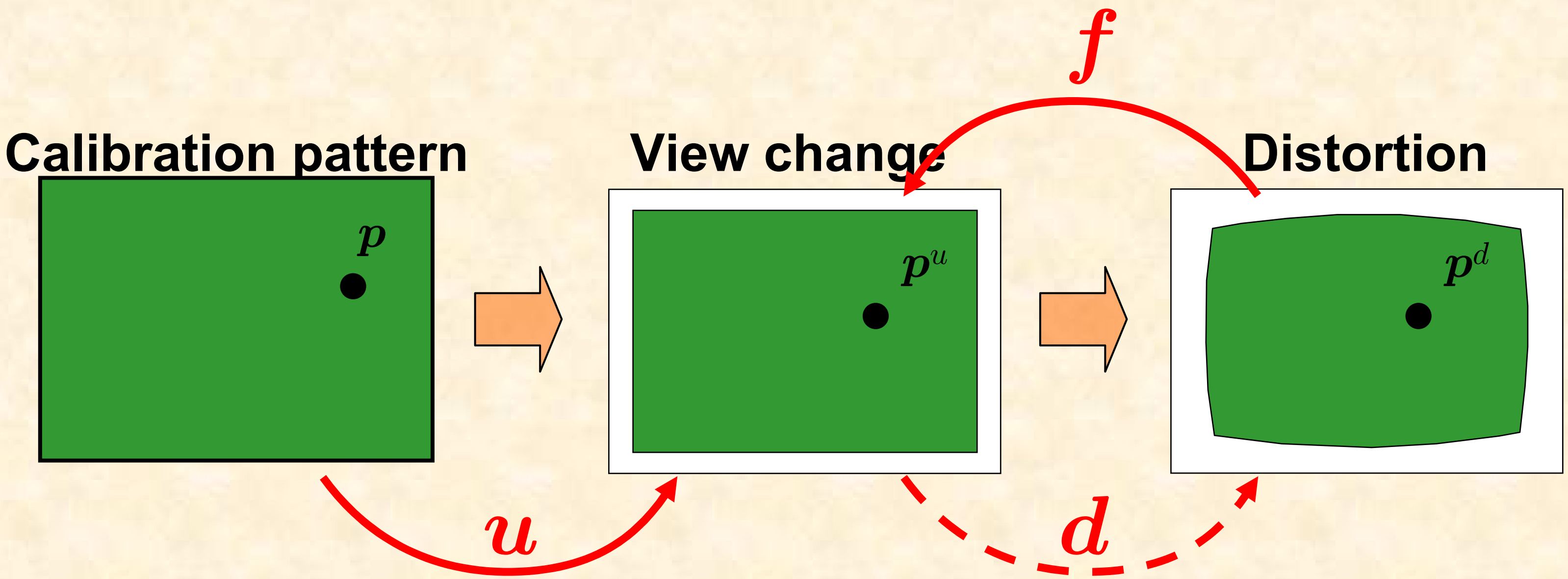
$$= \begin{pmatrix} \dfrac{x^d - c_x}{s_x}\left(1 + \kappa_1 R^2 + \kappa_2 R^4\right) \\ \left(y^d - c_y\right)\left(1 + \kappa_1 R^2 + \kappa_2 R^4\right) \end{pmatrix}$$

$$R = \sqrt{\left(\frac{x^d - c_x}{s_x}\right)^2 + (y^d - c_y)^2}$$

$$\boldsymbol{d}^{-1} = \boldsymbol{f}$$

$\boldsymbol{d}$ is not explicitly expressed by $\boldsymbol{p}^d$.
(implemented by an iterative procedure)

The Problem



$u()$ and $f()$ are differentiable.

$d()$ and $d(u())$ are not explicit.

How we get the gradient of $d(u())$?

