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Modulation of a chirp gravitational wave from a compact binary due to gravitational lensing

Kazuhiro Yamamoto

Department of Physical Science, Hiroshima University, Higashi-Hiroshima 739-8526, Japan

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A possible wave effect in the gravitational lensing phenomenon is discussed. We consider the interference of two coherent gravitational waves of slightly different frequencies from a compact binary, due to the gravitational lensing by a galaxy halo. This system shows the modulation of the wave amplitude. The lensing probability of such the phenomenon is of order 10^{-5} for a high-z source, but it may be advantageous to the observation due to the magnification of the amplitude.

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The wave effect in gravitational lensing phenomenon has been investigated by many authors (see e.g., [1,2] and references therein). This subject is recently revisited by several authors, motivated by a possible phenomenon which might be observed in the future gravitational wave experiments [3–12]. In these works, the authors focus on the diffraction in the wave effect, which is substantial for $\lambda \sim R_E$, where λ is the wave length and R_E is the Schwarzschild radius of the lens mass. However, in the present work, we focus on another different aspect of the wave effect, the modulation of superposed two waves, in the limit that the geometrical optics is valid $\lambda \ll R_E$. Some aspect of this effect has been investigated by the author and Tsunoda [13] as a lensing phenomenon by a cosmic string. Here we perform the similar analysis for the lensing by a galaxy halo. Throughout this paper, we use the convention $G = c = 1$.

We start by considering the superposition of two lensed waves with the amplitude A_1 and A_2 and the angular frequencies ω_1 and ω_2. Namely, we consider the wave expressed by

$$\mathcal{E} = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t + \delta), \tag{0.1}$$

where we assume that the phase δ involves the information of the path difference, and the difference of the frequencies ω_1 and ω_2 are due to the time delay effect. Assuming $A_2 = A_1 + \Delta A$ and $\Delta A < < A_1$, $\mathcal{E}$ reduces to

$$\mathcal{E} \simeq \sqrt{2(A_1^2 + A_2^2)}\Big[\cos\Big(\frac{(\omega_1 + \omega_2)t + \delta}{2}\Big) \times \cos\Big(\frac{(\omega_1 - \omega_2)t - \delta}{2}\Big) + \frac{\Delta A}{2A_1} \times \sin\Big(\frac{(\omega_1 + \omega_2)t + \delta}{2}\Big)\sin\Big(\frac{(\omega_1 - \omega_2)t - \delta}{2}\Big)\Big]. \tag{0.2}$$

This means that, if $\Delta A/A_1 \ll 1$, the wave amplitude modulates with the period

$$\mathcal{T} = \frac{4\pi}{(\omega_1 - \omega_2)} \simeq \frac{4\pi}{\Delta T}\Big(\frac{d\omega}{dt}\Big)^{-1}, \tag{0.3}$$

where ΔT is the time delay and $d\omega/dt$ is the change rate of ω for an observer.

We consider a gravitational wave from a binary of compact objects with equal mass M. By decreasing energy due to the gravitational wave radiation, the orbit of the binary changes. Thus the angular frequency ω changes. The rate of the change is estimated as [14],

$$\frac{d\omega}{dt} = 0.3 \times 10^{-8}(1 + z_s)^{5/3}\Big(\frac{M}{M_\odot}\Big)^{5/3}\Big(\frac{\omega}{\mathrm{rad/s}}\Big)^{11/3} \mathrm{rad/s^2}, \tag{0.4}$$

where z_s is the redshift of the source. Here note that ω and t are the angular frequency and the time of the observer. Now we consider a lens halo modeled by the singular isothermal sphere with the velocity dispersion σ. Then the time delay is

$$\Delta T = 2.7 \times 10^7(1 + z_l)\Big(\frac{\sigma}{200\ \mathrm{km/s}}\Big)^4\Big(\frac{D_{\mathrm{OL}}D_{\mathrm{LS}}}{H_0^{-1}D_{OS}}\Big) \times \Big(\frac{f_r - 1}{f_r + 1}\Big)\mathrm{s}, \tag{0.5}$$

where D_{OL}, D_{OS} and D_{LS} are the angular diameter distances following the usual convention [2], f_r is the flux ratio of the two waves, z_l is the redshift of the lens, and we have adopted the Hubble parameter $H_0 = 70$ km/s/Mpc. Then the period of the modulation is estimated as

$$\mathcal{T} \simeq \frac{3.5 \times 10^6}{(1 + z_l)(1 + z_s)^{5/3}}\frac{f_r + 1}{f_r - 1} \times \Big(\frac{M}{M_\odot}\Big)^{-5/3} \times \Big(\frac{\nu}{0.01\ \mathrm{Hz}}\Big)^{-11/3}\Big(\frac{\sigma}{200\ \mathrm{km/s}}\Big)^{-4}\Big(\frac{D_{\mathrm{OL}}D_{\mathrm{LS}}}{H_0^{-1}D_{OS}}\Big)^{-1} \mathrm{s}. \tag{0.6}$$

