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Analysis of mutual communication between qubits by capacitive coupling

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A behavior of a two-qubit system coupled by the electric capacitance has been studied quantum mechanically. We found that the interaction is essentially the same as the one for the dipole-dipole interaction; i.e., qubit-qubit coupling of the NMR quantum gate. Therefore, a quantum gate could be constructed by the same operation sequence for the NMR device if the coupling is small enough. The result gives an information to the effort of development of the devices assuming capacitive coupling between qubits.

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Toward the realization of quantum computers, various type of devices have been studied intensively in systems such as ion traps [1,2], NMR [3], linear optics [4,5], cavity QED with atoms [6], quantum dots in an optical cavity [7,8], and the Josephson junction [9,10], etc. In terms of the basic physics of the quantum gate, the quantum system must satisfy requirements that (1) the transition between two levels in each qubit has to be controlled independently for both phase and amplitude, (2) qubits must have suitable mutual interaction to construct a quantum gate. It has to be emphasized that the first requirement means that the two-qubit system has to have a function to switch interaction between qubits during the quantum gate operation.

Recently, an observation of Rabi oscillation in a two-qubit system using the Josephson-junction device operating in the charge regime has been reported [11]. It is an encouraging evidence of the existence of the capacitive interaction between qubits. However, the construction of the universal quantum gate is yet to be demonstrated both theoretically and experimentally, i.e., a way of switching interaction between qubits is necessary to be demonstrated. The switching could be realized by either an embedded mechanism in the device or by a sophisticated operation with proper qubit-qubit interaction. The first one is simple and straightforward to understand; however, it may be difficult in the Josephson-junction device in the charge regime or the exciton-photon device [12,13], since the device itself has to have a kind of mechanism to decouple two qubits.

For a later case, a typical and a widely used way by NMR devices is to utilize the dipole-dipole interaction between qubits which can be described by the Hamiltonian as

$$H_{\text{dipole}}=\Omega_i\sigma_x^i+\Omega_j\sigma_x^j+\omega_i\sigma_z^i+\omega_j\sigma_z^j+\omega_{ij}\sigma_z^i\sigma_z^j, \qquad (1)$$

where $\Omega_{i(j)}$ and $\omega_{i(j)}$ are the Rabi oscillation strength and the energy level of the quantum states in the $i(j)$th qubit, while the last term describes dipole-dipole coupling between the ith and jth qubits with the strength of ω_{ij}. The Pauli matrix $\sigma_z^{i(j)}$ stands for the magnetic or the electric dipole operator depending on the devices being considered.

The possibility of multiqubits coupling using Josephson-junction devices with interaction (1) has been discussed in Ref. [9] using an *LC*-oscillator mode coupling between qubits, however, the Josephson-junction device described in Ref. [11] and the exciton-photon device intend to use the capacitive coupling between two qubits rather than the dipole-dipole coupling. In these devices, the excited and the ground state in a qubit are characterized by the difference of the electric charge rather than the direction of the dipole moment so that the interaction of the two qubits is not described by the same Hamiltonian as (1).

The operation of a single qubit by the Josephson-junction devices in the phase regime has been reported [14] and the quantum-mechanical behavior of two-qubit coupling via capacitive coupling in the phase regime has been studied by several authors [15–17]. These works showed the way to construct the universal quantum gate with the Josephson-junction devices in the phase regime.

In this paper, the quantum-mechanical calculation of the behavior of the two-qubit system operating in the charge regime is reported. We show that the system has the same nature with the dipole-dipole coupling, therefore, the same operation with the NMR devices are applicable to construct quantum gates in weak-coupling regime

In order to see the behavior of the capacitive coupling between two qubits, we analyzed the wave function of a two-qubit system in the four-dimensional vector space; $\psi \equiv \varphi_1\otimes\varphi_2$, where the basis of the space is defined explicitly as

$$\begin{pmatrix} |1\rangle \\ |0\rangle \end{pmatrix}_1 \otimes \begin{pmatrix} |1\rangle \\ |0\rangle \end{pmatrix}_2 = \begin{pmatrix} |1\rangle|1\rangle \\ |1\rangle|0\rangle \\ |0\rangle|1\rangle \\ |0\rangle|0\rangle \end{pmatrix}. \qquad (2)$$