Implicit Function Theorem

$$\boldsymbol{F}(\boldsymbol{q},\boldsymbol{p}^d) \equiv \boldsymbol{p}^u - \boldsymbol{f}(\boldsymbol{p}^d,\boldsymbol{\theta}^d) = 0$$ is given : $\boldsymbol{q}=(\boldsymbol{p}^d,\boldsymbol{\theta}^d)$

If $\boldsymbol{F}(\boldsymbol{q},\boldsymbol{d}(\boldsymbol{q})) = 0$ for $\forall \boldsymbol{q}$, then

$\boldsymbol{p}^d = \boldsymbol{d}(\boldsymbol{q})$ is an implicit function defined by $\boldsymbol{F}(\boldsymbol{q},\boldsymbol{p}^d) = 0$

$$\left|\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}\right| \neq 0 \quad \Rightarrow \quad \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{q}} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}^d}^{-1} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{q}}$$

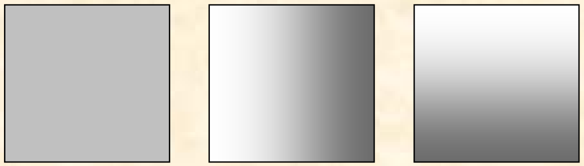
the gradient of $\boldsymbol{d}(\boldsymbol{u}())$ is available.

Modeling Illumination Change

Gain and bias are represented by spatial linear functions

$$H(I(\boldsymbol{p}), \boldsymbol{p}, \boldsymbol{\theta}^h)$$

$$= (\theta_1^h + \theta_2^h x + \theta_3^h y) I(\boldsymbol{p}) + (\theta_4^h + \theta_5^h x + \theta_6^h y)$$



$$\frac{\partial H}{\partial \boldsymbol{\theta}^h} = \left(I(\boldsymbol{p}) \;\; xI(\boldsymbol{p}) \;\; yI(\boldsymbol{p}) \;\; 1 \;\; x \;\; y \right)$$

Estimation

$$\min_{\boldsymbol{\theta}^u \boldsymbol{\theta}^d \boldsymbol{\theta}^h} \sum_i r_i^{\,2} \qquad r = I_1(\boldsymbol{p}) - H(I_2(\boldsymbol{d}(\boldsymbol{u}(\boldsymbol{p}))))$$

Gauss-Newton method :

Iterative nonlinear optimization
based on gradient method

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \delta \boldsymbol{\theta}$$

Estimates : $\boldsymbol{\theta}$

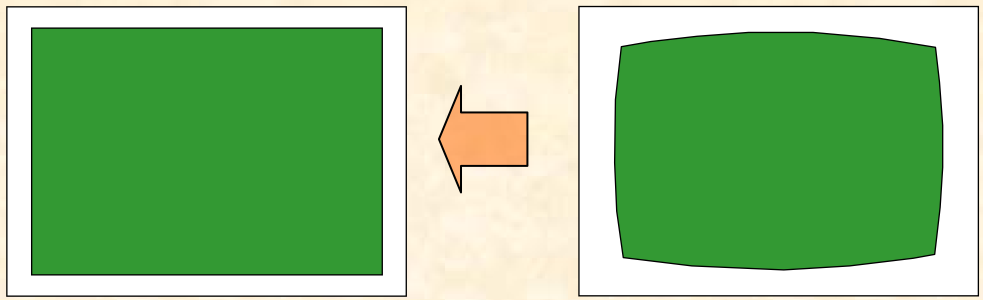
Update : $\delta\boldsymbol{\theta}$

Solving the system of equations :

$$\sum_l \sum_i \frac{\partial r_i}{\partial \theta_k} \frac{\partial r_i}{\partial \theta_l} \delta \theta_l = -\sum_i r_i \frac{\partial r_i}{\partial \theta_k}$$

Correction

Corrected image **Distorted image**



$I_2^{'}$ I_2

$$I_2^{'}(\boldsymbol{p}) = I_2(\boldsymbol{d}(\boldsymbol{p}\,;\boldsymbol{\theta}^d))$$

For all points $\boldsymbol{p}$ in the corrected image I'_2,

Intensity is calculated by $\boldsymbol{d}(\boldsymbol{p})$ in I_2

Experiment

The **calibration pattern** used for the experiment
(scanned photograph)

