Small-scale fluctuations in the cosmic x-ray background: A power spectrum approach

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(Received 1 May 1998; published 9 October 1998)

Equations to investigate fluctuations in cosmic x-ray background radiation due to pointlike sources at high-redshift are formulated in a systematic way. The angular power spectrum of x-ray background fluctuations is investigated from large scales to small scales in various cosmological models such as open universe models and models with the cosmological constant, assuming a simple evolution model of the sources. The effect of the epoch-dependent bias is demonstrated for small-angle fluctuations. The contribution from shot noise fluctuations is also discussed. [S0556-2821(98)04720-1]

PACS number(s): 98.80.Es, 98.65.Dx, 98.70.Vc

I. INTRODUCTION

The cosmic x-ray background (CXB) has been studied for a long time in astrophysics [1]. Recently several authors have discussed the possibility of the CXB fluctuations as a probe of the structure formation in the high redshift universe [2–8]. These works are motivated from observational results of the deep survey carried out by ROSAT, which resolved a significant fraction of the x-ray background into sources ($\sim 70\%$). It turns out to be that most of the sources are extra-galactic objects, i.e., active galactic nuclei (AGN) and other x-ray luminous galaxies in the high redshift universe. While the situation is not so clear in the harder band, something similar would be happening. Several x-ray mission projects are in progress [9] which will give us a fine solution of the source problem in the near future.

On the other hand, a scenario of the cosmic structure formation is now constrained from the observations, e.g., temperature anisotropies in the cosmic microwave background radiation (CMB) and large-scale distribution of galaxies. Future satellite experiments of CMB and survey projects of the large-scale distribution of galaxies will provide severer constraints on theoretical models of the large scale structure formation. At present the scenario introducing cold dark matter (CDM) seems to be successful though some modification may be required [10,11].

Formation processes of subgalactic objects, galaxies, AGNs and clusters at the high redshift universe are one of the hottest topics in the area of astrophysics and cosmology. Since these objects are parts of the large scale structure of the universe, we could expect that information about the structure formation in the high redshift universe would be obtained by investigating them.

Several works have been done to answer following questions: What kind of information about cosmic structure formation can be obtained from an analysis of the x-ray background fluctuations? Are they really a probe of cosmology? For example, Boughn *et al.* claimed that the cross correlation between the x-ray background fluctuations and the microwave background anisotropies can provide a constraint on a cosmological constant. Small-scale fluctuations in the x-ray background have been investigated in [5,6,12–14]. On the other hand, Lahav *et al.* [2] calculated the angular power spectrum of the fluctuations by a multipole expansion method. They mainly focused on large scale fluctuations and their formula is only limited to the case of the flat universe. Recently Treyer *et al.* have compared the theoretical models by Lahav *et al.* with the observational data of HEAO1 [3].

In this paper we expand Lahav *et al.*'s formula to more general cosmological models, i.e., open universe models and models with the cosmological constant, and investigate general behaviors of the angular power spectrum of fluctuations from large-scales to small-scales. The x-ray background fluctuation in the open universe has been considered in Ref. [15], and the similar expression was obtained. However, in that paper, the discussion was only focused on the large-scale fluctuations due to the source clustering, and the derivation of equations was very complicated. In this paper we formulate the treatment in a simple way. And the shot noise fluctuation is treated in a systematic way. The formulation developed in this paper will be useful for studying the clustering of high redshift sources not only for x-ray sources but also for other pointlike sources such as radio galaxies [16,17] and γ-ray bursts.

This paper is organized as follows. In Sec. II we derive expression for the angular two-point correlation function by using the multipole expansion method in a systematic way. In Sec. III the fluctuations in the x-ray background are investigated assuming a simple luminosity function of x-ray sources. The competition of the fluctuations due to the source clustering and the shot noise is also studied there. Section IV is devoted to summary and discussions. Throughout this paper we use the unit $c=1$.

II. FORMULATION

In this section we formulate equations to calculate background fluctuations which come from pointlike sources at high redshift. Essence of the formulation is divided into two parts. One is how the sources are distributed in the universe. This is the problem of source evolution and statistics of the distribution, which we describe in the latter part of this sec-

0556-2821/98/58(10)/103508(9)/$15.00

tion. The other part is how the sources are observed in the sky, which is obtained by solving light propagation in the expanding universe, if the distribution of the sources in the universe was given.

First we consider the light propagation. The photon intensity I_ν in an expanding universe is described by the radiative transfer equation:

$$\frac{\partial I_\nu}{\partial \eta}+\gamma^j\frac{\partial I_\nu}{\partial x^j}+\mathcal{H}\left[3I_\nu-\nu\frac{\partial I_\nu}{\partial \nu}\right]+\frac{d\gamma^j}{d\eta}\frac{\partial I_\nu}{\partial \gamma^j}=aj_\nu(\mathbf{x},\eta), \tag{2.1}$$

where a is the scale factor which is normalized to be unity at present, η is conformal time defined by $ad\eta=dt$ with the cosmic time t, $\mathcal{H}$ is the Hubble parameter defined as $\dot{a}/a$ with overdot denoting η differentiation, γ^i is the directional vector of the photon momentum, and $j_\nu(\mathbf{x})$ is a field of emissivity per unit comoving volume. Assuming that an (i)-th pointlike source is located at the coordinate $\mathbf{x}^{(i)}$ and has the power put out per unit frequency and per unit time $L_\nu^{(i)}(\eta)$, we write the field of emissivity as

$$j_\nu(\mathbf{x},\eta)=\frac{1}{4\pi a^3}\sum_i L_\nu^{(i)}(\eta)\delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)}). \tag{2.2}$$

We solve Eq. (2.1) with Eq. (2.2) in the Friedmann-Robertson-Walker space-time with the line element

$$ds^2=-dt^2+a^2\gamma_{ij}dx^idx^j, \tag{2.3}$$

where γ_{ij} is the three-metric on a space of constant negative curvature K:

$$\gamma_{ij}dx^idx^j=\frac{1}{-K}(d\chi^2+\sinh^2\chi(d\theta^2+\sin^2\theta d\phi^2)). \tag{2.4}$$

