Young Children's Knowledge and Strategies for Comparing Sizes

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This study examined the knowledge and strategies that young children used for comparing sizes of geometric figures. Sixty-nine children from the ages 3 to 6 were asked to compare sizes of geometric figures and their placement and adjustment strategies were observed. The children were also presented with strategies for comparing sizes and asked to choose the most effective one. As a result, children showed four different patterns of uses of strategies and judgments. Differences among children showing the four patterns (referred to as Clusters 1-4) were summarized as follows: (a) Children from Clusters 2 to 4 made correct judgments for the relative sizes of figures placed on one another, (b) children from Clusters 3 to 4 very often used the strategy of adjustment based on two dimensions, (c) only children in Cluster 4 very often used the strategies of superimposition and adjustment based on two dimensions at the same time and made more correct judgments for the relative areas of two figures; and (d) children in Cluster 4 selected as effective the strategy of adjusting figures based on two dimensions.

Size comparison is an important step toward measurement in mathematics. Young children acquire the knowledge of how to judge relative areas[1] through comparing things of different sizes in everyday life before entering school. Research indicates that children as young as 3 years may use different kinds of relative size standards (i.e., normative, perceptual, and functional) to assess the size of an object (Ebeling & Gelman, 1988, 1994; Gelman & Ebeling, 1989). Developmental psychologists widely agree that preschool children construct informal mathematical knowledge prior to formal schooling in arithmetic (e.g., Ginsburg, Klein, & Strarkey, 1998; Resnick, 1989). The informal mathematical knowledge provides a form of conceptual competence that may be an important resource for meaningful learning of concepts and cognitive procedures at school.

The present study is concerned with knowledge and strategies that young children of different ages use for comparing sizes. A majority of studies reported thus far have stressed a qualitative difference in use of knowledge for comparing sizes between young children and older children or adults. Some researchers suggest that preschoolers judge the sizes of geometric figures by only one dimension or one salient aspect of the stimulus (Bausano & Jeffrey, 1975; Maratsos, 1973; Piaget, Inhelder, & Szeminska, 1960; Raven & Gelman, 1984; Russell, 1975; Sena & Smith, 1990). Others indicate that even preschoolers integrate more than one aspect of stimuli for area judgment, but that the integration is based on an additive combination of the height and width (Anderson & Cuneo, 1978; Cuneo, 1980; Wilkening, 1979). More recently, it was suggested that infants use the total length of contour of stimuli to judge the relative amounts for both discrete and continuous quantities (Clearfied, & Mix, 1999; Mix, Levin, & Huttenlocher, 2001).

One problem in these studies is that they did not allow the children to interact actively with the figures to be compared in the experiments. In everyday life, however, young children acquire the knowledge of comparative sizes through interacting with objects at hand. For example, Bryant and Kopytynska (1983) presented 5- and 6-year-olds with a pair of black wooden blocks with either a 4 inch or a 6 inch hole in the top and a 10 inch stick with the center 2 inches painted yellow. When asked which block had the deeper hole, or whether the holes were of equal depth, children used the stick to measure the depth of the holes, regardless of whether or not they had been told to use the stick. Only through such an active interaction with objects could children rely on everyday knowledge of sizes to make their performance more effective. This is also suggested by a more recent study by Wolf (1995).

Wolf (1995) examined the effect of the direct handling of objects on estimation of sizes of Euclidean objects among young children. Five- to 6-year-old children in the experimental group were asked to estimate the size of stimulus materials (e.g., rectangular pieces of baking chocolate) on a graphic rating scale, after they were provided with an opportunity to play with the stimulus materials. While the children in the control group who had not received a handling opportunity estimated the stimulus sizes by an addition rule of Height + Width, the children in the experimental group used a normative multiplicative rule.

Miller (1989) argued that children's thinking about quantity is organized in terms of the procedures used to measure amount and the measurable attributes that these procedures quantify. In the case of area, the key procedure is regarded as overlaying objects with other objects or standard areas (Miller, 1989). An opportunity to interact actively with objects at hand allows a child to exercise the procedure of placing one object on another, which should tap the child's developing concept of area. In considering the important role the procedure of placing one object on another plays in young children's early area concept, it is important to make it clear how they come to use the procedure of placing one object on another spontaneously as a tool for judging relative areas. Some studies have examined the early strategies for size comparison.

