

# Internalizing Technological Externality under Default Risk

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## Abstract

In this short article, we investigate the effects of public policies on the market equilibrium resource allocation in an economy that is affected by two sources of market failure; (i) positive technological externalities, and (ii) indebted firms' incentive to default. The Priority Production System employed by the Japanese government during the reconstruction period after World War II could achieve the first-best outcome in such an environment.

## 1. Introduction

During the reconstruction period after World War II, the Japanese government employed the Priority Production System that was aimed to help the recovery of such key industries as coal, electricity, iron and steel. The recovery of these industries was thought to be a base for rebuilding the economy that had lost 1/4 of the national asset during the war (Nakamura (1995)). The Japanese real GDP in 1946 was only 55% of the 1936 level (Kousai (1981)).

The Priority Production System had two key components; (i) subsidizing the production costs of the target industries, and (ii) directly providing public fund through the Reconstruction Finance Bank that was owned by the Japanese government (Noguchi (1986)). In this short paper, we would like to show that the combination of these two policies together can achieve the first-best resource allocation in an imperfect economy where production technologies exhibit positive externalities and indebted firms have incentive to default. In other words, either one of these two policies alone can not achieve the first-best outcome in such an environment. When there are two independent sources of market failure, government needs at least two policy tools to correct them.

An intuitive explanation for this observation is given as follows. The firms' investment level tends to be smaller than the first-best level when there are

positive technological externalities. Then, a public subsidy aimed to induce the firms to invest more does not change indebted firms' incentive to default because they need to borrow more. Anticipating the firms' incentive to default, the creditors will not extend credit to the firms beyond the level that triggers default. Therefore, the government needs to subsidize not only the firms but also the creditors so as to induce the firms to invest more by internalizing the technological externalities, and to induce the creditors to lend more by shifting the default risk from the creditors to the government. The Priority Production System works as subsidy to both firms and creditors because the government is able to shift the default risk from private creditors to the nationally-owned Reconstruction Finance Bank.<sup>1</sup>

The remainder of the paper is organized as follows. In Section 2, we present a model economy that will be used to analyze the resource allocation under market imperfection and the effects of economic policies. In that section, we compare the first-best resource allocation with the market equilibrium without default risk and the market equilibrium with default risk. In Section 3, we analyze four types of economic policies; (i) subsidizing firms, (ii) subsidizing creditors, (iii) subsidizing both firms and creditors, and (iv) subsidizing firms and directly providing public fund to firms (the Priority Production System). Through the

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<sup>1</sup> There are extensive researches on the resource allocation in credit-constrained economies. See Chapter 6 of Obstfeld and Rogoff (1996), and Bernanke, Gertler, and Gilchrist (1999) for surveys of the literature. These researches also look at the implication of empirical monetary policies on the resource allocation in credit-constrained economies. There are not many researches, however, that look at the implication of normative fiscal policies in economies with default risk and technological external effect.

analysis, it will be shown that (iii) and (iv) achieve the first-best outcome, and the Priority Production System (iv) is able to provide the same incentive structure to firms and creditors as the subsidy to both firms and creditors policy (iii) does. Section 4 concludes.

## Section 2. Model

In this section we present a model economy in which production technologies exhibit positive externalities and indebted firms have incentive to default. Then, we describe the first-best resource allocation, the market equilibrium resource allocation without default risk, and the market equilibrium resource allocation with default risk.

In the model, there are many price-taking firms and creditors. There are two time periods. In the first period, the creditors plan disposition of their endowment. There are two investment opportunities available to the creditors; (i) a "storage technology", and (ii) lending to the firms. The storage technology transforms one unit of endowment into  $R$  units of the second period output. The creditors, hence, demand at least the interest rate  $R$  on the lending to the firms. In the second period, the firms choose investment  $K$  and produces output  $Y(K)$ . The production function  $Y(K)$  exhibits an positive externality

