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Wave effect in gravitational lensing by a cosmic string

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The wave effect in the gravitational lensing phenomenon by a straight cosmic string is investigated. The interference pattern is expressed in terms of a simple formula. We demonstrate that modulations of the interfered wave amplitude can be a unique signature of the wave effect. We briefly mention a possible chance of detecting the wave effect in future gravitational wave observatories.

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I. INTRODUCTION

The standard model of elementary particle theory involves the mechanism of spontaneous symmetry breaking, which predicts phase transitions in the early Universe. In general, such phase transitions produce topological defects, depending on the symmetry spontaneously broken. The cosmic string is the topological defect produced when a $U(1)$ symmetry is broken [1]. Historically, the cosmic strings were well investigated, motivated by a possible mechanism to explain the structure formation of the Universe [2]. However, this possibility is severely limited or rejected by the observation of large scale structure of the galaxies and the measurements of the cosmic microwave background (CMB) anisotropies [3,4]. Thus the cosmic strings cannot play a central role in structure formation, which constrains the cosmic string energy density. But, we also note that a cosmological model with a non-negligible defect contribution has not been rejected completely. It is argued that the model with a mixture of inflation and topological defects can be compatible with a CMB anisotropy data [5].

On the other hand, other possible roles of the cosmic strings are considered. For example, Sazhin *et al.* have reported detection of the source which is naturally explained by lensing effect of a cosmic string [6]. The gravitational lensing effect due to the cosmic strings has been investigated by several authors [7–11]. Actually the gravitational lensing phenomenon is known as a possible unique probe to detect the cosmic strings. The prospect to detect the cosmic string with wide field galaxy surveys is discussed [12,13]. In the present paper, we investigate the wave effect in the gravitational lensing by the cosmic string. Recently wave effect in gravitational lensing has been investigated, motivated by the gravitational wave observation [14–18]. These works focus on wave effect by isolated lenses (see also [19,20]). However, the wave effect in the lensing by the cosmic string has not been investigated, as far as we know.

This paper is organized as follows: In Sec. II we present a solution of the wave equation on the background spacetime with a straight static cosmic string. Then we show the condition that the interference occurs as a wave effect. In Sec. III, we demonstrate typical numbers of gravitational lensing by the cosmic string. We also discuss a possibility of detecting the interference in the lensing of gravitational waves from a compact binary in Sec. IV. Section V is devoted to conclusions. We use the convention $G=c=1$, otherwise we express them explicitly.

II. WAVE SOLUTION

The properties of a cosmic string in a vacuum spacetime was studied by Vilenkin [21]. He showed that the local nature of the solution is same as that of the Minkowski spacetime, but the conical singularity arises around the cosmic string. This is the origin of the deficit angle $\delta=8\pi\mu$ of the spacetime, where μ is the energy (line) density of the cosmic string. The energy density μ depends on the individual model of the cosmic string, however, the constraint on μ from cosmological observations of the galaxy large scale structure and the cosmic microwave background anisotropies is $\mu \lesssim 10^{-6}$ ([12] and references therein).

We consider the simple configuration (see Fig. 1): A straight string is located parallel to the z axis. The source is located on the coordinate $A=(-L\cos[\delta/2], L\sin[\delta/2],0)$ and $B=(-L\cos[\delta/2],-L\sin[\delta/2],0)$, where both A and B are identical because this spacetime has the conical singularity. Denoting the coordinate of an observer (x,y,z), the distance between $A(B)$ and the observer is

$$r_1=\sqrt{\left(x+L\cos\frac{\delta}{2}\right)^2+\left(y-L\sin\frac{\delta}{2}\right)^2+z^2}, \tag{1}$$

$$r_2=\sqrt{\left(x+L\cos\frac{\delta}{2}\right)^2+\left(y+L\sin\frac{\delta}{2}\right)^2+z^2}, \tag{2}$$

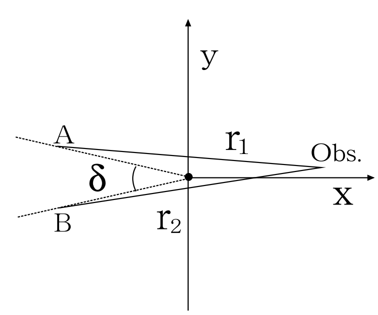

FIG. 1. Configuration of the lensing system: The cosmic string is located parallel to the z axis. This figure shows a projection on the x-y plane. The dashed two lines should be identified due to conical singularity with the deficit angle δ.

