Early growth of cosmological inhomogeneity at the horizon scale

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The weakly nonlinear evolution of collisionless matter in the expanding Universe is examined within general relativity. An approximate solution, whose perturbation scale is comparable to or larger than the Hubble radius, is obtained by series expansion with respect to the background scale factor. The solution represents that the early density contrast is controlled by the initial data at each Lagrangian position and the subsequent growth is modified by the presence of the magnetic part of the Weyl curvature, which is produced by the nonlinear coupling, even if it vanishes in the initial condition. The effect of the gravitomagnetic part on the dynamics is demonstrated in a concrete example. The dynamical role becomes less important as the perturbation scale increases beyond the horizon scale.

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I. INTRODUCTION

The evolution of density fluctuations is one of the important problems in cosmology. One approach is to use the linear perturbation around the uniform isotropic Universe. The linear theory has been extensively studied and is now well understood. The large-scale anisotropy of the cosmic background radiation (CBR) is of order 10^{-5}. The method is adequate during early times when the amplitude was very small. However, present local structure is inhomogeneous. The initial fluctuations became nonlinear at a certain stage.

Zel'dovich [1] examined the dynamics of an irrotational dust flow as a model of the large-scale structure. He formulated the motion under the Newtonian potential in the expanding Universe in terms of the Lagrangian coordinate. The model is valid before the pressure force becomes important, and can be used to connect the results of the linear stage to the nonlinear regime regarding the density contrast. The usefulness of the approximation is demonstrated, e.g., in Refs. [2,3]. The Lagrangian formalism is extended to include higher order corrections [4,5].

Recently, Matarrese *et al.* [6] proposed an approximate method to calculate the dynamics of dust in the expanding universe within the relativistic framework. If the so-called "magnetic" part of the Weyl curvature can be neglected, the dynamics can be described by the sixth-order ordinary differential equations. The motion of the dust is geodesic and the dynamics of the system can be determined only by the initial data at each Lagrangian position. That is, there is no force exerted to change the motion. This assumption may be too restrictive. They subsequently estimated the effect of the gravitomagnetic part in a qualitative way [7]. Croudace *et al.* [8] showed that nearly one-dimensional dust collapse is numerically unstable in the system where the gravitomagnetic part is approximately neglected. They also suggested that the instability is attributed to the neglect of the gravitomagnetic part. It is therefore important to clarify the role of the gravitomagnetic part in the dynamics. The post-Newtonian formalism is used and the relation between the Newtonian and the general relativistic dynamics is examined in Ref. [9]. The approximation is useful if the characteristic scale of the spatial variation is smaller than the Hubble radius. The opposite limit is also sometimes drawing attention, i.e., "the long wavelength limit" or "anti-Newtonian approximation" [10]. The relevant physical situation is as follows. Because of the large expansion during the inflationary epoch, the perturbation scale has been stretched out beyond the horizon. The perturbation subsequently has reentered the horizon and has started to form the Zel'dovich's pancake.

In this paper, the role of the gravitomagnetic part in the dynamics is examined in such a situation. It is adequate to approximate the fluid as collisionless dust, because the pressure force is negligible there. If the initial data are given by a scalar function, then the density perturbation can be determined by the function. The tensor modes corresponding to the degree of the gravitational wave vanish for the initial condition. The nonlinear coupling, however, produces the modes at a later time, which subsequently affect the density growth. This picture is general, but when and how this occurs depends on the initial data. We will consider a simple model and demonstrate the effect. In Sec. II, the basic equations governing the dynamics of the collisionless dust in the expanding universe are summarized. We consider a spatially flat Universe, i.e., an Einstein–de Sitter universe, as the background. An approximate solution is obtained by the power series of the scale factor in Sec. III. The solution is valid until the mildly nonlinear stage. The solution describes how the gravitomagnetic part is produced and affects the density evolution. The result is applied to a simple case in Sec. IV. Finally, the conclusion is given in Sec. V.