$$Y(K)=K^\alpha(\bar{K})^\beta \quad (2.1)$$

where  $0 < \alpha < 1$ ,  $0 < \beta < 1$ , and  $\bar{K}$  is the average investment. If the firms choose to default, the creditors are able to confiscate  $\eta \times 100\%$  of the firms' output, where  $0 < \eta < 1$ . Denote the firms' non-default payoff as  $\Pi_n \equiv Y(K) - RK$  and the default payoff as  $\Pi_d \equiv (1 - \eta)Y(K)$ . The firms default if  $\Pi_n < \Pi_d$ . Otherwise, they do not default. This decision rule is summarized as

$$\eta Y \begin{cases} \geq \\ < \end{cases} RK \rightarrow \begin{cases} \text{non-default} \\ \text{default} \end{cases} . \quad (2.2)$$

In (2.2),  $\eta Y$  is the cost of default, and  $RK$  is the cost of non-default to the firms.

Suppose the creditors lend  $D$  to the firms in the first period. If the firms do not default, the creditors' payoff is  $[(RD)/R] - D$ . If the firms default, it is  $[(\eta Y)/R] - D$ . In a market equilibrium,  $D = K = \bar{K}$  holds. The

firms and the creditors share the same belief about the average investment  $\bar{K}$  in the equilibrium.

### 2-1. The First-Best Resource Allocation

The first-best resource allocation in this economy is a solution to the following problem.

$$\text{Max}_K \quad K^{\alpha+\beta} - RK \quad . \quad (2.3)$$

Assume  $0 < \alpha + \beta < 1$  so that the social optimum exists. It can be shown that the investment and the firms' payoff in the first-best resource allocation are

$$\hat{K} \equiv [(\alpha + \beta)/R]^\gamma \quad , \quad (2.4)$$

$$\hat{\Pi} \equiv (\hat{K})^{\alpha+\beta} - R\hat{K} = [(\alpha + \beta)/R]^\gamma \times [(1/(\alpha + \beta)) - 1]R \quad (2.5)$$

where  $\gamma \equiv 1/[1 - (\alpha + \beta)]$ .

### 2-2. Market Equilibrium without Default Risk

In this subsection, we describe the market equilibrium without default risk so as to separate the effects of the two sources of market failure on resource allocation. Assume that the firms always honor the debt contract. Then the investment is a solution to the following problem.

$$\text{Max}_K \quad Y - RK \quad (2.6)$$

subject to (2.1), given  $\{R, \bar{K}\}$ . The solution to this problem is

$$K^* \equiv [\alpha(\bar{K})^\beta / R]^{1/(1-\alpha)} \quad (2.7)$$

By substituting the equilibrium condition  $\bar{K} = K^*$  into (2.7), it can be shown that the investment and the firms' payoff in the market equilibrium without default risk are

$$K_e^* \equiv (\alpha/R)^\gamma \quad (2.8)$$

$$\Pi_e^* \equiv (K_e^*)^{\alpha+\beta} - RK_e^* = (\alpha/R)^\gamma [(1/\alpha) - 1]R \quad (2.9)$$

where  $\gamma$  is the same as before, and the subscript "e" in (2.8) and (2.9) imply the "value in equilibrium".

### 2-3. Market Equilibrium with Default Risk

Given the belief about the average investment  $\bar{K}$ , suppose the creditors lend  $D = K^* = [\alpha(\bar{K})^\beta / R]^{1/(1-\alpha)}$  of

(2.7) to the firms. The difference between the non-default payoff and the default payoff for the firms is shown to be

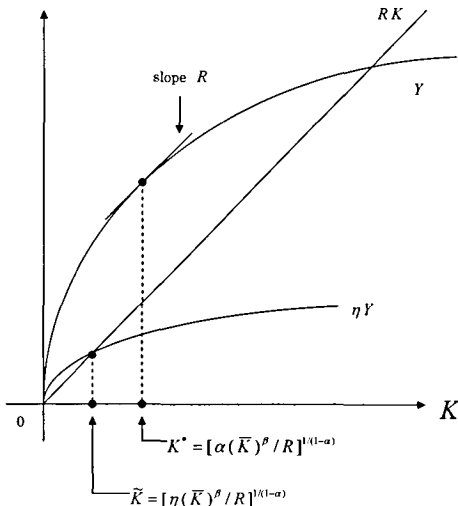
$$\Pi_N - \Pi_D = K^* R [(\eta / \alpha) - 1]. \quad (2.10)$$

