The Secret of Rotating Object Images

Using Cyclic Permutation for View-based Pose Estimation

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The Key: Cyclic Permutation

The relationship between images

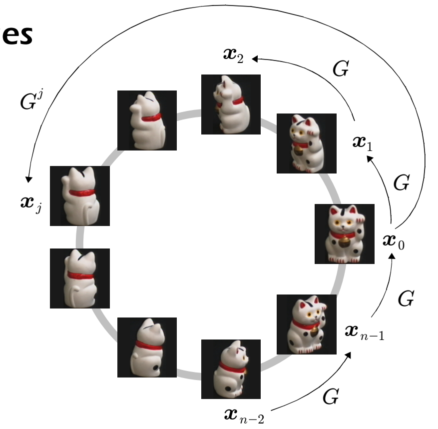
$$\boldsymbol{x}_{j+1 \bmod n} = G \boldsymbol{x}_j$$

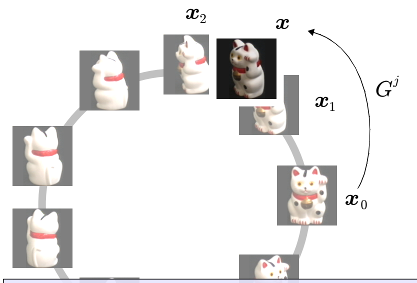
A matrix G transforms an image to another.

$$\boldsymbol{x}_1 = G \boldsymbol{x}_0$$
$$\boldsymbol{x}_2 = G \boldsymbol{x}_1$$
$$\vdots$$
$$\boldsymbol{x}_0 = G \boldsymbol{x}_{n-1}$$

G^j transforms x_0 to x_j directly

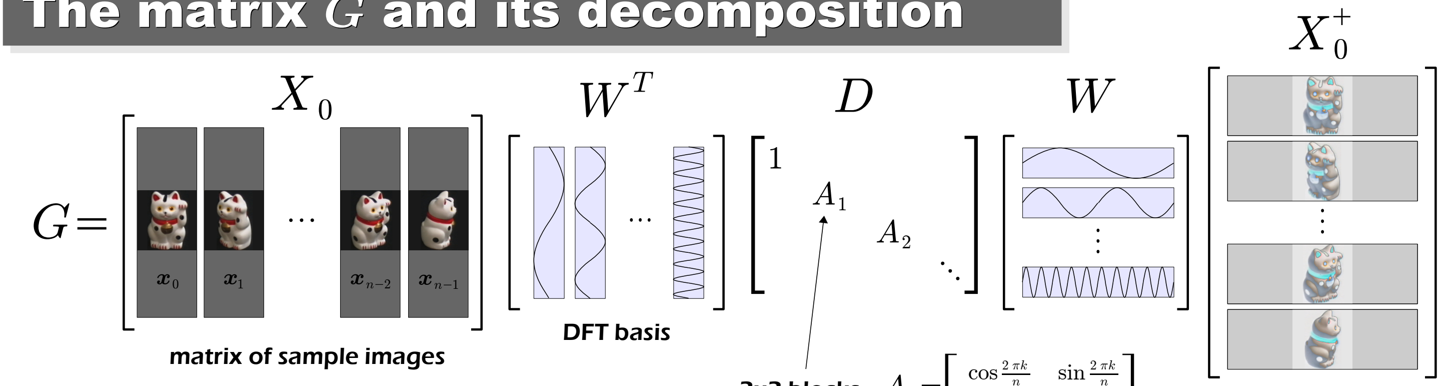
$$\boldsymbol{x}_j = G^j \boldsymbol{x}_0$$





A novel image x is represented as $\boldsymbol{x} = G^j \boldsymbol{x}_0$ for some j

The matrix G and its decomposition



matrix of sample images

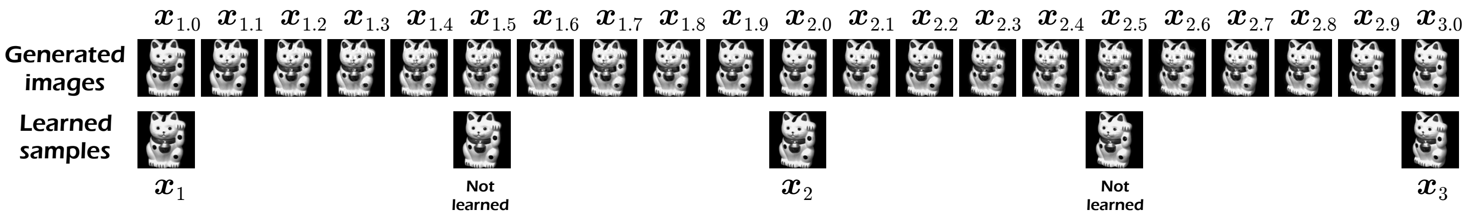
DFT basis

2x2 blocks $A_k = \begin{bmatrix} \cos\frac{2\pi k}{n} & \sin\frac{2\pi k}{n} \\ -\sin\frac{2\pi k}{n} & \cos\frac{2\pi k}{n} \end{bmatrix}$

Pseudo inverse

View Generation

Learn finite samples, Generate many images : $\boldsymbol{x}_{1+0.1j} = G^{0.1j} \boldsymbol{x}_1 = X_0 W^T D^{0.1j} W X_0^+ \boldsymbol{x}_1$



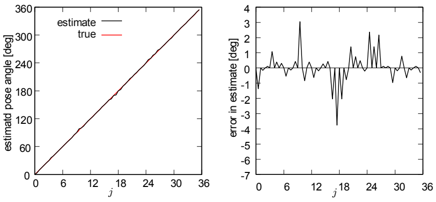
Pose Estimation

Compare images projected onto the subspace :

$$\boldsymbol{x} = G^j \boldsymbol{x}_0$$
$$\boldsymbol{x} = X_0 W^T D^j W X_0^+ \boldsymbol{x}_0$$
$$W X_0^+ \boldsymbol{x} = D^j W X_0^+ \boldsymbol{x}_0$$
$$\boldsymbol{x}' = D^j \boldsymbol{x}_0'$$

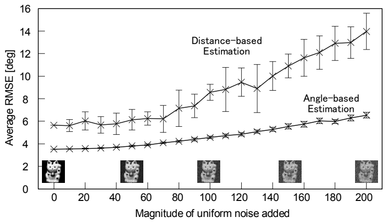
Find j such that two images in the subspace are close to each other