Miller (1984) asked children at each of ages 3 and 5 years, and grades 2 and 4 to divide materials evenly among two, three, or four puppet turtles. The materials included "candies," strips of clay "spaghetti," clay squares of "fudge," and glasses of "kool-aid," which emphasized number, length, area, and volume, respectively. The children's measurement strategies were observed. Miller found that strategies employed in measuring area and length were similar to each other. A dominant strategy in preschoolers was to cut the material into arbitrary pieces and count them to ensure the same number of pieces. Another strategy that increased with age was to cut the material directly into fractions of approximately equal size. Use of units of constant size was rare in preschool children. Hiebert and Tonnessen (1977) also asked young children to divide a quantity (area, length, and number) equally and obtained a similar result.

The results of these studies are interesting, because they reveal that young children can adopt systematic procedures to determine equality. But the results did not provide the data explaining what role the procedure of putting one thing on another plays in early judgment of areas. Children did not use the strategy of putting one thing on another, perhaps because the task of children was to divide "fudge" (Miller, 1984) or "pie" (Hiebert & Tonnessen, 1977) equally (i.e., children would not place some fudge or pie on other fudge or pie to compare the amounts).

Another study concerning early strategies for size comparison was Yuzawa, Bart, Kinne, Sukemune, & Kataoka (1999). They tested the effect of paper folding on size comparison strategies in young American and Japanese children. Four-, 5-, and 6-year-old children were asked to judge the relative sizes of two figures, and their strategies were observed. It was found that whether children paid attention only to one dimension or to two dimensions of the geometric figures was dependent on which strategies they used. When they placed the two figures on one side or side by side, children tended to adjust the figures only by one dimension or by no dimension. However, when they used the strategy of placing one figure on another, children tended to adjust the two figures in terms of two dimensions (i.e., height and width).

Yuzawa et al. (1999) argued that for the comparison of sizes of geometric figures preschoolers are beginning to use the strategy of placing one figure on another, which might be related with the development of a two-dimensional concept of area. But their focus was more on the effect of paper folding on strategy changes and the strategy differences between American and Japanese children. Thus, the present study was designed to extend the findings of Yuzawa et al. and to clarify what strategies and knowledge children of ages from 3 to 6 years would use for size comparison.

The main question to be addressed in the present study was how the strategies that children used spontaneously for comparing the sizes of two geometric figures would be related with the children's judgment and ages. Yuzawa et al. (1999) observed that children used different placement and adjustment strategies for size comparison. Placement referred to the arrangement of two figures. There were three possibilities of placement strategies when children compared two geometric figures. First, two figures were placed with the figures touching, or without a side of the figures touching (referred to as "side by side", see C-1 to C-3, Figure 1). Furthermore, when two figures were touching, there were two cases: In one case, one figure was placed on the other (referred to as "one on another", see A-1 to A-3, Figure 1), and in another case, two figures were aligned just with sides touching (referred to as "on one side", see B-1 to B-3, Figure 1).

On the other hand, adjustment referred to the directional manipulation of two figures. There were also three possibilities of adjustment strategies. When children dealt with two-dimensional figures such as triangle, the first possibility was that they paid attention to the two dimensions (i.e., the height and width): The two figures were adjusted so that they looked as similar as possible (referred to as "general shape" see A-1 to C-1). The second possibility was that children paid attention only to one dimensions: The two figures were adjusted so that they could be compared along the dimension of one side (referred to as "one side " see A-2 to C-2). The third possibility was that there were no adjustments (referred to as "no adjustment" see A-3 to C-3).

Our focus was on the placement strategy of "one on another" and the adjustment strategy of "general shape." It was indicated that "one on another" was related with "general shape," and that the uses of these strategies were related to correct judgment (Yuzawa et al., 1999). However, it was not clear how the relationship between these strategies changed with ages. For example, would these strategies be used at the same time, or would one of the strategies be used earlier than the other? Children could pay attention to the overall shapes of the figures and adjust them by two dimensions without placing one figure on another; Whereas, if the figures were placed on each other, children would find it easy to adjust the figures by the overall shapes, because they just have to pay attention to the areas of one figure not covered by another figure. Therefore, it was predicted that the use of "general shape" would precede or at least accompany the use of "one on another." The present study identified the patterns with which these strategies were used and examined how the patterns were related with correct judgment and ages of children.