II. RELATIVISTIC DYNAMICS OF DUST

In this section, we shall summarize the basic equations which describe the evolution of dust. (See, e.g., Ref. [7].)

Following Ellis [11], equations can be written in terms of the observable fluid and geometric quantities: the mass density ρ, the volume expansion θ, the shear tensor σ_{ij}, and the electric and magnetic parts of the Weyl tensor, E_{ij} and H_{ij}, respectively. These tensors σ_{ij}, E_{ij}, and H_{ij} are symmetric traceless tensors. We shall use the normalized quantities Δ, ϑ, s^i_j, e^i_j, $\mathcal{H}^i_j$, which vanish in the background, as

$$\Delta = (\rho/\rho_0 - 1)/a, \quad \vartheta = (3t/2a)(\theta - \theta_0),$$
$$s^i_j = (3t/2a)\sigma^i_j, e^i_j = (3t^2/2a)E^i_j, \quad \mathcal{H}^i_j = (3t^2/2a)H^i_j, \quad (1)$$

where ρ_0, θ_0 are the background density and volume expansion, respectively, and a is the scale factor in the Einstein–de Sitter universe. They are given by

$$\rho_0 = 1/(6\pi G t^2), \quad \theta_0 = 2/t,$$

$$a = a_0(t/t_0)^{2/3} = B^{-1/3}t^{2/3}, \quad (2)$$

where $B = t_0^2/a_0^3$. Throughout this paper, we use comoving synchronous coordinates,

$$ds^2 = -dt^2 + a^2 h_{ij} dq^i dq^j. \quad (3)$$

The metric tensor h_{ij} evolves as

$$h'_{ij} = s^k_j h_{ik} + s^k_i h_{kj} + \tfrac{2}{3}\vartheta h_{ij}, \quad (4)$$

where the prime means a derivative with respect to a.

The full set of equations for the variables in Eq. (1) consists of two different kinds: propagation equations and constraint equations. The propagation equations are

$$\Delta' + \frac{1}{a}(\Delta + \vartheta) + \Delta\vartheta = 0, \quad (5)$$

$$\vartheta' + \frac{3}{2a}(\Delta + \vartheta) + \tfrac{1}{3}\vartheta^2 + s^i_j s^j_i = 0, \quad (6)$$

$$s^{i\prime}_j + \frac{3}{2a}(s^i_j + e^i_j) + \tfrac{2}{3}\vartheta s^i_j + s^i_k s^k_j - \tfrac{1}{3}\delta^i_j s^k_l s^l_k = 0, \quad (7)$$

$$e^{i\prime}_j + \frac{1}{a}(s^i_j + e^i_j) + \vartheta e^i_j + \Delta s^i_j - \tfrac{5}{2}e^i_k s^k_j - \tfrac{1}{2}s^i_k e^k_j + \delta^i_j e^k_l s^l_k - \frac{3t}{4a^2}h_{jk}(\epsilon^{kpq}\mathcal{H}^i_{p;q} + \epsilon^{ipq}\mathcal{H}^k_{p;q}) = 0, \quad (8)$$

$$\mathcal{H}^{i\prime}_j + \frac{1}{a}\mathcal{H}^i_j + \vartheta\mathcal{H}^i_j - \tfrac{5}{2}\mathcal{H}^i_k s^k_j - \tfrac{1}{2}s^i_k\mathcal{H}^k_j + \delta^i_j\mathcal{H}^k_l s^l_k + \frac{3t}{4a^2}h_{jk}(\epsilon^{kpq}e^i_{p;q} + \epsilon^{ipq}e^k_{p;q}) = 0, \quad (9)$$

where ϵ^{ijk} is the Levi-Civita tensor. The quantities in Eq. (1) have to satisfy the constraint equations

$$\mathcal{H}^i_j = \frac{t}{2a}h_{jk}(\epsilon^{kpq}s^i_{p;q} + \epsilon^{ipq}s^k_{p;q}), \quad (10)$$

$$s^i_{j;i} = \tfrac{2}{3}\vartheta_{,j}, \quad (11)$$

$$\mathcal{H}^i_{j;i} = \frac{2a^2}{3t}h_{jk}h_{lm}\epsilon^{kpq}s^l_p e^m_q, \quad (12)$$

$$e^i_{j;i} = \tfrac{2}{3}\Delta_{,j} - \frac{2a^2}{3t}h_{jk}h_{lm}\epsilon^{kpq}s^l_p\mathcal{H}^m_q. \quad (13)$$

