Low scale seesaw model and lepton flavor violating rare B decays

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We study lepton flavor number violating rare B decays, $b \to s l_h^{\pm} l_l^{\mp}$, in a seesaw model with low scale singlet Majorana neutrinos motivated by the resonant leptogenesis scenario. The branching ratios of inclusive decays $b \to s l_h^{\pm} \bar{l}_l^{\mp}$ with two almost degenerate singlet neutrinos at TeV scale are investigated in detail. We find that there exists a class of seesaw model in which the branching fractions of $b \to s\tau\mu$ and $\tau \to \mu\gamma$ can be as large as 10^{-10} and 10^{-9} within the reach of Super B factories, respectively, without being in conflict with neutrino mixings and mass-squared difference of neutrinos from neutrino data, invisible decay width of Z, and the present limit of $\mathrm{Br}(\mu \to e\gamma)$. In the model, the lepton number asymmetries are vanishing.

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I. INTRODUCTION

Thanks to the worldwide collaborative endeavor for B factories and neutrino experiments, our understanding of the flavor mixing phenomena in the quark sector as well as in the lepton sector has been dramatically improved over the past few years. Even though discoveries of neutrino oscillations in solar, atmospheric, and reactor experiments gave robust evidence for the existence of nonzero neutrino masses, we do not yet understand mechanisms of how to generate the masses of neutrinos and why those masses are so small. The most attractive proposal to explain the smallness of neutrino masses is the seesaw mechanism [1–4] in which superheavy singlet particles are introduced. One of the virtues of the seesaw mechanism is to provide us with an elegant way to achieve the observed baryon asymmetry in our universe via the related leptogenesis [5]. However, the typical seesaw scale is of order 10^{10}–10^{14} GeV, which makes it impossible to probe the seesaw mechanism at collider experiments in a foreseeable future. Moreover, the leptogenesis at such a high energy scale meets a serious problem, the so-called gravitino problem, when it realizes in supersymmetric extensions of the standard model. Thus, it may be quite desirable to achieve the seesaw mechanism as well as the leptogenesis at a rather low energy scale. In these regards, scenarios of the resonant leptogenesis, in which singlet neutrinos with masses of order 1–10 TeV are introduced, have been recently proposed [6–13] Interestingly enough, the amplitudes of some flavor violating processes, which are highly suppressed in usual seesaw models with superheavy singlet neutrinos, may be enhanced with such low scale singlet neutrinos. Thus, it is worthwhile to examine how we can probe a seesaw model with low scale singlet neutrinos at collider experiments and to find some experimental evidence for the seesaw model via probe of lepton flavor violating processes.

In this paper, within the context of a low scale seesaw model based on $SU(2)_L \times U(1)$ [14], we study quark and lepton flavor violating (QLFV) rare decay processes such as $b \to s l_h^{\pm} l_l^{\mp}$ and $l_h \to l_l\gamma$, where l_h and l_l denote heavy and light charged leptons, respectively. After the quark flavor changing neutral currents (FCNC) $b \to s l^+ l^-$ had been discovered in B factories [15], it has been naturally expected that the next generation experiments could probe well the lepton flavor violating (LFV) processes [1,16–18], which may eventually uncover the mechanism for the generation of small neutrino masses and the leptogenesis as well. Therefore, the precise predictions for those processes based on well-motivated scenarios are very useful to find out if such scenarios can describe the nature correctly or not. Here we focus on the seesaw model with two almost degenerate singlet neutrinos at TeV scale. In the context of the seesaw mechanism, lowering the singlet mass scale leads to an undesirable enhancement of the light neutrino masses unless the Dirac neutrino couplings are guaranteed to be naturally small. However, as shown in [19,20], despite of the low mass scale of singlet Majorana neutrinos we can obtain light neutrino mass spectrum consistent with the current neutrino data by tuning the phase of the Yukawa-Dirac mass terms so that the two degenerate singlets contribute to the low energy effective Majorana mass terms destructively and the lepton number is approximately conserved. Interestingly enough, in such a scenario the sizable LFV processes and the suppression of the lepton number violation required in the effective Majorana mass for light neutrinos may coexist. We will show that the amplitudes of quark FCNC and LFV processes can be enhanced by considering some specific structure of Yukawa-Dirac and singlet Majorana mass

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matrices. We will obtain rather stringent constraints on QLFV processes by taking the constraints arising from the invisible decay of Z boson, neutrino mass-squared differences and lepton flavor mixings measured at neutrino experiments and the experimental constraints of LFV processes such as $l_h \to l_l \gamma$. About the leptogenesis of the model, we show that the lepton number asymmetries are vanishing in the present model.

The paper is organized as follows: In Sec. II, we present the lepton flavor mixings of seesaw models with an arbitrary number of singlet Majorana neutrinos. The analytical expressions for the branching fractions of the QLFV rare B decays are presented. In Sec. III, we discussed the constraint on the model from the Z invisible decay width and charged current lepton universality. Based on the constraints, the model independent bound on the branching fractions is obtained. In Sec. IV, we study the low mass scale scenarios for singlet neutrinos and build the models in which the quark FCNC and LFV processes are enhanced. We give the predictions for the QLFV processes by taking into account the various constraints. In Sec. V, we summarize the results and discuss the leptogenesis of the model.

II. LEPTON FLAVOR MIXING OF SEESAW MODEL AND QLFV RARE B DECAY

The lepton favor mixings of seesaw model are described in detail in Refs. [21,22]. Here we extend the model to the case with arbitrary number of singlet neutrinos and introduce a convenient decomposition of the Yukawa-Dirac mass term which is useful to our study. Let us start with a seesaw model described by following Lagrangian,

$$\mathcal{L}_m = -y_\nu^{ik} \bar{L}_i N_{R_k} \tilde{\phi} - y_l^i \bar{L}_i l_{R_i} \phi - \tfrac{1}{2} \bar{N}^c_{R_k} M_k N_{R_k} + \text{h.c.}, \tag{1}$$

and the neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}, \tag{2}$$

where M is a $N \times N$ real diagonal singlet Majorana neutrino mass matrix and m_D is a Dirac-Yukawa mass term. Then, $(3+N) \times (3+N)$ neutrino mass matrix can be diagonalized by the mixing matrix $\mathcal{V}$ as,

$$M_\nu^{\text{Diag}} = \mathcal{V}^\dagger M_\nu \mathcal{V}^*. \tag{3}$$

If the seesaw condition, i.e. $\frac{m_D}{M} < 1$, is satisfied, $(3+N) \times (3+N)$ unitary matrix $\mathcal{V}$ can be approximately parametrized as,

$$\mathcal{V} = \begin{pmatrix} V & m_D \frac{1}{M} \\ -\frac{1}{M} m_D^\dagger V & (1_N - \frac{1}{M} {m_D}^\dagger m_D \frac{1}{M})^{1/2} \end{pmatrix}, \tag{4}$$

where $\mathcal{V}$ satisfies unitarity to the order of $\frac{m_D^2}{M^2}$, and 1_N denotes an $N \times N$ unit matrix. Here the 3×3 submatrix V is not exactly unitary and can be written [22] as

$$V = \left(1 - m_D \frac{1}{M^2} m_D^\dagger\right)^{1/2} V_0, \tag{5}$$

where V_0 is a unitary matrix which diagonalizes the effective Majorana mass term,

$$m_{\text{eff}} = -m_D \frac{1}{M} m_D^T, \qquad V_0^\dagger m_{\text{eff}} V_0^* = \text{Diag}[n_1, n_2, n_3], \tag{6}$$

where n_i are the masses of three light neutrinos.