An additional question in the present study was whether children could evaluate which strategy would be useful for comparing sizes. Siegler and Crowley (1994) indicated that 5-year-olds who did no yet use the min strategy for adding numbers (e.g., to solve 2 + 3, they represent the larger addend by simply saying it and then count from it the number of times indicated by the smaller addend) judged it to be smarter than an equally novel illegitimate strategy. However, other research (e.g., Baroody, 1984) showed a result to the contrary, suggesting that young children have difficulty recognizing an advanced strategy to be legal. Thus, it was not clear how children's metacognitive judgment of strategies was related with their use of the strategies. The present study addressed this question by examining how children using or not using "one on another" or "general shape" would evaluate these strategies.

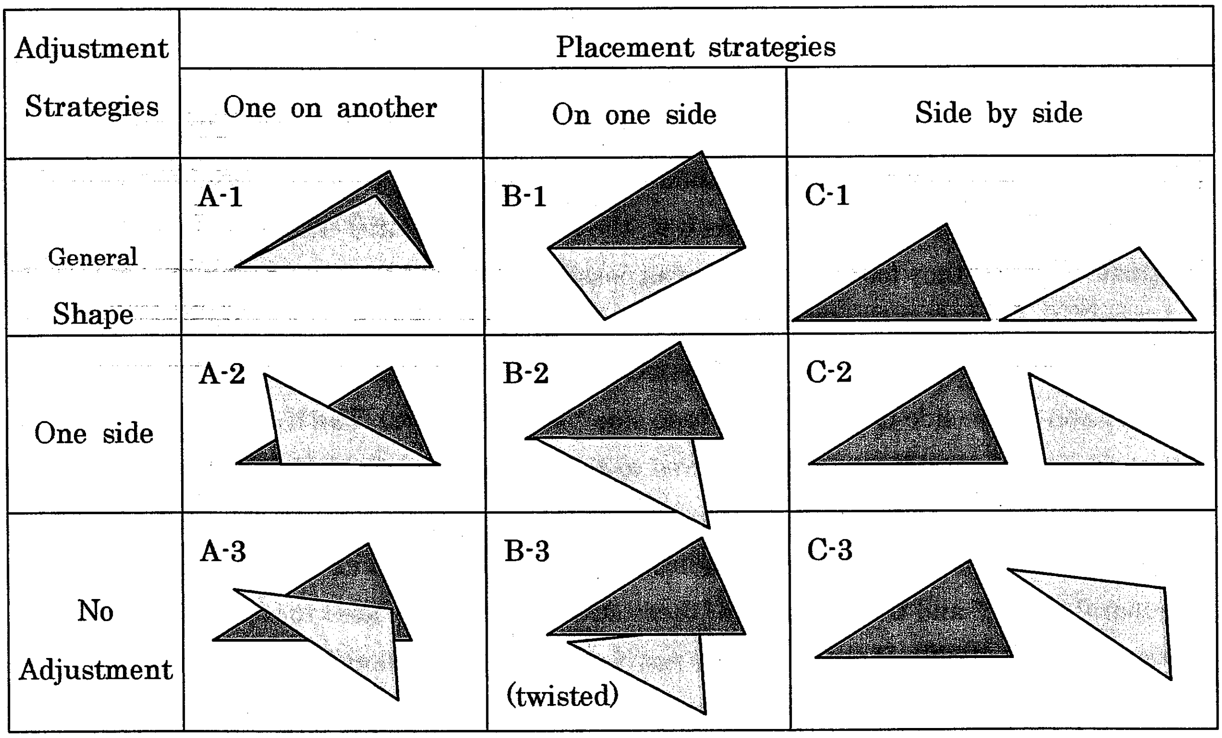

Figure 1. Schematic illustrations of each code of strategies

Method

Participants

Sixty-nine 3- to 6-year-olds (mean age = 4.75 years, range = 3.33 to 6.25) participated in the experiment. The children attended a nursery school in a middle-sized city in Japan. The children were all Japanese from predominantly middle-class families. They spoke Japanese. The numbers of male and female participants were 34 and 35 respectively.