Since we are interested in an economy where the two sources of market failure are operational, we assume  $\eta < \alpha$  in the following. In this case, if the creditors lend  $D = K^*$  to the firms, the firms choose to default because  $\Pi_N < \Pi_D$ . By (2.1) and (2.2), given  $\bar{K}$ , the investment level which makes the firms indifferent between non-default and default ( $\Pi_N = \Pi_D$ ) is shown to be

$$\tilde{K} \equiv [\eta(\bar{K})^\beta / R]^{1/(1-\alpha)}. \quad (2.11)$$

If the creditors lend more than  $\tilde{K}$ , the firms default, and the creditors' payoff  $[(\eta Y)/R] - D$  becomes negative. Anticipating the firms' incentive to default, the creditors will not lend more than  $\tilde{K}$ . This situation is depicted in Figure 1. In the figure, the ex-ante optimal investment  $K^*$  for the firms equates the marginal return on investment  $dY/dK$  to the interest rate  $R$ . By (2.2) the investment level  $\tilde{K}$  that leaves the firms indifferent between non-default and default is determined at the intersection of  $\eta Y$  and  $RK$ . On the horizontal axis, for  $K \in [0, \tilde{K}]$ , the firms do not default because  $\eta Y \geq RK$ . For  $K \in (\tilde{K}, \infty)$ , the firms default because  $\eta Y < RK$ .

Figure 1



By substituting  $\bar{K} = \tilde{K}$  into (2.11), the investment and the firms' payoff in the market equilibrium with

default risk are

$$\tilde{K}_\varepsilon \equiv (\eta / R)^\beta \quad (2.12)$$

$$\begin{aligned} \tilde{\Pi}_\varepsilon &\equiv (\tilde{K}_\varepsilon)^{\alpha+\beta} - R \tilde{K}_\varepsilon \\ &= (\eta / R)^\beta [(1/\eta) - 1] R \end{aligned} \quad (2.13)$$

By (2.4), (2.8), and (2.12), it can be shown that

$$\tilde{K}_\varepsilon < K^* < \hat{K}. \quad (2.14)$$

By (2.5), (2.9), and (2.13), it can be shown as well that

$$\tilde{\Pi}_\varepsilon < \Pi^* < \hat{\Pi}. \quad (2.15)$$

(2.15) is proved as follows. At  $\beta = 0$ ,  $\hat{\Pi} / \Pi^* = 1$ . In addition,  $\partial(\hat{\Pi} / \Pi^*) / \partial \beta > 0$ . Therefore,  $\hat{\Pi} / \Pi^* > 1$  if  $\beta > 0$ . Similarly, by using  $\alpha \equiv \eta + \varepsilon$ , it can be shown that  $\Pi^* / \tilde{\Pi}_\varepsilon = 1$  at  $\varepsilon = 0$ , and  $\partial(\Pi^* / \tilde{\Pi}_\varepsilon) / \partial \varepsilon > 0$ . Therefore,  $\Pi^* / \tilde{\Pi}_\varepsilon > 1$  if  $\varepsilon > 0$ .

The equilibrium social welfare in this economy is equal to the firms' payoff because the creditors' payoff is always zero. (2.15) implies that the equilibrium social welfare without default risk is smaller than the first-best level due to the technological externality. In addition, when  $\alpha > \eta$ , the equilibrium social welfare is even smaller because the firms' incentive to default is operational.

### 3. Economic Policies

In this section, we analyze the effects of economic policies on the market equilibrium resource allocation with default risk. We consider four policies; (i) subsidizing firms, (ii) subsidizing creditors, (iii) subsidizing both firms and creditors, and (iv) subsidizing firms and directly providing public fund to firms. We are interested in if these policies are able to achieve the first-best resource allocation by providing appropriate incentive to firms and creditors.