For convenience, we introduce the following parametrization for m_D [23],

$$\begin{aligned} m_D &= (\mathbf{m_{D1}}, \mathbf{m_{D1}}, \ldots, \mathbf{m_{DN}}) \\ &= (\mathbf{u_1 u_2} \ldots \mathbf{u_N}) \begin{pmatrix} m_{D1} & 0 & 0 & 0 \\ 0 & m_{D2} & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & m_{DN} \end{pmatrix}, \end{aligned} \tag{7}$$

where N unit vectors are introduced as

$$\mathbf{u}_{\mathbf{I}} = \frac{\mathbf{m_{DI}}}{m_{DI}}, \tag{8}$$

with $m_{DI} = |\mathbf{m_{DI}}|$. By introducing the parameters of mass dimension $X_I = \frac{m_{DI}^2}{M_I}$ $(I = 1 \sim N)$, m_{eff} can be written as

$$m_{\text{eff}} = -\sum_{i=1}^{N} \mathbf{u_i} X_i \mathbf{u_i^T}. \tag{9}$$

The charged current is associated with the $3 \times (3+N)$ submatrix of $\mathcal{V}$. The deviation from the unitarity of the 3×3 matrix V is given as

$$\begin{aligned} \sum_{a=1}^{3} V_{ia} V_{ja}^* &= \delta_{ij} - \sum_{I=1}^{N} \frac{X_I}{M_I} u_{iI} u_{jI}^*, \\ \sum_{i=e,\mu,\tau} V_{ia} V_{ib}^* &= \delta_{ab} - \sum_{I=1}^{N} (V_0^\dagger U)_{bI} \frac{X_I}{M_I} (U^\dagger V_0)_{Ia}, \end{aligned} \tag{10}$$

where $U = (\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_N})$.

Now we study the QLFV rare B decay processes $b \to s l_h^\pm l_l^\mp$ in the context of the seesaw model we consider. We denote l_h as a heavy lepton and l_l as a light lepton, and the possible combinations for $(l_h^\pm, l_l^\mp)$ are $(\tau^\pm, \mu^\mp)$, $(\mu^\pm, e^\mp)$, and $(\tau^\pm, e^\mp)$. By separating the contributions from the light neutrinos with masses $n_a (a = 1\text{–}3)$ and ones from the heavy neutrinos with masses M_{A-3} $(A = 4, 5, \ldots, 3+N)$, we can express the amplitude for $b \to s l_h^- l_l^+$ as

$$T = \frac{\sqrt{2}G_F\alpha_{\rm QED}}{4\pi s_W^2}(\bar{u}_s\gamma_\mu L u_b)(\bar{u}_h\gamma^\mu L v_l) \times \sum_{i=u\,c\,t} V_{ib}V_{is}^*\Big(\sum_{a=1}^{3} V_{ha}V_{la}^* E(x_i, y_a) + \sum_{A=4}^{(3+N)} \mathcal{V}_{hA}\mathcal{V}_{lA}^* E(x_i, y_{A-3})\Big), \tag{11}$$

where u_b, u_s, u_h, and v_l denote the spinor of bottom quark, strange quark, heavy lepton, and light antilepton, respectively, and $L = \frac{1-\gamma_5}{2}$, $x_i = \frac{m_i^2}{M_W^2}$, $y_a = \frac{n_a^2}{M_W^2}$, $y_A = \frac{M_A^2}{M_W^2}$. E is an Inami-Lim [24] function presented as

$$E(x, y) = -\Big(1 + \frac{xy}{4}\Big)\frac{f(x) - f(y)}{x - y} + \Big(1 - \frac{7xy}{4}\Big) \times \frac{xf'(x) - yf'(y)}{x - y}, \tag{12}$$

$$\text{with} \quad f(x) = \frac{x\log[x]}{x-1}.$$

We may neglect the up-quark loop contribution because of the smallness of $V_{ub}V_{us}^*$. By using the unitarity relations for leptonic and quark sectors

$$\sum_{a=1}^{3} V_{ha}V_{la}^* = -\sum_{A=4}^{3+N}\mathcal{V}_{hA}\mathcal{V}_{lA}^*, \qquad V_{tb}V_{ts}^* = -V_{cb}V_{cs}^*, \tag{13}$$

we can simplify Eq. (11) as follows:

$$T = \frac{\sqrt{2}G_F}{4\pi}\frac{\alpha_{\rm QED}}{s_W^2}(\bar{u}_s\gamma_\mu L u_b) \times (\bar{u}_h\gamma^\mu L v_l)V_{tb}V_{ts}^*\sum_{A=4}^{3+N}\mathcal{V}_{hA}\mathcal{V}_{lA}^*(\bar{E}(x_t, x_c, y_{A-3})), \tag{14}$$

In Eq. (14), we only keep the contributions of the top quark, charm quark, and heavy neutrinos, and

$$\bar{E}(x_t, x_c, y_{A-3}) = E(x_t, y_{A-3}) - E(x_t, 0) - E(x_c, y_{A-3}) + E(x_c, 0) \simeq x_t y_{A-3}\Big(\frac{3}{4(1-x_t)(1-y_{A-3})} - \frac{(x_t^2 - 8x_t + 4)\log(x_t)}{4(x_t-1)^2(x_t - y_{A-3})} - \frac{(y_{A-3}^2 - 8y_{A-3} + 4)\log(y_{A-3})}{4(y_{A-3}-1)^2(y_{A-3} - x_t)}\Big), \tag{15}$$

which agrees with the results in Refs. [14,25]. The branching fraction of $b \to s l_h^- l_l^+$ can be easily obtained as

$$\mathrm{Br}(b \to s l_h^- l_l^+) = \mathrm{Br}(b \to c\bar{\nu}_e e)\frac{|V_{tb}V_{ts}|^2}{|V_{cb}|^2}\frac{P(\frac{m_h}{m_b})}{P(\frac{m_c}{m_b})}\Big(\frac{\alpha_{\rm QED}}{8\pi s_W^2}\Big)^2\Big|\sum_{A=4}^{3+N}\mathcal{V}_{hA}\mathcal{V}_{lA}^*\bar{E}(x_t, x_c, y_{A-3})\Big|^2, \tag{16}$$

where the phase factor P is given by $P(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4\log(x)$ and m_h denotes the heavier lepton (l_h) mass. For numerical computation, we take $\mathrm{Br}(b \to c\bar{\nu}_e e) = 0.107$, $m_b = 4.75$ (GeV), $m_c = 1.25$ (GeV), $\alpha_{\rm QED} = \frac{1}{137}$, $m_\tau = 1.78$ (GeV), $m_\mu = 0.106$ (GeV), $m_W = 80.4$ (GeV). The branching fractions are then given as

$$\begin{aligned}\mathrm{Br}(b \to s\tau^- e^+) &= 1.0\times 10^{-7}|S(\tau, e)|^2,\\ \mathrm{Br}(b \to s\tau^-\mu^+) &= 1.0\times 10^{-7}|S(\tau, \mu)|^2,\\ \mathrm{Br}(b \to s\mu^- e^+) &= 2.8\times 10^{-7}|S(\mu, e)|^2,\end{aligned} \tag{17}$$

where $S(h, l)$ is the suppression factor defined by

$$S(h, l) = \sum_{A=4}^{3+N}\mathcal{V}_{hA}\mathcal{V}_{lA}^*\bar{E}(x_t, x_c, y_{A-3}) = \sum_{I=1}^{N}\frac{X_I}{M_I}\mathbf{u}_{hI}\mathbf{u}^*{}_{lI}\bar{E}(x_t, x_c, y_I). \tag{18}$$