image in the subspace
$\boldsymbol{x}' = W X_0^+ \boldsymbol{x}$

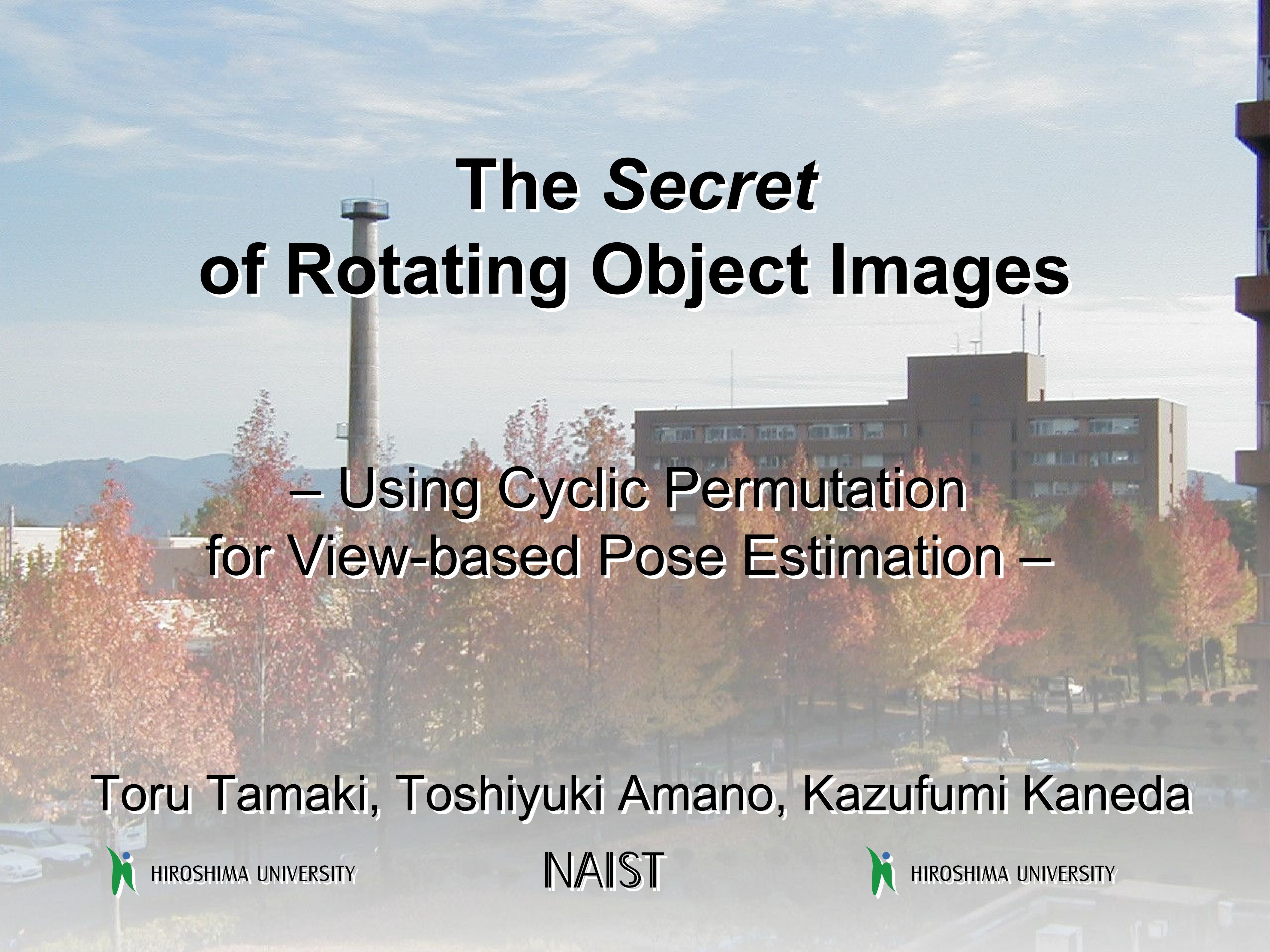


Target :
obj4 in COIL-20

Error :
Max 3.76[deg]
RMSE 1.23[deg]



Estimation error under uniform noise for 20 objects in COIL-20



The *Secret* of Rotating Object Images

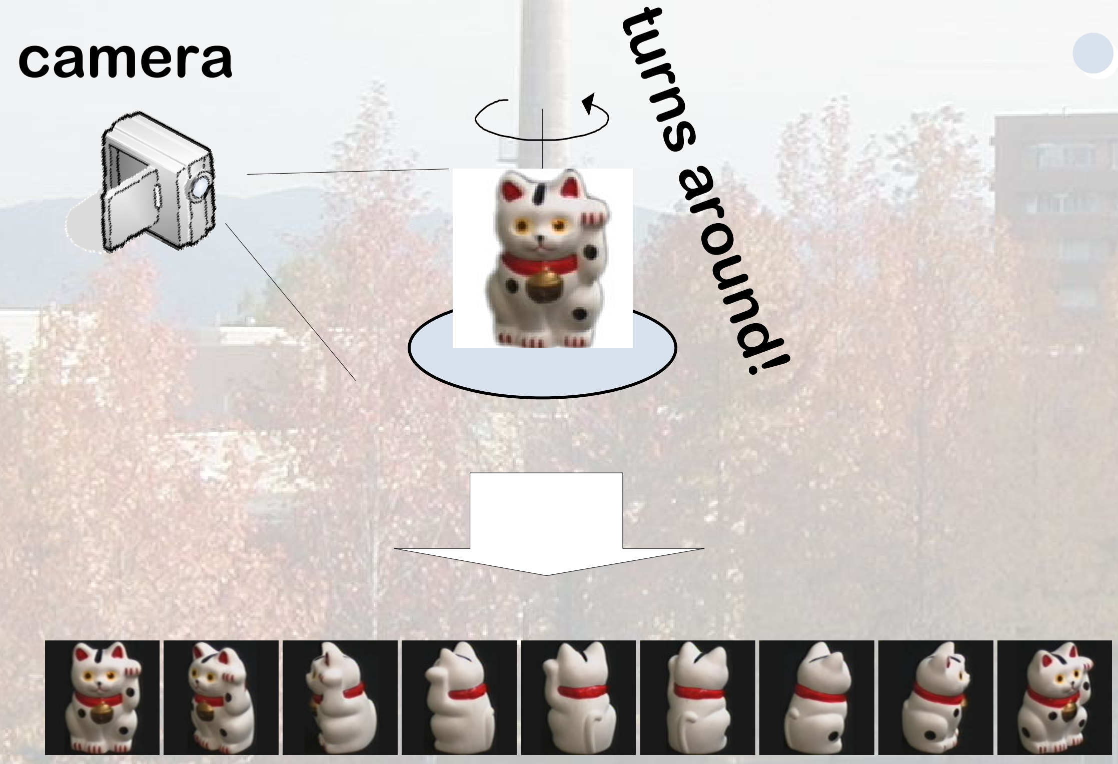
– Using Cyclic Permutation for View-based Pose Estimation –

