Pushing Manipulation for Multiple Objects

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This paper discusses the manipulation of multiple objects by pushing. When multiple objects are placed on the flat floor and manipulated simultaneously by pushing, relative motion between two objects such as sliding and rotation may occur. The set of the pusher motion manipulating objects stably is obtained as an intersection of multiple sets of pusher motion where each set prevents the relative motion at one of the contact areas. Experimental results are also included to show the effectiveness of our idea. [DOI: 10.1115/1.2199857]

1 Introduction

Human beings can grasp and manipulate objects skillfully. Corresponding to the given task, they usually choose the most efficient strategy unconsciously. For example, let us imagine the case where there are many pencils randomly placed on a floor. Instead of picking them up one by one, a person usually cleans up the pencils by sweeping them together on the floor. This is because the person knows through his/her experience that handling multiple objects simultaneously leads to an efficient means of saving time.

For the pushing manipulation, there are several previous works [1–9], while all of them handling a single object. Can we apply these results to the case of multiple objects? If not, how should we cope with the issue of pushing manipulation for multiple objects? This work is motivated by these questions. When manipulating multiple objects by pushing, an object may sometimes slip on the edge of another object, otherwise the object may rotate around the vertex of another object. However, if relative motion occurs at one of the contact points, it becomes difficult to predict the motion of the object and becomes difficult to manipulate them. Knowing the difficulty of pushing manipulation for multiple objects, we obtain the set of pusher motion that keep the objects fixed relative to each other and to the pusher.

Now we will explain the basic idea of the approach proposed in this paper. Figure 1(*a*) shows the overview of the pushing manipulation of two objects. To apply the method for stably manipulating a single object by pushing, we regard the manipulation of two objects as the combination of two systems, i.e., the manipulation of the system composed of objects 1 and 2 by a pusher as shown in Fig. 1(*b*) and the manipulation of object 1 by the system composed of object 2 and the pusher as shown in Fig. 1(*c*). Now, to stably manipulate two objects, we show that the set of the pusher motion is obtained as an intersection of two sets of pusher motions for stably manipulating a single object. Also, to obtain the set of the pusher motion, we apply the effective center of friction (ECOF) [8] proposed by Lynch. To show the effectiveness of our proposed approach, we show some simulation and experimental results.

2 Relevant Work

As for the pushing manipulation, Mason and Salisbury [1] derived a simple rule for determining the direction of rotation of a pushed object where it depends only on the location of the COF. Lynch and Mason [9] discussed the controllability of pushing manipulation. Yoshikawa and Kurisu [3] and Lynch and Mason [4] proposed a method for estimating the COF experimentally by observing the motion of the pushed object. Mason and Brost [5], Jia and Erdmann [6], and Peshkin and Sanderson [7] discussed the bounds on the rotation rate of an object pushed by point contact. Goyal, Ruina, and Papadopoulos [10] studied the limit surface characterization of the friction force under the assumption of the known friction force distribution. Alexander and Maddocks [11] studied the possible motion of a pushed object when the friction force distribution is not known. Lynch [8] studied stable push where the pusher keeps contact with the object during the pushing manipulation. He also proposed a pushing manipulation without using visual feedback. The mechanics of pushing manipulation can also be seen in the textbooks such as [12].

As for the manipulation of multiple objects by a robotic hand, the authors studied the lifting of two objects [13] and the rolling manipulation [14]. Recently, Harada, Nishiyama, Murakami, and Kaneko [15] and Bernheisel and Lynch [16,17] studied the pushing manipulation of multiple objects.

3 Definitions

To simplify the discussion, the following assumptions are imposed:

(1) The objects are placed on the horizontal plane;
(2) all contact between two objects is line contact with friction in two-dimensional (2D). We consider 2D force vectors within the horizontal plane and one-dimensional (1D) moment perpendicular to the plane;
(3) the shape of an object is polygonal;
(4) the motion of the objects is quasi-static, and the forces acting on an object always balance;
(5) the dynamic and the static coefficient of friction acting between the object and the floor are the same;
(6) the friction force distribution between the object and the floor is known;
(7) The object does not rotate about the center of pressure between an object and the floor.