If $\mathcal{H}^i_j = 0$, s^i_j and e^i_j can be diagonalized at the same time. The dynamical system is reduced to the six-order ordinary differential equations for the six variables $\Delta, \vartheta, s^1_1, s^2_2, e^1_1, e^2_2$ and $s^3_3 = -s^1_1 - s^2_2$, $e^3_3 = -e^1_1 - e^2_2$.

III. APPROXIMATE SOLUTION

In the linear regime, the metric for the scalar perturbations is given by a single gauge-invariant potential Φ_H [12]. In the comoving synchronous gauge, we have

$$h_{ij} = (1 + \tfrac{10}{3}\Phi_H)\delta_{ij} + \frac{3t_0^2 a}{a_0^3}\Phi_{H,ij}. \quad (14)$$

We may neglect the potential itself in practical problem, but not the spatial derivative of it [6,7]. With the definition, $\varphi(q^i) = -(3t_0^2/2a_0^3)\Phi_H$, we take, as the initial condition,

$$h_{ij} = \delta_{ij} - 2a\varphi_{ij}. \quad (15)$$

The indices of φ mean the derivative with respect to q^i and can be raised and lowered by η_{ij}, η^{ij}, where η_{ij} and η^{ij} are the metrics of three-dimensional Euclidean space, e.g., $\varphi^i_{jk} = \eta^{il}\partial_l\partial_j\partial_k\varphi$, which will be used later. Equations (5)–(13) admit the following solution by the expansion series near $a \sim 0$:

$$\Delta = \sum_{n=1}\Delta_{(n)}a^{n-1}, \quad \vartheta = \sum_{n=1}\vartheta_{(n)}a^{n-1},$$

$$s^i_j = \sum_{n=1}s^i_{(n)j}a^{n-1}, \quad e^i_j = \sum_{n=1}e^i_{(n)j}a^{n-1},$$

$$\mathcal{H}^i_j = \frac{t}{2a}\sum_{n=1}\mathcal{H}^i_{(n)j}a^{n-1}, \quad (16)$$

where the expansion coefficients such as $\Delta_{(n)}$ are functions of q^i. The growing and decaying solutions in the linear regime are given, e.g., by $\Delta \propto$ const, and $\Delta \propto a^{-5/2}$ in our convention (1). The only growing mode solutions are examined here. The order n in this paper does not refer to n-fold multiplication of φ, i.e., perturbation amplitude. The number n is related to the order of the gradient expansion method [13], i.e., the order n means $2n$ times spatial derivatives of the expansion coefficient. Equations are grouped into the terms with the same number of spatial derivatives in the gradient expansion method. The range of the validity of the expansion series will be discussed later.

Corresponding to the initial condition given by the scalar function $\varphi(q^i)$, the growing solution of the linearized equations can be given by

$$\Delta_{(1)} = -\vartheta_{(1)} = K, \quad -s^i_{(1)j} = e^i_{(1)j} = \varphi^i_j - \tfrac{1}{3}K\delta^i_j, \tag{17}$$
$$\mathcal{H}^i_{(1)j} = 0,$$

where $K = \mathrm{tr}[\varphi^i_j]$. The magnetic part of the Weyl tensor vanishes for the initial condition as expected, because the tensor perturbations are decoupled with the density contrast within the linearized equations. See, e.g., standard textbooks or Ref. [14] for the linearized equations of (5)–(13).