Toru Tamaki, Toshiyuki Amano, Kazufumi Kaneda

HIROSHIMA UNIVERSITY NAIST HIROSHIMA UNIVERSITY

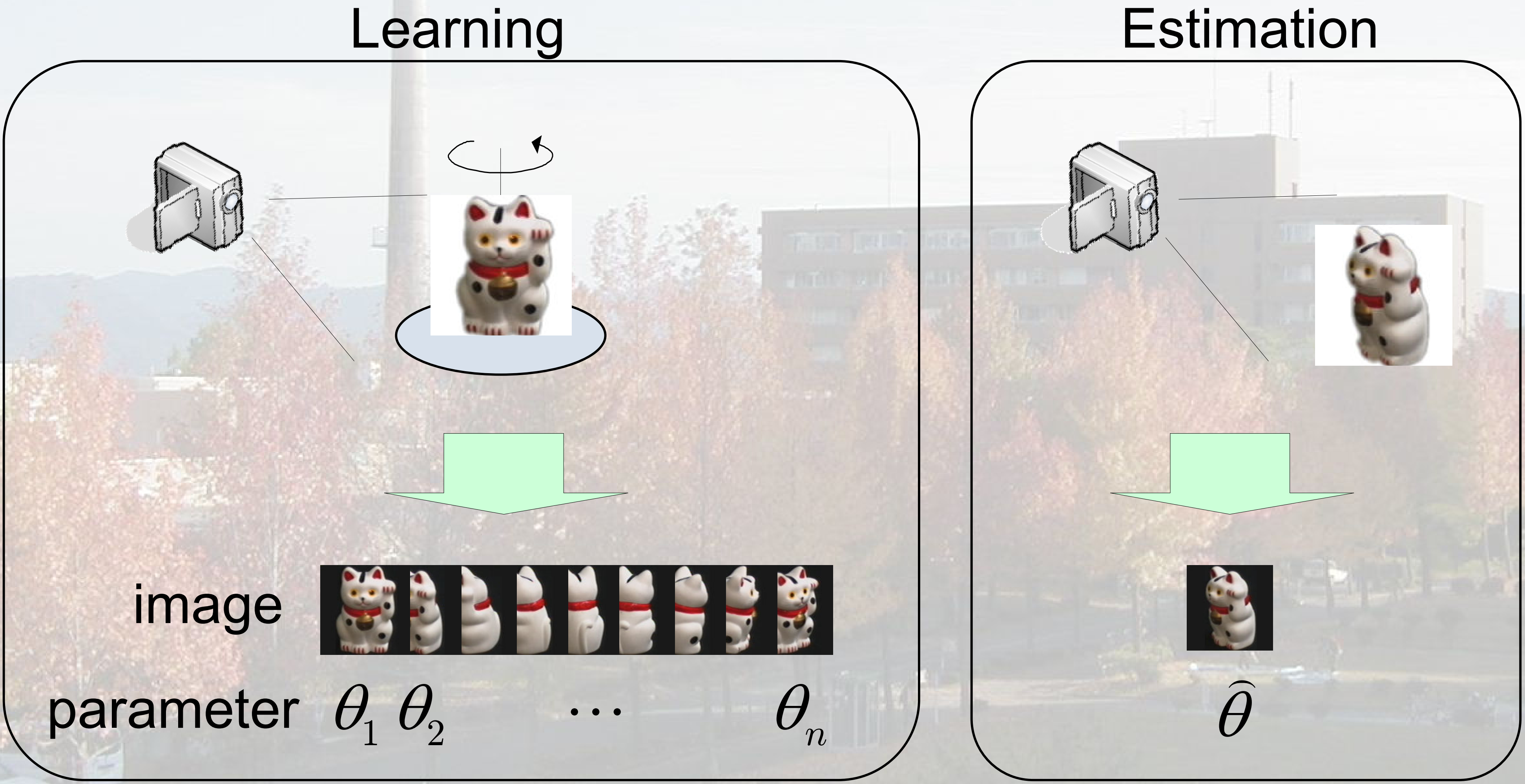
When an object rotates…

- Images are taken.
- They make a manifold…?



What are these images?

View-based pose estimation



Learning

image

parameter θ_1 θ_2 $\cdots$ θ_n

Estimation

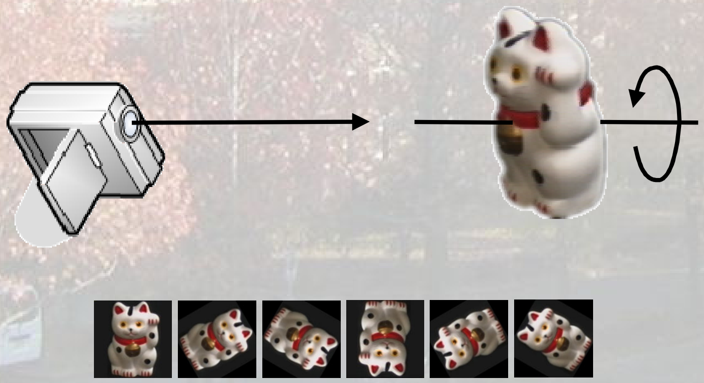
$\hat{\theta}$

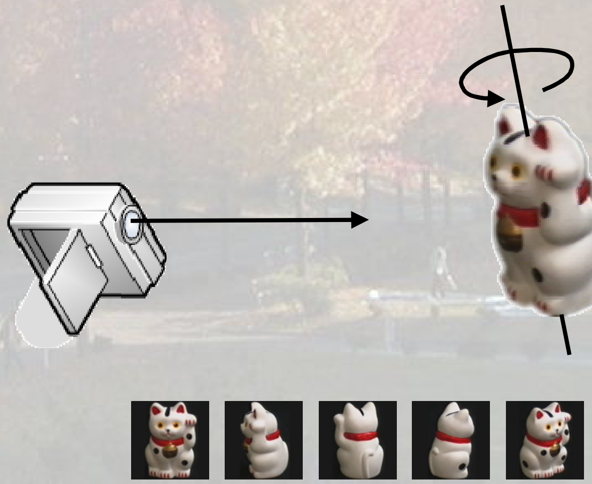
- Learns the relationship between images (input) and parameters (output)

In and Off: 1DOF rotations

- in-plane rotation
  - about the optical axis of the camera
  - appearance rotates in the image plane

- off-the-plane rotation
  - about any axis in 3D
  - appearance changes due to the object geometry





In and Off: 1DOF rotations

- **in-plane rotation**
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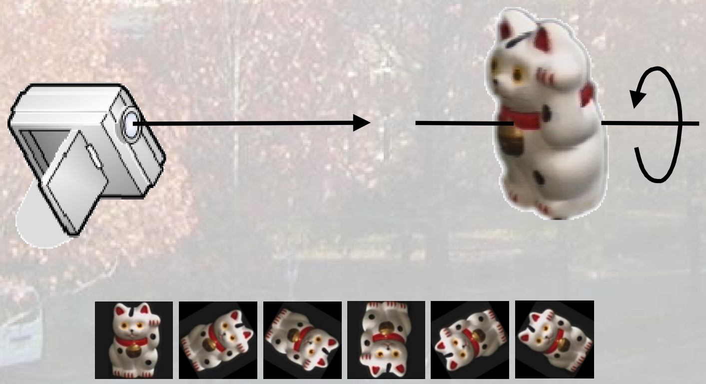
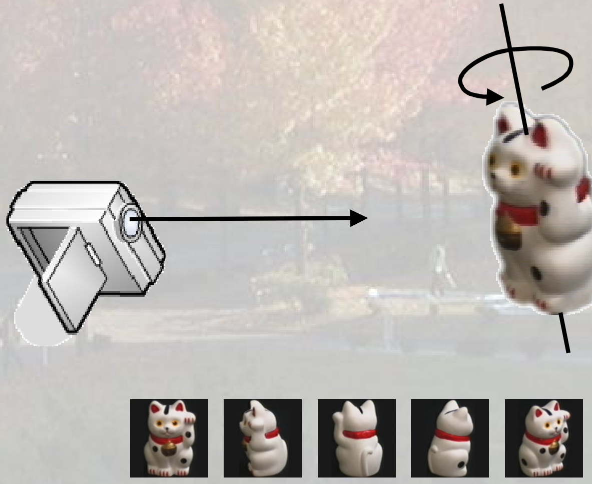


- Analitical solutions
  - Eigenimages are obtained by DFT.
    - (Uenohara et al., 1998)
    - (Chang et al., 2000)
    - (Park, 2002)
    - (Jorgan et al., 2003)
    - (Sengel et al., 2005)
  - But, hopeless for extending off-the-plane rotation.

In and Off: 1DOF rotations

- Pose estimation
  - Parametric Eigenspace method
    - (Murase et al., 1995)
  - linear regression
    - (Okatani et al., 2000)
    - (Amano et al., 2007)
  - kernel methods
    - (Melzer et al., 2003)
    - (Ando et al., 2005)
  - Manifold learning

- **off-the-plane rotation**
  - about any axis in 3D
  - appearance changes due to the object geometry

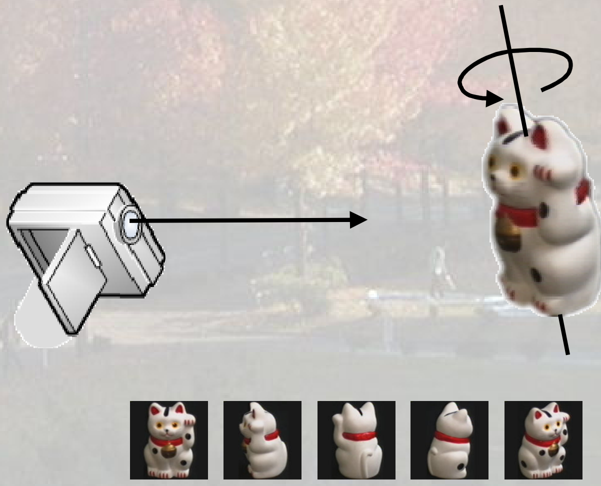


In and Off: 1DOF rotations

- Questions:
  - Can we represent these images analytically?
  - and How?