Assumption 6 means that the friction force distribution has been estimated experimentally,[1] and the change of the friction coefficient on the floor is not large. Neither the COF nor the ECOF can be defined unless Assumption 7 is satisfied. We note that we obtain a necessary condition on the pusher motion to keep the pushed objects stationary relative to each other and the pusher [9]. Because of the possible ambiguity of solutions of rigid-body me-

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[1]For example, in Ref. [3], they estimated the friction force distribution by using the least-squares method. Also, we show the extension to the case where the friction distribution is unknown in Sec. 4.5.

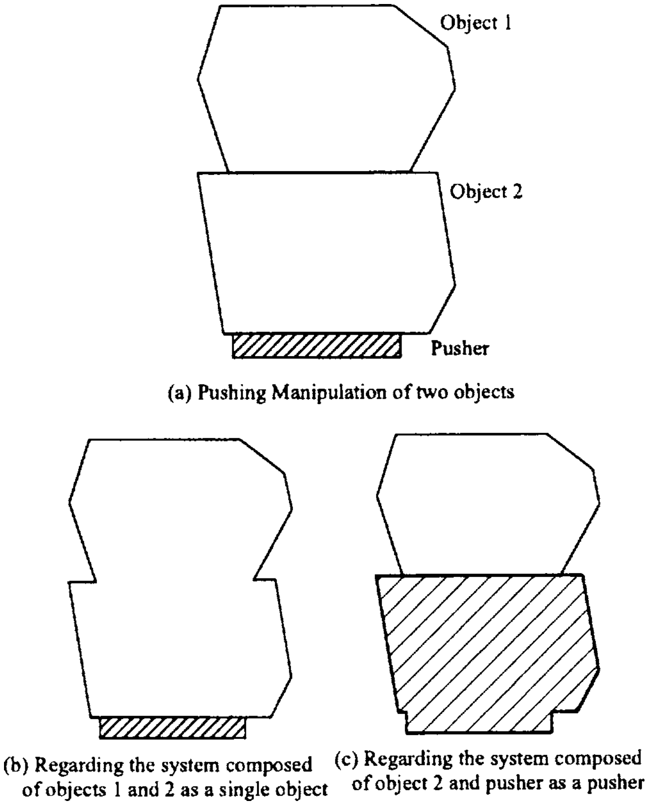


Fig. 1 Explanation of the proposed method

chanics problems with friction, however, this condition is not sufficient to guarantee that the objects will not slide or roll relative to each other and the pusher.

4 Formulation

In this section, we will obtain some conditions for the pusher motion to push a chain of n objects stably. We first show the mathematical condition where each edge contact can support forces through the edge and inside the friction cone. By discretizing the COR space, we then show the condition for the pusher motion to stably push two objects. We further show that the proposed method can by simplified by introducing some assumptions.

4.1 Model of the System. Figure 2 shows the model used in this research where two polygonal objects are pushed by a pusher whose translational/rotational velocities are controlled. Let $\boldsymbol{f}_C$ be the contact force which object 2 exerts on object 1 at the point $\boldsymbol{x}_C$