In order to take the limit of the flat universe, let us first introduce a radial coordinate r instead of χ by $\chi=\sqrt{-K}r$ and take the limit $K\rightarrow 0$. The scale factor a is governed by the Friedmann equation

$$\mathcal{H}^2=\left(\frac{\dot{a}}{a}\right)^2=H_0^2\left(\frac{1}{a}\Omega_0+\Omega_K+a^2\Omega_\Lambda\right), \tag{2.5}$$

where H_0 is the Hubble parameter with $H_0=100\,h$ km/s/Mpc, Ω_0 is the density parameter, $\Omega_\Lambda\equiv\Lambda/3H_0^2$ which is the density parameter of the cosmological constant Λ, and $\Omega_K\equiv 1-\Omega_0-\Omega_\Lambda$ which describes the spatial curvature of the universe.

Employing a new variable $f=I_\nu/\nu^3$, we can rewrite Eq. (2.1) as

$$\frac{\partial f}{\partial \eta}+\gamma^j\frac{\partial f}{\partial x^j}-\mathcal{H}\nu\frac{\partial f}{\partial \nu}+\frac{d\gamma^j}{d\eta}\frac{\partial f}{\partial \gamma^j}=\frac{a}{4\pi(a\nu)^3}\sum_i L_\nu^{(i)}(\eta)\delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)}). \tag{2.6}$$

Introducing $\nu_0(=\nu a)$ instead of ν, Eq. (2.6) reduces to

$$\frac{\partial f}{\partial \eta}+\gamma^j\frac{\partial f}{\partial x^j}+\frac{d\gamma^j}{d\eta}\frac{\partial f}{\partial \gamma^j}=\frac{a}{4\pi\nu_0^3}\sum_i L_{\nu\rightarrow\nu_0/a}^{(i)}(\eta)\delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)}). \tag{2.7}$$

Operating $\int_{\nu_1}^{\nu_2}d\nu_0\nu_0^3$ on both sides of Eq. (2.7), we have

$$\frac{\partial I}{\partial \eta}+\gamma^j\frac{\partial I}{\partial x^j}+\frac{d\gamma^j}{d\eta}\frac{\partial I}{\partial \gamma^j}=\frac{a}{4\pi}\sum_i \delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)})\int_{\nu_1}^{\nu_2}d\nu_0 L_{\nu\rightarrow\nu_0/a}^{(i)}(\eta), \tag{2.8}$$

where $I=\int_{\nu_1}^{\nu_2}d\nu_0\nu_0^3 f$. Since Eq. (2.8) is rewritten as

$$\frac{dI(\eta,\mathbf{x},\vec{\gamma})}{d\eta}=\frac{\partial I}{\partial \eta}+\gamma^j\frac{\partial I}{\partial x^j}+\frac{d\gamma^j}{d\eta}\frac{\partial I}{\partial \gamma^j}=\frac{a}{4\pi}\sum_i \delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)})K^{(i)}(\eta)L^{(i)}(\eta), \tag{2.9}$$

where the luminosity of the i-th source is defined by

$$L^{(i)}(\eta)=\int_{\nu_1}^{\nu_2}d\nu L_\nu^{(i)}(\eta), \tag{2.10}$$

and $K^{(i)}(\eta)$ is defined by[1]

$$K^{(i)}(\eta)=\frac{1}{L^{(i)}(\eta)}\int_{\nu_1}^{\nu_2}d\nu_0 L_{\nu\rightarrow\nu_0/a}^{(i)}(\eta), \tag{2.11}$$

then by integrating Eq. (2.9), we get

$$I(\eta_0,\mathbf{x}_0,\vec{\gamma})=\frac{1}{4\pi}\sum_i\int d\eta a(\eta)K^{(i)}(\eta)L^{(i)}(\eta)\times\delta^{(3)}(\mathbf{x}(\eta,\vec{\gamma})-\mathbf{x}^{(i)}), \tag{2.12}$$

where $\mathbf{x}(\eta,\vec{\gamma})$ stands for a photon path.

Because $\Sigma_i\delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)})$ is regarded as a number density field and we replace it with

$$\sum_i \delta^{(3)}(\mathbf{x}-\mathbf{x}^{(i)})\Rightarrow\int d\log Ln(L,\eta,\mathbf{x}), \tag{2.13}$$

where $n(L,\eta,\mathbf{x})$ is the luminosity function which denotes a number of sources per unit comoving volume and per unit $\log L$. Then we rewrite Eq. (2.12) as

[1] $K^i(\eta)/a(\eta)$ denotes the usual K correction.

$$I(\eta_0,\mathbf{x}_0,\vec{\gamma})=\frac{1}{4\pi}\int d\,\log LL\int d\eta a(\eta)K(L,\eta)\times n(L,\eta,\mathbf{x}(\eta,\vec{\gamma})), \tag{2.14}$$

where $K(L,\eta)$ is defined for a source with luminosity L in same way as Eq. (2.11).

Here we mention the range of η integration in Eq. (2.14). The range of η integration depends on observational situation and strategy. In this paper we focus on background fluctuations. In order to reduce the shot noise fluctuations, which are described later in this paper, we assume that bright nearby sources are removed from an observed map. Darker and darker sources we get rid of, more and more distant sources are removed. This flux cutoff limit is denoted by S_c. The local luminosity of a source L is related with the observed flux S and the distance, which we describe by the conformal time η in stead of the redshift, as

$$S=\frac{aLK(L,\eta)}{4\pi D(\eta)^2}, \tag{2.15}$$

where $D(\eta)=(-K)^{-1/2}\sinh(\sqrt{-K}(\eta_0-\eta))$ and η_0 is the conformal time at the present epoch. Note that the distance D is related with the luminosity distance d_L by $D(\eta)=d_L/(1+z)$ [18]. In case of observations with the flux cutoff limit S_c, the range of η integration must be $0\leqslant\eta\leqslant\eta_c$, where η_c is determined by solving Eq. (2.15) with setting $S=S_c$. Thus η_c is a function of L and S_c, in general. As we will see in the next section, the removal of bright sources is an important factor in predicting the fluctuations.