III. THE CONSTRAINTS FROM INVISIBLE DECAY WIDTH OF Z BOSON AND CHARGED CURRENT UNIVERSALITY

As can be seen above, the QLFV processes depend on $\frac{X_I}{M_I} = \frac{m_{DI}^2}{M_I^2}$, $\mathbf{u_{hI}}\mathbf{u_{lI}^*}$, and M_I in $\bar{E}$. If $\frac{X_I}{M_I}$ is not very small, there will be a chance to detect the QLFV processes at B factories near future. First, we can roughly estimate the branching fractions of the QLFV processes. We notice that a model independent constraint on $\sum_{I=1}^{N}\frac{X_I}{M_I}$ can be obtained from the invisible decay width of Z and a charged currents lepton universality test [26–28] because $Z \to \nu_a\bar{\nu}_b$ coupling is suppressed compared with the standard model prediction as

$$\frac{g}{2\cos\theta_W}\sum_{a,b=1}^{3} Z_{ba}Z^\mu\bar{\nu}_b\gamma_\mu\frac{(1-\gamma_5)}{2}\nu_a, \tag{19}$$

where Z_{ba} is related to the violation of the unitarity of V,

$$Z_{ba} = \sum_{i=e\mu\tau} V_{ib}^* V_{ia} = \delta_{ab} - (V_0^\dagger U)_{bI}\frac{X_I}{M_I}(U^\dagger V_0)_{Ia}. \tag{20}$$

In the model under consideration, the effective number of light neutrinos N_ν is given by

$$N_\nu = \sum_{a,b=1}^{3} |Z_{ba}|^2 = 3 - 2\sum_{I=1}^{N} \frac{X_I}{M_I}. \tag{21}$$

From the experimental result [29] $N_\nu = 2.984 \pm 0.008$, we can obtain the following bound,

$$\sum_{I=1}^{N} \frac{X_I}{M_I} = 0.008 \pm 0.004, \tag{22}$$

If the bound is dominated by N degenerate singlet heavy neutrinos with $X_1 \sim X_2 \sim X_N$, $\frac{m_{DI}}{M_I} \simeq \frac{0.1}{\sqrt{N}}$, which is achieved in the case of low scale singlet neutrinos.

From the fact that the lepton flavor universality of charged current is generally violated in the seesaw model, one can obtain experimental bounds on $\epsilon_i \equiv \sum_{I=1}^{N} |u_{iI}|^2 \frac{X_I}{M_I}$ ($i = e, \mu, \tau$) from the lepton universality test of charged current interactions. The deviation from the universality is expressed in terms of the flavor dependent coupling g_i defined as

$$\frac{g_i^2}{g^2} = \sum_{a=1}^{3} |V_{ia}|^2 = 1 - \epsilon_i. \tag{23}$$

Using the definition of g_i above, for instance, the decay width of W into charged lepton l_i and neutrino is given by

$$\begin{aligned}\sum_{a=1}^{3} \Gamma[W \to l_i \bar{\nu}_a] &= \frac{g^2 M_W}{48\pi} \sum_{a=1}^{3} |V_{ia}|^2 \left(1 - \frac{m_i^2}{M_W^2}\right)^2 \left(1 + \frac{m_i^2}{2M_W^2}\right) \\ &= \frac{g_i^2 M_W}{48\pi} \left(1 - \frac{m_i^2}{M_W^2}\right)^2 \left(1 + \frac{m_i^2}{2M_W^2}\right),\end{aligned} \tag{24}$$

where we ignore the neutrino masses. The experimental bounds on $\epsilon_\mu - \epsilon_e$ and $\epsilon_\tau - \epsilon_e$ were obtained from the ratios of the branching fractions of W decays and also from the ratios of the branching fractions of $\tau \to \mu(e)\nu\bar{\nu}$ and $\mu \to e\nu\bar{\nu}$ decays. The constraints obtained from τ and μ leptonic decays (W leptonic decays) are summarized in Eq. (13) [Eq. (6)] of [28],

$$\begin{aligned}\epsilon_\mu - \epsilon_e &= 0.0002 \pm 0.0042(0.002 \pm 0.022), \\ \epsilon_\tau - \epsilon_e &= -0.0008 \pm 0.0044(-0.058 \pm 0.028).\end{aligned} \tag{25}$$

The off-diagonal elements of the correlation matrix of $\epsilon_\mu - \epsilon_e$ and $\epsilon_\tau - \epsilon_e$ are 0.51 and 0.44, respectively [28]. In Fig. 1, we show the constrains Eq. (25) on the plane $(\epsilon_\mu - \epsilon_e, \epsilon_\tau - \epsilon_e)$ by taking into account the correlations. In the figure, we also show the constraint obtained from Eq. (22) for the case that ϵ_e is vanishing. The constraint of Eq. (22) is written with ϵ_i as

$$(\epsilon_\mu - \epsilon_e) + (\epsilon_\tau - \epsilon_e) = (0.008 \pm 0.004) - 3\epsilon_e. \tag{26}$$

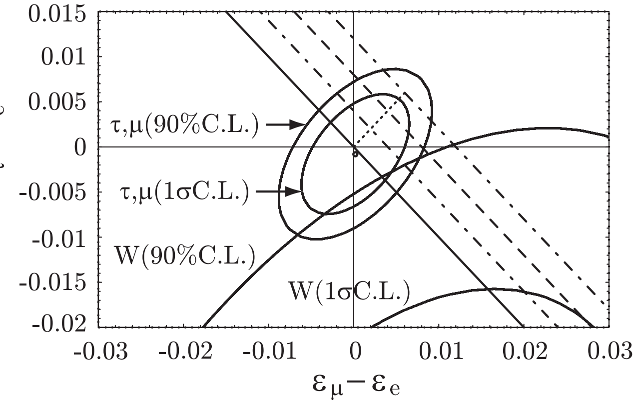

FIG. 1. The experimentally allowed region of $(\epsilon_\mu - \epsilon_e, \epsilon_\tau - \epsilon_e)$ is shown. τ, μ denote the constraints obtained from τ μ leptonic decays. W denotes the bound obtained from $W \to l_i\bar{\nu}$ decays. The invisible decay width constraints Eq. (26) is also shown for $\epsilon_e = 0$. The dashed line corresponds to $N_\nu = 2.984$. The solid straight line corresponds to $N_\nu = 3$, and the dash-dotted lines correspond to 1σ bound ($N_\nu = 2.984 \pm 0.008$). The dotted line corresponds to the prediction of the Class B model (NH) case. (See Sec. V in text.)

From Fig. 1, we can see that the constraints obtained from τ and μ leptonic decays are consistent with the Z invisible decay width constraint within 1σ CL under the assumption $\epsilon_e = 0$ while the constraints obtained from W decays are not consistent and $\epsilon_\tau - \epsilon_e < 0$ seems to be required in this case. We come back to the lepton universality constraints when we consider the specific structure of the Yukawa-Dirac mass term in the following sections.