respectively. We assume that the amplitude of the wave field is expressed as

$$\mathcal{E}=A_0\frac{\cos(\omega t-kr_1)}{r_1}\Theta(y+x\tan\delta/2)+A_0\frac{\cos(\omega t-kr_2)}{r_2}\Theta(y-x\tan\delta/2), \tag{3}$$

where $\Theta(X)$ is the Heaviside function and A_0 is a constant. The interference occurs in the region $y\geqslant -x\tan\delta/2$ and $y\leqslant x\tan\delta/2$. Within this region we have

$$\mathcal{E}\simeq\frac{2A_0}{r}\cos\left[\left(\omega t-k\frac{r_1+r_2}{2}\right)\right]\cos\left[\frac{k(r_1-r_2)}{2}\right] \tag{4}$$

with $r=\sqrt{(x+L)^2+z^2}$. Thus the amplitude of the wave is determined by the phase $k(r_1-r_2)/2$ because of the interference. Using Eqs. (1) and (2), the path difference is

$$r_1-r_2\simeq 2y\sin[\delta/2]\frac{L}{\sqrt{(x+L)^2+z^2}}=2y\sin[\delta/2]\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}, \tag{5}$$

where we assumed $y\ll z,x,L$, in the last equality, we used θ to denote the angle between the string direction and the line of sight, and $D_{\rm LS}$ and $D_{\rm OS}$ to denote angular diameter distances from observer to source and string to source, respectively. The condition that the wave amplitude has a maximum is

$$k(r_1-r_2)/2=n\pi \tag{6}$$

with integer $n=0,\pm1,\pm2,\dots,\pm n_{\rm max}$, where $n_{\rm max}$ is the largest integer satisfying

$$\pi|n|\leqslant kx\sin^2[\delta/2]\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}. \tag{7}$$

III. TYPICAL NUMBERS

In this section we estimate quantitatively physical numbers of the gravitational lensing system by the cosmic string. It is well known that the angular separation of two images is

$$\Delta\alpha=2\sin[\delta/2]\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}\simeq 2''\left(\frac{\delta}{10^{-5}}\right)\left(\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}\right). \tag{8}$$

The path difference causes time delay, defined by $T_{dl}=|r_1-r_2|/c$. To estimate its typical number, we set $y=(1+z_L)D_{\rm LO}\sin[\delta/2]/2$ as a typical value in Eq. (5), where $D_{\rm LO}$ is the angular diameter distance between the string and observer. Then, we have

$$T_{dl}=\frac{(1+z_L)\sin^2[\delta/2]D_{\rm LO}}{c}\left(\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}\right)\simeq 10^7(1+z_L)\left(\frac{\delta}{10^{-5}}\right)^2\left(\frac{D_{\rm LO}}{cH_0^{-1}}\right)\times\left(\frac{D_{\rm LS}\sin\theta}{D_{\rm OS}}\right)\ \text{sec} \tag{9}$$

with the Hubble constant $H_0=70$ km/s/Mpc.

On the other hand, Eq. (6) is the condition for the maximum peak of the wave. Therefore the wave effect is characterized by the length between two neighboring peaks,

$$\Delta y=\frac{\pi}{k\sin[\delta/2]}\left(\frac{D_{\rm OS}}{D_{\rm LS}\sin\theta}\right)\simeq 3\times10^{15}\left(\frac{\rm Hz}{\nu}\right)\left(\frac{10^{-5}}{\delta}\right)\times\left(\frac{D_{\rm OS}}{D_{\rm LS}\sin\theta}\right)\ \text{cm}, \tag{10}$$

where ν is the frequency of the wave. Thus Δy can be an astronomical distance and the observed amplitude of the wave changes in this periodic interval due to the wave effect. If the string moves in a direction perpendicular to the line of sight direction, the amplitude of the wave will modulate periodically. Assuming that the cosmic string moves in the direction of y axis with a speed near the velocity of light, the typical period of the modulation is roughly estimated as

$$\mathcal{T}_v=\frac{\Delta y}{c}\simeq 10^7\left(\frac{0.01\text{Hz}}{\nu}\right)\left(\frac{10^{-5}}{\delta}\right)\left(\frac{D_{\rm OS}}{D_{\rm LS}\sin\theta}\right)\ \text{sec}. \tag{11}$$

The statistical probability of gravitational lensing by a cosmic string can be estimated as follows: The probability for lensing for a single source at the redshift z_S due to an infinitely long cosmic string located at the redshift z_L is [12]

$$P(z_S,z_L)\sim\frac{2\delta}{\pi^2}\frac{D_{\rm LS}}{D_{\rm OS}}\simeq 2\times10^{-6}\left(\frac{\delta}{10^{-5}}\right)\left(\frac{D_{\rm LS}}{D_{\rm OS}}\right). \tag{12}$$

Thus the probability for a cosmic string is very small. However, numerical simulations of string networks show the possibility that more strings can exist within the horizon effectively [10,22].