- What is *the key* to understand?
  - in terms of “linear”

- **off-the-plane rotation**
  - about any axis in 3D
  - appearance changes due to the object geometry



The Key : Cyclic Permutation.

The relationship between images:

$$\boldsymbol{x}_{j+1 \bmod n} = G\boldsymbol{x}_j$$

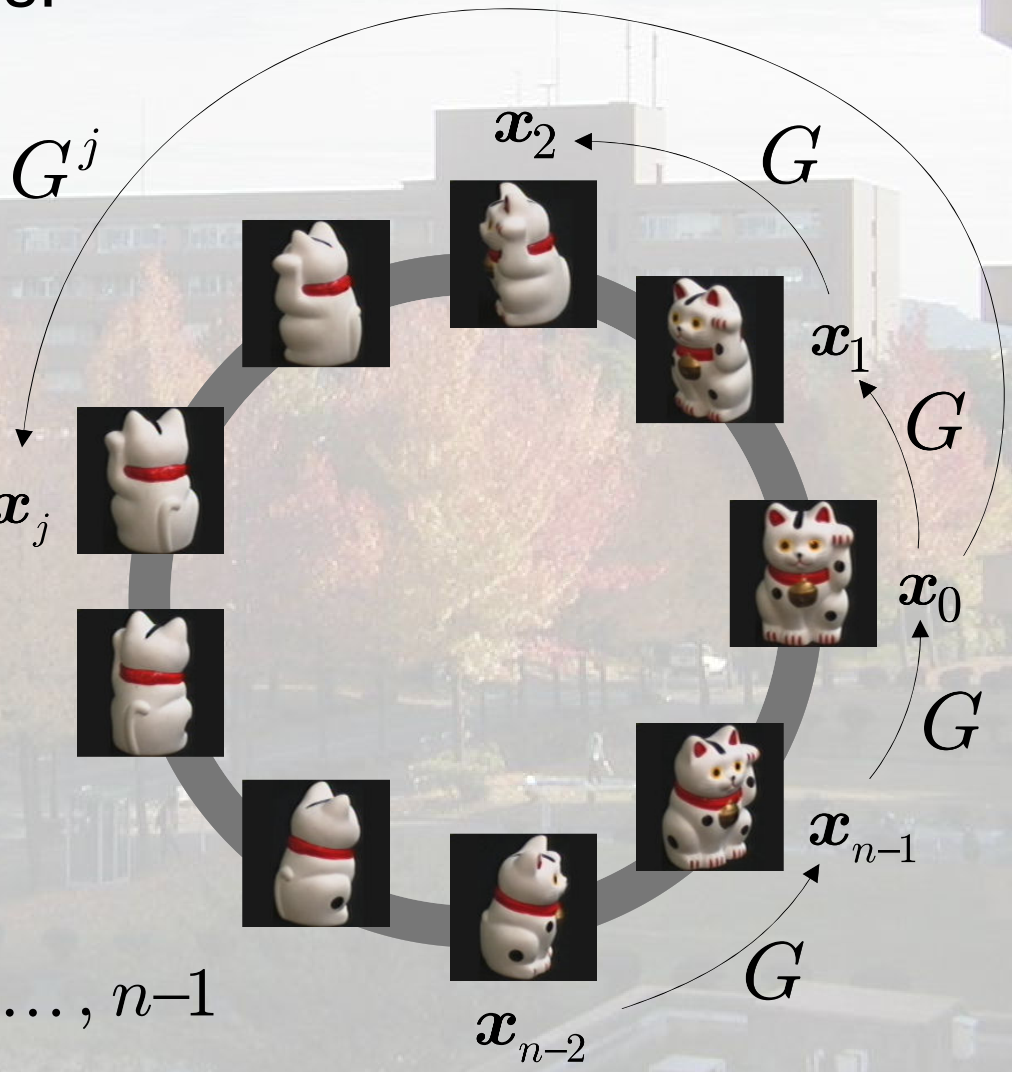
A matrix G transforms an image to another:

$$\begin{aligned} \boldsymbol{x}_1 &= G\boldsymbol{x}_0 \\ \boldsymbol{x}_2 &= G\boldsymbol{x}_1 \\ &\vdots \\ \boldsymbol{x}_0 &= G\boldsymbol{x}_{n-1} \end{aligned}$$

G^j transforms $\boldsymbol{x}_0$ to $\boldsymbol{x}_j$ directly.

$$\boldsymbol{x}_j = G^j\boldsymbol{x}_0$$

$j = 0, 1, 2, \ldots, n-1$



Why Cyclic Permutation?