Stimuli

Five pairs of geometric figures were used for stimuli in the experiment: one pair of circles (diameters: 10 cm vs. 12 cm), one pair of rectangles (height x length: 5 x 7 cm vs. 6 x 7 cm), and three pairs of triangles including a Congruent pair with the heights and lengths the same (base: 10 cm, one side: 6 cm, the angle between the base and the side: 0.91 rad), a Same-Base pair with the heights different and the bases the same (base: 10 cm, one side: 6 cm vs. 7cm, the angle between the base and the side: 0.91 rad), and a Same-Height pair with the heights the same and the bases different (base: 10 cm vs. 11cm, one side: 6 cm, the angle between the base and the side: 0.91 rad). The figures were made of cardboard. Of the two geometric figures in each pair, one was red and the other was green. The sizes of the figures were decided so that young children could handle the figures easily.

Tasks

There were three types of tasks: a Size comparison task, a Superimposition task, and a Choice task.

Size Comparison task. The Size Comparison task consisted of trials for the five pairs of stimuli. This task was to examine what strategies young children used spontaneously to make judgment for the relative sizes of geometric figures.

On a trial of the Size Comparison task, an experimenter sat at a table in front of a child and handed a pair of figures directly to the child. Then, the experimenter asked the child, "Here are two things, a red one and a green one. Are these two things the same size or different sizes?" The child was allowed to handle the pair freely. If the child responded that they were the same size, then the experimenter proceeded to the next trial. If the child responded that they had different sizes, then the experimenter asked the child, "Which is bigger, the red one or the green one?" The procedure was repeated for each of the five trials. The order of the five trials was randomized for each child.

Superimposition task. The Superimposition task was to examine whether or not young children could judge the relative sizes if an experimenter placed one figure on another and adjusted the figures by two dimensions. The task consisted of three trials for the three pairs of triangles (i.e., a Congruent pair, a Same-Base pair, and a Same-Height pair). On each trial, an experimenter showed a pair of triangles to a child, and placed the smaller triangle on the bigger one with the base toward the child. Then the child was asked to judge the relative sizes of the triangles. The experimenter asked the child, "Are these two things the same size or different sizes?" If the child responded that they were the same size, then the experimenter proceeded to the next trial. If the child responded that they had different sizes, then the experimenter asked the child again, "Which is bigger, the red one or the green one?"

Choice task. The Choice task was to examine how children would evaluate the placement strategy of superimposition and the adjustment strategy based on two dimensions. This task consisted of five trials for two placement sets and three adjustment sets. One placement set included three Same-Base pairs of triangles. Each of these three pairs was arranged in the same way as A-1, B-1, and C-1 in Figure 1, respectively. Another placement set included three Same-Height pairs of triangles arranged also in the same way as A-1, B-1, and C-1 in Figure 1. Each of the three adjustment sets included three Same-Base pairs of triangles: (a) In one set, three pairs were arranged in the same way as A-1, A-2, and A-3, (b) in a second set, three pairs were arranged in the same way as B-1, B-2, and B-3; and (c) in a third set, three pairs were arranged in the same way as C-1, C-2, and C-3. On a trial of the strategy choice task, an experimenter showed a pair of triangles to a child and said, "We want to know whether these two things are the same size or different sizes. There are three ways we can compare the sizes." Then the experimenter demonstrated three different strategies one by one, and placed in front of the child three pairs of triangles arranged in the same ways as demonstrated. The child was asked, "Which is the best way to compare the sizes of two things?" The child just selected one strategy, and the experimenter started the next trial. The procedure was repeated on each trial for the five sets of triangles. The order of demonstrations of the three different strategies, the positions of the three pairs in front of a child, and the order of the five sets were randomized for each child.

Procedure

There were two conditions under which the three types of tasks were administered. In one condition (SC condition), children received the Size Comparison task first, then the Choice task, and finally the Superimposition task. In the other condition (CS condition), children received the Choice task first, then the Size Comparison task, and finally the Superimposition task. Two conditions were used, because performance on either the Choice task or the Size Comparison task might influence children's spontaneous use or selection of strategies on the other task. Half of the children were randomly assigned to one of the two conditions. To make a control for the preference for a particular color, half of the children received one of the two stimulus sets whose figure colors (i.e., red and green) were reversed for all of the pairs. The experiment was conducted in a quiet room of the nursery school. Experimenters were Japanese. It took approximately 15-20 minutes for a child to complete all of the tasks.