#### 3-1. Subsidizing Firms

In this policy, the government subsidizes the firms' investment cost. The subsidy is financed by imposing lump-sum tax on the firms. If the firms do not default, their payoff is

$$\Pi_N = Y - (R - G_f)K - T, \quad (3.1)$$

and if the firms default, their payoff becomes

$$\Pi_D = (1 - \eta)Y + G_f K - T. \quad (3.2)$$

In (3.1) and (3.2),  $G_f$  is the subsidy rate, and  $T$  is the lump-sum tax. The government's budget constraint is

$$G_f K = T. \quad (3.3)$$

Consider the following problem;

$$\begin{aligned} \text{Max}_K \quad & \Pi_N \text{ subject to (2.1), given } \{\bar{K}, R, G_f, T\}. \\ & (3.4) \end{aligned}$$

The solution to this problem is

$$K = [\alpha (\bar{K})^\beta / (R - G_f)]^{1/(1-\alpha)}. \quad (3.5)$$

By substituting the equilibrium condition  $\bar{K} = K$ , (3.5)

becomes

$$K = [\alpha / (R - G_f)]^\gamma. \quad (3.6)$$

This  $K$  of (3.6) is equal to the first-best investment  $\hat{K}$  of (2.4) when the government set the subsidy rate at

$$G_f = R\beta / (\alpha + \beta). \quad (3.7)$$

Suppose the government announces  $\{G_f, T\}$  where  $G_f$  is given by (3.7), and  $T$  is given by

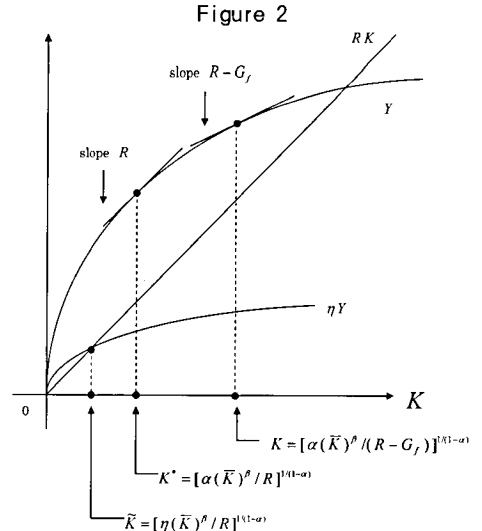
$$\begin{aligned} T = G_f \hat{K} &= \left( \frac{R\beta}{\alpha + \beta} \right) \left( \frac{\alpha + \beta}{R} \right)^\gamma \\ &= \beta \left( \frac{\alpha + \beta}{R} \right)^{\gamma-1}. \end{aligned} \quad (3.8)$$

The firms and the creditors take  $\{\bar{K}, R, G_f, T\}$  as given. If the creditors lend the ex-ante optimal  $D = K$  of (3.5), the firms choose to default because, by (3.1) and (3.2),

$$\begin{aligned} \Pi_N - \Pi_D &= \eta Y - RK = \\ & K \left[ R \left( \frac{\eta}{\alpha} - 1 \right) - \frac{\eta}{\alpha} G_f \right] < 0. \end{aligned} \quad (3.9)$$

Anticipating the firms' incentive to default, the creditors will not lend any more than  $\tilde{K}$  of (2.11) that leaves the firms indifferent between default and non-default. When the incentive to default is operational, the subsidy which induces the firms to borrow and invest more does not change the firms' incentive to default. Therefore, this policy can not achieve the first-best resource allocation. This situation is depicted by Figure 2. In the figure, the ex-ante optimal investment  $K$  of (3.5) equates the marginal return on investment  $dY/dK$  to the subsidized cost of investment  $R - G_f$ . Because this investment level is larger than  $K^*$  of (2.7), the subsidy to the firms does not change the firms' incentive to default. The market equilibrium resource allocation under this policy is the same as before. The equilibrium investment is  $\tilde{K}$ , of (2.8), and

the social welfare is  $\tilde{\Pi}_s$  of (2.9).