- Pose Estimation
  - Find j such that

$$\boldsymbol{x} = G^j \boldsymbol{x}_0$$

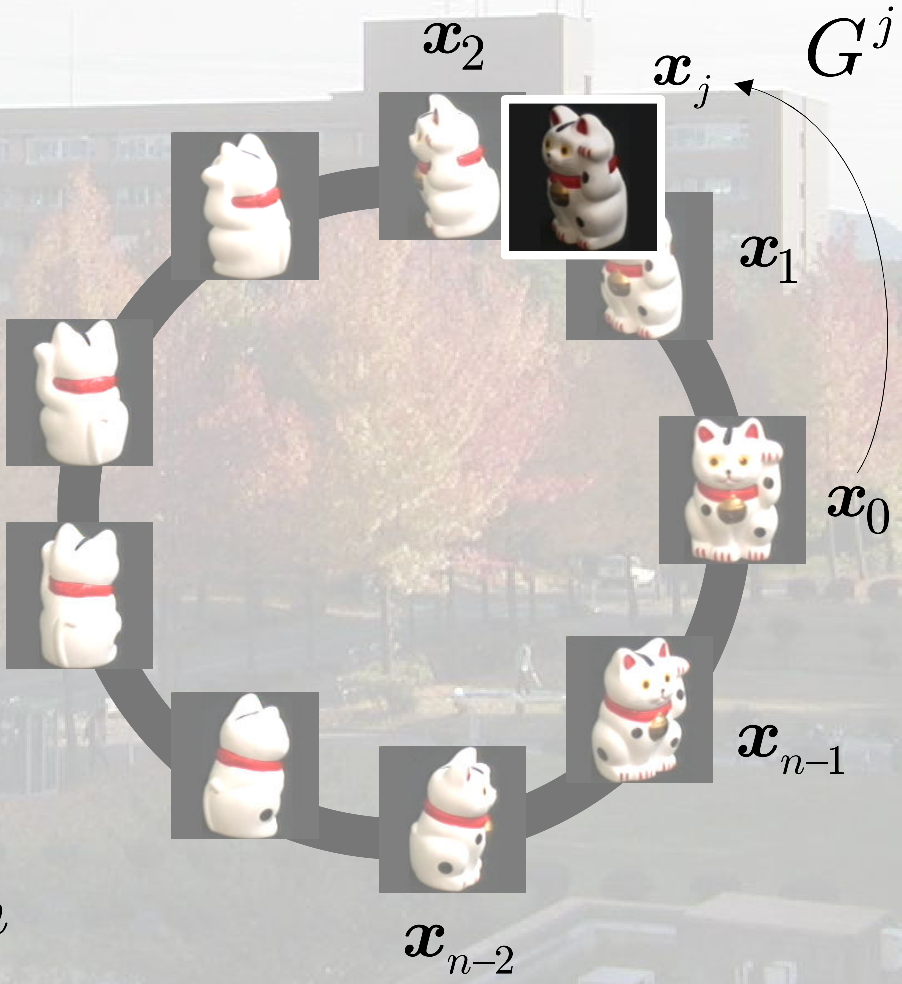
for a given image $\boldsymbol{x}$

- View Generation
  - Create an image $\boldsymbol{x}_j$

$$\boldsymbol{x}_j = G^j \boldsymbol{x}_0$$

for given j

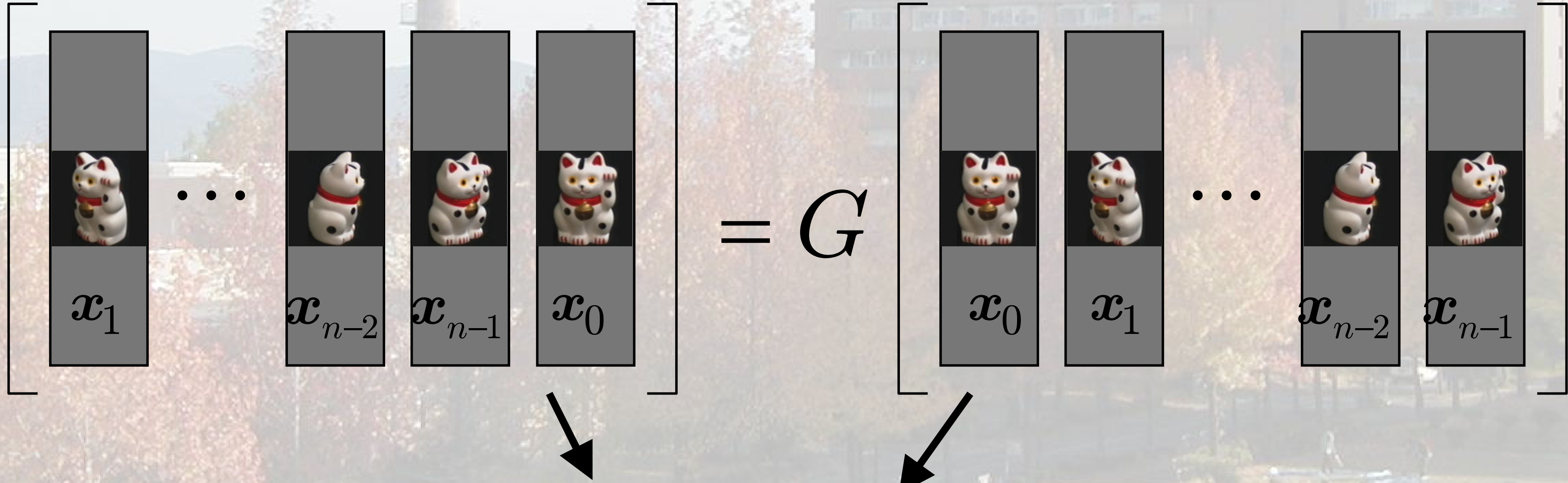
$0 \leq j < n$



Obtaining the matrix G …

Matrix representation

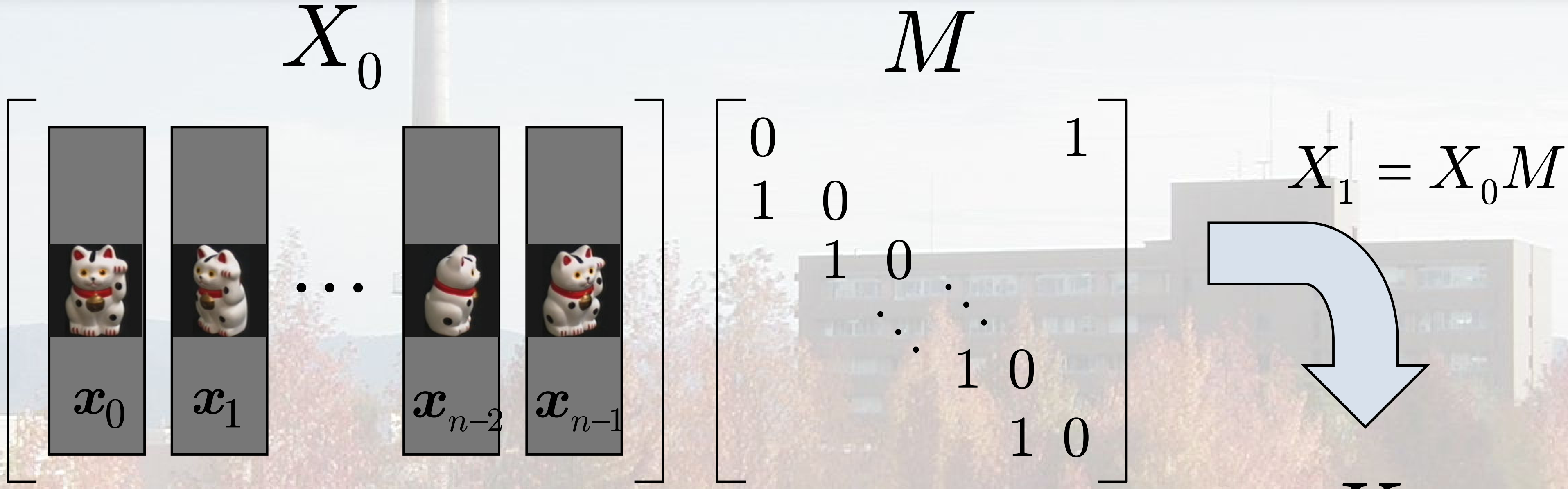
$$\left[\boldsymbol{x}_1\ \boldsymbol{x}_2 \ldots \boldsymbol{x}_{n-1}\ \boldsymbol{x}_0\right] = G\left[\boldsymbol{x}_0\ \boldsymbol{x}_1 \ldots \boldsymbol{x}_{n-2}\ \boldsymbol{x}_{n-1}\right]$$



$$X_1 = GX_0 \Rightarrow G = X_1X_0^+$$

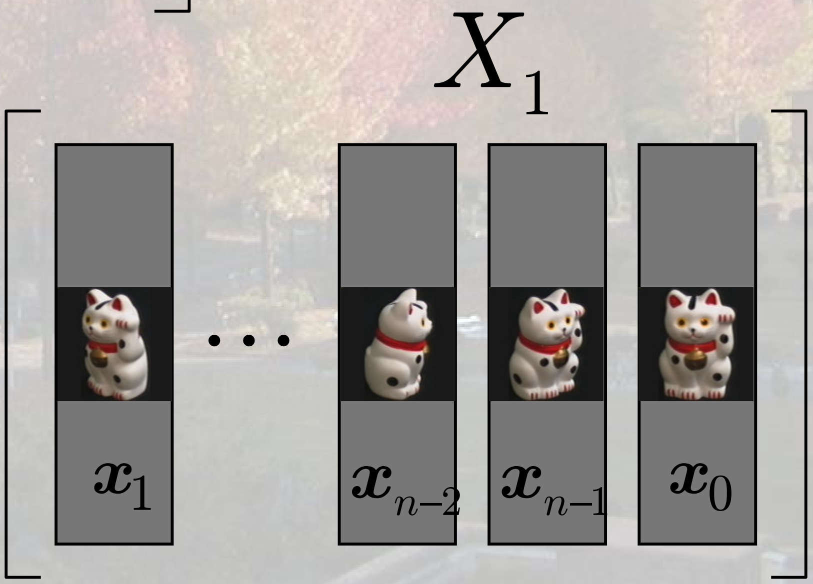
Using pseudoinverse

with Column permutation.



$$M = \begin{bmatrix} 0 & & & & & 1 \\ 1 & 0 & & & & \\ & 1 & 0 & \ddots & & \\ & & \ddots & \ddots & & \\ & & & 1 & 0 & \\ & & & & 1 & 0 \end{bmatrix}$$

$$X_1 = X_0 M$$



$$G = X_1 X_0^+ = X_0 M X_0^+$$

But,

Does really G transform $\boldsymbol{x}_0$ to $\boldsymbol{x}_1$?

$$\boxed{G \ = \ X_0 M X_0^+}$$

$$\begin{tikzcd}
X_1 & X_0 \arrow[l, "G"']
\end{tikzcd}$$

Yes, it does !