In order to discuss statistics of the fluctuations we define the angular two-point correlation function:

$$\begin{aligned}C(\theta)&=\int\int\frac{d\Omega_{\vec{\gamma}_1}}{4\pi}\frac{d\Omega_{\vec{\gamma}_2}}{2\pi}\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\theta)\\&\quad\times I(\eta_0,\mathbf{x}_0,\vec{\gamma}_1)I(\eta_0,\mathbf{x}_0,\vec{\gamma}_2)\\&=\int\int\frac{d\Omega_{\vec{\gamma}_1}}{4\pi}\frac{d\Omega_{\vec{\gamma}_2}}{2\pi}\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\theta)\\&\quad\times\frac{1}{4\pi}\int d\,\log L_1L_1\int_0^{\eta_c}d\eta_1a_1K(L_1,\eta_1)\\&\quad\times n(L_1,\eta_1,\mathbf{x}(\eta_1,\vec{\gamma}_1))\\&\quad\times\frac{1}{4\pi}\int d\,\log L_2L_2\int_0^{\eta_c}d\eta_2a_2K(L_2,\eta_2)\\&\quad\times n(L_2,\eta_2,\mathbf{x}(\eta_2,\vec{\gamma}_2)),\end{aligned} \tag{2.16}$$

where $a_1=a(\eta_1)$ and $a_2=a(\eta_2)$.

Theoretically we can only predict the ensemble average of fluctuations. To obtain the ensemble average of the two-point correlation function, we need the ensemble average $\langle n(L_1,\eta_1,\mathbf{x}_1)n(L_2,\eta_2,\mathbf{x}_2)\rangle$ from Eq. (2.16). In the case of pointlike sources, it is well known that the ensemble average of a product of the density field has three terms, i.e., homogeneous term, clustering term and shot term [19,20]. As we are dealing with the density field as a function of luminosity and time, the relation $\langle n(L_1,\eta_1,\mathbf{x}_1)n(L_2,\eta_2,\mathbf{x}_2)\rangle$ is not trivial in a strictly sense. However, assuming that the luminosity of sources is statistically independent of their positions relative to the other sources, we set the following relation [12,13]:

$$\begin{aligned}&\langle n(L_1,\eta_1,\mathbf{x}_1)n(L_2,\eta_2,\mathbf{x}_2)\rangle\\&\quad=\overline{n}(L_1,\eta_1)\overline{n}(L_2,\eta_2)+\overline{n}(L_1,\eta_1)\\&\qquad\times\overline{n}(L_2,\eta_2)\xi(\eta_1,\eta_2;\mathbf{x}_1-\mathbf{x}_2)\\&\qquad+\overline{n}(L_1,\eta_1)\delta(\log L_1-\log L_2)\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_2),\end{aligned} \tag{2.17}$$

where $\overline{n}(L,\eta)$ is the spatially averaged quantity of $n(L,\eta,\mathbf{x})$. The first term of the right-hand side of Eq. (2.17) is the isotropic background component, the second term describes the clustering of the sources, the third term is called the shot term, which arises from the discreteness of the sources [19]. Then the ensemble average $\langle C(\theta)\rangle$ has three terms, i.e., isotropic background term, clustering term and shot noise term, which we write as $\langle C(\theta)\rangle=\langle C^{\mathrm{ISO}}\rangle+\langle C^{\mathrm{CL}}(\theta)\rangle+\langle C^{\mathrm{SN}}(\theta)\rangle$.

A. Isotropic background

First we consider the isotropic background term. From Eq. (2.16) and the first term of the right-hand side (RHS) of Eq. (2.17), we easily get

$$\begin{aligned}\langle C^{\mathrm{ISO}}\rangle&=\left(\frac{1}{4\pi}\int d\,\log LL\int_0^{\eta_c}d\eta aK(L,\eta)\overline{n}(L,\eta)\right)^2\\&=(I^{(0)})^2,\end{aligned} \tag{2.18}$$

where we used

$$\int\int\frac{d\Omega_{\vec{\gamma}_1}}{4\pi}\frac{d\Omega_{\vec{\gamma}_2}}{2\pi}\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\theta)=1. \tag{2.19}$$

B. Fluctuations from source clustering

The correlation function of the source clustering term is

$$\begin{aligned}\langle C(\theta)^{\mathrm{CL}}\rangle&=\int\int\frac{d\Omega_{\vec{\gamma}_1}}{4\pi}\frac{d\Omega_{\vec{\gamma}_2}}{2\pi}\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\theta)\\&\quad\times\frac{1}{4\pi}\int d\,\log L_1L_1\int_0^{\eta_c}d\eta_1a_1\\&\quad\times K(L_1,\eta_1)\overline{n}(L_1,\eta_1)\\&\quad\times\frac{1}{4\pi}\int d\,\log L_2L_2\int_0^{\eta_c}d\eta_2a_2\\&\quad\times K(L_2,\eta_2)\overline{n}(L_2,\eta_2)\\&\quad\times\xi(\eta_1,\eta_2;\mathbf{x}_1(\eta_1,\vec{\gamma}_1)-\mathbf{x}_2(\eta_2,\vec{\gamma}_2)).\end{aligned} \tag{2.20}$$

To relate the correlation function of x-ray sources with that of cold dark matter (CDM), we write

$$\xi(\eta_1,\eta_2;\mathbf{x}_1-\mathbf{x}_2)=b_X(\eta_1)b_X(\eta_2)\xi(\eta_1,\eta_2;\mathbf{x}_1-\mathbf{x}_2)_{\rm CDM}, \tag{2.21}$$

where $b_X(\eta)$ is the bias factor,[2] the CDM density correlation function is $\xi(\eta_1,\eta_2;\mathbf{x}_1-\mathbf{x}_2)_{\rm CDM}=\langle\delta_{\rm c}(\eta_1,\mathbf{x}_1)\delta_{\rm c}(\eta_2,\mathbf{x}_2)\rangle$, and $\delta_{\rm c}(\eta,\mathbf{x})$ is the CDM density contrast. While the nonlinearity of the CDM density contrast may be effective, however, we only consider linear theory in this paper. In this case $\xi(\eta_1,\eta_2;\mathbf{x}_1-\mathbf{x}_2)_{\rm CDM}$ is proportional to $D_1(\eta_1)D_1(\eta_2)$, where $D_1(\eta)$ is the linear growth rate normalized to unity at present.