### 3-2. Subsidizing Creditors

When the creditors lend no more than the amount that leaves the firms indifferent between default and non-default, the government may be able to induce the creditors to lend more by subsidizing the default-loss. Assume that the subsidy is financed by imposing lump-sum tax on the firms. In this policy, the firms' non-default payoff is

$$\Pi_N = Y - RK, \quad (3.10)$$

and the firms' default payoff is

$$\Pi_D = (1 - \eta)Y - T \quad (3.11)$$

where  $T$  is the lump-sum tax. On the other hand, the creditors' non-default payoff is

$$[(RD)/R] - D \quad (3.12)$$

and the creditors' default payoff is

$$[(\eta Y + G_c D)/R] - D \quad (3.13)$$

where  $G_c$  is the subsidy rate to the creditors. When the firms default, the following budget constraint applies to the government.

$$G_c D = T \quad (3.14)$$

Consider the following problem;

$$\begin{aligned} \text{Max}_K \quad & \Pi_N \text{ subject to (2.1), given } \{\bar{K}, R, G_c, T\}. \\ & (3.15) \end{aligned}$$

The solution to this problem is the same as (2.7)  $K^* = [\alpha (\bar{K})^\beta / R]^{1/(1-\alpha)}$ , and it becomes (2.8)  $K^* = (\alpha / R)^\gamma$  in equilibrium where  $\bar{K} = K^*$  holds. The government set the subsidy rate  $G_c$  so as to induce the creditors to lend  $D = K^*$  to the firms. Because the creditors' default

payoff is zero at such  $G_c$ , the creditors will lend

$$D = [\eta (\bar{K})^\beta / (R - G_c)]^{1/(1-\alpha)} \quad (3.16)$$

given  $\{\bar{K}, R, G_c\}$ . By equating (3.16) and (2.8), and by using the equilibrium condition  $\bar{K} = D$ , the subsidy rate is shown to be

$$G_c = R(1 - \eta / \alpha). \quad (3.17)$$

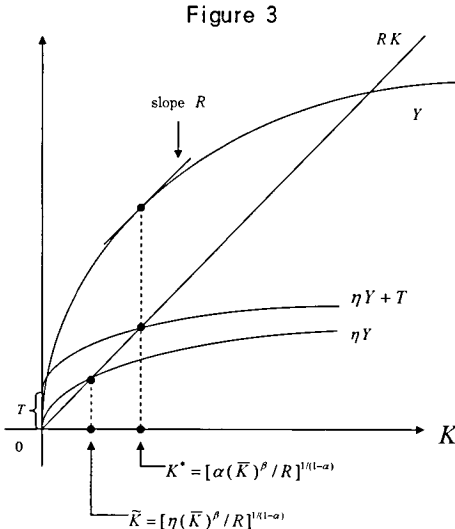
Suppose the government announces  $\{G_c, T\}$  where  $G_c$  is given by (3.17), and  $T$  is given by

$$T = G_c K_c^* = R(1 - \eta / \alpha) (\alpha / \eta)^{\alpha-1} \quad (3.18)$$

In the market equilibrium under this policy, the creditors lend  $D = K_c^*$  to the firms, and the firms do not default because, by (3.10) and (3.11),

$$\begin{aligned} \Pi_N - \Pi_D &= \eta Y + T - RK \\ &= \eta (K_c^*)^{\alpha+\beta} + G_c K_c^* - RK_c^* \\ &= K_c^* [\eta (R/\alpha) + R(1 - \eta/\alpha) - R] = 0. \end{aligned} \quad (3.19)$$

In the first line of (3.19),  $\eta Y + T$  is the cost of default and  $RK$  is the cost of non-default for the firms. This policy is able to induce the creditors to lend more because it affects the firms' incentive to default by increasing the cost of default from  $\eta Y$  to  $\eta Y + T$ . This situation is depicted in Figure 3. The figure shows that the policy expands the range of investment such that the firms choose not to default from  $K \in [0, \tilde{K}]$  to  $K \in [0, K^*]$



The market equilibrium resource allocation under this policy is the same as that without default risk. The equilibrium investment is  $K_c^*$  of (2.8), and the social welfare is  $\Pi_c^*$  of (2.9). Although the policy is able to improve the resource allocation, the outcome is still

inferior to the first-best because the policy does not internalize the production externality.