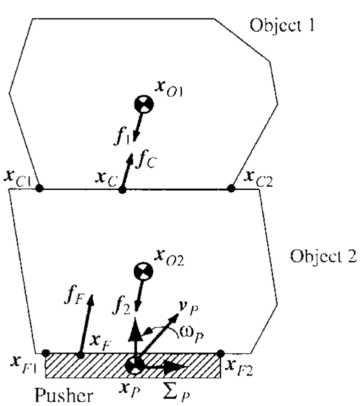


Fig. 2 Model of the system

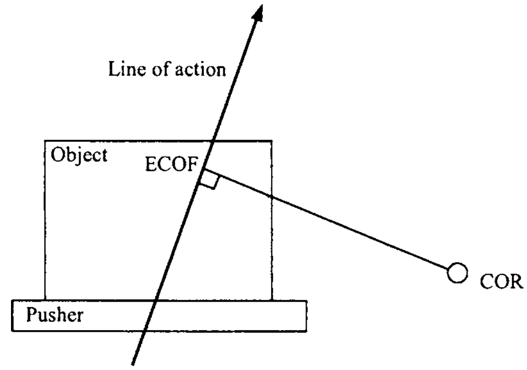


Fig. 3 Geometrical interpretation of the ECOF

where the line of action passes through. Also, let $\boldsymbol{f}_F$ be the contact forces which the pusher exerts on object 2 at the point $\boldsymbol{x}_F$ where the line of action passes through. The position vectors are defined by:

$$\boldsymbol{x}_C = \lambda_C \boldsymbol{x}_{C1} + (1-\lambda_C)\boldsymbol{x}_{C2}, \quad 0 \leqslant \lambda_C \leqslant 1 \tag{1}$$

$$\boldsymbol{x}_F = \lambda_F \boldsymbol{x}_{F1} + (1-\lambda_F)\boldsymbol{x}_{F2}, \quad 0 \leqslant \lambda_F \leqslant 1 \tag{2}$$

where $\boldsymbol{x}_{Cj}$ and $\boldsymbol{x}_{Fj}$ $(j=1,2)$ denote the position vectors at the edge of the contact segments between objects 1 and 2 and the pusher and object 2, respectively. Also, the inequalities $0 \leqslant \lambda_C \leqslant 1$ and $0 \leqslant \lambda_F \leqslant 1$ indicate that $\boldsymbol{x}_C$ and $\boldsymbol{x}_F$, respectively, are included in each edge.

We consider the friction force between each object and the floor acts through the effective center of friction (ECOF) proposed in [8]. The ECOF can be defined for general motion of the objects including rotation while the conventional COF cannot. As shown in Fig. 3, the point along the line of action nearest the COR becomes the ECOF, and the position of the ECOF is obtained in the Appendix . Note that the ECOF velocity is antiparallel to the frictional force.

4.2 Contact Forces for Maintaining Contact. As for the contact between two objects, the relative motion does not occur if the contact force is included strictly inside of the friction cone and if the line of action passes strictly inside of the contact segment. We consider extending this idea to the manipulation of two objects by pushing.

Let $\boldsymbol{x}_{Oi}$ and $\boldsymbol{f}_i$ $(i=1,2)$ be the ECOF position and the friction force acting between the object and the floor, respectively. The relationship of the forces acting on each object can be given by

$$\boldsymbol{w}_1 = \begin{bmatrix} \boldsymbol{f}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_C \\ (\boldsymbol{x}_C - \boldsymbol{x}_{O1}) \otimes \boldsymbol{f}_C \end{bmatrix} \tag{3}$$

$$\boldsymbol{w}_2 = \begin{bmatrix} \boldsymbol{f}_2 \\ 0 \end{bmatrix} - \begin{bmatrix} \boldsymbol{f}_C \\ (\boldsymbol{x}_C - \boldsymbol{x}_{O2}) \otimes \boldsymbol{f}_C \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_F \\ (\boldsymbol{x}_F - \boldsymbol{x}_{O2}) \otimes \boldsymbol{f}_F \end{bmatrix} \tag{4}$$

where $\otimes$ denotes the vector product in the 2D space defined by $\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right] \otimes \triangleq [-b \ a]$. $\boldsymbol{w}_i$ $(i=1,2)$ denotes the total wrench vector of the object i.