The Inami-Lim function $\bar{E}(\frac{M_I^2}{m_W^2})$ is shown in Fig. 2. The typical values for the Inami-Lim function are $|\bar{E}| \sim 2.8$–8.1 for $M = 200$–2000 (GeV). Finally, from the fact that the factor $u_{hI}u_{lI}^*$ depends on the flavor structure of m_D, the following relation

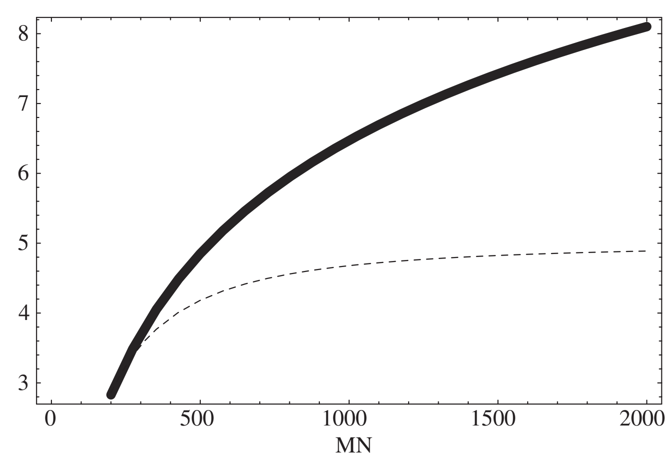

FIG. 2. Inami-Lim functions $-\bar{E}$ (thick solid line) and $10 \times F_2$ (dashed line) as functions of the singlet neutrino mass M_N (GeV).

$$2|u_{hI}u_{lI}| \leq |u_{hI}|^2 + |u_{lI}|^2 \leq \sum_{i=e,\mu,\tau} |u_{iI}|^2 = 1, \tag{27}$$

leads us to the constraint, $|u_{hI}u_{lI}|^2 \leq 0.25$. Then, the numerical value of $S(h, l)$ for the case with N degenerate M_i and X_i is

$$|S(h, l)|^2 \leq \left(N\frac{X}{M}\right)^2 \bar{E}\left(\frac{M^2}{M_W^2}\right)^2 0.25 \simeq 2 \times 10^{-4}\text{–}2 \times 10^{-3}, \tag{28}$$

where we denote $M_i \equiv M(i = 1 \sim N)$ and $X_i \sim X(i = 1 \sim N)$ and we use $N\frac{X}{M} \leq 0.012$. The upper bound of the branching fractions for $b \to s l_h^{\mp} l_l^{\pm}$ is roughly predicted to be 10^{-11}–10^{-10} for 200 (GeV) $< M <$ 2000 (GeV). The branching fraction for $l_h \to l_l \gamma$ is

$$\mathrm{Br}\,(l_h \to l_l \gamma) = \frac{\alpha^3}{256\pi^2 s_W^4} \frac{m_h^4}{M_W^4} \frac{m_h}{\Gamma_h} |G|^2. \tag{29}$$

Numerically computing the prefactors, the branching fractions are given by

$$\begin{aligned} \mathrm{Br}(\tau \to \mu\gamma) &= 5.4 \times 10^{-4} |G(\tau, \mu)|^2, \\ \mathrm{Br}(\tau \to e\gamma) &= 5.4 \times 10^{-4} |G(\tau, e)|^2, \\ \mathrm{Br}(\mu \to e\gamma) &= 3.1 \times 10^{-3} |G(\mu, e)|^2, \end{aligned} \tag{30}$$

where G is a suppression factor defined by

$$G = \sum_{\alpha=4}^{3+N} V_{l\alpha} V_{h\alpha}^* F_2(y_\alpha) = \sum_{I=1}^{N} u_{lI} u_{hI}^* \frac{X_I}{M_I} F_2(y_I), \tag{31}$$

where $y_I = \frac{M_I^2}{m_W^2}$ and F_2 is the Inami-Lim function, $F_2(y) = -\frac{2y^3+5y^2-y}{4(1-y)^3} - \frac{3y^3 \log y}{2(y-1)^4}$, as shown in Fig. 2.

IV. LEPTONIC FCNC AND QLFV RARE DECAYS WITH LOW MASS SCALE SINGLET MAJORANA NEUTRINOS

We now predict the branching fractions of the QLFV processes more concretely. As previously discussed, the branching fractions can be enhanced for rather large values of X_I/M_I. The large values of X_I/M_I are realized for low scale of M_I, which may not generally be consistent with neutrino data. The large values of X_I/M_I can be consistent with neutrino data when the contributions of X_I to m_{eff} in Eq. (9) are canceled. Such a cancellation can be achieved by taking two almost degenerate small $M_{1,2}$ and tuning the relative phase between $\mathbf{u_1}$ and $\mathbf{u_2}$ so that those two terms contribute to m_{eff} destructively while keeping X_3 so small that its contribution to m_{eff} is suppressed. Thus, we need some specific flavor structure of the Yukawa-Dirac mass term in order to obtain an enhancement of the branching fractions. Let us assume $X_1 \sim X_2 \gg X_3$ so as for $X_{12} \equiv X_1 - X_2$ and X_3 to be of order the light neutrino mass-squared differences $\sqrt{\Delta m_{\mathrm{sol}}^2}$ or $\sqrt{\Delta m_{\mathrm{atm}}^2}$. The relative phase of the Yukawa-Dirac mass term from two singlet neutrinos N_1 and N_2 is tuned as $\mathbf{u_2} = i\mathbf{u_1}$. Then, m_{eff} becomes

$$m_{\mathrm{eff}} = -(\mathbf{u_1}\mathbf{u_1^T} X_{12} + \mathbf{u_3}\mathbf{u_3^T} X_3). \tag{32}$$

We further assume the orthogonality of $\mathbf{u_1}$ and $\mathbf{u_3}$, i.e. $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} = 0$, so that $\mathbf{u_1}$ and $\mathbf{u_3}$ can be directly related to the Maki-Nakagawa-Sakata (MNS) matrix. Then, there exists a massless state due to the alignment of $\mathbf{u_1}$ and $\mathbf{u_2}$, which is assigned to $n_1 = 0$ for normal hierarchy and $n_3 = 0$ for inverted hierarchy. The other two masses are given by X_3 or $|X_{12}|$.

In Table I, we classify the assignment of mass spectrum (n_1, n_2, n_3) and the unitary part of the mixing matrix V_0, where p is a diagonal Majorana phase which is irrelevant to the QFLV and LFV processes and thus we omit it from now on. Notice that V_0 is identical to the MNS matrix if we neglect its deviation from V_{MNS}, $V_0 - V_{\mathrm{MNS}} = \mathcal{O}(\frac{m_D^2}{M^2})$. In fact, since the QLFV and LFV processes are already in the order of $\frac{m_D^2}{M^2}$ at the leading order, the difference can be safely ignored. The flavor dependence of the amplitudes for the QFLV and LFV processes is then extracted in terms of the

TABLE I. The assignment of mass spectrum and MNS matrix.

	(n_1, n_2, n_3)	$V_0 = (\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$	flavor dependence of $b \to s l_h^- l_l^+$
Normal	$(0\sqrt{\Delta m_{\mathrm{sol}}^2}, \sqrt{\Delta m_{\mathrm{atm}}^2 + \Delta m_{\mathrm{sol}}^2})$		
Class A	$(0, \lvert X_{12}\rvert, X_3)$	$(\mathbf{u_1^*} \times \mathbf{u_3^*}, \mathbf{u_1}, \mathbf{u_3})p$	$\mathbf{v_{h2}}\mathbf{v_{l2}^*}$
Class B	$(0, X_3, \lvert X_{12}\rvert)$	$(\mathbf{u_1^*} \times \mathbf{u_3^*}, \mathbf{u_3}, \mathbf{u_1})p$	$\mathbf{v_{h3}}\mathbf{v_{l3}^*}$
Inverted	$(\sqrt{\Delta m_{\mathrm{atm}}^2 - \Delta m_{\mathrm{sol}}^2}, \sqrt{\Delta m_{\mathrm{atm}}^2}, 0)$		
Class A	$(\lvert X_{12}\rvert, X_3, 0)$	$(\mathbf{u_1}, \mathbf{u_3}, \mathbf{u_1^*} \times \mathbf{u_3^*})p$	$\mathbf{v_{h1}}\mathbf{v_{l1}^*}$
Class B	$(X_3, \lvert X_{12}\rvert, 0)$	$(\mathbf{u_3}, \mathbf{u_1}, \mathbf{u_1^*} \times \mathbf{u_3^*})p$	$\mathbf{v_{h2}}\mathbf{v_{l2}^*}$

TABLE II. The combinations of $\mathbf{v}_{ij}$ relevant to the flavor dependence of QLFV and LFV decays.