$$\begin{tikzcd}
X_1 & M \arrow[l, "X_0"'] & I \arrow[l, "M"'] & X_0 \arrow[l, "X_0^+"']
\end{tikzcd}$$

$$\begin{bmatrix} 0 & & & & & 1 \\ 1 & 0 & & & & \\ & 1 & 0 & \ddots & & \\ & & \ddots & \ddots & & \\ & & & 1 & 0 & \\ & & & & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

Block diagonalization

$$G = X_0 M X_0^+ = X_0 \begin{bmatrix} 0 & & & & & 1 \\ 1 & 0 & & & & \\ & 1 & 0 & \ddots & & \\ & & \ddots & \ddots & & \\ & & & 1 & 0 & \\ & & & & 1 & 0 \end{bmatrix} X_0^+$$

$$= X_0 \, W^T \underbrace{\begin{bmatrix} 1 & & & \\ & \boxed{A_1} & & \\ & & \boxed{A_2} & \\ & & & \ddots \end{bmatrix}}_{D} W \, X_0^+$$

DFT basis

2x2 blocks
(rotation matrix)

Decomposing the matrix G

$$G = X_0 W^T \underset{D}{\begin{bmatrix} 1 & & & \\ & A_1 & & \\ & & A_2 & \\ & & & \ddots \end{bmatrix}} W X_0^+$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$G = U_2 \quad D \quad U_1$$

$$\boxed{G^j = U_2 \quad D^j \quad U_1}$$

View generation

Equations for leaning samples

$$\boldsymbol{x}_j = G^j \boldsymbol{x}_0 = U_2 D^j U_1 \boldsymbol{x}_0 \qquad j = 0, 1, 2, \ldots, n–1$$

Extend j to arbitrary number

$$\boldsymbol{x}_j = G^j \boldsymbol{x}_0 = U_2 D^j U_1 \boldsymbol{x}_0 \qquad 0 \leq j < n$$

Using only finite number of images $\boldsymbol{x}_0$, $\boldsymbol{x}_1$, …, $\boldsymbol{x}_{n\text{-}1}$,

Generating $\boldsymbol{x}_j$ for any j .

Pose estimation

Equations for leaning samples

$$\boldsymbol{x}_j = G^j \boldsymbol{x}_0 = U_2 D^j U_1 \boldsymbol{x}_0 \qquad j = 0, 1, 2, \ldots, n–1$$

Estimating j of arbitrary number

$$\boldsymbol{x} = G^j \boldsymbol{x}_0 = U_2 D^j U_1 \boldsymbol{x}_0 \qquad 0 \leq j < n$$

$$U_1 \boldsymbol{x} = U_1 U_2 D^j U_1 \boldsymbol{x}_0$$

$$U_1 \boldsymbol{x} = D^j U_1 \boldsymbol{x}_0$$

$$\boldsymbol{x}' = D^j \boldsymbol{x}'_0$$

Compare two images
projected in the subspace

$$\boldsymbol{x}'_j = U_1 \boldsymbol{x}_j$$

An image
in the subspace

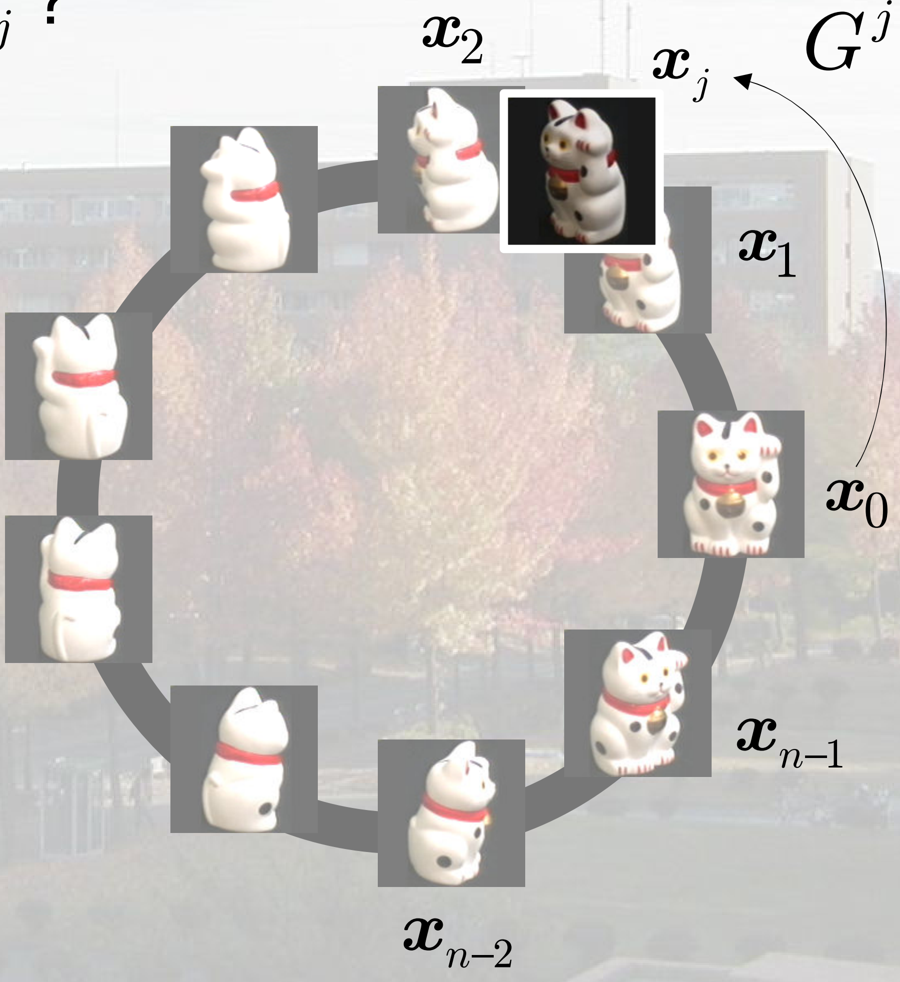
What’s the power of G?

Does G^j really transform $\boldsymbol{x}_0$ to $\boldsymbol{x}_j$?

$$G^j = U_2 D^j U_1$$

Can G^j really produce the back from the front?

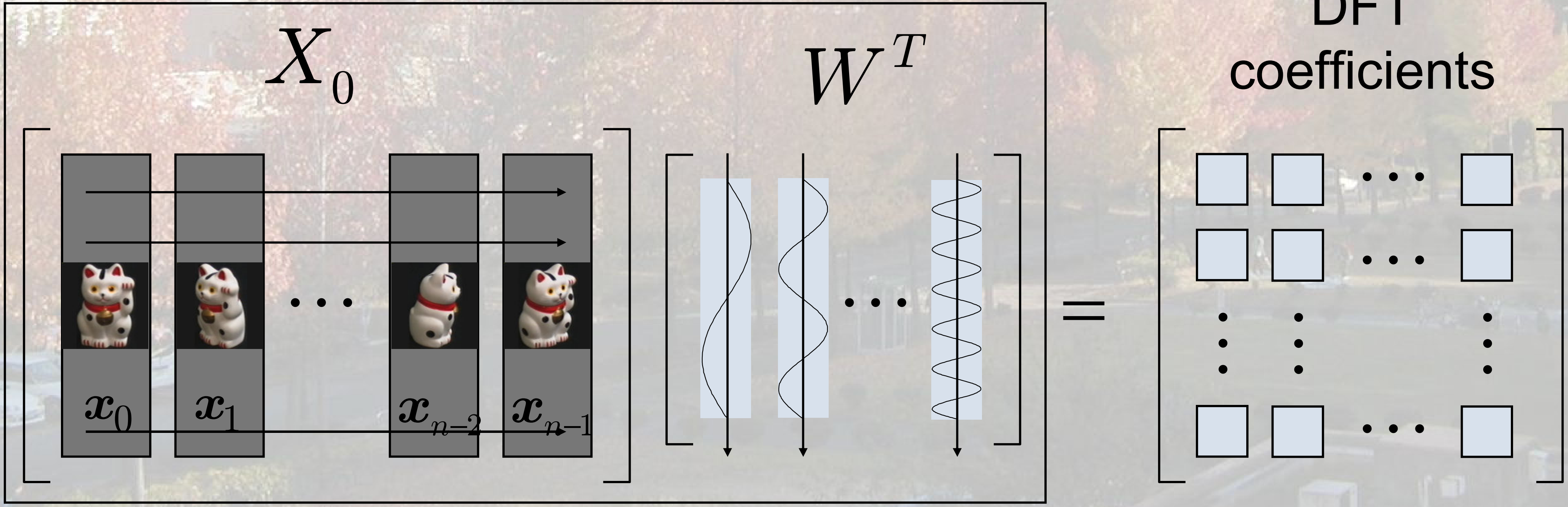
$$= G^j$$



Pixel-wise DFT

$$G = \boxed{X_0 W^T} \overset{D}{\begin{bmatrix} 1 & & & \\ & A_1 & & \\ & & A_2 & \\ & & & \ddots \end{bmatrix}} W X_0^+$$