Substituting Eqs. (1) and (2) into Eqs. (3) and (4) and assuming the balance of forces $(\boldsymbol{w}_i = \boldsymbol{o})$, the contact forces and the position of the zero moment points can be obtained as follows:

$$\boldsymbol{f}_C = -\boldsymbol{f}_1 \tag{5}$$

$$\lambda_C = \frac{\boldsymbol{r}_{C21} \otimes \boldsymbol{f}_1}{(\boldsymbol{r}_{C21} - \boldsymbol{r}_{C11}) \otimes \boldsymbol{f}_1} \tag{6}$$

$$\boldsymbol{f}_F = -\boldsymbol{f}_1 - \boldsymbol{f}_2 \tag{7}$$

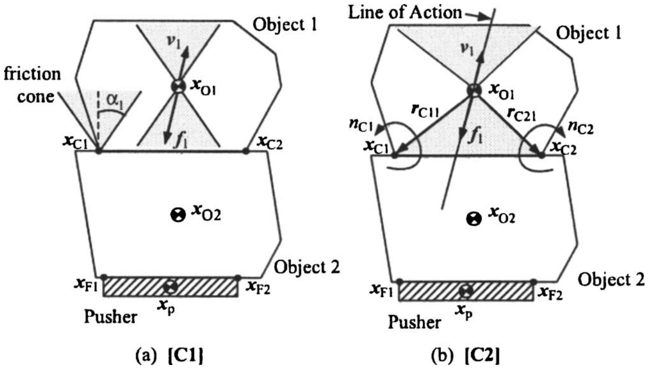


Fig. 4 Physical interpretation of [C1], [C2]

$$\lambda_F = \frac{(\boldsymbol{r}_{C21} - \boldsymbol{r}_{C22}) \otimes \boldsymbol{f}_1 + \boldsymbol{r}_{F22} \otimes (\boldsymbol{f}_1 + \boldsymbol{f}_2)}{(\boldsymbol{r}_{F22} - \boldsymbol{r}_{F12}) \otimes (\boldsymbol{f}_1 + \boldsymbol{f}_2)} \tag{8}$$

where $\boldsymbol{r}_{Cij}=\boldsymbol{x}_{Ci}-\boldsymbol{x}_{Oj}$ $(i,j=1,2)$ and $\boldsymbol{r}_{Fi2}=\boldsymbol{x}_{Fi}-\boldsymbol{x}_{O2}$ $(i=1,2)$. From these equations, the contact forces and the position of the zero moment points can be obtained uniquely as a function of the friction forces.

Now, we can find the following observations:

[C1] Let $\boldsymbol{v}_1$ be the velocity vector of object 1 at $\boldsymbol{x}_{O1}$. From Eq. (5), since $\boldsymbol{v}_1$ is antiparallel to $\boldsymbol{f}_1$, the slip may not occur between the objects 1 and 2 if the direction of $\boldsymbol{v}_1$ is included strictly inside of the friction cone between the objects 1 and 2 (Fig. 4(*a*)).

[C2] Let n_{C1} and n_{C2} be the moment which object 2 exerts to object 1 at each end of the contact segment. Rewriting Eq. (6) by using n_{C1} and n_{C2}, we obtain

$$0 < \frac{n_{C2}}{n_{C2} - n_{C1}} < 1 \tag{9}$$

As shown in Fig. 4(*b*), both $n_{C1}>0$ and $n_{C2}<0$ are satisfied if the line of action passes through strictly inside of the contact segment between the objects 1 and 2.

[C3] Let $\boldsymbol{v}_G$ be the velocity vector of objects at the ECOF of the system of two objects $\boldsymbol{x}_G$. From Eq. (7), since $\boldsymbol{v}_G$ is antiparallel to $\boldsymbol{f}_1+\boldsymbol{f}_2$, the slip may not occur between the object 2 and the pusher if the direction of $\boldsymbol{v}_G$ is included strictly inside of the friction cone between the object 2 and the pusher (Fig. 5(*a*)).