$\mathbf{v}_{\mu 2}\mathbf{v}_{e2}^*$	$c_{13}s_{\rm sol}c_{\rm sol}c_{\rm atm}$	$0.33c_{13}$
$\mathbf{v}_{\tau 2}\mathbf{v}_{\mu 2}^*$	$-c_{\rm atm}s_{\rm atm}c_{\rm sol}^2$	-0.35
$\mathbf{v}_{\tau 2}\mathbf{v}_{e2}$	$-c_{13}s_{\rm atm}c_{\rm sol}s_{\rm sol}$	$-0.33c_{13}$
$\mathbf{v}_{\mu 3}\mathbf{v}_{e3}^*$	$c_{13}s_{\rm atm}s_{13}\exp(i\delta)$	$0.71s_{13}c_{13}\exp(i\delta)$
$\mathbf{v}_{\tau 3}\mathbf{v}_{\mu 3}^*$	$c_{13}^2s_{\rm atm}c_{\rm atm}$	$0.5c_{13}^2$
$\mathbf{v}_{\tau 3}\mathbf{v}_{e3}$	$c_{13}c_{\rm atm}s_{13}\exp(i\delta)$	$0.71s_{13}c_{13}\exp(i\delta)$
$\mathbf{v}_{\mu 1}\mathbf{v}_{e1}^*$	$-c_{13}c_{\rm sol}s_{\rm sol}c_{\rm atm}$	$-0.33c_{13}$
$\mathbf{v}_{\tau 1}\mathbf{v}_{\mu 1}^*$	$-s_{\rm sol}^2c_{\rm atm}s_{\rm atm}$	-0.16
$\mathbf{v}_{\tau 1}\mathbf{v}_{e1}^*$	$c_{13}c_{\rm sol}s_{\rm sol}s_{\rm atm}$	$0.33c_{13}$

mixing angles of the neutrino oscillation from $V_0 = V_{\rm MNS}$:

$$V_0 = \begin{pmatrix} c_{13}c_{\rm sol} & c_{13}s_{\rm sol} & 0 \\ -s_{\rm sol}c_{\rm atm} & c_{\rm atm}c_{\rm sol} & c_{13}s_{\rm atm} \\ s_{\rm sol}s_{\rm atm} & -s_{\rm atm}c_{\rm sol} & c_{13}c_{\rm atm} \end{pmatrix} + s_{13}\begin{pmatrix} 0 & 0 & \exp(-i\delta) \\ -c_{\rm sol}s_{\rm atm}\exp(i\delta) & -s_{\rm sol}s_{\rm atm}\exp(i\delta) & 0 \\ -c_{\rm sol}c_{\rm atm}\exp(i\delta) & -s_{\rm sol}c_{\rm atm}\exp(i\delta) & 0 \end{pmatrix}, \tag{33}$$

where we take $s_{\rm sol} = 0.56$, $c_{\rm sol} = 0.84$, and $s_{\rm atm} = c_{\rm atm} = \sqrt{1/2}$.

In Table II, we present the relevant combinations of $\mathbf{v}_{ij}$ which correspond to the flavor dependence shown in the fourth column of Table I. The value of s_{13} is very small and thus we ignore the terms of order $\mathcal{O}(s_{13}^2)$. Then, the suppression factor $S(h, l)$ is approximately given by

$$S(h, l) \simeq \left(\frac{X}{M_1}\bar{E}(x_t, x_c, y_1) + \frac{X}{M_2}\bar{E}(x_t, x_c, y_2)\right)\mathbf{v}_{\mathbf{h}\alpha}\mathbf{v}_{\mathbf{l}\alpha}^*, \tag{34}$$

where α denotes the index depending on the class and neutrino mass hierarchy, $X_1 \simeq X_2 \equiv X$. And the term proportional to $\frac{X_3}{M_3}$ is not relevant at all and thus ignored. By using Eq. (34), the ratios of the branching fractions are given by

$$\frac{\mathrm{Br}(b \to s\tau^\pm e^\mp)}{\mathrm{Br}(b \to s\tau^\pm \mu^\mp)} = \left|\frac{\mathbf{v}_{\tau\alpha}\mathbf{v}_{e\alpha}^*}{\mathbf{v}_{\tau\alpha}\mathbf{v}_{\mu\alpha}^*}\right|^2, \tag{35}$$

$$\frac{\mathrm{Br}(b \to s\mu^\pm e^\mp)}{\mathrm{Br}(b \to s\tau^\pm \mu^\mp)} = \frac{P(\frac{m_\mu}{m_b})}{P(\frac{m_\tau}{m_b})}\left|\frac{\mathbf{v}_{\mu\alpha}\mathbf{v}_{e\alpha}^*}{\mathbf{v}_{\tau\alpha}\mathbf{v}_{\mu\alpha}^*}\right|^2, \tag{36}$$

where $P(\frac{m_\mu}{m_b})$ and $P(\frac{m_\tau}{m_b})$ are phase space factors and $\frac{P(m_\mu/m_b)}{P(m_\tau/m_b)} = 2.74$.

In Table III, the numerical results of the ratios of the branching fractions given in Eqs. (35) and (36) are presented. It can be seen that in the Class B model for the normal hierarchical (NH) case, only $b \to s\tau^-\mu^+$ can be much larger than the other channel because of the absence of the suppression factor s_{13}^2 for $\tau\mu$ final states, while the branching fractions of the different channels in models except Class B for the NH case are within a factor of 10. Furthermore, as discussed in Ref. [14], there is strong correlation between QLFV processes and LFV radiative decays $l_h \to l_l\gamma$. Experimentally, there are stringent bounds as $\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [30], $\mathrm{Br}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$ [31], and $\mathrm{Br}(\tau \to e\gamma) < 1.1 \times 10^{-7}(3.9 \times 10^{-7})$ [32,33]. The bounds on the LFV

TABLE III. Ratios of the branching fractions of $b \to s l_h^- l_l^+$. $P_\mu \equiv P(\frac{m_\mu}{m_b})$ and $P_\tau \equiv P(\frac{m_\tau}{m_b})$.

	Class A NH (Class B IH)	Class B NH	Class A IH
$\frac{\mathrm{Br}(b\to s\tau^\pm e^\mp)}{\mathrm{Br}(b\to s\tau^\pm\mu^\mp)}$	$(\frac{s_{\rm atm}c_{\rm sol}s_{\rm sol}}{c_{\rm atm}s_{\rm atm}c_{\rm sol}^2})^2 = 0.89$	$(\frac{c_{\rm atm}s_{13}}{s_{\rm atm}c_{\rm atm}})^2 = 2.0s_{13}^2$	$(\frac{c_{\rm sol}s_{\rm sol}s_{\rm atm}}{s_{\rm sol}^2c_{\rm atm}s_{\rm atm}})^2 = 4.5$
$\frac{\mathrm{Br}(b\to s\mu^\pm e^\mp)}{\mathrm{Br}(b\to s\tau^\pm\mu^\mp)}$	$(\frac{s_{\rm sol}c_{\rm sol}c_{\rm atm}}{c_{\rm atm}s_{\rm atm}c_{\rm sol}^2})^2\frac{P_\mu}{P_\tau} = 2.4$	$(\frac{s_{\rm atm}s_{13}}{s_{\rm atm}c_{\rm atm}})^2\frac{P_\mu}{P_\tau} = 5.5s_{13}^2$	$(\frac{c_{\rm sol}s_{\rm sol}c_{\rm atm}}{s_{\rm sol}^2c_{\rm atm}s_{\rm atm}})^2\frac{P_\mu}{P_\tau} = 12.0$

TABLE IV. Ratios of the branching fractions of $l_h \to l_l\gamma$.