Now we proceed the expansion to examine the effect of gravitomagnetic part to the density contrast. The gravitomagnetic part at the second-order level, $\mathcal{H}^i_{(2)j}$, does not vanish in general. It affects the shear and the gravitoelectric part at the third-order level, $s^i_{(3)j}, e^i_{(3)j}$, and the density and the expansion at the fourth-order level, $\Delta_{(4)}, \vartheta_{(4)}$. We calculate these quantities influenced by the presence of gravitomagnetic part. The results are

$$\Delta_{(2)} = K^2 - \tfrac{4}{7}M, \tag{18}$$

$$\vartheta_{(2)} = -K^2 + \tfrac{8}{7}M, \tag{19}$$

$$s^i_{(2)j} = (\tfrac{1}{3}K^2 + \tfrac{10}{21}M)\delta^i_j - \tfrac{12}{7}K\varphi^i_j + \tfrac{5}{7}\varphi^i_k\varphi^k_j, \tag{20}$$

$$e^i_{(2)j} = -(\tfrac{1}{3}K^2 + \tfrac{26}{21}M)\delta^i_j + \tfrac{20}{7}K\varphi^i_j - \tfrac{13}{7}\varphi^i_k\varphi^k_j, \tag{21}$$

$$\mathcal{H}^i_{(2)j} = \tfrac{12}{7}[\bar{\epsilon}^{ipq}(\varphi^m_{mp}\varphi_{qj} + \varphi^q_{jm}\varphi^m_p) + \bar{\epsilon}_{jpq}(\varphi^{mp}_m\varphi^{iq} + \varphi^{iq}_m\varphi^{mp})], \tag{22}$$

$$\Delta_{(3)} = K^3 - \tfrac{8}{7}KM - \tfrac{10}{21}P, \tag{23}$$

$$\vartheta_{(3)} = -K^3 + \tfrac{12}{7}KM + \tfrac{10}{7}P, \tag{24}$$

$$s^i_{(3)j} = -\tfrac{1}{16}Bw^i_j + (\tfrac{1}{3}K^3 + \tfrac{11}{21}MK + \tfrac{92}{21}P)\delta^i_j - (\tfrac{46}{21}K^2 + \tfrac{25}{7}M)\varphi^i_j + \tfrac{39}{7}K\varphi^i_k\varphi^k_j - \tfrac{92}{21}\varphi^i_k\varphi^k_l\varphi^l_j, \tag{25}$$

$$e^i_{(3)j} = \tfrac{7}{48}Bw^i_j - (\tfrac{1}{3}K^3 + \tfrac{137}{63}KM + \tfrac{92}{9}P)\delta^i_j + (\tfrac{46}{9}K^2 + \tfrac{179}{21}M)\varphi^i_j - \tfrac{43}{3}K\varphi^i_k\varphi^k_j + \tfrac{92}{9}\varphi^i_k\varphi^k_l\varphi^l_j, \tag{26}$$

$$\Delta_{(4)} = K^4 - \tfrac{12}{7}K^2M + \tfrac{292}{693}M^2 - \tfrac{6460}{4851}PK + \tfrac{2}{77}BQ, \tag{27}$$

$$\vartheta_{(4)} = -K^4 + \tfrac{16}{7}K^2M - \tfrac{5008}{4851}M^2 + \tfrac{16600}{4851}PK - \tfrac{8}{77}BQ, \tag{28}$$

where $\bar{\epsilon}^{ijk}$ and $\bar{\epsilon}_{ijk}$ are the Levi-Civita tensor in the Euclidean space, and