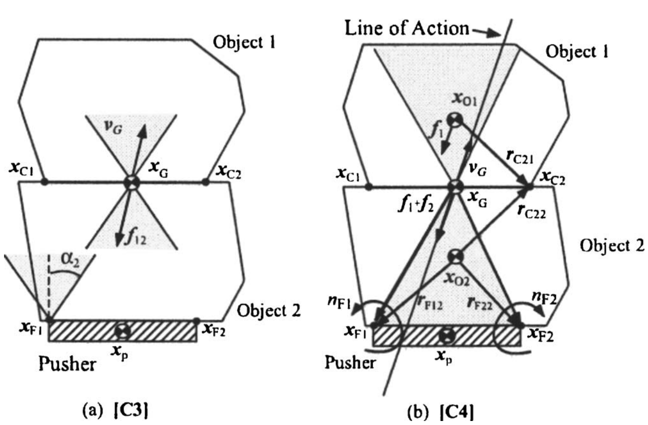


Fig. 5 Physical interpretation of [C3], [C4]

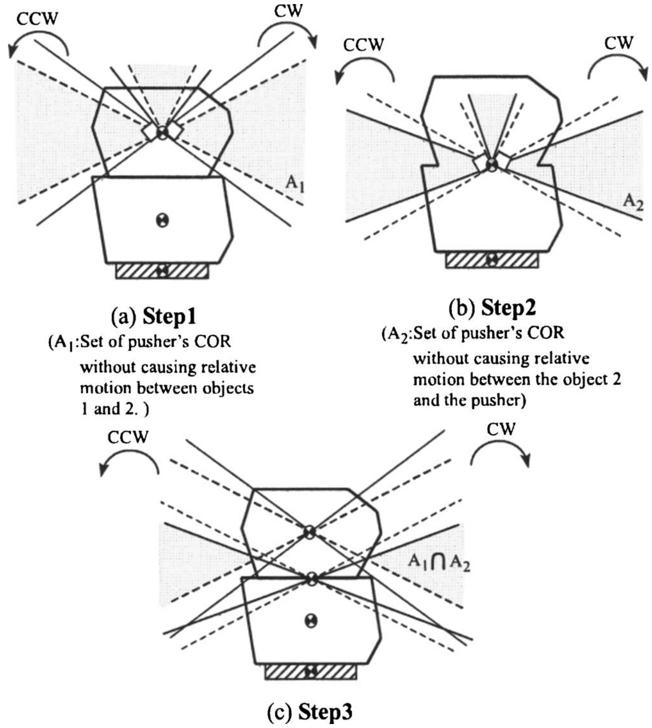


Fig. 6 Region of the center of rotation

[C4] Let n_{F1} and n_{F2} be the moment which the pusher exerts to the object 2 at each end of the contact segment. Rewriting Eq. (8) by using n_{F1} and n_{F2}, we obtain

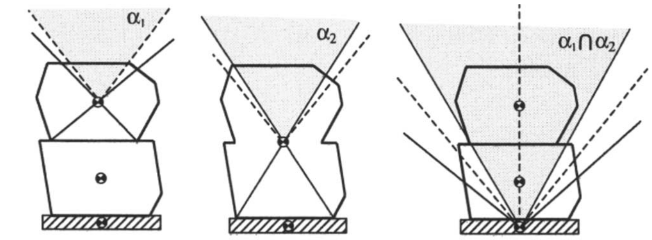
$$0 < \frac{n_{F2}}{n_{F2} - n_{F1}} < 1 \tag{10}$$

As shown in Fig. 5(*b*), both $n_{F1}>0$ and $n_{F2}<0$ are satisfied if the line of action passes through strictly inside of the contact segment between the objects 2 and the pusher.

4.3 The Set of Pusher Velocity. We now propose the algorithms to obtain the set of the stable pusher's motion. First, we show the method for obtaining the region stable pusher's COR. In this method, by discretizing the COR space, we check whether the objects are stably manipulated or not for the given position of the COR.

Algorithm 1—The set of pusher COR.
[**Step1**] Discretize the region on the floor where the COR might exist.
[**Step2**] Select one of the discretized area on the floor. For the Given position of the COR included in the area, obtain the position of the ECOF.
[**Step3**] Obtain $\boldsymbol{v}_1$ subject to **[C1]** and **[C2]**.
[**Step4**] Obtain $\boldsymbol{v}_G$ subject to **[C3]** and **[C4]**.
[**Step5**] From **Steps 3** and **4**, check whether or not the contact can be maintained without causing relative motion for a given COR. Repeat the above steps for all discretized area on the floor.