We next consider to express the CDM density contrast in terms of the density power spectrum. In order to express Gaussian random process of the CDM density field, we introduce probability variables $a_{\omega lm}$, which satisfy

$$\langle a_{\omega lm}\rangle=0,\quad \langle a_{\omega lm}a^*_{\omega' l' m'}\rangle=\delta(\omega-\omega')\,\delta_{ll'}\delta_{mm'}. \tag{2.22}$$

For the condition of reality of density field we assume $a^*_{\omega lm}=a_{\omega l-m}$. Then we can write the CDM density fluctuation fields as

$$\delta_{\rm c}(\eta,\mathbf{x})=\int_0^\infty d\omega\sum_{lm} a_{\omega lm}\delta_\omega(\eta)\mathcal{Y}_{\omega lm}(\mathbf{x}), \tag{2.23}$$

where $\delta_\omega(\eta)$ is the root-mean-square of the coefficient associated with the orthonormalized harmonics in the hyperbolic universe

$$\begin{aligned}\mathcal{Y}_{\omega lm}(\mathbf{x})&=(-K)^{3/4}W_{\omega l}(\chi)Y_{lm}(\Omega)\\ &=(-K)^{3/4}\left|\frac{\Gamma(i\omega+l+1)}{\Gamma(i\omega)}\right|\\ &\quad\times\frac{P^{-l-1/2}_{i\omega-1/2}(\cosh\chi)}{\sqrt{\sinh\chi}}Y_{lm}(\Omega),\end{aligned} \tag{2.24}$$

which satisfies

$$\begin{aligned}\int&(-K)^{-3/2}\sinh^2\chi d\chi d\Omega\,\mathcal{Y}_{\omega lm}(\chi,\Omega)\mathcal{Y}^*_{\omega' l' m'}(\chi,\Omega)\\ &=\delta(\omega-\omega')\,\delta_{ll'}\delta_{mm'},\end{aligned} \tag{2.25}$$

where $\Gamma(z)$ is the gamma function and $P^\nu_\mu(z)$ is the Legendre function. Here ω is a nondimensional wave number and $K(\omega^2+1)$ is the eigenvalue of the scalar harmonics $\mathcal{Y}_{\omega lm}(\mathbf{x})$ in the hyperbolic universe (2.4) [21–23]. Then we have

$$\begin{aligned}\langle\delta_{\rm c}(\eta_1,\mathbf{x}_1)\delta_{\rm c}(\eta_2,\mathbf{x}_2)\rangle=\sum_{lm}\int_0^\infty d\omega\,\delta_\omega(\eta_1)\mathcal{Y}_{\omega lm}(\chi_1,\Omega_1)\\ \times\delta_\omega(\eta_2)\mathcal{Y}^*_{\omega lm}(\chi_2,\Omega_2).\end{aligned} \tag{2.26}$$

From a straightforward calculation using the relations

$$\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\theta)=\sum_{l=0}\frac{2l+1}{2}P_l(\vec{\gamma}_1\cdot\vec{\gamma}_2)P_l(\cos\theta), \tag{2.27}$$

$$P_l(\vec{\gamma}_1\cdot\vec{\gamma}_2)=\frac{4\pi}{2l+1}\sum_{m=-l}^{m=l}Y_{lm}(\Omega_{\vec{\gamma}_1})Y^*_{lm}(\Omega_{\vec{\gamma}_2}), \tag{2.28}$$

where $Y_{lm}(\Omega_{\vec{\gamma}})$ is the spherical harmonics in an unit sphere, Eq. (2.20) reduces to[3]

$$\langle C(\theta)^{\rm CL}\rangle=\sum_l\frac{2l+1}{4\pi}C_l^{\rm CL}P_l(\cos\theta), \tag{2.29}$$

with

$$\begin{aligned}C_l^{\rm CL}=\int_0^\infty d\omega(-K)^{3/2}\biggl(\frac{1}{4\pi}\int d\log LL\int_0^{\eta_c}d\eta aK(L,\eta)\\ \times\overline{n}(L,\eta)b_X(\eta)\delta_\omega(\eta)W_{\omega l}(\Delta\eta)\biggr)^2,\end{aligned} \tag{2.30}$$

where $\Delta\eta=\sqrt{-K}(\eta_0-\eta)$ and $W_{\omega l}(\chi)$ is defined in Eq. (2.24). This expression is rewritten as [15]

$$\begin{aligned}C_l^{\rm CL}=\frac{2}{\pi}\int_0^\infty d\tilde{k}\tilde{k}^2M_l\biggl(\frac{1}{4\pi}\int d\log LL\int_0^{\eta_c}d\eta aK(L,\eta)\\ \times\overline{n}(L,\eta)b_X(\eta)\delta_\omega(\eta)X^l_\omega(\Delta\eta)\biggr)^2,\end{aligned} \tag{2.31}$$

where $M_l=(\tilde{k}^2-K)\cdots(\tilde{k}^2-l^2K)/(\tilde{k}^2-K)^l$, $\tilde{k}=\sqrt{-K}\omega$, and

$$X^l_\omega(\chi)=\left(\frac{\pi(\omega^2+1)^l}{2}\right)^{1/2}\frac{P^{-l-1/2}_{i\omega-1/2}(\cosh\chi)}{\sqrt{\sinh\chi}}. \tag{2.32}$$

In the limit of flat universe, i.e., $K\to 0$, the function $X^l_\omega(\chi)$ reduces to the spherical Bessel function [21–23].