The time evolution of each qubit can be described by Schrödinger's equation as $i\hbar(d\varphi_i/dt)=H_i\varphi_i$, where H_i is the Hamiltonian of ith qubit and its explicit form is $H_i=\begin{pmatrix} \Delta_i & a_i \\ \alpha_i & -\Delta_i \end{pmatrix}$ with Δ_i and a_i being the energy level and the Rabi oscillation strength of the qubit. It is assumed, as in all proposed devices, that the Rabi oscillation in the qubit can be controlled by changing the energy level Δ_i via external parameters such as voltages applied to the device. Using these Hamiltonians, the time evolution of the two-qubit system ψ in the four-dimensional space is described as $i\hbar(d\psi/dt)$

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$=(H_1\otimes I+I\otimes H_2+H_{12})\psi$, where H_{12} stands for the interaction between two qubits. As for the H_{12}, we assume that the electric charge appears only for the excited state so that an additional energy is put only when both qubits are in the excited state [$|1\rangle|1\rangle$ state in Eq. (2)] [18]. The Hamiltonian to describe the system is

$$H_{\text{cap}}=\begin{pmatrix} \Delta_1+\Delta_2 & a_2 & a_1 & 0 \\ a_2 & \Delta_1-\Delta_2 & 0 & a_1 \\ a_1 & 0 & -\Delta_1+\Delta_2 & a_1 \\ 0 & a_1 & a_1 & -\Delta_1-\Delta_2 \end{pmatrix} + \begin{pmatrix} \Delta_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

where Δ_{12} in the second term is the coupling energy between two qubits. This assumption contrasts to the dipole-dipole interaction described in Hamiltonian (1). In fact, in the four-dimensional vector space, Hamiltonian (1) can be expressed as

$$H_{\text{dipole}}=\begin{pmatrix} \omega_1+\omega_2 & \Omega_2 & \Omega_1 & 0 \\ \Omega_2 & \omega_1-\omega_2 & 0 & \Omega_1 \\ \Omega_1 & 0 & -\omega_1+\omega_2 & \Omega_2 \\ 0 & \Omega_1 & \Omega_2 & -\omega_1-\omega_2 \end{pmatrix} + \begin{pmatrix} \omega_{12} & 0 & 0 & 0 \\ 0 & -\omega_{12} & 0 & 0 \\ 0 & 0 & -\omega_{12} & 0 \\ 0 & 0 & 0 & \omega_{12} \end{pmatrix}, \tag{4}$$

where the second term is a four-dimensional expression of the dipole-dipole interaction. We see the coupling energies are added symmetrically to the diagonal elements of the Hamiltonian and they also change their sign as the direction of a dipole flips. This feature plays an essential role to switchoff the interaction between qubits effectively by the refocusing operation.

In order to further see characteristics of the capacitive coupling, it is useful to rewrite Hamiltonian (3) in a two-component form as

$$H'_{\text{cap}}=\frac{\Delta_{12}}{4}I+a_1\sigma_x^1+a_2\sigma_x^2+\left(\Delta_1+\frac{\Delta_{12}}{4}\right)\sigma_z^1 + \left(\Delta_2+\frac{\Delta_{12}}{4}\right)\sigma_z^2+\frac{\Delta_{12}}{4}\sigma_z^1\sigma_z^2. \tag{5}$$

A comparison of Hamiltonians (1) and (5) clearly shows similarities and differences of the two couplings schemes. Both of the two have the same type of dipole coupling term as seen in the last term of the Hamiltonians. On the other hand, in Hamiltonian (5), the coupling energy Δ_{12} is added to the energy level of each state as shown in the fourth and the fifth term. As a result, the energy level $E_{1(2)}$ of the quantum state in the 1(2)th qubit can be expressed as

$$E_{1(2)}=\Delta_{1(2)}+\Delta_{12}/4\pm\Delta_{12}/4, \tag{6}$$

where the first two terms represent modified but fixed energy level of the quantum state in the qubit, while the last term is the contribution from the dipole-type coupling in Hamiltonian (5). The sign of the last term depends on the relative states of the two qubits. This fact shows that interaction of the capacitive coupling can be described essentially by the same form with the dipole-dipole couplings. Therefore, it may be possible to perform the same quantum gate operations that have been applied on devices of the dipole-dipole coupling such as NMR devices.