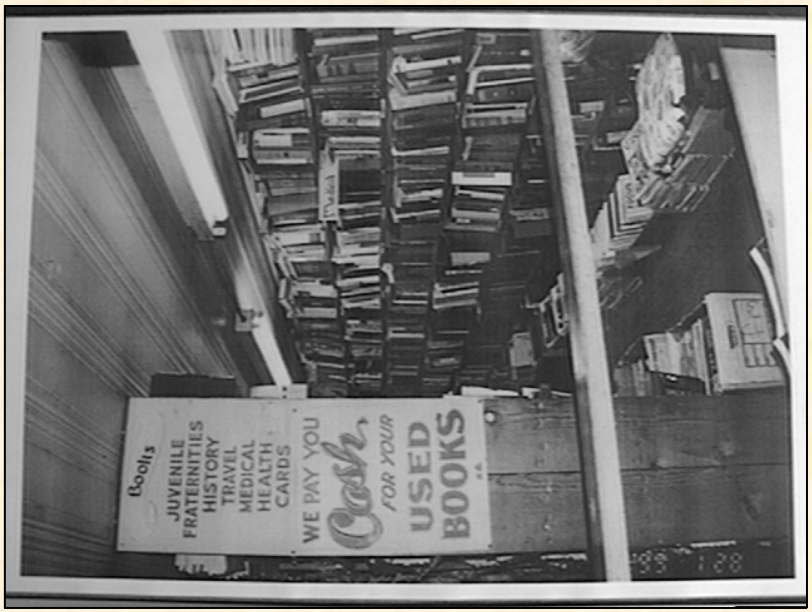
Experiment

Placed in front of the camera
almost parallel to the image plane

Correction of calibration pattern image

Captured image
With distortion

Corrected image



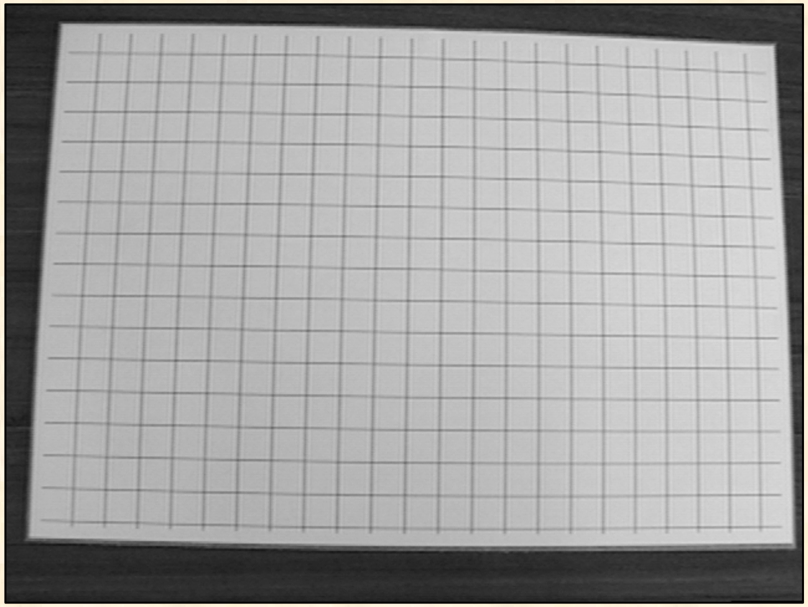
I_2

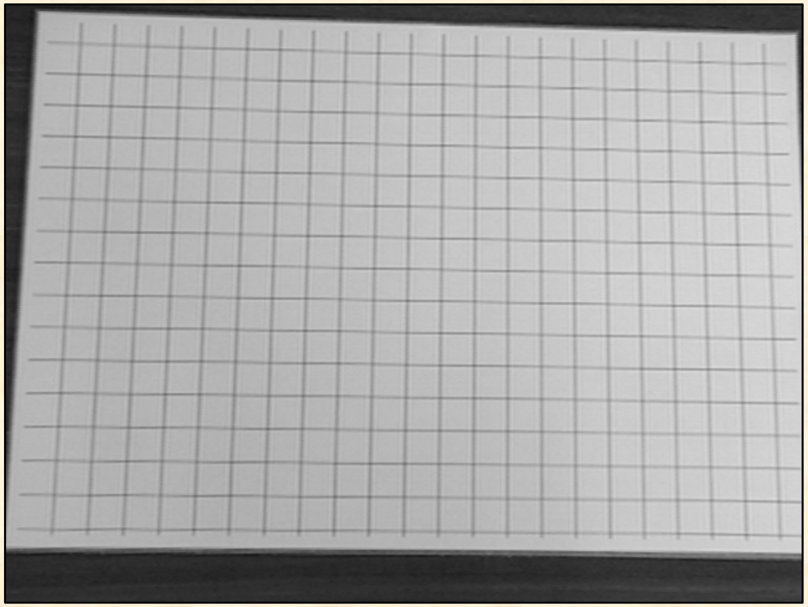
$I_2^{'}$

Correction of grid pattern

Captured image
With distortion

Corrected image