C. Random fluctuations

Finally we consider random fluctuations from the shot term. The ensemble average of the two-point correlation function is

[2]The bias factor could be a function of luminosity L in general. However we ignore the dependence of luminosity for simplicity.

[3]Throughout this paper we consider an ideal detector with infinite small angular resolution. The effect of a finite beam width and a limited observational area can be taken into account by multiplying a window function in the similar way as the case of CMB temperature anisotropies.

$$\langle C(\theta)^{\rm SN}\rangle=\int\int \frac{d\Omega_{\vec{\gamma}_1}}{4\pi}\frac{d\Omega_{\vec{\gamma}_2}}{2\pi}\delta(\vec{\gamma}_1\cdot\vec{\gamma}_2-\cos\,\theta)$$
$$\times\frac{1}{4\pi}\int d\,\log\,L_1L_1\int_0^{\eta_c}d\eta_1a_1K(L_1,\eta_1)$$
$$\times\frac{1}{4\pi}\int d\,\log\,L_2L_2\int_0^{\eta_c}d\eta_2a_2K(L_2,\eta_2)$$
$$\times\overline{n}(L_1,\eta_1)\delta(L_1-L_2)$$
$$\times\delta^{(3)}(\mathbf{x}_1(\eta_1,\vec{\gamma}_1)-\mathbf{x}_2(\eta_2,\vec{\gamma}_2)). \tag{2.33}$$

As the delta function is expressed

$$\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_2)=(-K)^{3/2}\frac{\delta(\chi_1-\chi_2)}{\sinh^2\,\chi_1}\,\delta^{(2)}(\vec{\gamma}_1-\vec{\gamma}_2), \tag{2.34}$$

then Eq. (2.33) leads to

$$C^{\rm SN}(\theta)=\sum_l \frac{2l+1}{4\pi}C_l^{\rm SN}P_l(\cos\,\theta), \tag{2.35}$$

with

$$C_l^{\rm SN}=\int d\,\log\,LL^2\int_0^{\eta_c}d\eta\left(\frac{1}{4\pi}aK(L,\eta)\right)^2$$
$$\times\overline{n}(L,a)\,\frac{-K}{\sinh^2\,\Delta\eta}, \tag{2.36}$$

where $\Delta\eta=\sqrt{-K}(\eta_0-\eta)$. Here we used Eqs. (2.27) and (2.28). This spectrum is independent of l, which implies that this fluctuation is a white noise type. And this means $\langle C(\theta)^{\rm SN}\rangle\propto\delta(\theta)$.

III. FLUCTUATIONS IN A SIMPLE MODEL

In this section we discuss characteristic behavior of fluctuations by considering a simple evolution model of x-ray sources. The evolution and population of x-ray sources at high redshift are less well understood. Here we consider a model of a single class of sources, i.e., the case that the sources have same luminosity profile at a cosmic time. In this case the luminosity function is given by

$$\overline{n}(L,\eta)=\overline{n}(a)\delta(\log\,L-\log\,\mathcal{L}), \tag{3.1}$$

where $\mathcal{L}$ is the luminosity of a source and $\overline{n}$ is the averaged number density of sources. We assume that the sources are distributed to the redshift $z_{\rm max}$. Furthermore we assume power-law evolution of the source number density and the luminosity:

$$\overline{n}(a)=n_0a^{-d},\quad \mathcal{L}=L_0a^{-e}, \tag{3.2}$$

where n_0 and L_0 are constants. We further assume the power-law frequency dependence $L_\nu\propto\nu^{-\alpha}$, then we have

$$K(L,\eta)=a(\eta)^\alpha. \tag{3.3}$$

As for the bias, we take into account the epoch-dependent bias by setting [3,24]

$$b_X(\eta)=b_{X0}+z[b_{X0}-1], \tag{3.4}$$

where z is the redshift and b_{X0} is a constant. Although this simple formula might be applicable only in the Einstein–de Sitter universe, we use it even for other cosmological models for simplicity.

A. Fluctuations from source clustering

The above simplification leads to the following simple expressions for the isotropic background and the angular power spectrum of source clustering:

$$I^{(0)}=\frac{n_0L_0}{4\pi}\int_{\eta_i}^{\eta_c}d\eta a^{1-d-e+\alpha}, \tag{3.5}$$

$$C_l^{\rm CL}=\frac{2}{\pi}\int_0^\infty d\tilde{k}\tilde{k}^2M_l\left(\frac{n_0L_0}{4\pi}\int_{\eta_i}^{\eta_c}d\eta a^{1-d-e+\alpha}b_X(\eta)\right.$$
$$\left.\times\delta_\omega(\eta)X_\omega^l(\Delta\eta)\right)^2, \tag{3.6}$$

where η_i is the conformal time at the redshift $z_{\rm max}$. Note also that the relative amplitude of fluctuations $C_l^{\rm CL}/(I^{(0)})^2$ is independent of n_0L_0. The evolution of the x-ray sources are less understood. Let us consider the case that the volume emissivity evolves with the time, and we set $p\equiv d+e=3$ for simplicity [2]. We adopt this value and $\alpha=0.4$ in the rest of this paper [2].