DFT basis



DFT coefficients

Reconstruction by DFT basis

$j = 0$

$$G^0 = X_0 W^T \overset{D^0}{\begin{bmatrix} 1 & & & \\ & A_1^0 & & \\ & & A_2^0 & \\ & & & \ddots \end{bmatrix}} W X_0^+$$

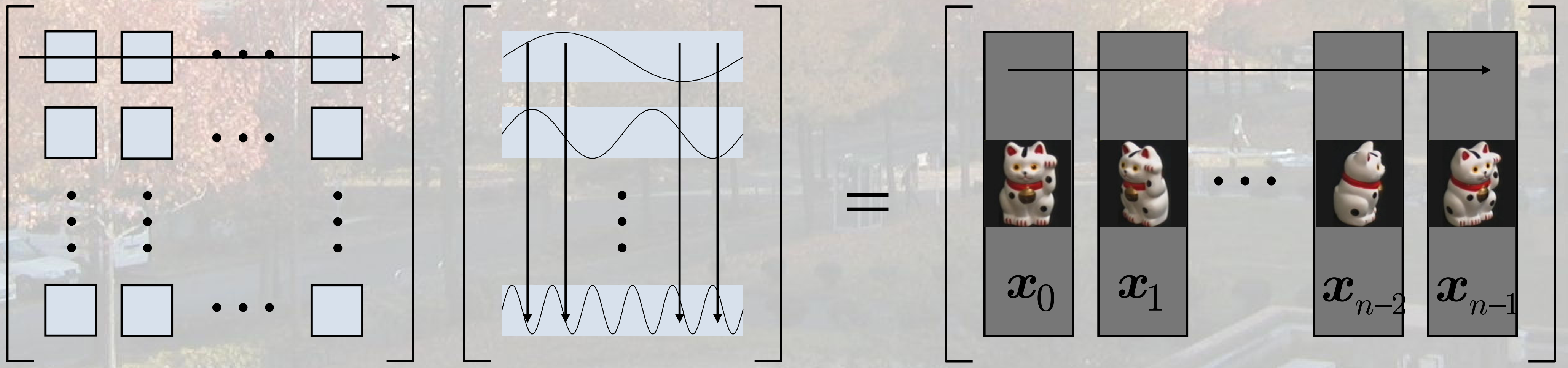
DFT basis

DFT coefficients

W

$D^0 = I$

X_0



Reconstruction by DFT basis

$j = 0$

$$G^0 = X_0 W^T \overset{D^0}{\begin{bmatrix} 1 & & & \\ & A_1^0 & & \\ & & A_2^0 & \\ & & & \ddots \end{bmatrix}} W X_0^+$$

DFT basis

X_0

$$\begin{bmatrix} \boxed{1} \\ \boxed{0} \\ \vdots \\ \boxed{0} \end{bmatrix} = X_0^+ \boldsymbol{x}_0$$

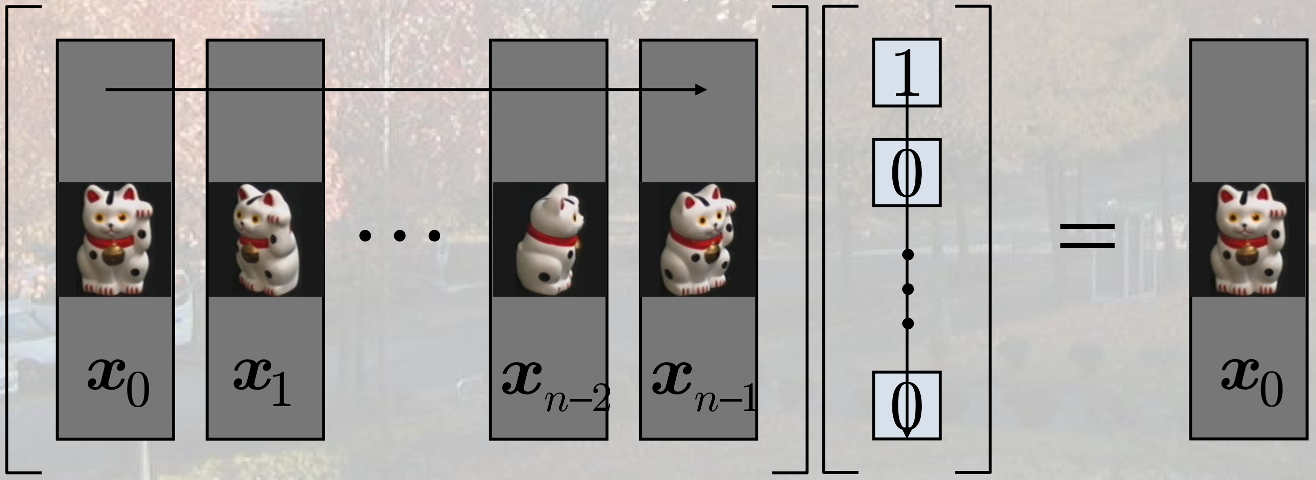
Reconstruction by DFT basis

$j = 0$

DFT basis

$$G^0 = X_0 W^T \overbrace{\begin{bmatrix} 1 & & & \\ & A_1^0 & & \\ & & A_2^0 & \\ & & & \ddots \end{bmatrix}}^{D^0} W X_0^+$$