	Class A NH (Class B IH)	Class B NH	Class A IH
$\frac{\mathrm{Br}(\tau\to e\gamma)}{\mathrm{Br}(\tau\to\mu\gamma)}$	$(\frac{s_{\rm atm}c_{\rm sol}s_{\rm sol}}{c_{\rm atm}s_{\rm atm}c_{\rm sol}^2})^2 = 0.89$	$(\frac{c_{\rm atm}s_{13}}{s_{\rm atm}c_{\rm atm}})^2 = 2.0s_{13}^2$	$(\frac{c_{\rm sol}s_{\rm sol}s_{\rm atm}}{s_{\rm sol}^2c_{\rm atm}s_{\rm atm}})^2 = 4.5$
$\frac{\mathrm{Br}(\mu\to e\gamma)}{\mathrm{Br}(\tau\to\mu\gamma)}$	$(\frac{s_{\rm sol}c_{\rm sol}c_{\rm atm}}{c_{\rm atm}s_{\rm atm}c_{\rm sol}^2})^2\frac{\tau_\mu}{\tau_\tau}(\frac{m_\mu}{m_\tau})^5 = 5.0$	$(\frac{s_{\rm atm}s_{13}}{s_{\rm atm}c_{\rm atm}})^2\frac{\tau_\mu}{\tau_\tau}(\frac{m_\mu}{m_\tau})^5 = 11s_{13}^2$	$(\frac{c_{\rm sol}s_{\rm sol}c_{\rm atm}}{s_{\rm sol}^2c_{\rm atm}s_{\rm atm}})^2\frac{\tau_\mu}{\tau_\tau}(\frac{m_\mu}{m_\tau})^5 = 25$

processes stringently constrain the branching fractions for QLFV processes.

With $M_1 = M_2 = M \ll M_3$ and $X_1 \simeq X_2 = X \gg X_3$, for the Class A and Class B (IH) model, we predict

$$\mathrm{Br}(\mu \to e\gamma) = 3.1 \times 10^{-3}\left(\frac{2X}{M}\right)^2 (0.33)^2 F_2(y)^2 \geq 4.4 \times 10^{-10}, \tag{37}$$

where we have used $\frac{2X}{M} \geq 0.004$ and $M \geq 200$ (GeV). Therefore, the Class A and Class B (IH) models are excluded if we apply a 1σ lower bound as $3 - N_\nu \geq 0.008$. If the bound on $\frac{X}{M}$ in Eq. (22) is not taken into account, one can obtain from Eqs. (17) and (30)

$$\begin{aligned}
\mathrm{Br}(b \to s\tau^-\mu^+) &= 1.9 \times 10^{-4}\mathrm{Br}(\tau \to \mu\gamma)\frac{|S(\tau\mu)|^2}{|G(\tau\mu)|^2} \leq 1.3 \times 10^{-11}\left|\frac{S(\tau\mu)}{G(\tau\mu)}\right|^2,\\
\mathrm{Br}(b \to s\tau^- e^+) &= 1.9 \times 10^{-4}\mathrm{Br}(\tau \to e\gamma)\frac{|S(\tau e)|^2}{|G(\tau e)|^2} \leq 2.0 \times 10^{-11}\left|\frac{S(\tau e)}{G(\tau e)}\right|^2,\\
\mathrm{Br}(b \to s\mu^- e^+) &= 9.0 \times 10^{-5}\mathrm{Br}(\mu \to e\gamma)\frac{|S(\mu e)|^2}{|G(\mu e)|^2} \leq 1.1 \times 10^{-15}\left|\frac{S(\mu e)}{G(\mu e)}\right|^2.
\end{aligned} \tag{38}$$

While Eq. (38) depends neither on the mass spectrum of heavy Majorana neutrinos nor on the flavor structure of Yukawa-Dirac mass terms, the ratio of $|\frac{S}{G}|$ depends on the details of them. For the present case with $M_1 = M_2 = M \ll M_3$ and $X_1 = X_2 = X \gg X_3$, the ratio $|S(h, l)/G(h, l)|$ is simply given as,

$$\begin{aligned}
\left|\frac{S(h,l)}{G(h,l)}\right|^2 &= \left|\frac{\bar{E}(x_t, x_c, y)}{F_2(y)}\right|^2\\
&= 98(M = 200\ \mathrm{GeV})\text{–}285(M = 2000\ \mathrm{GeV}).
\end{aligned} \tag{39}$$

Therefore, the upper bounds on the branching fractions given in Eq. (38) are translated to

$$\begin{aligned}
\mathrm{Br}(b \to s\tau^- e^+) &\leq (1.3\text{–}3.6) \times 10^{-9},\\
\mathrm{Br}(b \to s\tau^-\mu^+) &\leq (2.0\text{–}5.0) \times 10^{-9},\\
\mathrm{Br}(b \to s\mu^- e^+) &\leq (1.1\text{–}3.0) \times 10^{-13}.
\end{aligned} \tag{40}$$

The range of the upper bounds corresponds to $M = 200$–2000 (GeV). We note that the upper bound of the branching fractions of QLFV processes for Class A (NH, IH) and Class B (IH) models are 10^{-14}–10^{-13} (see Table III) and the branching fractions for LFV processes is 10^{-13}–10^{-12} (see Table IV). As we have already shown in Eq. (37), the Class A model and Class B models for the IH case cannot satisfy the upper limit of the branching fraction of $\mu \to e\gamma$ and the 1σ constraint from the effective light neutrinos number N_ν simultaneously. This is because the former requires very small $\frac{X}{M}$, while the latter requires larger $\frac{X}{M}$. Below, we show that the Class B model for the NH case may satisfy both constraints. Furthermore, the model predicts the large branching fractions of $\tau \to \mu\gamma$ and $b \to s\tau\mu$ which are within the reach of near future Super B factories [34,35]. If we take into account the constraints coming from $\mathrm{Br}(\mu \to e\gamma)$ and the effective number of light neutrinos N_ν, we can have the parameter regions consistent with the present bounds only for the Class B model with the NH case. In this class, the stringent experimental limit on $\mathrm{Br}(\mu \to e\gamma)$ is not effective on $\mathrm{Br}(\tau \to \mu\gamma)$ and $\mathrm{Br}(b \to s\tau\mu)$, because the former process is proportional to s_{13}^2 and thus ignorable, but the latter processes are not suppressed by the factor.