The angular power spectrum is shown in Fig. 1 for various cosmological models. In Figs. 1(a) and 1(b), the three panels correspond to the different cosmological models, a ‘‘standard’’ CDM model (SCDM), a CDM model with the cosmological constant (ΛCDM) and an open CDM model (OCDM) [25]. In each panel three lines stand for the different parameters for removal of bright sources. For simplicity we take this effect into account by specifying the distance of redshift $z_{\rm min}$ instead of specifying S_c. We assumed that the sources of $0\leqslant z\leqslant z_{\rm min}$ were removed. In the figure three lines correspond to the cases $z_{\rm min}$=0, 0.02, 0.01. Figure 1(a) is the case of no bias by setting $b_{X0}=1$ in Eq. (3.4). In Fig. 1(b) we chose $b_{X0}=1.6$ as a case of the epoch-dependent bias, where the bias factor becomes significant at $z\gtrsim1$. The CDM density power spectrum is normalized as $\sigma_8=1$.

The first notable feature is that the amplitude of fluctuations strongly depends on $z_{\rm min}$ for low multipoles, i.e., large-angle scales. As $z_{\rm min}$ controls distance to which nearby sources are removed, the decrease of the low multipoles is due to the removal of the sources. This implies that the nearby sources dominantly contribute to the low multipoles [15]. On the contrary, the high multipoles do not depend on $z_{\rm min}$.

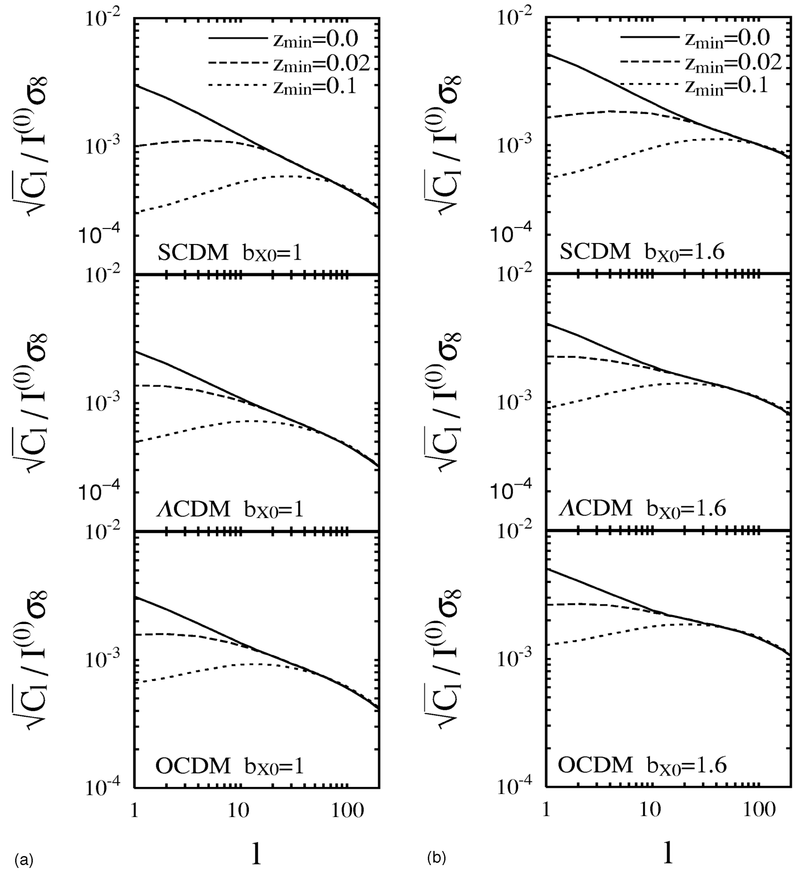

FIG. 1. The angular power spectrum of fluctuations due to source clustering for various cosmological models. The cosmological parameters are taken as $h=0.5$ and $\Omega_0=1$ for the SCDM model, and $h=0.7$ and $\Omega_0=0.3$ for the ΛCDM and OCDM models. The parameters $p \equiv d+e=3$ and $z_{\rm max}=3$ are taken. In each panels three lines correspond to $z_{\rm min}=0$, 0.02, 0.1, respectively. (a) is the case of no bias by setting $b_{X0}=1$ in Eq. (3.4). In (b) $b_{X0}=1.6$ is chosen as a case of epoch-dependent bias. Note that the area under the lines do not directly describe the amount of observed fluctuations since $\sqrt{l(2l+1)C_l}$ is the one in case of the logarithmic interval of x-axis [see Eq. (2.29)]. Roughly speaking, we need to multiply l. Therefore there are larger fluctuations on smaller scales. On the other hand, the contribution from the shot noise does not depend on l in these figures.

In order to understand dominant contributors of sources to fluctuations in more detail, we show the amplitude of the multipoles $l=1$, 10, 30, 100, as a function of $z_{\rm min}$ in Fig. 2. The large value of $z_{\rm min}$ taken in this figure might be unrealistic from the observational view point for an all sky survey. However this figure is instructive to understand what sources are dominant contributors to the fluctuations of each angular size. The steep decrease of the low multipole at $z_{\rm min} \lesssim 0.1$ indicates that low redshift sources sensitively contribute to the large-angle fluctuations. On the other hand the higher multipole ($l=100$) does not sensitively depend on $z_{\rm min}$ and it is not almost affected by the removal of the sources for $z_{\rm min} \lesssim 2$. This means that the high redshift sources of $z \gtrsim 1$ are the dominant contributors to the small-scale fluctuations $l \gtrsim 100$. It is shown that the amplitude of the fluctuations increases for $z_{\rm min} \gtrsim 2$. The width of the range in which sources are distributed becomes thin when $z_{\rm min}$ becomes near $z_{\rm max}$. This effect makes the amplitude of the fluctuations increase since the fluctuations suffer less significant thickness damping, that is contributions from sources at different redshifts cause the cancellation. We think that the increase of the fluctuations at $z_{\rm min} \gtrsim 2$ in Fig. 2 are due to this effect.

Comparing Figs. 1(a) and 1(b), evolution of the bias derives significant difference on small scales. For the low multipoles, since the nearby sources of low redshift dominantly contribute to the fluctuations, the (low multipole) amplitude scales by the bias factor at present epoch b_{X0}. However the amplitude on small scales is significantly increased for the epoch-dependent bias model. This is understood that the high redshift sources are the dominant contributors to the small-angle fluctuations and the amplitude is increased due to the large value of the bias factor at high redshift.