Data coding

Experimenters recorded children's responses to the tasks. All of the responses were also videotaped. The responses to the Size Comparison task and the Superimposition task were coded as correct or incorrect. On a trial for figures of different sizes, the response was regarded as correct only if children were accurate in their answer to both questions: (a) whether the two figures were the same size or different sizes, and (b) which figure was larger.

Children's strategies for the Size Comparison task were also coded in terms of placement and adjustment. Figure 1 shows schematic examples of each code of the placement and the adjustment. Placement referred to the arrangement of two figures. Placement strategies were coded as "one on another," "on one side," or "side by side." First, when a child placed one figure on the other, the strategy was coded as "one on another." Second, when a child aligned two figures with the sides touching, the strategy was coded as "on one side." Finally, when a child placed two figures without a side of the figures touching, the strategy was coded as "side by side."

Adjustment referred to the directional manipulation of two figures. Adjustment strategies were coded as "general shape," "one side," or "no adjustment." First, when a child adjusted the figures so that they looked as similar as possible, the strategy was coded as "general shape." Second, when a child adjusted the figures so that they could be compared along the dimension of one side, the strategy was coded as "one side." Finally, when a child did not adjust the figures, the strategy was coded as "no adjustment."

For each of the five pairs of figures, placement strategies were coded, and for each of the four pairs except for the circle one, adjustment strategies were coded. When children used more than one placement strategy for one pair, all of the placement strategies were recorded, and an adjustment strategy was coded for each of the placement strategies.

To examine the reliability of the coding, a graduate student majoring in psychology watched the video tapes and coded all of the responses. The agreements on the coding between experimenters and the graduate student were 98.3% concerning the placement strategies and 95.9% concerning the adjustment strategies. In case of disagreements, one of the experimenters reviewed the video tapes to decide which was correct.

Results

First, we tested for effects related to gender and task order on correct judgments and use or selections of strategies in the Size Comparison task, the Superimposition task, and the Choice task. Then, we classified children into several groups based on their different responses to the Size Comparison task and the Superimposition task and examined these groups' response patterns and differences in ages. Finally, we examined the relationships between the groups and the evaluations of strategies in the Choice task. Because triangle pairs were used in common across the three tasks, only the data about three triangle pairs were used for analyses concerning the Size Comparison task.

The effects of gender and task order on correct judgments and strategy uses

We made initial analyses to determine whether or not there were differences related to gender and presentation order. *T* tests were carried out with the independent variable gender or task order (i.e., SC condition or CS condition) for the following dependent variables separately: (a) the number of correct responses for three triangle pairs in the Size Comparison task; (b) the number of uses of the "one on another" placement strategy for three triangle pairs in the Size Comparison task; (c) the number of uses of the "general shape" adjustment strategy for three triangle pairs in the Size Comparison task; (d) the number of correct responses in the Superimposition task; (e) the number of selections of the "one on another" placement strategy in the Choice task; and (f) the number of selections of the "general shape" adjustment strategy in the Choice task. There were no significant differences except for (e) related to presentation order (0.64 for SC condition and 0.33 for CS condition, $t(67) = 2.14$, $p < .05$). The reason for the difference was not clear, but the numbers of selections of "one on another" in the Choice task were generally very small, and the effects of presentation order on analyses in the following sections were negligible.

Response patterns of size comparison strategies

We used a cluster analysis by k-means method (STATISTICA Release 5.1) to classify children into several groups based on their patterns of responses to the Size Comparison task and the Superimposition task. The analysis was carried out using the following four variables: the number of correct responses, and the numbers of uses of the "one on another" placement strategy and the "general shape" adjustment strategy for three triangle pairs in the Size Comparison task, and the number of correct responses in the Superimposition task. The cluster analysis was exploratory, and we decided that four clusters obtained were most meaningful, because the four clusters had clearly different patterns in terms of the four variables and ages. Table 1 shows mean scores of the variables in children of the four clusters. Table 1 also shows mean ages of months, the first quartile (Q1), and the third quartile (Q3) of ages of children of the four clusters.