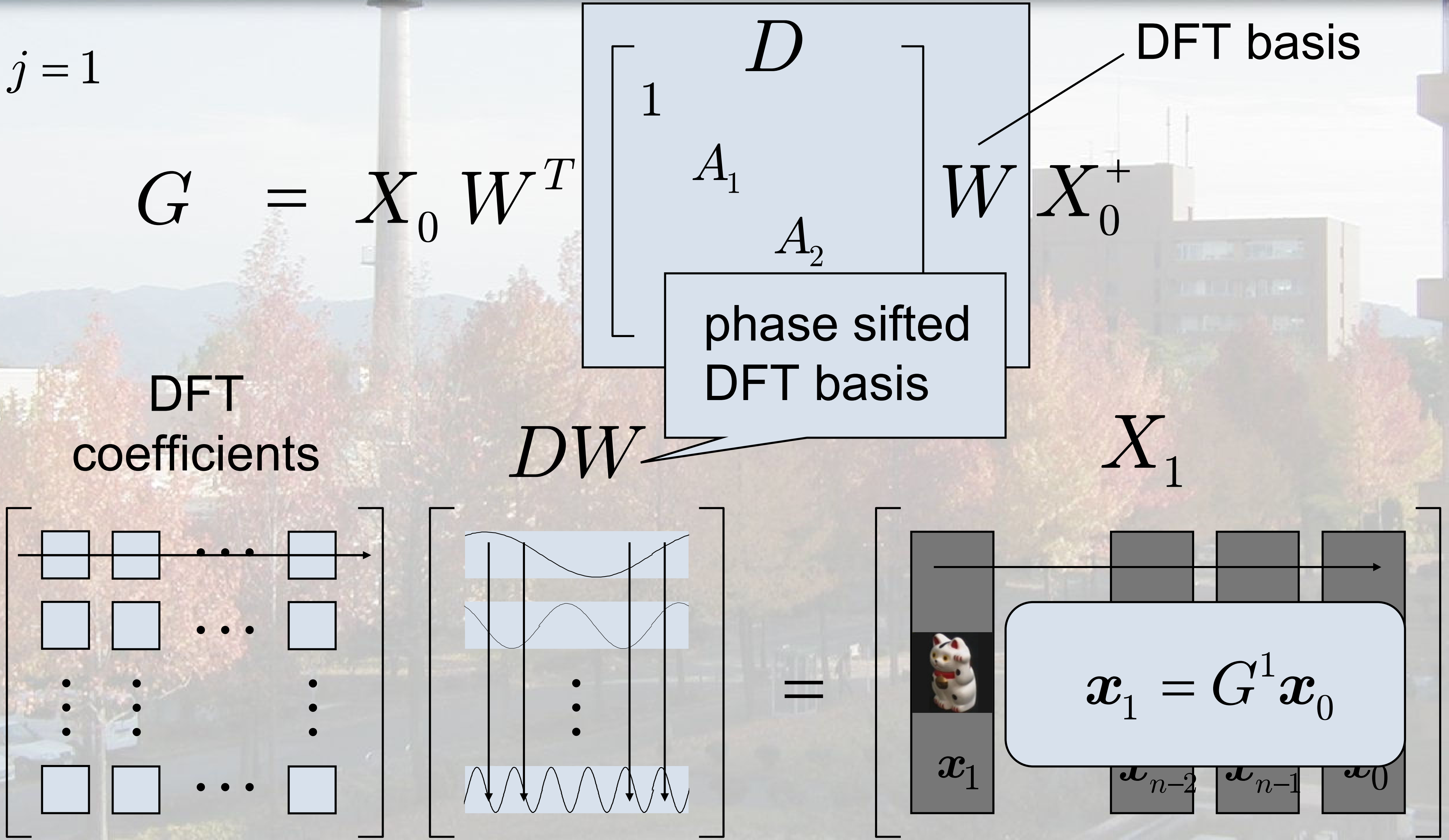
X_0



$$\boldsymbol{x}_0 = G^0 \boldsymbol{x}_0$$

Phase shift of DFT basis

$j = 1$



$$G = X_0 W^T \overset{D}{\begin{bmatrix}1&&\\&A_1&\\&&A_2\end{bmatrix}} W X_0^+$$

DFT basis

phase sifted DFT basis

DFT coefficients

$$\text{DFT coefficients} \; DW = X_1$$

$$\boldsymbol{x}_1 = G^1 \boldsymbol{x}_0$$

Continuous phase shift

$0 \le j < n$

$$G^j = X_0 W^T \overbrace{\begin{bmatrix} 1 & & \\ & A_1^j & \\ & & A_2^j \end{bmatrix}}^{D^j} W X_0^+$$

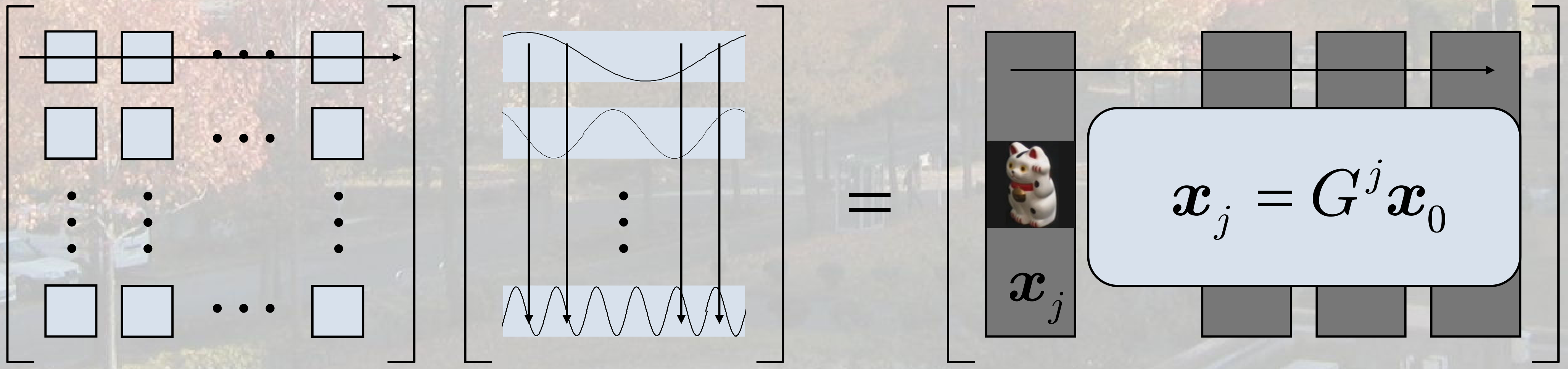
DFT bases

phase sifted DFT bases

DFT coefficients

$D^j W$

X_j



$$x_j = G^j x_0$$

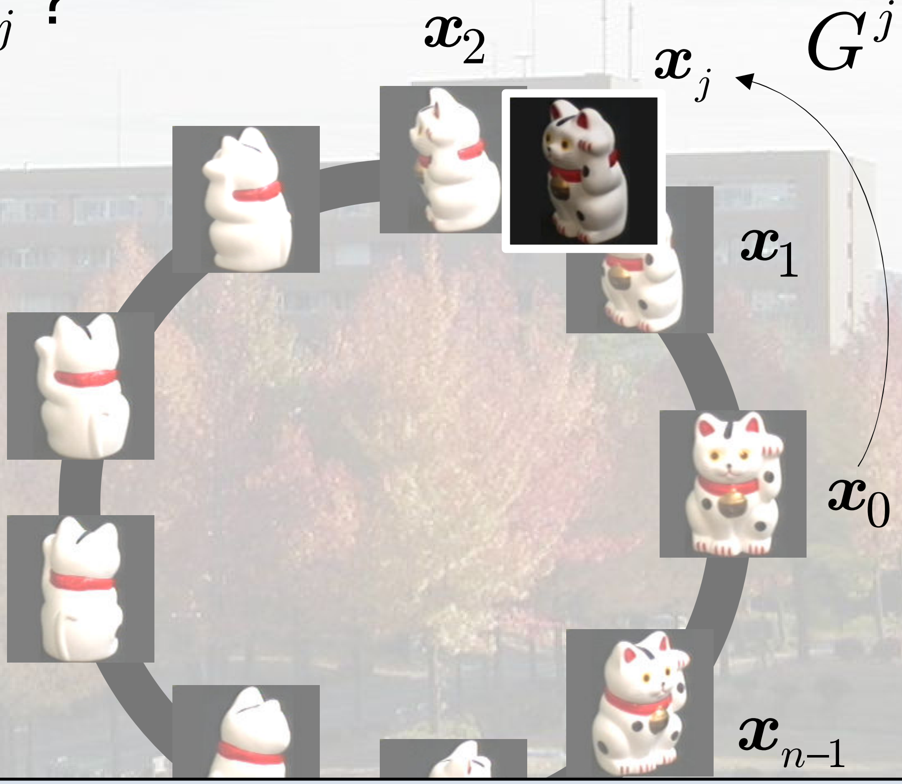
This is the power of G!

Does G^j really transform $\boldsymbol{x}_0$ to $\boldsymbol{x}_j$?

$$G^j = U_2 D^j U_1$$

Can G^j really produce
the back from the front?

$= G^j$



Yes, it does by pixel-wise DFT!

Conclusions

- Introduced cyclic permutation to represent images of rotationg object.
- Applied to view generation and pose estimation.
- Closely related to DFT in pixel-wise for generating novel image by $x_j = G^j x_0$.