$$M = (\varphi^i_i\varphi^j_j - \varphi^i_j\varphi^j_i)/2, \tag{29}$$

$$P = \det[\varphi^i_j], \tag{30}$$

$$Q = \varphi^i_j(\varphi^k_l\varphi^{jl}_{ik} - 4\varphi^j_k\varphi^{kl}_{il} + 4\varphi^j_{ik}\varphi^{kl}_l - 3\varphi^{kl}_i\varphi^j_{kl} - \varphi^k_{ik}\varphi^{jl}_l) + K(2\varphi^i_j\varphi^{jk}_{ik} - \varphi^j_{ij}\varphi^i_k + \varphi^i_{jk}\varphi^{jk}_i) + (K^2 - 4M)\varphi^{ij}_{ij}, \tag{31}$$

$$w^i_j = [\bar{\epsilon}^{ipq}\mathcal{H}_{(2)jp,q} + \bar{\epsilon}_{jpq}\mathcal{H}^{ip,q}_{(2)}]. \tag{32}$$

We can easily discriminate the terms coming from the gravitomagnetic part in these expressions. They contain the third- or fourth-order derivatives of φ. In other words, the system can be determined only by the initial deformation tensor φ^i_j if the gravitomagnetic term is neglected. The typical ratio of the gravitomagnetic term to other terms is $(B\partial^2_q\varphi_{ij})/(\varphi_{ij})^2 \sim t^2/(l^2a\varphi_{ij})$, where l is perturbation scale of φ_{ij}. The ratio of the wavelength of the perturbation to the horizon is therefore determined by the constant B in the approximate solution. Small values of B correspond to the perturbations beyond the horizon scale. In the limit of $B \to 0$, we can eliminate the gravitomagnetic terms in $s^i_{(3)j}, e^i_{(3)j}, \Delta_{(4)}, \vartheta_{(4)}$. As pointed out in Ref. [7], this limit corresponds to the physical situation in which the perturbation scale is much larger than the Hubble radius, $l^2 \gg t^2$, even though $a\varphi_{ij}$ is small. Spatial gradients play no role in this case. However, the dynamical role is not negligible for the perturbations with the subhorizon scale. For such perturbations, i.e., large B, a is limited to sufficiently small value for the convergence of the expansion series (16). That is, such perturbations behave as waves, so that our approximation breaks down soon and different treatment is necessary. We therefore assume that the perturbation scale is not smaller than the Hubble radius, and that B is smaller than a critical value.

IV. EFFECT OF THE MAGNETIC PART

Different degrees of freedom, i.e., information about higher derivatives of φ, are necessary to evaluate the gravitomagnetic term. We cannot evaluate the effect of the term unless the function is specified. One trivial example is $\varphi_{ijk} = \varphi_{ijkl} = 0$, in which the contribution from the gravitomagnetic part vanishes in $s^i_{(3)j}, e^i_{(3)j}, \Delta_{(4)}, \vartheta_{(4)}$. In order to demonstrate the effect of the gravitomagnetic term, we adopt a simple function near the origin:

$$\varphi = -X_1(q^1)X_2(q^2)X_3(q^3), \tag{33}$$

where $X_i(q^i) = 1 - \lambda_i(q^i)^2/2 + \lambda_i^2 d(q^i)^4/8 + \cdots$, and λ_i and d are positive constants. This potential gives the local density maximum at the origin and the matter near the origin collapses anisotropically with the rate λ_i along the axis q^i. The function X_i is the approximation of the cosine type, $\cos\sqrt{\lambda_i}q^i$ for $d = 1/3$, and of the Gaussian type, $\exp[-\frac{1}{2}\lambda_i(q^i)^2]$ for $d = 1$, near the origin. In this way, a class of the potential can be described in the form (33) near the density peak. Although it is possible to calculate the evolution at every Lagrangian point, the motion at the density peak is the most interesting. Therefore, we concentrate on the behavior at the Lagrangian coordinate $q^i = 0$ from now on. The evolution at the linear stage is given as

$$\Delta_{(1)} = -\vartheta_{(1)} = \lambda_1 + \lambda_2 + \lambda_3 > 0,$$

$$-s^i_{(1)j} = e^i_{(1)j} = \frac{1}{3}\mathrm{diag}(2\lambda_1 - \lambda_2 - \lambda_3, -\lambda_1 + 2\lambda_2 - \lambda_3, -\lambda_1 - \lambda_2 + 2\lambda_3). \quad (34)$$