Table 1

Mean scores of the variables in children of the four clusters

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
n	10	18	23	18
Mean Months of Age	49	56	59	62
Q1	45	48	55	52
Q3	55	63	63	72
Superimposition Task				
Correct Response[1]				
M	0.80_a	2.56_b	2.83_{bc}	2.17_{bd}
SD	0.42	0.51	0.39	1.10
Size Comparison Task				
General Shape				
M	0.50_a	0.39_a	2.78_b	2.94_b
SD	0.85	0.50	0.80	0.94
One on Another				
M	0.20_a	0.11_a	0.13_a	2.50_b
SD	0.63	0.47	0.34	0.51
Correct Response[1]				
M	1.60_a	1.67_a	1.78	2.33_b
SD	0.52	0.59	0.67	0.91

Note. Values enclosed in parentheses represent standard deviations. Means in the same row that do not share subscripts differ at $p < .05$ in the Tukey's HSD tests.

[1]Score range: 0-3

In order to test differences among these four clusters, we conducted one-way ANOVAs for the four variables. First, concerning correct responses to the Superimposition task, the effect of Cluster was significant ($F(3, 65) = 22.03$, $p < .01$, $\eta^2 = .50$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Cluster 1 with Clusters 2, 3, and 4 and the difference between Clusters 3 and 4 were significant. Second, concerning "general shape" in the Size Comparison task, the effect of Cluster was significant ($F(3, 65) = 53.68, p < .01, \eta^2 = .71$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Clusters 1 and 2 with Clusters 3 and 4 were significant. Third, concerning "one on another" in the Size Comparison task, the effect of Cluster was significant ($F(3, 65) = 111.16, p < .01, \eta^2 = .84$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Clusters 1, 2, and 3 with Cluster 4 were significant. Fourth, concerning correct responses to the Size Comparison task, the effect of Cluster was significant ($F(3, 65) = 3.70, p < .05, \eta^2 = .15$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Clusters 1 and 2 with Cluster 4 were significant.

Table 2

The numbers of children who made correct responses or used each strategy on all the trials of Superimposition task and Size Comparison task

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
n	10	18	23	18
Superimposition Task				
Correct Response	0	10	19	11
Size Comparison Task				
Correct Response	0	0	2	10
Placement Strategies				
One on Another	0	0	0	9
On One Side	2	3	14	3
Side by Side	6	10	5	0
Adjustment Strategies				
General Shape	0	0	14	12
One Side	1	0	0	2
No Adjustment	6	10	0	0

Children in Cluster 1 made few correct responses to the Superimposition task, and rarely used the "one on another" placement strategy or the "general shape" adjustment strategy in the Size Comparison task. Next, children in Cluster 2 made correct responses to the Superimposition task, but rarely used the "one on another" placement strategy or the "general shape" adjustment strategy in the Size Comparison task. Children in Cluster 3 made correct responses to the Superimposition task and very often used the "general shape" adjustment strategy, but rarely used the "one on another" placement strategy in the Size Comparison task. Finally, children in Cluster 4 made correct responses to the Superimposition task, very often used both "one on another" placement strategy and "general shape" adjustment strategy in the Size Comparison task, and made more correct responses to the Size Comparison task.

Because of some bias of distributions toward none or all in the variables, we also made analyses based on responses of individual children. We counted children who made correct responses or used each of placement and adjustment strategies on all the three trials of Superimposition Task or Size Comparison Task. Note that the probability of making correct responses by chance on all the three trials was less than .05 (the probabilities for a Congruent pair, a Same-Base pair, and a Same-Height pair were .05, .25, and .25, respectively, and the multiplication of these probabilities was .031). Table 2 shows the numbers of those children among the four clusters. The results concerning correct responses, the "one on another" placement strategy, and the "general shape" adjustment strategy were congruent with those in Table 1: the proportions of children concerning correct responses to Superimposition task increased from Cluster 1 to Cluster 2 ($p = .003$ by Fisher's exact test), those concerning correct responses to Size Comparison task increased from Cluster 3 to Cluster 4 ($p = .001$ by Fisher's exact test), those concerning the "one on another" placement strategy increased from Cluster 3 to Cluster 4 ($p = .000$ by Fisher's exact test), and those concerning the "general shape" adjustment strategy increased from Cluster 2 to Cluster 3 ($p = .000$ by Fisher's exact test)[2]. Moreover, relatively many children were included in Clusters 1 and 2 who used the "side by side" placement strategy and the "no adjustment" strategy on all the three trials of Size Comparison task. On the other hand, many children in Cluster 3 used the "on one side" placement strategy on all the three trials of Size Comparison task.