While the ECOF position is a function of the COR position, the COF position is not. Thus we can simplify the algorithm 1 in case where the difference between the COF and the ECOF is small enough. Their difference becomes smaller as the radius of rotation becomes larger. In the next algorithm, we do not need to discretize the COR space and do not need to calculate the ECOF position for the given COR position.



(a) **Step1**
(α1:Set of pusher's velocity without causing relative motion between objects 1 and 2.)

(b) **Step2**
(α2:Set of pusher's velocity without causing relative motion between the object 2 and the pusher)

(c) **Step3**

Fig. 7 Region of the pusher's velocity for translational motion

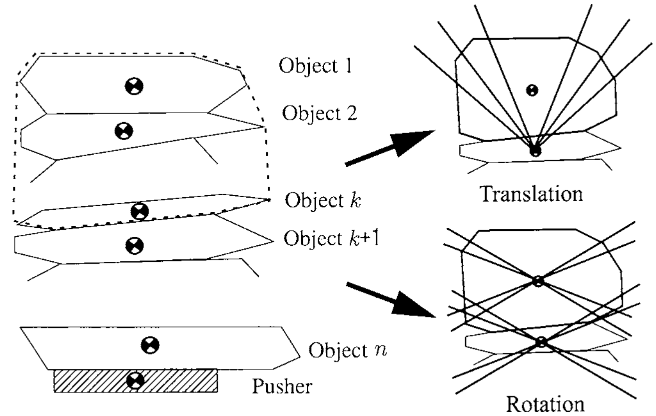


Fig. 8 Model composed of *n* objects

Algorithm 2—The set of pusher COR using COF.
[**Step1**] Obtain the set of $\boldsymbol{v}_1$ subject to **[C1]** and **[C2]**, and rotate it ±90 deg around the COF of object 1 (Fig. 6(*a*)).
[**Step2**] Obtain the set of $\boldsymbol{v}_G$ subject to **[C3]** and **[C4]**, and rotate it ±90 deg around the COF of the system of two objects (Fig. 6(*b*)).
[**Step3**] Obtain the common set of **Steps 1** and **2** (Fig. 6(*c*)).

Also, we can further simplify the algorithm if we consider only the case where the objects translate. When the objects translate, the COF position can be defined.

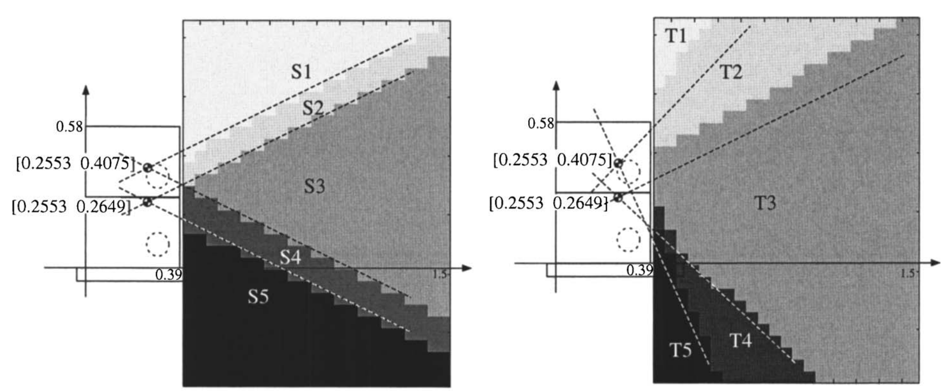
Algorithm 3—The set of pusher translational velocity.
[Step1] Obtain the set of $\boldsymbol{v}_1$ subject to **[C1]** and **[C2]** (Fig. 7(*a*)).
[Step2] Obtain the set of $\boldsymbol{v}_G$ subject to **[C3]** and **[C4]** (Fig. 7(*b*)).
[Step3] Obtain the common set of **Steps 1** and **2** (Fig. 7(*c*)).