If the distortion is one dimensional, the solution of Eqs. (5)–(13) can be written as the Szekeres solution [15]:

$$\Delta = -\vartheta = 3s^1_1 = 3s^2_2 = -\frac{3}{2}s^3_3 = -3e^1_1 = -3e^2_2 = \frac{3}{2}e^3_3 = \frac{f}{1-af}, \quad (35)$$

where f is a constant given at each Lagrangian position and the other components vanish. The caustic, i.e., the shell-crossing singularity, corresponds to $af = 1$. Croudace *et al.* [8] showed that the Szekeres solution is unstable in the system without the gravitomagnetic part. They showed that the deviation from the exact solution goes to minus infinity in one component of the electric part, e^1_1, and to plus infinity in e^2_2, as the solution approaches the singularity. They also conjectured that the gravitomagnetic part might suppress the instability. We will consider this problem in terms of the expansion solution. For small af, the function in (35) can be written as $\frac{f}{1-af} = f(1 + af + (af)^2 + \cdots)$, and we compare the analytic form with the expansion one. If completely planar initial condition is imposed, i.e., $\lambda_1 = \lambda_2 = 0$, then the gravitomagnetic part term always vanishes and Eqs. (18)–(28) reduce to the Szekeres solution with $f = \lambda_3$. The initial condition is relaxed to include the small deviation from the symmetric case, i.e., nearly one-dimensional collapse, $f = \lambda_3 \gg \lambda_1 > \lambda_2 = 0$. Keeping the lowest order of $\xi = \lambda_1/\lambda_3$, we have the approximate solution as

$$\begin{aligned}
s^1_{(1)1} &= \frac{f}{3}(1-2\xi), \quad s^2_{(1)2} = \frac{f}{3}(1+\xi),\\
s^1_{(2)1} &= \frac{f^2}{3}(1-\tfrac{12}{7}\xi), \quad s^2_{(2)2} = \frac{f^2}{3}(1+\tfrac{24}{7}\xi),\\
s^1_{(3)1} &= \frac{f^3}{3}[1-2\xi+\tfrac{9}{14}(1+3d)B\xi],\\
s^2_{(3)2} &= \frac{f^3}{3}[1+\tfrac{32}{7}\xi-\tfrac{9}{14}(1+3d)B\xi],\\
e^1_{(1)1} &= -\frac{f}{3}(1-2\xi), \quad e^2_{(1)2} = -\frac{f}{3}(1+\xi), \qquad (36)\\
e^1_{(2)1} &= -\frac{f^2}{3}(1-\tfrac{20}{7}\xi), \quad e^2_{(2)2} = -\frac{f^2}{3}(1+\tfrac{40}{7}\xi),\\
e^1_{(3)1} &= -\frac{f^3}{3}[1-\tfrac{122}{21}\xi+\tfrac{3}{2}(1+3d)B\xi],\\
e^2_{(3)2} &= -\frac{f^3}{3}[1+\tfrac{200}{21}\xi-\tfrac{3}{2}(1+3d)B\xi].
\end{aligned}$$

The shear and electric part tensors can be diagonalized in this model up to this order, and $e^3_{(n)3} = -e^1_{(n)1} - e^2_{(n)2}$, $s^3_{(n)3} = -s^1_{(n)1} - s^2_{(n)2}$. If a small ξ is allowed, the numerical coefficients in front of ξ increase with the order n and have definite sign in each series. When summing up the series, the terms with ξ cannot be ignored for later time. This tendency is significant in the electric part terms. The sign in front of ξ is opposite in $e^1_{(n)1}$ and $e^2_{(n)2}$, so that the deviation in these components increases toward different direction, i.e., to minus infinity and to plus infinity. In this way, we can understand the reason why the deviation from the planar symmetric case increases as a increases. The expansion solution up to the third order suggests that the planar collapse solution is unstable in the system without the gravitomagnetic part. One interesting thing is that the gravitomagnetic term has the opposite sign in $s^1_{(3)1}, s^2_{(3)2}, e^1_{(3)1}, e^2_{(3)2}$. Hence, the gravitomagnetic part will cancel or weaken the instability somewhat. This is true in the third-order level, but it is necessary to extend to the higher order or to simulate numerically in order to address this problem completely.