In Fig. 3, we have shown the correlation of branching fractions between $\mu \to e\gamma$ and the other LFV and QLFV processes in the Class B model for the NH case. The

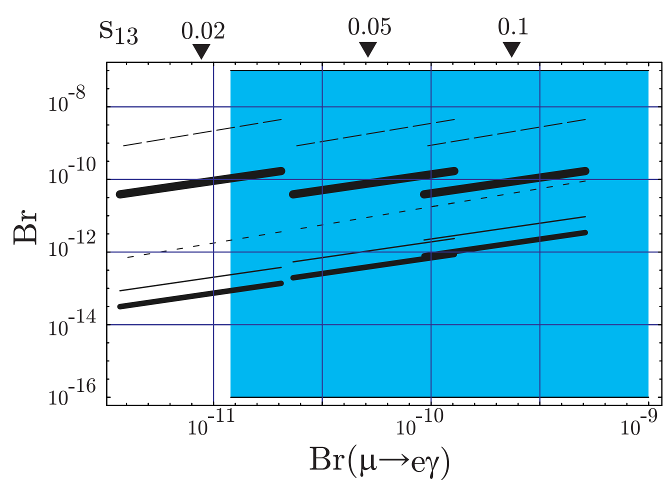

FIG. 3 (color online). Correlation between the branching fraction for $\mu \to e\gamma$ and the branching fractions for $b \to s\tau\mu$ (thick solid line) $b \to s\tau e$ (solid line) $b \to s\mu e$ (thin solid line), $\tau \to \mu\gamma$ (long dashed line) $\tau \to e\gamma$ (dashed line) for the Class B model with the NH case. From left to right, the lines correspond to $s_{13} = 0.02$, 0.05, 0.1, respectively. The shaded region is excluded by the current bound on $\mathrm{Br}(\mu \to e\gamma)$.

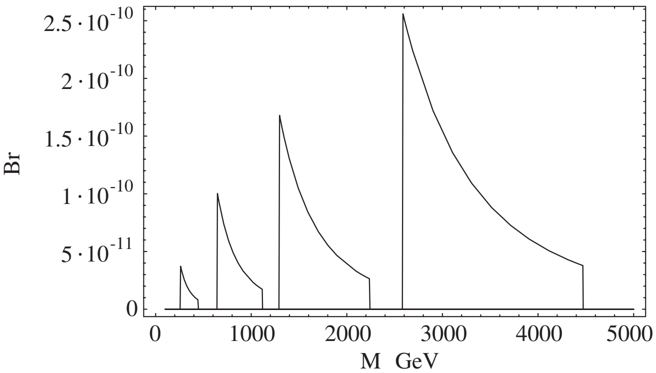

FIG. 4. $\mathrm{Br}(b \to s\tau\mu)$ vs M for Class B (NH). From left to right, the curves correspond to $m_{D1} = 20, 50, 100, 200$ (GeV), respectively.

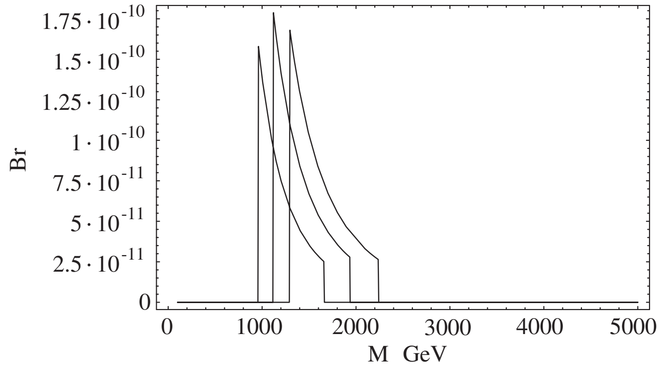

FIG. 6. $\mathrm{Br}(b \to s\tau\mu)$ vs M for Class B (NH). We fix $m_{D1} = 100$ (GeV). From right to left, the curves correspond to the ratio $R = \frac{M_2}{M}$ 1, 2, 10, respectively.

numerical results in Fig. 3 are obtained as follows: We first set $X_1 \sim X_2 = X$ and $M_2 = M_1 = M$. From the constraint given in Eq. (22), the allowed range of M is

$$\frac{m_{D1}}{\sqrt{\sigma_{\max}}}\sqrt{2} < M < \frac{m_{D1}}{\sqrt{\sigma_{\min}}}\sqrt{2}. \tag{41}$$

Equation (22) corresponds to $\sigma_{\max} = 0.012$ and $\sigma_{\min} = 0.004$. When we fix m_{D1}, the allowed range of M is determined. By varying M within the above range, we plot the correlation between $\mathrm{Br}(\mu \to e\gamma)$ and the other five QLFV and LFV branching fractions. Here, s_{13} is a free parameter and is chosen to be 0.02, 0.05, and 0.1. m_{D1} is chosen to be 100 (GeV). As can be seen in Fig. 3, the present upper limit on $\mathrm{Br}(\mu \to e\gamma)$ gives a very tight bound on s_{13}, typically smaller than 0.02. With this small s_{13}, $\tau \to e\gamma$ and $b \to s\tau(\mu)e$ are also severely suppressed. Only $b \to s\tau\mu$ and $\tau \to \mu\gamma$ are free from the suppression and the former branching fraction can be as large as 10^{-10} and the latter can be 10^{-9}. They are independent on small s_{13}.

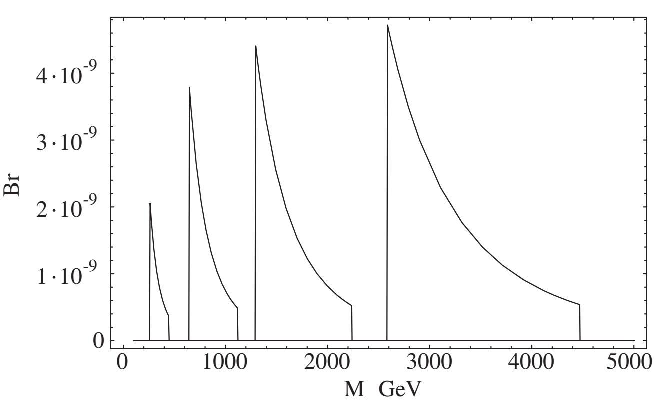

FIG. 5. $\mathrm{Br}(\tau \to \mu\gamma)$ vs M for Class B (NH). From left to right, the curves correspond to $m_{D1} = 20, 50, 100, 200$ (GeV), respectively.

We show in Fig. 4 and 5 the dependence of the branching fractions of $b \to s\tau\mu$ and $\tau \to \mu\gamma$ on m_{D1} and the heavy Majorana neutrino mass for the exact degenerate case, i.e. $M_1 = M_2 = M$. We fix m_{D1} to 20, 50, 100, 200 (GeV). Although the branching fractions become small as m_{D1} becomes small, the change of the branching fractions is within a factor 10. We also consider the nondegenerate case for Majorana neutrino masses ($M_1 \equiv M \neq M_2$) while keeping the degeneracy $X_1 \sim X_2 = X$. By setting $M_2 \equiv RM_1$ and $M_1 \equiv M$, the dependence of the branching fraction $b \to s\tau\mu$ on the ratio R is studied. The allowed range of the lightest heavy Majorana neutrino mass M of Eq. (41) is modified as $\frac{m_{D1}}{\sqrt{\sigma_{\max}}}\sqrt{1+\frac{1}{R}} < M < \frac{m_{D1}}{\sqrt{\sigma_{\min}}}\sqrt{1+\frac{1}{R}}$. From Fig. 6, we find that the lower limits of M become smaller as R becomes larger. However, the branching fraction does not change so significantly.

V. SUMMARY AND DISCUSSION

As shown, the contributions of the singlet Majorana neutrinos to QLFV and LFV decays can be significant in the low scale seesaw model motivated by resonant leptogenesis. The branching fractions of inclusive decays $b \to s l_h^{\pm} l_l^{\mp}$ in the seesaw model considered in this paper depend on the suppression factor $\frac{m_D}{M}$ which is arisen from the mixing between the singlet heavy neutrinos with three light neutrinos, and can be as large as about 10% without being in conflict with the neutrino mass-squared differences from neutrino data and the current bound on the invisible decay width of the Z boson.