It is notable that the dependence on the cosmological models is quite weak even on small scales. Because we have employed σ_8 normalization, the shape of the density power spectrum of the OCDM model is almost same as the ΛCDM model. The difference between these two models is small at a few×10% level. However the amplitude of fluctuations of the OCDM model is slightly larger than the Λ model on small scales. This would be understood as follows. When we see a same physical size in the open universe, the angle size becomes smaller compared with the case of the flat universe due to the curvature of background geometry. Therefore the large-scale fluctuations in the open model shift to the smaller scale. This is the same situation as the case of CMB anisotropies.

B. Shot noise vs source clustering

In this subsection we discuss the shot noise fluctuations comparing them with the fluctuations due to the source clustering. The shot noise fluctuations are random fluctuations originated from discreteness of the sources. The spectrum is the white noise type and the angular power spectrum does not depend on l. In our simple model, Eq. (2.36) reduces to

$$C_l^{\rm SN} = \frac{n_0 L_0^2}{(4\pi)^2} \int_{\eta_i}^{\eta_c} d\eta (a^{1-e+\alpha})^2 a^d \frac{-K}{\sinh^2 \Delta\eta}. \qquad (3.7)$$

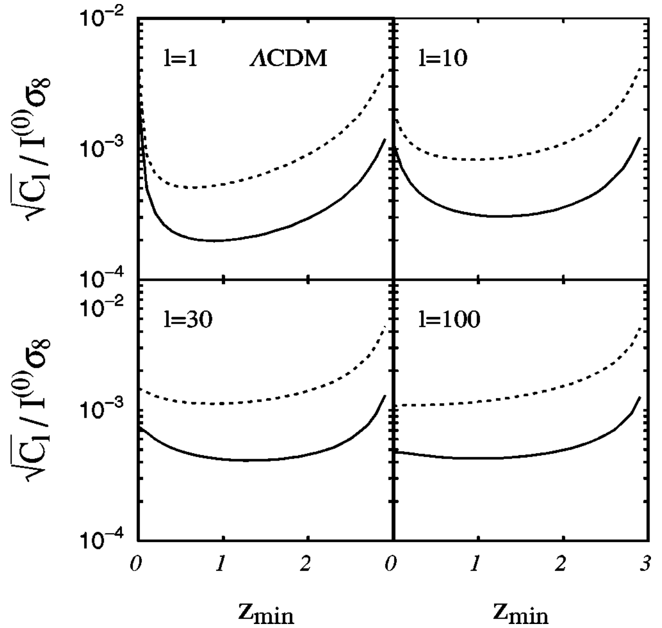

FIG. 2. $z_{\min}$ dependence of multipoles for the ΛCDM model. The four panels correspond to the multipoles $l=1$, 10, 30, 100, respectively. The solid line represents the case $b_{X0}=1$ and the dashed line does the case $b_{X0}=1.6$. The model parameters are same as that in Fig. 1(a).

Assuming $\eta_c \simeq \eta_0$ and $(\eta_0-\eta_c)/\eta_0 \ll 1$, we approximately rewrite it as

$$C_l^{\rm SN} \simeq \frac{n_0 L_0^2}{(4\pi)^2}\int_{\eta_i}^{\eta_c}\frac{d\eta}{(\eta_0-\eta)^2} \simeq \frac{n_0 L_0^2}{(4\pi)^2}\frac{1}{\eta_0-\eta_c}, \quad (3.8)$$

$$S_c \simeq \frac{L_0}{4\pi(\eta_0-\eta_c)^2}, \quad (3.9)$$

where the second equation is derived from Eq. (2.15). Then we have

$$C_l^{\rm SN} \simeq S_c{}^{1/2} n_0 \left(\frac{L_0}{4\pi}\right)^{3/2}. \quad (3.10)$$

This approximation is valid for the case that the removed bright sources are located at low redshift $z \ll 1$. Using the value $I^{(0)}=5.2\times10^{-8}$ erg s^{-1} cm^{-2} str^{-1} in 2–10 keV band [26], we have

$$\frac{\sqrt{C_l^{\rm SN}}}{I^{(0)}} = 1.2\left(\frac{S_c}{\text{erg s}^{-1}\,\text{cm}^{-2}}\right)^{1/4} \times\left(\frac{L_0}{3\times10^{43}\ \text{erg s}^{-1}}\right)^{3/4} \times\left(\frac{n_0}{3\times10^{-5}\ \text{Mpc}^{-3}}\right)^{1/2}. \quad (3.11)$$

From Eq. (3.9) and $\eta_0-\eta\simeq z/H_0$, which is obtained from the Friedmann equation, we estimate the redshift $z_{\min}$ to which the bright sources are removed as