Finally, in order to test differences in age among the four clusters, we conducted a one-way ANOVA for age. The effect of Cluster was significant ($F(3, 65) = 7.56$, $p < .01$, $\eta^2 = .26$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Clusters 1 with Clusters 3 and 4 and the difference between Clusters 2 and 4 were significant. Although the mean ages increased gradually from Cluster 1 to Cluster 4, the variations of age in clusters were fairly large and the ranges of ages of adjacent clusters overlapped with each other.

Evaluations of the strategies by children in each cluster

Finally, we examined how children in each cluster evaluated the "one on another" placement strategy and the "general shape" adjustment strategy in the Choice task. Table 3 shows the frequencies with which children in each cluster selected "one on another" as the best strategy on two trials for the placement sets and those with which children in each cluster selected "general shape" as the best strategy on three trials for the adjustment sets in the Choice task.

Table 3

Mean numbers of selection of "one on another" and "general shape" in the Choice task among children of each cluster

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
N	10	18	23	18
One on another[1]				
M	0.30	0.44	0.43	0.72
SD	0.67	0.51	0.51	0.75
General shape[2]				
M	1.10_a	1.11_a	1.48	2.22_b
SD	0.537	0.96	0.85	1.06

Note. Means in the same row that do not share subscripts differ at $p < .05$ in the Tukey's HSD tests.

[1]Score range: 0-2, [2]Score range: 0-3,

Table 4

The numbers of children who selected "one on another" or "general shape" on all the trials of the Choice task

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
N	10	18	23	18
One on another	1	0	0	3
General shape	0	1	3	10

In order to test differences among the four clusters, we conducted one-way ANOVAs for these frequencies. First, concerning the selections of "one on another," the effect of Cluster was not significant ($F(3, 65) = 0.32$, *ns.*). Second, concerning the selections of "general shape," the effect of Cluster was significant ($F(3, 65) = 5.54$, $p<.01$, $\eta^2 = .20$), and post hoc comparisons using Tukey's HSD test indicated that the differences of Clusters 1 and 2 with Cluster 4 were significant.

Furthermore, we made analyses based on responses of individual children. We counted children who selected "one on another" and "general shape" on all the trials of Choice task. Note that the probabilities of selecting "one on another" and "general shape" by chance on all the trials were .027 and .09, respectively. Table 4 shows the numbers of the children in each cluster. The overall proportions of children were significantly different by the four clusters for "general shape," but not for "one on another" (p = .000 and .061 for "general shape" and "one on another" respectively, calculated by the SPSS exact tests, Release 12.0.1J). The difference of proportions between Clusters 3 and 4 was statistically significant (p = .006 by Fisher's exact test) when the significance level was adjusted by the Ryan's procedure with α .05.

Discussion

The main question in the present study was how the strategies that children used spontaneously for comparing the sizes of two geometric figures would be related with the children's judgment and ages. In order to address this question, we classified children into four groups that made different patterns of responses to the Size Comparison task and the Superimposition task.

Children in Cluster 1 made few correct judgments about the relative areas of the figures placed on each other in the Superimposition task, whereas children in Cluster 2 made correct judgments. The results suggested that children in Cluster 2 should have the knowledge that a figure that includes the other has the larger area. However, these children rarely used the "one on another" placement strategy or the "general shape" adjustment strategy in the Size Comparison task. They did not apply the knowledge actively to the task.

It should be noted that even children in Cluster 1 made correct judgments in the Size Comparison task above the chance level ($t(9) = 3.87$, $p < .01$), although no individual children in Cluster 1 made correct responses reliably on all the trials. One explanation was that the dimension in which the two figures differed in terms of length (e.g., the height of a Same-Base pair) was salient to children and that they paid attention to that dimension, which happened to lead to a correct response. However, this possibility might be low, because children in Cluster 1 did not adjust the figures in terms of two dimensions. If the two figures were not adjusted in terms of two dimensions, both height and base of the two figures would look different in length and children could not decide which dimension was salient.