### 3-3. Subsidizing both Firms and Creditors

So far, we saw in Section 3-1 that the subsidy to the firms induces them ex-ante to choose the first-best investment level, but fails ex-post to induce them not to default. We saw as well in Section 3-2 that the subsidy to the creditors induces them to lend more by affecting the firms' incentive to default, but fails to induce the firms to choose the first-best investment level. Then, it is natural to combine these two policies because the policies together may be able to induce the firms to choose the first-best investment and not to default so that the creditors will lend enough to achieve the first-best resource allocation.

In this policy, the firms' non-default payoff is

$$\Pi_N = Y - RK + G_f K - T_N, \quad (3.20)$$

and the firms' default payoff is

$$\Pi_D = (1 - \eta)Y + G_f K - T_D. \quad (3.21)$$

In (3.20) and (3.21),  $T_N$  and  $T_D$  are the semi-lump-sum taxes in a sense that they are not continuous functions of the firms' action, but dependent on the firms' discrete choice {non-default, default}. The creditors' non-default payoff is

$$[(RD/R)] - D \quad (3.22)$$

and the creditors' default payoff is

$$[(\eta Y + G_c D)/R] - D \quad (3.23)$$

Finally, the government's non-default budget constraint is

$$G_f K = T_N \quad (3.24)$$

and the government's default budget constraint is

$$G_f K + G_c D = T_D. \quad (3.25)$$

The firms' incentive to non-default or default is described by

$$\begin{aligned} \Pi_N - \Pi_D \quad \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} 0 \\ \rightarrow \left\{ \begin{array}{l} \text{non-default} \\ \text{default} \end{array} \right\}. \end{aligned} \quad (3.26)$$

By (3.20) and (3.21), (3.26) is rewritten as

$$\begin{aligned} \eta Y + T_D \quad \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} RK + T_N \\ \rightarrow \left\{ \begin{array}{l} \text{non-default} \\ \text{default} \end{array} \right\}. \end{aligned} \quad (3.27)$$

The left-hand side of (3.27) is the firms' cost of default, and the right-hand side is the cost of non-default. Consider the following problem.

$$\text{Max}_K \quad \Pi_N = Y - RK + G_f K - T_N \quad (3.28)$$

subject to (2.1), given  $\{\bar{K}, R, G_f, T_N, T_D\}$ . The solution to this problem is the same as (3.5)  $K = [\alpha(\bar{K})^\beta / (RG_f)]^{1/(1-\alpha)}$ , and it becomes (3.6)  $K = [\alpha / (R - G_f)]^\gamma$  in equilibrium where  $\bar{K} = K$  holds. As we saw in Section 3-2, the government can induce the firms ex-ante to choose the first-best investment  $\hat{K}$  of (2.4) by setting the subsidy rate at  $G_f = R\beta / (\alpha + \beta)$  of (3.7). On the other hand, the government will set the subsidy rate  $G_c$  to the creditors so as to induce them to lend  $D = \hat{K}$ . Because the creditors' default payoff is zero at such  $G_c$ , the creditors will lend

$$D = [\eta(\bar{K})^\beta / (R - G_c)]^{1/(1-\alpha)} \quad (3.29)$$

given  $\{\bar{K}, R, G_c\}$ . By equating (2.4) and (3.29), and by using the equilibrium condition  $\bar{K} = D$ , the subsidy rate is shown to be