To realize the operation discussed above, the most important condition on the parameters is the strength of the qubit-qubit coupling Δ_{12}. Since the energy level of the quantum state in the qubit $E_{1(2)}$ depends on the state of the neighboring qubit as expressed by the $\pm$ sign in Eq. (6), the quantum state of a qubit affects the condition of Rabi oscillation of neighboring qubit. This fact does not allow the independent control of each qubit. However, if the Δ_{12} is smaller enough than the $a_{1(2)}$, the condition of Rabi oscillation can be virtually independent of neighboring qubit as typical width of Rabi resonance is its strength $a_{1(2)}$.

It has to be remembered that the situation described above is the same for the interaction with Hamiltonian (1); i.e., for NMR devices. The quantum operations demonstrated using the NMR devices always have been performed in the weak-coupling regime. Since the spin-spin coupling of the NMR devices is so small the condition of weak coupling has been satisfied without any special treatment.

It is now known that the capacitive coupling interaction also has the dipole-dipole feature as shown in Hamiltonian (5), so that devices with capacitive coupling could be operative as a universal quantum gate utilizing the same operation sequence on the NMR devices.

In order to see feasibility of the quantum computation with weak capacitive coupling, we performed a numerical calculation of a quantum gate operation using Hamiltonian (3). As an example of the two-qubit operation, we tried a controlled-NOT operation by the procedure commonly used in the NMR devices [19], which is schematically expressed in Fig. 1. It has to be noted that even though it is a single controlled-NOT operation, it consists of all necessary operations for general quantum gate operations. The controlled-NOT operation described in Fig. 1 is expressed in terms of a unitary transformation as

$$\psi_i\Rightarrow\psi_f=\sqrt{-i}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\psi_i, \tag{7}$$

after subtracting overall phase factor. In the calculation, the initial state was chosen as $\psi_i=(1,0,0,0)$ on the basis shown

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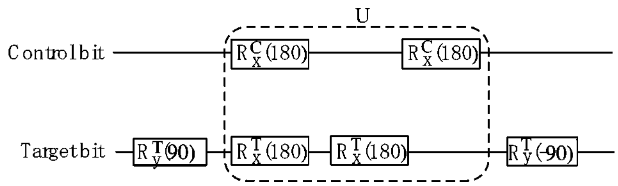

FIG. 1. The controlled-NOT operation for an NMR device described in Ref. [19]. $R_i^{C(T)}(\theta)$ stands for θ rotation around the i axis on the control (C) or target (T) bit. The first and the last $R_y^T(90)$ stand for Rabi oscillation of 90° while a series of operation denoted as U is a phase operation on the qubits, showing that the operation includes all components necessary to construct the general quantum operations.

in Eq. (2) and the expected final state is $\psi_f = (0, e^{-1/4\pi}, 0, 0)$. As the result of the calculation, we plotted, in Fig. 2, the amplitude and the phase of the $|1\rangle|0\rangle$ state as a function of Δ_{12} normalized to $a_{1(2)}$, where we expect 1.0 and $-1/4\pi$ for the amplitude and the phase, respectively. In the calculation, we also have to consider treatment of the Rabi oscillation strength $a_{1(2)}$ in a qubit. Depending on the devices, we can assume that parameter $a_{1(2)}$ exists throughout the operation or appears only when the device is on Rabi oscillation. Since these two different conditions for $a_{1(2)}$ could affect the precision of the quantum gate operation, we evaluated both cases.

It can be concluded that the results are reasonably close to the expected values and are stable up to $\Delta_{12} \approx 0.1 a_{1(2)}$. As for the treatment of $a_{1(2)}$, some deviation from the ideal value is seen if the $a_{1(2)}$ is on throughout the calculations, particularly for the phase of the state. It is preferable to control $a_{1(2)}$ as is realized in the NMR devices; however, the deviation appears to be at an acceptable level. It is also worthwhile mentioning that $\Delta_{12} \approx 0.1 a_{1(2)}$ is much larger than those of typical NMR devices. It indicates that the devices with capacitive coupling potentially have advantage of larger signal than those of the NMR devices.