IV. WAVE EFFECT IN GRAVITATIONAL WAVES

Now let us consider a gravitational wave from a massive binary with an equal mass M as a source. We assume that the binary angular frequency $\bar{\omega}$ corresponds to the frequency of the gravitational wave ν of an observer by $\bar{\omega}=2\pi\nu(1+z_S)$. The lifetime of the binary system T_{lf} must be longer than the time delay T_{dl} so that the interference occurs. The binary lifetime is estimated [23]

$$T_{lf}\sim 0.02M^{-5/3}\bar{\omega}^{-8/3}\simeq\frac{1.3\times10^5}{(1+z_S)^{8/3}}\left(\frac{M}{M_\odot}\right)^{-5/3}\left(\frac{\nu}{\rm Hz}\right)^{-8/3}\ \text{sec}. \tag{13}$$

Then, the condition $T_{lf}>T_{dl}$ yields

$$M < \frac{120}{(1+z_L)^{3/5}(1+z_S)^{8/5}}\left(\frac{\nu}{0.01\ \ \mathrm{Hz}}\right)^{-8/5}\left(\frac{\delta}{10^{-5}}\right)^{-6/5} \times\left(\frac{D_{\mathrm{LO}}}{cH_0^{-1}}\right)^{-3/5}\left(\frac{D_{\mathrm{LS}}\sin\theta}{D_{\mathrm{OS}}}\right)^{-3/5} M_\odot . \tag{14}$$

The DECIGO planed as a future gravitational wave observatory has an excellent sensitivity around $\nu = 0.1$ Hz [24]. For $\nu = 0.01$ Hz and $\delta = 10^{-5}$, Eq. (14) yields the maximum mass of the binary larger than $1 M_\odot$. Thus the neutron stars (NS-NS) binary can be the source for the wave effect.

Due to emission of the gravitational waves, the binary orbit changes, then the angular frequency of the wave changes. Namely, waves with different angular frequencies interfere because of the time delay effect. This causes another modulation of the wave amplitude as a wave effect, whose period is approximately written as

$$\mathcal{T}_p = \frac{4\pi}{(r_1 - r_2)d\omega/dt} \simeq \frac{1.5\times 10^6}{(1+z_L)(1+z_S)^{5/3}}\left(\frac{\nu}{0.01\ \ \mathrm{Hz}}\right)^{-11/3}\left(\frac{M}{M_\odot}\right)^{-5/3} \times\left(\frac{10^{-5}}{\delta}\right)^{2}\left(\frac{cH_0^{-1}}{D_{\mathrm{LO}}}\right)\left(\frac{D_{\mathrm{OS}}}{D_{\mathrm{LS}}\sin\theta}\right)\ \ \mathrm{sec}. \tag{15}$$

For $\nu \gtrsim 0.01$ Hz and $M \sim M_\odot$, $\mathcal{T}_p$ is shorter than the period of the modulation due to the string motion $\mathcal{T}_v$.

V. CONCLUSION

In this work we have investigated wave effect in gravitational lensing by a cosmic string. Our investigation is limited to the simplest case of the static and straight cosmic string. In this case the interference of the wave field is expressed in a very simple form (5). The condition that the interference occurs is Eq. (6). When the string moves in the direction perpendicular to the line of sight direction, the wave amplitude for an observer will modulate by a wave effect in gravitational lensing. A chirp gravitational wave from a compact binary at a cosmological distance is a possible source in which we might detect the wave effect. However, concerning this source, the change of orbit due to gravitational wave emission causes another modulation of the wave amplitude. These modulations can be a unique signature of the cosmic string when a detector with sufficient angular resolution and sensitivity is assumed. Due to the same mechanism, a similar modulation can appear in a chirp gravitational wave by the gravitational lensing by a galaxy halo [25]. However, in this case, the flux ratio of two waves must be almost 1 for the clear modulation. Namely, a configuration of a lensing system near the Einstein ring must be required. This means that the lensing probability is relatively small and the flux is amplified by the magnification effect. Thus, in principle, it can be discriminated whether the lens causing the modulating signal is a cosmic string or a galaxy halo.

In the present work, we have considered an idealized situation. We have assumed a point source, neglecting the finite size effect of the source. For the gravitational wave from a compact binary, however, the assumption will be plausible. We have considered the interference of two waves with the same amplitude. The change of the orbit of compact binary results in a difference between the two wave amplitudes due to the time delay effect. However, it can be negligible for a low frequency binary $\nu \sim 0.01$ Hz with $M \sim M_\odot$.

Various sources of gravitational waves and their detectability have been considered (e.g., [26] for a recent review). Binaries of massive objects will be the most stable source of a periodic wave, and NS-NS binary is the most promising source. The ground based detectors, such as LIGO and VIRGO, have the sensitivity in the high frequency band, $1\ \mathrm{Hz} \lesssim f \lesssim 10^4\ Hz$, which corresponds to that of gravitational waves from the NS-NS binary at final coalescing stage. On the other hand, the gravitational wave observatories in space, such as LISA and DECIGO, will detect gravitational waves in the low frequency band, $10^{-4}\ \mathrm{Hz} \lesssim f \lesssim 1$ Hz. The modulation will not be observed by the facility with the high frequency band because it appears in stable and periodic gravitational waves. However, the facility with the low frequency band might have a chance to observe the modulation, if the sensitivity was sufficient. The detectability of a signal depends on the distance of a source and a method of data analysis, therefore more details are needed for the definite conclusion. Actually, when the angular resolution is not sufficient, modulations of gravitational waves can appear due to superposition of waves from different individual binary sources.

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