$$G_c = R[1 - \eta / (\alpha + \beta)]. \quad (3.30)$$

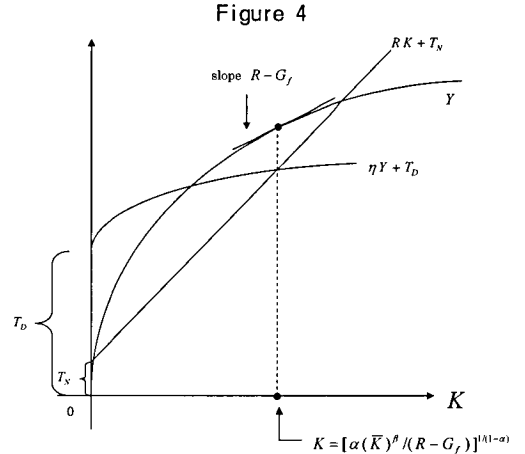
Suppose the government announces  $\{G_f, G_c, T_N, T_D\}$ , where  $G_f$  is given by (3.7),  $G_c$  is given by (3.30),  $T_N$  is given by (3.8), and  $T_D$  is given by

$$\begin{aligned} T_D &= G_f K + G_c D \\ &= \left[ \frac{R\beta}{\alpha + \beta} + R \left( 1 - \frac{\eta}{\alpha + \beta} \right) \right] \left( \frac{\alpha + \beta}{R} \right)^\gamma. \end{aligned} \quad (3.31)$$

In the market equilibrium, this policy achieves the first-best resource allocation because the creditors lend  $D = \hat{K}$  to the firms, and firms do not default because, by (2.4), (3.8), and (3.31),

$$\begin{aligned} \Pi_N - \Pi_D &= (\eta Y + T_D) - (RK + T_N) \\ &= \hat{K} \left[ \eta \left( \frac{R}{\alpha + \beta} \right) - R + R \left( 1 - \frac{\eta}{\alpha + \beta} \right) \right] = 0. \end{aligned} \quad (3.32)$$

This situation is depicted in Figure 4. The figure shows that the government induces the firms ex-ante to choose the first-best investment  $\hat{K}$  by the subsidy rate  $G_f$ . The figure also shows that the government induces the firms ex-post not to default by equating the cost of non-default  $RK + T_N$  and the cost of default  $\eta Y + T_D$  at  $K = \hat{K}$ .



### 3-4. Subsidizing Firms and Directly Providing Public Fund to Firms

As we stated in Section 1, the Priority Production System employed by the Japanese government during the economic recovery after WWII had two major components; subsidizing the production cost of the target firms, and directly providing public fund to the firms through a nationally-owned bank. In this section, we show these two policy measures together achieve the first-best resource allocation because they can provide the same incentive structure to the firms and the creditors as the policy in Section 3-3 (subsidizing both firms and creditors) does.

The policy in this section is described as follows. In the first period, the government borrows  $D_c$  from the creditors at the interest rate  $R$ . The government lends  $D_s$  to the firms at the interest rate  $R$ . In the second period, the government subsidize the firms' investment cost at the subsidy rate  $G_f$ . The firms invest  $K$  and produce output  $Y$ . If the firms choose not to default, the firms repay  $RD_s$  to the government, and the government repays  $RD_c$  to the creditors. On the other hand, if the firms choose to default, the government confiscates  $\eta \times 100\%$  of the firms' output  $Y$  and repays  $RD_c$  to the creditors. The government also imposes lump-sum tax on the firms to balance its budget constraint if necessary. In this policy, the firms' non-default payoff is

$$\Pi_N = Y - RK + G_f K - T_N, \quad (3.33)$$

and the firms' default payoff is

$$\Pi_N = (1 - \eta)Y + G_f K - T_D \quad (3.34)$$

where  $T_n$  and  $T_d$  are the lump-sum taxes. On the other hand, regardless of the firms' action {non-default, default}, the creditors' payoff is always equal to

$$[(RD_c)/R] - D_c \quad (3.35)$$

because the default risk is absorbed by the government. The government's non-default budget constraint is

$$RD_c + G_f K = T_n + RD_g, \quad (3.36)$$

and the government's default budget constraint is

$$RD_c + G_f K = T_d + \eta Y. \quad (3.37)$$

In (3.36) and (3.37), the left-hand side is the government's expenditure, and the right-hand side is the revenue.