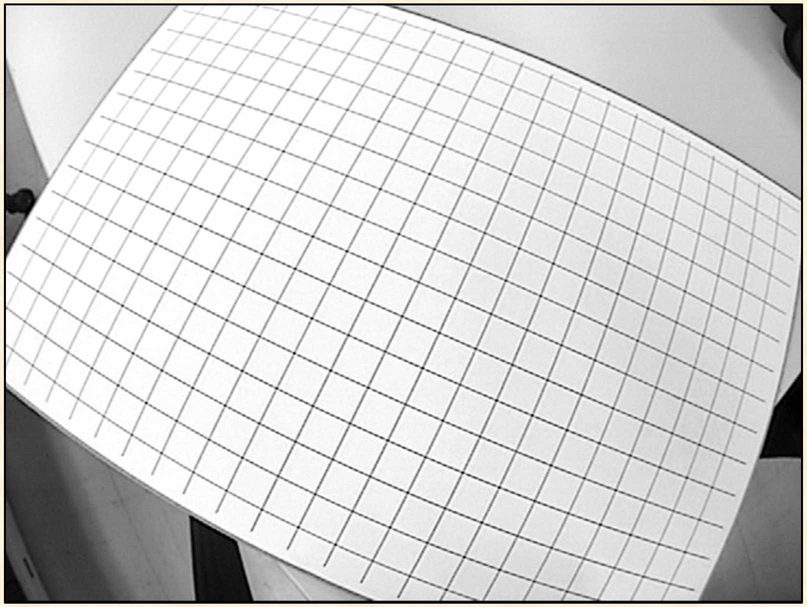




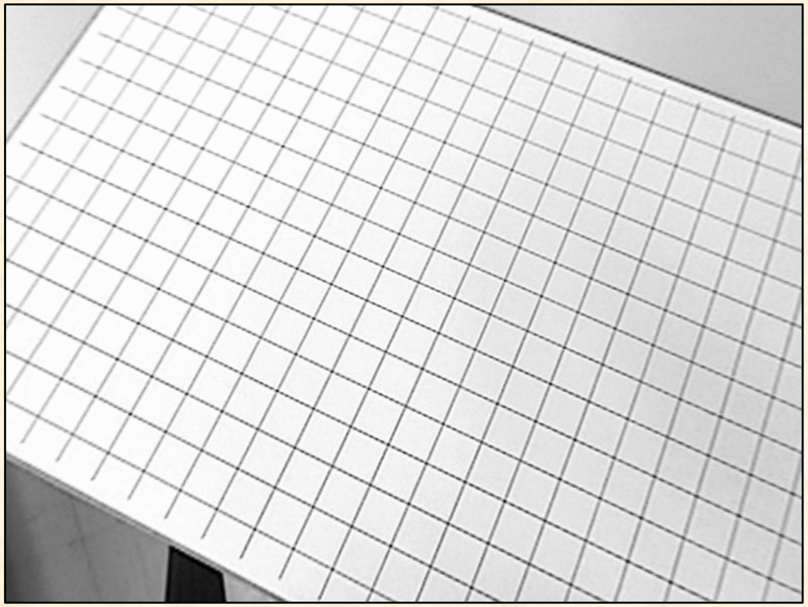
Corrected by using the same parameters with the previous slide

Correction of severe distortion

distorted image
taken by wide-angle lens



Corrected image



Optimization doesn't converge

Unless we use appropriate initial values

Zoom Lens Calibration

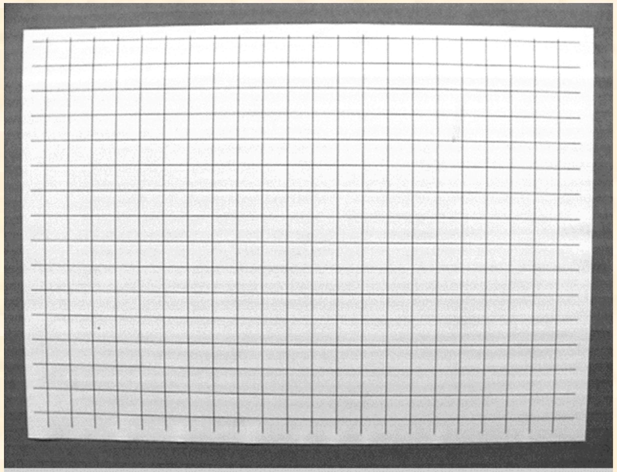
- Zoom range:

  $F_{num.}$ = 5.4~64.8 mm

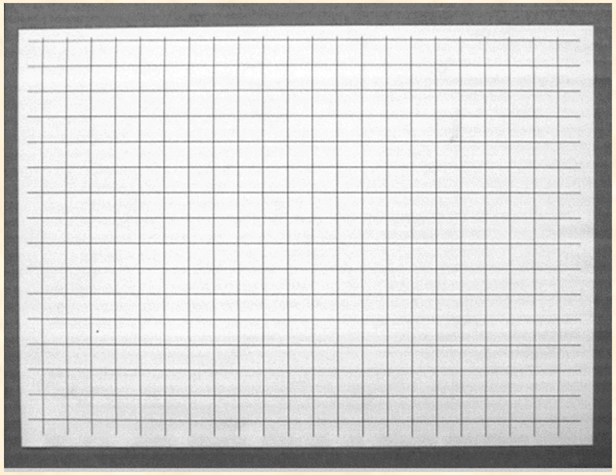
  Angle = 48.8~4.4 deg

  Zoom = 0~1023

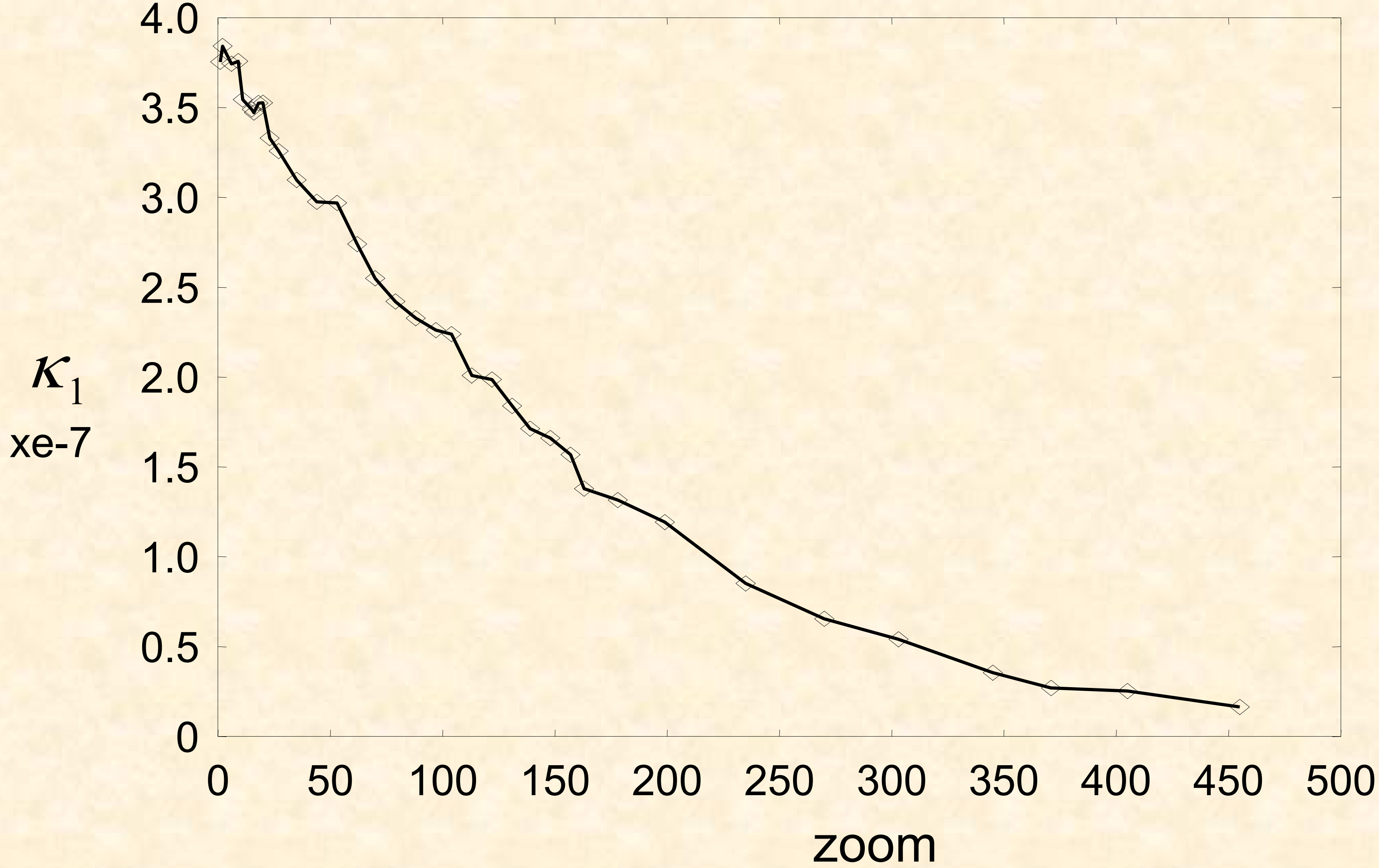
Wide view angle（zoom=0)