We have classified four classes of the model along with the light neutrino mass spectrum and the assignment of the mixing matrix V_0, and studied how the ratios of the branching fractions for the various channels of QLFV and LFV decays along with lepton flavors could be distinctively predicted in each class. We have found that only the Class B for the NH case presented in Table I survives the current limit on $\mathrm{Br}(\mu \to e\gamma)$ and the invisible decay width of the Z boson. One may check if the model is consistent with the experiments of lepton universality tests. The Class B for the NH case predicts

$$\epsilon_e = \frac{2X}{M} s_{13}^2 \leq 5 \times 10^{-6} \qquad (s_{13} \leq 0.02),$$
$$\epsilon_\mu = \frac{2X}{M} s_{\rm atm}^2 c_{13}^2 \simeq 0.004, \tag{42}$$
$$\epsilon_\tau = \frac{2X}{M} c_{\rm atm}^2 c_{13}^2 \simeq 0.004,$$

where we use $\frac{2X}{M} = 0.008$. The model predicts very small ϵ_e and $\epsilon_\mu = \epsilon_\tau$ which are shown in Fig. 1 with dotted line. The model is consistent with the constraint of the Z invisible decay width and the lepton universality constraints from τ and μ decays while it may not be consistent with the lepton universality constraints determined by W decays. In this class, the branching fractions of $b \to s\tau\mu$ and $\tau \to \mu\gamma$ are predicted to be as large as 10^{-10} and 10^{-9}, respectively. Such large branching fractions can be tested in the future B factory experiments. The enhancement of the branching fractions of QFLV and LFV is originated from the one-loop Feynman diagrams in which the heavy Majorana neutrinos contribute to its nonsupersymmetric radiative correction.

We comment on the leptogenesis of the model. The total lepton number asymmetry generated from heavy Majorana decays is proportional to $\mathrm{Im}\{(\mathbf{u_i^\dagger} \cdot \mathbf{u_j})^2\}$ $(i \neq j)$. In the model described in the previous sections, the asymmetry cannot be generated because the imaginary parts are vanishing. One can show the individual lepton number asymmetries are also vanishing. In order to generate the lepton number, we must relax the relations $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} = 0$ and/or $\mathbf{u_1} = i\mathbf{u_2}$. By relaxing the first relation, one may generate the total lepton number asymmetry. However, the total lepton asymmetry from the lighter states N_1 and N_2 decays is estimated to be 10^{-13}–10^{-16} and is too small to account for the baryon number of the universe. The further details of the leptogenesis for the case without imposing the relations is out of the scope of the present paper and the leptogenesis including a single lepton number generation will be studied elsewhere.

We finally comment on how our prediction of (Q)LFV processes may be changed if we relax the relation $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} = 0$ as

$$\mathbf{u}_{\mathbf{1}}^\dagger \cdot \mathbf{u_3} = \pm \sin\beta, \tag{43}$$

where $\sin\beta > 0$ and we assume that there is no CP violation and take $\mathbf{u_1}$ and $\mathbf{u_3}$ as real. We also assume $X_{12} > X_3$ and study the NH case. If we relax the condition, the simple relation $\mathbf{u_1} = \mathbf{v_3}$ which holds for the Class B NH case will be changed to

$$\mathbf{u}_{\mathbf{1}} = \mathbf{v_3} \cos\theta \mp \mathbf{v_2} \sin\theta, \tag{44}$$

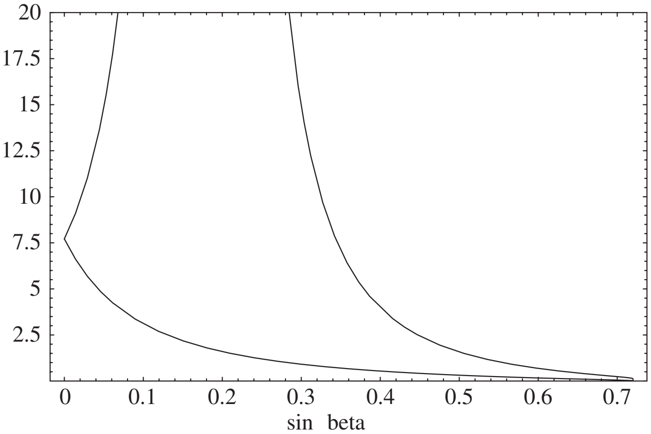

FIG. 7. $\frac{\mathrm{Br}(b\to s\tau\mu)}{\mathrm{Br}(\mu\to e\gamma)}$ vs $\sin\beta$. The upper curve corresponds to $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} > 0$ and the lower curve corresponds to $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} < 0$. $s_{13} = 0.02$.

with $\sin2\theta = \frac{X_3}{n_h - n_l} \sin2\beta$ and X_3 can be written as $X_3 = \frac{n_h+n_l}{2} - \sqrt{(\frac{n_h+n_l}{2})^2 - \frac{n_h n_l}{\cos\beta^2}}$ where $n_h = \sqrt{\Delta m_{\rm atm}^2 + \Delta m_{\rm sol}^2}$ and $n_l = \sqrt{\Delta m_{\rm sol}^2}$ for the NH case. Now the ratio of the branching fractions of $b \to s\tau\mu$ and $\mu \to e\gamma$ is written as a function of $\sin\beta$,

$$\frac{\mathrm{Br}(b \to s\tau\mu)}{\mathrm{Br}(\mu \to e\gamma)} \simeq 6 \times 10^{-3} \Big(\frac{\mathbf{u}_{\tau1}}{\mathbf{u}_{e1}}\Big)^2$$
$$= 6 \times 10^{-3} \Big(\frac{\cos\theta c_{\rm atm} \pm \sin\theta s_{\rm atm} c_{\rm sol}}{\cos\theta s_{13} \mp \sin\theta s_{\rm sol}}\Big)^2, \tag{45}$$

where the ratio of the Inami-Lim functions $(E/F_2)^2 = 191$ corresponding to $M = 1000$ GeV. The ratio of the branching fractions in Eq. (45) is about 7.5 with $\beta = 0$ and $s_{13} = 0.02$. $\mathrm{Br}(b \to s\mu\tau)$ is almost constant and is (6–7) $\times 10^{-11}$ with $2\frac{X}{M} = 0.008$. In Fig. 7, we have shown the ratio for $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} > 0$ and $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} < 0$. When $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} < 0$, as β increases, the ratio decreases. Therefore for large $\sin\beta$, $\mathrm{Br}(\mu \to e\gamma)$ exceeds the experimental upper limit. Only small $\sin\beta \leq 0.1 \sim 0.2$ is allowed. With the small $\sin\beta$, $\mu \to e\gamma$ is still suppressed and $\mathrm{Br}(b \to s\tau\mu)$ is larger than $\mathrm{Br}(\mu \to e\gamma)$. For $\mathbf{u_1^\dagger} \cdot \mathbf{u_3} > 0$, as $\sin\beta$ increases the ratio becomes infinite at the point $\mathbf{u}_{e1} = 0$ where the denominator of $(\mathbf{u}_{\tau1}/\mathbf{u}_{e1})$ vanishes. At this point $\mu \to e\gamma$ is extremely suppressed while $b \to s\tau\mu$ is not changed from the $\beta = 0$ case. Therefore, by considering the constraint of $\mu \to e\gamma$, a large $b \to s\tau\mu$ branching fraction and the suppression of $\mu \to e\gamma$ still happen for the cases studied here.

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