Next we consider the effect of the gravitomagnetic part on the evolution of density contrast. For the initial condition (33), the solutions at the origin are given by

$$\begin{aligned}
\Delta_{(2)} &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \tfrac{10}{7}(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1),\\
\Delta_{(3)} &= \lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \tfrac{13}{7}[(\lambda_1+\lambda_2)\lambda_3^2 + (\lambda_2+\lambda_3)\lambda_1^2 + (\lambda_3+\lambda_1)\lambda_2^2] + \tfrac{44}{21}\lambda_1\lambda_2\lambda_3,\\
\Delta_{(4)} &= \lambda_1^4 + \lambda_2^4 + \lambda_3^4 + \tfrac{16}{7}[(\lambda_1+\lambda_2)\lambda_3^3 + (\lambda_2+\lambda_3)\lambda_1^3 + (\lambda_3+\lambda_1)\lambda_2^3] + [\tfrac{2074}{693} + \tfrac{12}{77}B(1-d)](\lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2)\\
&\quad + [\tfrac{14260}{4851} - \tfrac{12}{77}B(1-d)](\lambda_1+\lambda_2+\lambda_3)\lambda_1\lambda_2\lambda_3 - \tfrac{8}{77}B[\lambda_1^2(\lambda_2-\lambda_3)^2 + \lambda_2^2(\lambda_3-\lambda_1)^2 + \lambda_3^2(\lambda_1-\lambda_2)^2]. \qquad (37)
\end{aligned}$$

All terms except the terms proportional to B in $\Delta_{(4)}$ are positive. As mentioned before, there is no force to stop the collapse in the system without the gravitomagnetic part. The gravitomagnetic part terms affect the evolution of $\Delta_{(4)}$ except the spherically symmetric case, $\lambda_1 = \lambda_2 = \lambda_3$, and the planar symmetric case, $\lambda_1 = \lambda_2 = 0$. The terms are not a definite sign, but the terms become negative and hence suppress the collapse in the Gaussian peak, $d = 1$. In this way, the gravitomagnetic parts in general affect the density contrast in the fourth-order level and the effect is significant for the perturbation with subhorizon scale, which corresponds to larger B.

V. CONCLUSION

In this paper, the dynamical role of the magnetic part of the Weyl tensor was examined in the model of the expanding Universe. We assumed that the density perturbation with long wavelength is given and the gravitomagnetic part vanishes at the linear stage. The gravitomagnetic part is produced in general, and affects the shear and gravitoelectric part at the third-order level, and the density contrast at the fourth-order level in the expansion series solution. The effect of the gravitomagnetic part is therefore crucial for the late stage.

We evaluated the effect at the Lagrangian position corresponding to the local density peak. The model shows that the planar collapse is unstable in the approximate system without the gravitomagnetic term, but that the gravitomagnetic term suppresses the growth of the instability at the lowest order. In this way, nearly planar collapse may be realized if the gravitomagnetic part is correctly taken into account. We have to pay attention to avoiding spurious solutions, if the gravitomagnetic part is approximately ignored.

Finally, Zel'dovich solution is also interesting from the viewpoint of the Lagrangian maps. The structure of the first collapsed object is related to the singularity theory of the maps [16]. The early evolution can be well described by the Zel'dovich model, but higher order corrections like the gravitomagnetic part are necessary for the later stages. If we add them to the Zel'dovich solution, the behavior near the singularity will be modified. Further consideration is therefore necessary to know the structure near the singularity.

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