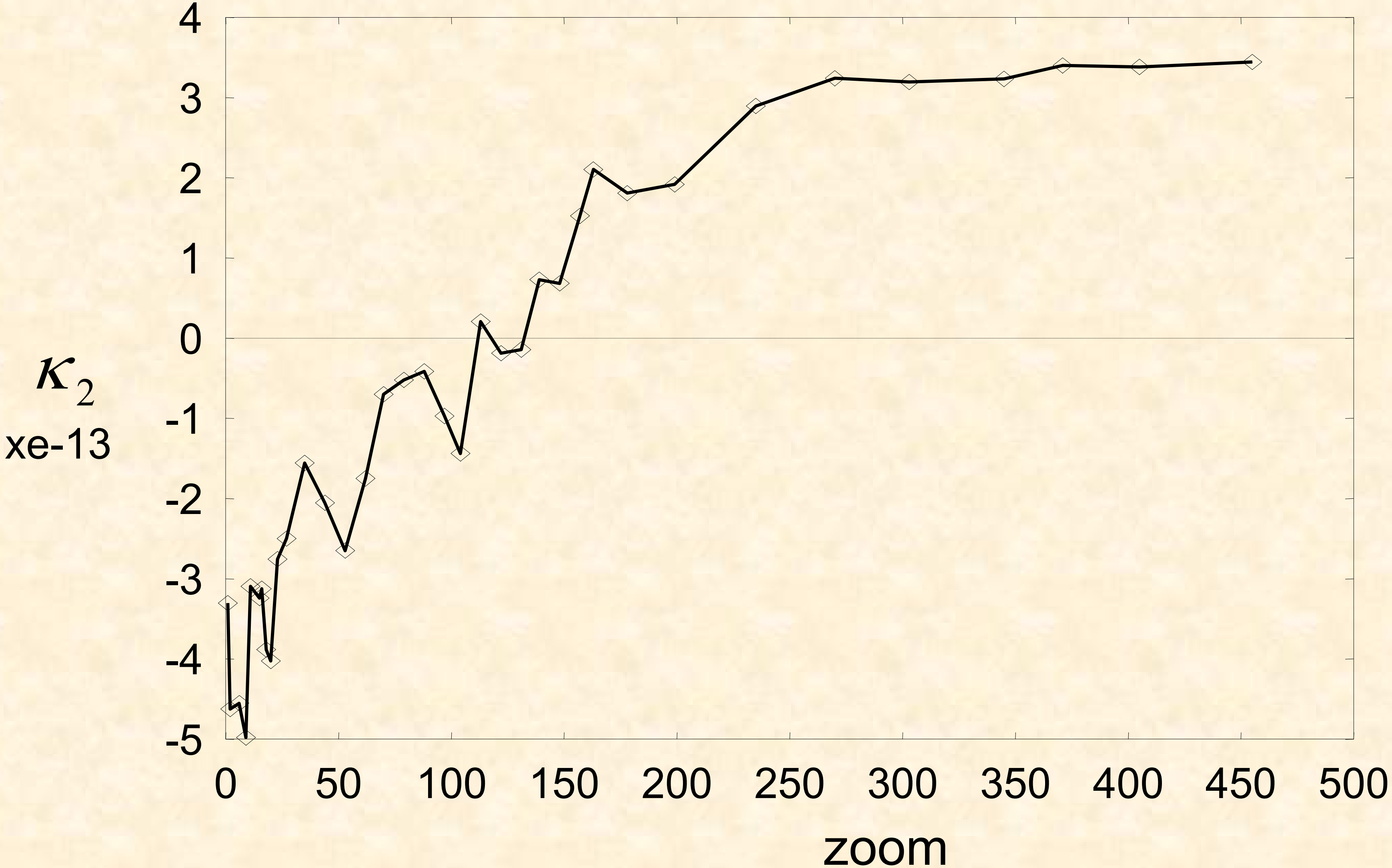
Middle view angle (zoom=455)