We note that this algorithm is an extension of the cone of pure forces [2] to multiple objects.

4.3.1 Extension to n Objects. In the proposed algorithms for obtaining the set of the pusher's motion, assuming the system composed of two objects, we first consider the effect of the friction force of object 1 acting on the contact segment between objects 1 and 2, and then consider the effect of the friction force of the system composed of two objects acting on the contact segment between the pusher and object 2. We extend the algorithms to the system composed of n objects as shown in Fig. 8. Regarding the system composed of objects from 1 to k as a single object, we consider the effect of the friction force of the system of objects acting on the contact segment between objects k and $k+1$. Regarding the pusher as object $n+1$, we obtain the set of the pusher's motion as a common set of regions obtained for $k=1,\ldots,n$. Thus, we can obtain the set of the pusher's motion for systems composed of n objects where they are serially connected.

4.4 Discussion. Lynch et al. [2] also proposed an algorithm to determine the set of the COR named "STABLE" for a single object. They assume that the pressure distribution between the object and the floor is unknown and obtain the region of the COR for maintaining contact between the object and the pusher for all possible pressure distributions.

We further consider extending our algorithm to the case where the friction force distribution between the object and the floor is unknown. To obtain the region of the pusher's velocity and the COR, we consider taking the common space of regions, each of which is obtained by using the method for a single object. Therefore, we can apply, for example, "STABLE" to the pushing manipulation of multiple objects maintaining contact at each contact point. In such a case, we obtain the region of the pusher's velocity by taking the common space of regions, each of which is obtained by using the "STABLE."



(a) Region of the pusher COR with respect to slip ([C1] and [C3])
(S3:Set of COR without slip)

(b) Region of the pusher COR with respect to rotation([C2] and [C4])
(T3:Set of COR without rotation)

Fig. 9 Region of the center of rotation

Fig. 10 Experimental setup

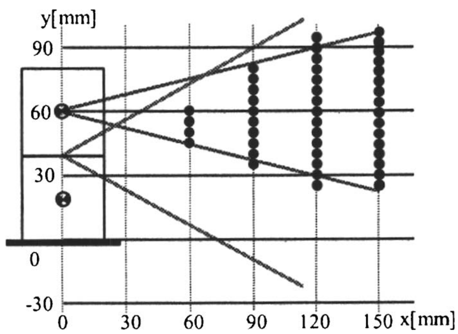


Fig. 11 Region of the COR obtained experimentally

5 Example

We first calculated the set of stable COR. The two objects have the same shape and the friction force distribution [3].[2] Figures 9(*a*) and 9(*b*) show the set of the COR obtained by using *Algorithm 1*. To compare *Algorithms 1* with *2*, we also show the result of *Algorithm 2* with the dashed lines. These sets can be obtained by simply drawing lines from the COF.

We then performed experiments on pushing manipulation. We first confirmed *Algorithm 2* by experiment. We used a rubber-covered plate as a pusher attached at the tip of the 3-DOF planar manipulator as a pusher, as shown in Fig. 10. We used a rectangular parallelepiped object made of steel as an object whose friction angle was $\alpha_1=14$ deg and $\alpha_2=29$ deg. Since the friction coefficient was considered to be uniform, the COF was assumed to be under the geometrical center. The results of experiment are shown in Fig. 11, where the position of the pusher's COR for maintaining contact is plotted against the region obtained by *Algorithm 2*. From the figure, we can see that the region of the COR without causing relative motion matches well with the experimental results.

Then, we performed the pushing manipulation of two objects. The snapshot of the experiment are shown in Fig. 12, where Fig. 12(*a*): Before starting the experiment, from Figs. 12(*b*)–12(*d*): Rotational motion of the objects, Fig. 12(*e*): Translational motion of the objects, and Fig. 12(*f*): At the desired position of the objects. We confirmed that the direction of translational velocity and the

[2]Due to static indeterminacy of the underconstrained support forces, there is no way to ensure that the actual objects have the same friction force distribution. We also note that the friction force distribution calculated in [3] includes negative support forces.