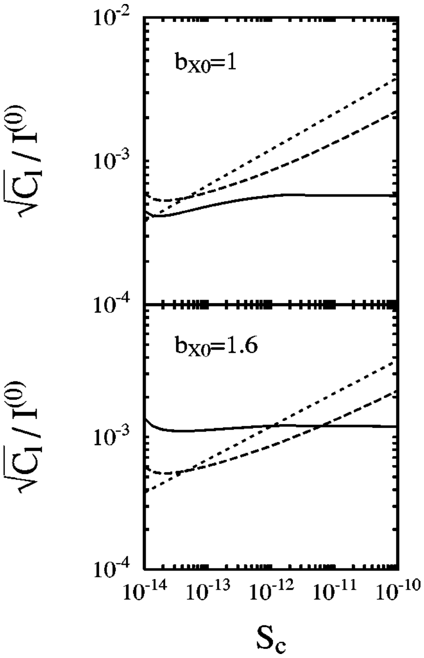

FIG. 3. Comparison of fluctuations due to source clustering and shot noise term as a function of flux cutoff limit S_c. The upper panel is the case of no bias by setting $b_{X0}=1$ in Eq. (3.4). In the lower panel $b_{X0}=1.6$ is chosen as a case of epoch-dependent bias. Each panel shows the multipole $l=100$. The solid line shows the clustering fluctuations and the dashed line does the shot noise fluctuations. For the clustering fluctuations we used the SCDM model with $h=0.5$ and $\Omega_0=1$. Here we normalized the CDM density fluctuations by $\sigma_8=0.5$. In order to calculate the shot noise term, we need to specify the values of e and d, separately. Though there are many uncertainties in the evolution model of the sources, let us take the values $e=1.4$ and $d=1.6$. In this case we have $1+\alpha-e=0$, which allow us some analytic calculations when solving Eq. (2.15). We also used the parameters $L_0=3\times10^{43}$ erg/s, $n_0=10^{-5}$/Mpc^3. We set $z_{\max}$=3.2 in order to be $I^{(0)}=5.2\times10^{-8}$ erg/s/cm^2 [26]. In this case of the parameters the estimation for the shot noise fluctuation is $\sqrt{C_l^{\rm SN}}/I^{(0)}\simeq 0.7 S_c{}^{1/4}/(\text{erg/s/cm}^2)$. The dashed line shows the result obtained by calculating Eq. (3.7). The dotted line is the result obtained by extrapolating the estimation by Lahav *et al.* [2].

$$z_{\min} \simeq \left(\frac{L_0}{S_c}\frac{H_0^2}{4\pi}\right)^{1/2} \simeq 0.03h\left(\frac{L_0}{3\times10^{43}\ \text{erg s}^{-1}}\right)^{1/2} \times\left(\frac{S_c}{3\times10^{-11}\ \text{erg s}^{-1}\ \text{cm}^{-2}}\right)^{-1/2}. \quad (3.12)$$

As we see from Eq. (3.10), the shot noise fluctuations are controlled by the flux cutoff limit S_c. When S_c is decreased the shot noise fluctuations reduce too. As we saw in the previous subsection, the amplitude of the low multipole fluctuations due to the source clustering decreases when S_c is decreased. However, the higher multipoles are constant even when S_c is decreased. The epoch-dependence of the bias factor makes the amplitude of small-scale fluctuations significant. Now let us compare these two fluctuations, the source clustering and the shot noise. In Fig. 3 the fluctuations due to source clustering are compared with the shot noise fluctuations as a function of S_c. In this figure we focused on the multipole of $l=100$. The upper panel is the case of no bias by setting $b_{X0}=1$ in Eq. (3.4). In the lower panel we chose $b_{X0}=1.6$. The large difference between these two panels, which is due to the high bias factor at high redshift, implies that the bias is a very important factor besides the evolution of the luminosity function of sources when predicting the fluctuations due to source clustering.

IV. SUMMARY AND DISCUSSION

In this paper we have formulated equations to investigate the x-ray background fluctuations in various cosmological models in a systematic way. We have made full use of the harmonic expansion method in terms of multipole components in both open (hyperbolic) and flat universes to obtain the angular power spectrum of fluctuations. The fluctuations due to the source clustering and the shot noise can be predicted if the evolution of the luminosity function and the bias factor are given. We have calculated the angular power spectrum for the x-ray background fluctuations in various cosmological models based on a simple x-ray source model. As Lahav *et al.* pointed out [2], the fluctuations predicted in the simple model strongly depend on the evolution parameter of sources, and rather weakly depend on the cosmological models. Moreover the large-angle (low multipole of $l\lesssim 10$) fluctuations are strongly affected by the flux cutoff parameter for removal of nearby bright sources [15,2]. This is because the nearby sources of low redshift are the dominant contributors to the large-angle fluctuations. Therefore when one makes the comparison between the theoretical prediction and the observational data on large angular scales, the procedure of removing bright sources has to be very carefully taken into account.

In this paper, we have developed the formulation which is useful to calculate the angular power spectrum. In the observational point of view, however, we may not have to derive C_l in small scales [6,3]. The description by $\langle C(\theta)\rangle$ is usually used when one compares the theoretical prediction with observational data (e.g., [13,7]). Of course, we can easily derive $\langle C(\theta)\rangle$ from C_l by Eq. (2.29) with a window function. Moreover, our approach is useful to understand the physical mechanism which determines the behaviors of fluctuations on various scales. This formulation is not limited to the x-ray background fluctuations but it can be easily generalized to calculate the clustering of pointlike sources at high redshift.

In predicting the x-ray background fluctuations the evolution of luminosity function is an important factor. Our analysis shows that the evolution of the bias factor is also a very important factor especially on small scales. This is because the small-scale fluctuations contain information of clustering in the high redshift universe. In order to use the x-ray background fluctuations as a probe of large scale cosmological density fluctuations at high redshift, the knowledge of the evolution of the bias factor and the luminosity function is necessary factor, because the dependence of cosmology on the x-ray background fluctuations is rather weak. Conversely, which suggests that the x-ray background is a good probe for the bias mechanism of high redshift sources [27,28] when the cosmological parameters are fixed. The future x-ray mission projects will give us fine solution for the evolution of x-ray sources. In that case the x-ray background will be a possible probe to investigate the clustering and formation process of x-ray sources in the high redshift universe.

ACKNOWLEDGMENTS

One of the authors (K.Y.) thanks Professor Y. Kojima for comments and encouragement. He is grateful to Professor K. Tomita and Professor J. Yokoyama and people in Yukawa Institute for Theoretical Physics, Kyoto University, where part of the work was done. He also thanks Professor Y. Suto, Professor N. Gouda, and Professor O. Lahav for useful comments and discussions. We acknowledge helpful conversations on the subject of this paper with Professor M. Sasaki, Professor K. Mitsuda, Dr. T. Matsubara, Professor T. Ohsugi, N. Sugiura, and Y. Miura. This work was supported by the Grants-in-Aid for Scientific Research of Ministry of Education, Science and Culture of Japan (No. 09740203).

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