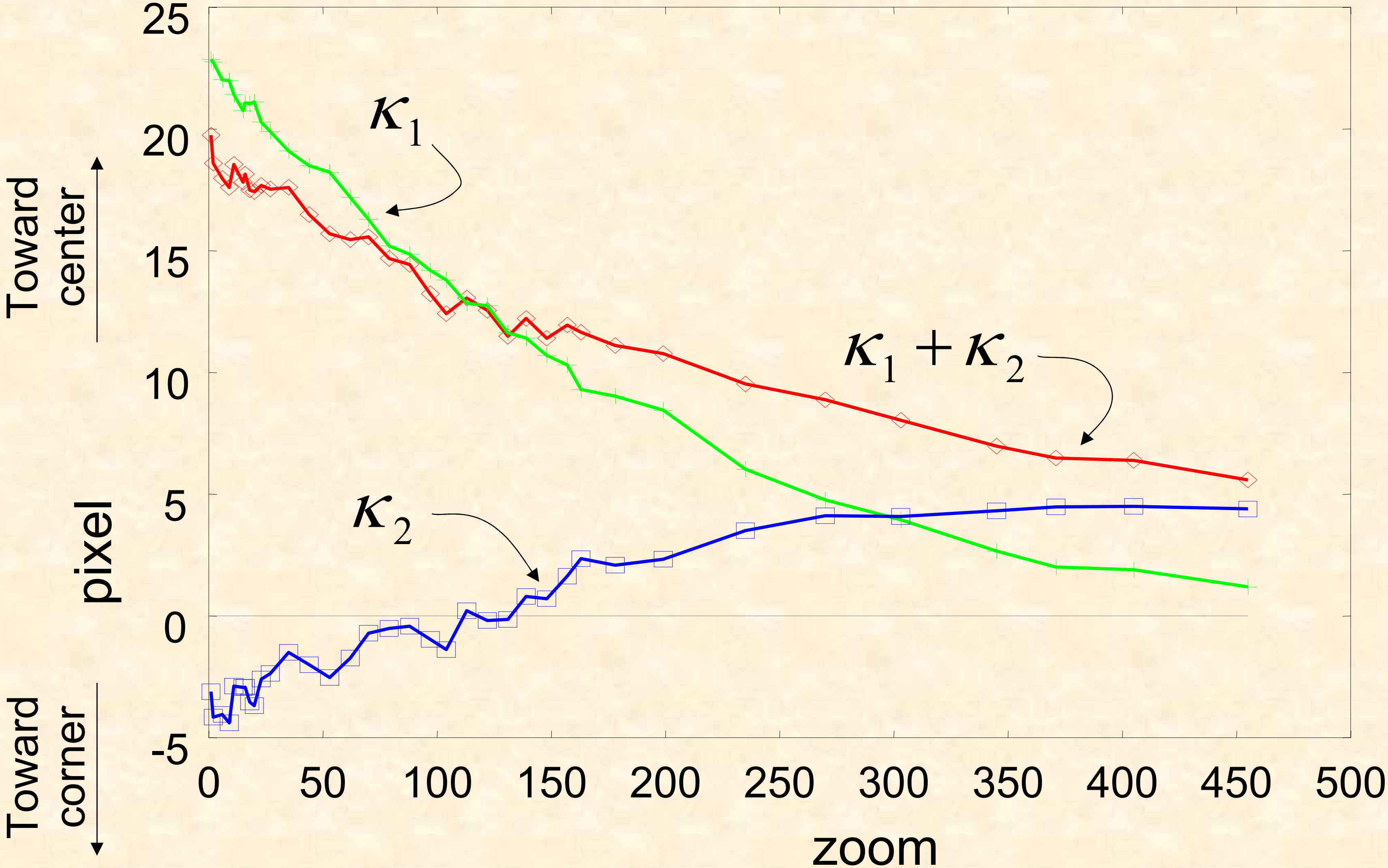
Change of κ_1



Change of κ_2



Total Displacement
(at the corner of image)



Conclusions

Proposed a new method for **correcting distortion** without point-to-point correspondences.

Based on **image registration :** nonlinear optimization by Gauss-Newton Method minimizing intensity residuals between pattern and image.

Implicit function theorem was introduced to calculate the Jacobian of the distortion function

experimental results of real image and real camera with *qualitative* evaluation.