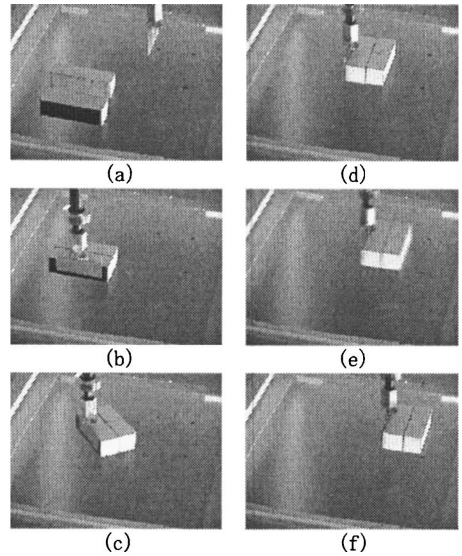


Fig. 12 Experimental results

position of COR are included in the sets obtained by Algorithms 1 and 2, respectively. We can see that the pushing manipulation of two objects is realized without causing relative motion at any contact segment.

6 Conclusion

In this paper, we discussed the pushing manipulation for multiple objects. We proposed three algorithms for obtaining the sets of the stable pusher motion. Also, when obtaining the sets of the pusher motion, we applied the ECOF. By experiments, we show that the two objects can be stably manipulated by pushing.

Since we assumed that the motion of the objects is quasi-static, the analysis of dynamics during the manipulation will be a future research topic.

Acknowledgment

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Appendix: Derivation of the ECOF

In this section, we will obtain the position of the ECOF by using the screw theory.

We first review the screw theory. Let f be the magnitude of the force acting along line l, and let n be the magnitude of the moment about l. Let $\boldsymbol{q} \in R^3$ be the unit direction vector of line l, and $\boldsymbol{p} \in R^3$ is the position vector along line l. Let $\boldsymbol{q}_0$ be given by

$$\boldsymbol{q}_0 = \boldsymbol{p} \times \boldsymbol{q} \tag{A1}$$

The ordered pair $(\boldsymbol{p}, \boldsymbol{q})$ is 6 Plücker coordinates. By using Plücker coordinates, the screw coordinates of the wrench are given by [12]

$$\boldsymbol{w} = f\boldsymbol{q} \tag{A2}$$

$$\boldsymbol{w}_0 = f\boldsymbol{q}_0 + fp\boldsymbol{q} \tag{A3}$$

where p is the pitch defined by $p=n/f$. Point $\boldsymbol{r}$ on the screw nearest the origin is given by

$$\boldsymbol{r} = \frac{\boldsymbol{w} \times \boldsymbol{w}_0}{\boldsymbol{w}^T \boldsymbol{w}} \tag{A4}$$

Then we consider the pushing manipulation of an object. Assuming that the origin of the base coordinate system be the COR, the screw coordinates of the wrench are given by

$$\boldsymbol{w} = \begin{bmatrix} 0 & \text{sgn}(\dot{\theta}) \\ -\text{sgn}(\dot{\theta}) & 0 \\ 0 & 0 \end{bmatrix} \int_R \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} \mu_i p(\boldsymbol{x}) dA \tag{A5}$$

$$\boldsymbol{w}_0 = \begin{bmatrix} 0 \\ 0 \\ -\text{sgn}(\dot{\theta}) \end{bmatrix} \int_R \boldsymbol{x}^T \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} \mu_i p(\boldsymbol{x}) dA \tag{A6}$$

where $\text{sgn}(\theta)$, $p(\boldsymbol{x})$, and μ_i denote the direction of rotation of the object, the pressure at the point $\boldsymbol{x}$, and the friction coefficient between the object and the floor, respectively. Substituting Eqs. (A5) and (A6) into Eq. (A4), we obtain the position of the ECOF.

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