Now let us consider the probability of such a lensing phenomenon. The optical depth is estimated by [2]

$$\tau(z_s) = \int_0^{z_s} dz_l \frac{dt}{dz_l}\int dM \pi a^2(\sigma[M], z_l, z_s) \times (1 + z_l)^3 \frac{dn(M, z_l)}{dM}, \tag{0.7}$$

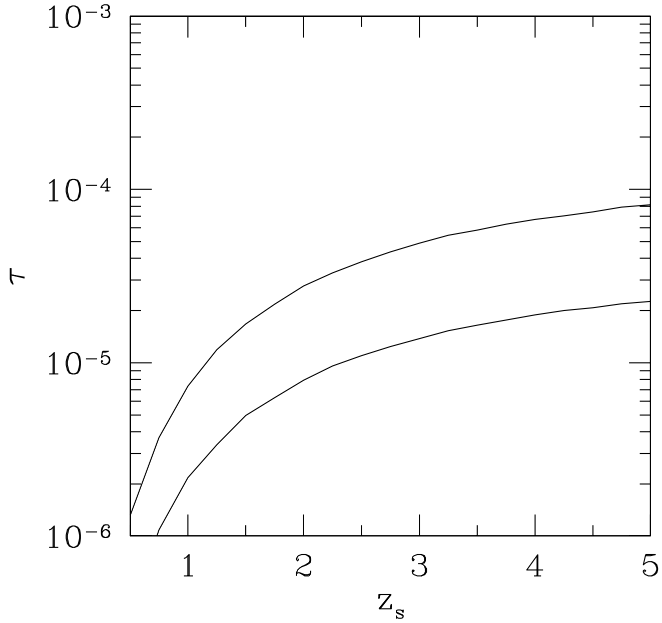

FIG. 1. Optical depth τ as the function of the redshift of the source z_s. The upper and lower curves correspond to $f_r \leq 1.4$ and $f_r \leq 1.2$, respectively. We integrated τ under the condition $\Delta T \leq 10^8$ sec.

where $\pi a^2(\sigma, z_l, z_s)$ is the cross section of the lensing event for a halo and dn/dM is the mass function. We have computed the optical depth by the numerical integration, adopting the analytic method for the mass function based on the Press-Schechter formalism [15,16]. Here we assumed the concordance cosmological model: The flat universe with the matter density parameter $\Omega_m = 0.3$ and the Harrison Zeldovich spectrum $n = 1$ normalized as $\sigma_8 = 0.9$.

The wave amplitudes A_1 and A_2 must be almost the same for the modulation formula (0.2), which is satisfied when the flux ratio f_r is nearly equal 1. Figure 1 demonstrates the optical depth τ as the function of the source redshift z_s. The upper and lower curves assume $f_r < 1.4$ and $f_r < 1.2$, respectively. In the computation of τ, we set the condition that the time delay ΔT is less than 10^8 s. It is clear that the optical depth is typically of order $10^{-6} \sim 10^{-4}$ for a source at redshift $z_s \gtrsim 1$.

The sum of the amplification factor of 2 images is written as $\mu_1 + \mu_2 = 2(f_r + 1)/(f_r - 1)$ with the flux ratio f_r for the singular isothermal sphere model [2]. The energy flux is in proportion to the square of the amplitude of the gravitational wave. Therefore the amplification of the gravitational wave amplitude due to the gravitational lensing can be written as $\sqrt{2(f_r + 1)/(f_r - 1)}$. If the flux ratio is very close to 1, the magnification factor can be large. It might allow us to observe the gravitational sources at very high redshift.

In conclusion, when a detector with sufficient angular resolution and sensitivity is assumed, the periodic modulation in the gravitational wave can be a signature of the interference by the lensing phenomenon. The probability of the lens phenomenon is small, being typically of order $10^{-6} \sim 10^{-5}$ ($10^{-5} \sim 10^{-4}$) for a source at redshift $z_s \sim 1(\sim 5)$. However, the flux of such a lensed source is amplified by the magnification effect. Therefore such a lensed source might be advantageous to be observed in the future gravitational wave experiments (see e.g., [17] and references therein).

In the present work, however, an idealized situation is assumed. We assumed a point source, which is a relevant assumption for the compact binary as a gravitational wave source. It might not be realistic to model the lensing halos with the spherical symmetry, however, we believe that this will not affect our conclusion qualitatively because we worked under the condition that the usual geometrical optics is valid.

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