Another explanation was that children used contour length to judge the relative amounts of areas as infants do (Clearfied, & Mix, 1999; Mix et al, 2001). In the Size Comparison task the larger figure always had the lengthier contour. This explanation might be congruent with the fact that children in Cluster 1 did not make correct judgment for the relative sizes of the figures placed on each other. When two figures were placed on one other, the contour of one figure was concealed under the other figure, which might have hindered correct judgment based on contour lengths. On the other hand, children in Cluster 2 could rely on the knowledge that a figure that includes the other has the larger area, when they make judgment for the figures placed on each other. The possibility of these explanations should be tested in future study.

Children in Cluster 3 very often used the "general shape" adjustment strategy, but rarely used the "one on another" placement strategy in the Size Comparison task. The fact that children adjusted the figures in terms of two dimensions suggested that they started to pay attention to the two dimensions of the figures. However, the use of "general shape" did not lead to an increase in correct judgments, which suggested that they did not make judgment based on two dimensions of the figures.

Finally, children in Cluster 4 very often used both "one on another" placement strategy and "general shape" adjustment strategy in the Size Comparison task. In addition, they made more correct responses to the Size Comparison task. Only in this group children applied the knowledge that a figure that includes the other has the larger area, and made correct judgments based on the knowledge. The spontaneous uses of the "one on another" placement strategy seemed to be a developmental cornerstone for area judgment, which was also suggested by Yuzawa, Bart, & Yuzawa (2000). Yuzawa et al. indicated that children who used the strategy of placing one object on another spontaneously to compare the sizes of geometric figures internalized the procedure: They were able to choose a target figure that was equal to standard rectangles in area just by looking, which was difficult for children who did not use the strategy.

Concerning the relationship among the clusters of response patterns and age, children might not necessarily progress sequentially from Cluster 1 to Cluster 4. Although the mean ages increased gradually from Cluster 1 to Cluster 4, the ranges of ages of the clusters overlapped fairly with each other. It was indicated that there was considerable variation in the age at which children used new strategies for problems in domains such as arithmetic (e.g., Siegler, 1996). However, some sequential relations among knowledge and strategies that were characteristic of the clusters were evident. First, children who often used the "general shape" adjustment strategy or the "one on another" placement strategy almost always made correct judgments in the Superimposition task, but children who made correct judgments in the Superimposition task did not necessarily use the "general shape" adjustment strategy or the "one on another" placement strategy. These patterns suggested that the knowledge that a figure that includes the other has the larger area should be a basis for spontaneous uses of the strategies of "one on another" and "general shape." In fact, the differences in average ages between Cluster 1 and Clusters 3 and 4 were significant.

Second, children who often used the "one on another" placement strategy also often used the "general shape" adjustment strategy, but children who often used the "general shape" adjustment strategy did not necessarily use the "one on another" placement strategy. It was suggested that the use of "general shape" should precede the use of "one on another." This relation between strategies could be explained as follows: Children could pay attention to the overall shapes of the figures and adjust them by two dimensions without placing one figure on another; Whereas, if the figures were placed on each other, children would find it easy to adjust the figures by the overall shapes, because they just have to pay attention to the areas of one figure not covered by another figure. Therefore, the use of "general shape" might precede the use of "one on another."

An additional question in the present study was whether children could evaluate which strategy would be useful for comparing sizes. It was indicated that only children in Cluster 4 who used both "one on another" placement strategy and "general shape" adjustment strategy selected "general shape" as the best strategy. However, these children did not select "one on another" as the best strategy. The results suggested that the uses of strategies precede the metacognitive awareness that the strategies are useful. The results were congruent with the finding that young children have difficulty recognizing an advanced strategy to be legal (Baroody, 1984), but children's use of strategies seemed to improve their metacognitive judgment of the strategies.

Finally, the findings in the present study have some implications for education, because it is necessary for educators to provide children with a task that supports children's learning more sophisticated strategies. Educators are generally interested in children's making correct judgment, which was indicated to be related with using the "one on another" placement strategy for comparing relative areas. However, using the "one on another" placement strategy seemed to follow learning the "general shape" adjustment strategy and the knowledge that a figure that includes the other has the larger area. Therefore, educators should pay attention first to children's learning the "general shape" adjustment strategy and the knowledge of relative sizes of areas placed on each other.

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