Suppose the government borrows  $D_c = \hat{K}$  of (2.4) from the creditors, lends  $D_g = \hat{K}$  to the firms, and subsidizes the firms' investment cost at the subsidy rate  $G_f = R\beta / (\alpha + \beta)$  of (3.7). The government sets the lump-sum taxes  $T_n$  and  $T_d$  so as to satisfy the budget constraints (3.36) and (3.37) as follows. By (3.36),

$$\begin{aligned} T_n &= RD_c + G_f K - RD_g \\ &= (R + G_f - R) = \beta [(\alpha + \beta)/R]^{1-\gamma} \end{aligned} \quad (3.38)$$

which is equal to (3.8), and by (3.37),

$$\begin{aligned} T_d &= RD_c + G_f K - \eta Y = R\hat{K} + G_f \hat{K} - \eta(\hat{K})^{\alpha+\beta} \\ &= \left[ \frac{R\beta}{\alpha+\beta} + R \left( 1 - \frac{\eta}{\alpha+\beta} \right) \right] \left( \frac{\alpha+\beta}{R} \right)^\gamma \end{aligned} \quad (3.39)$$

which is equal to (3.31). Then the firms invest  $\hat{K}$ , and choose not to default because  $\Pi_n - \Pi_d = (\eta Y + T_d) - (RK + T_n)$  becomes zero as (3.32) of Section 3-3 showed. Because the policy variables  $\{G_f, T_n, T_d\}$  are the same as those of Section 3-3, Figure 4, which depicts the effects of subsidy to firms and creditors, also applies to the policy of this subsection. Therefore, the combination of the subsidy to firms and the direct provision of public fund achieves the first-best resource allocation by providing the same incentive structure to the firms and the creditors as the policy in Section 3-3 does.

#### 4. Conclusion

In this paper, we analyzed the effects of public policies on market equilibrium resource allocation in

an economy with two sources of market failure; (i) positive externalities in production technologies, and (ii) indebted firms' incentive to default. We saw that a combination of subsidy to firms and subsidy to creditors can achieve the first-best resource allocation by providing an appropriate incentive structure to the firms and the creditors. Either one of these two policy measures alone can not achieve the first-best outcome. When there are two independent sources of market failure, the government needs at least two policy tools to correct the market failure. We saw as well that a combination of subsidy to firms and direct provision of public fund to firms, which is regarded as the Priority Production System employed by the Japanese government during the reconstruction period after WWII, also achieves the first-best outcome because such a policy is able to provide the same incentive structure as a combination of subsidy to firms and subsidy to creditors does.

External effects play important roles in the recent developments in economic theories such as growth, environment, and public policy analysis. In this paper, we assumed that there is a positive technological external effect, and the social production function exhibits a decreasing returns to scale, i.e.,  $\alpha + \beta < 1$  in (2.1), so that the socially optimal first-best resource allocation exists. On the other hand, it is known in endogenous growth models that an increasing returns to scale in social production function can be a main engine of growth in competitive economies with positive technological externalities. In these models, the existence of a social optimum is assured by imposing restrictions on parameters so that the social welfare is bounded at the first-best resource allocation. (See Romer (1986), and Lucas (1988).) We may extend our model to investigate the effects of economic policies in endogenously growing economies where the technologies exhibit positive externalities, the social production function exhibits an increasing returns to scale, and indebted firms have incentive to default.<sup>2</sup> Because the market equilibrium investment level is known to be smaller than the socially optimal level in endogenous growth models with positive technological externalities, we expect to

see the same problems arise when indebted firms have incentive to default. That is, subsidy to firms alone can not achieve the first-best resource allocation. The government needs more than two policy tools for correcting the market failure.

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<sup>2</sup> See Zagler and Durnecker (2003) for the survey of the economic policy analysis in endogenous growth models.