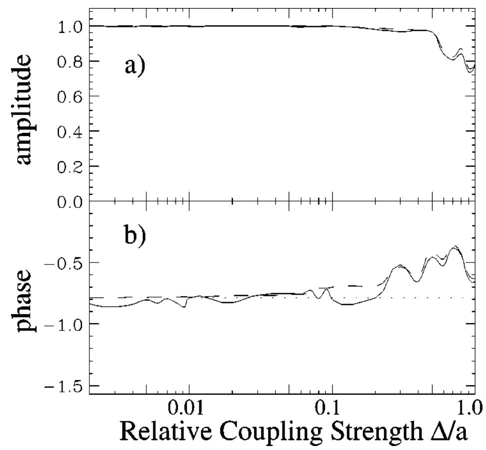

FIG. 2. Δ_{12} dependence of the amplitude (a) and phase (b) of the $|1\rangle|0\rangle$ state. The expected value for the amplitude is 1.0 and for the phase is $-1/4\pi$ (indicated by dots), respectively. The solid line is for the case in which the Rabi oscillation strength $a_{1(2)}$ is on throughout the operation and the dashed line is for the case in which $a_{1(2)}$ is on only for the Rabi oscillation stage.

In summary, we analyzed the interaction between two charge qubits via capacitive coupling. We conclude that the coupling is essentially the same as the dipole-dipole-type interaction and the device can be operative as a quantum gate if the coupling is smaller enough than the Rabi oscillation strength. This fact gives important information for the development of those devices such as Josephson-junction or exciton-photon devices.

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[1] C. Monroe, D. Meekhof, B. King, W. Itano, and D. Wineland, Phys. Rev. Lett. **75**, 4724 (1995).
[2] C. Sackett, D. Kielpinski, B. King, C. Langer, V. Meyer, C. Myatt, Q. Turchette, W.M. Itano, D.J. Wineland, and C. Monroe, Nature (London) **404**, 256 (2000).
[3] L. Vandersypen, M. Steffen, G. Breyta, C. Yannoni, M. Sherwood, and I.L. Chuang, Nature (London) **414**, 883 (2001).
[4] E. Knill, R. Lafalamme, and G. Milburn, Nature (London) **409**, 46 (2001).
[5] S. Takeuchi, Phys. Rev. A **62**, 032301 (2000).
[6] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, and H.J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).
[7] M.S. Sherwin, A. Imamoglu and T. Montroy, Phys. Rev. A **60**, 3508 (1999).
[8] X.Q. Li and Y.J. Yan, Phys. Rev. B **65**, 205301 (2002).
[9] Y. Makhlin, G. Schon, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[10] Y. Nakamura, Yu. Pashkin, and J. Tsai, Nature (London) **398**, 786 (1999).
[11] Yu. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. Averin, and J. Tsai, e-print cond-mat/0212314.
[12] M. Yamanishi and Y. Kadoya, in *Proceedings of the Third Conference on Quantum Information and Technology*, edited by N. Inomoto (Institute of Electronics, Information and Communication Engineers, Tokyo, 2000).
[13] Y. Hokomoto, Y. Kadoya, and M. Yamanishi, Appl. Phys. Lett. **74**, 3839 (1999).
[14] J.M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).
[15] A. Blais, A.M. van den Brink, and A.M. Zagoskin, e-print cond-mat/0207112.
[16] P.R. Johnson, F.W. Strauch, A.J. Dragt, R.C. Ramos, C.J. Lobb, J.R. Anderson, and F.C. Wellstood, Phys. Rev. B **67**, 020509 (2003).

[17] F.W. Strauch, P.R. Johnson, A.J. Dragt, C.J. Lobb, J.R. Anderson, and F.C. Wellstood, e-print quant-ph/0303002.
[18] In general, the difference of the electric charge between the excited and the ground state is essential and it is not necessary for the ground state to be electrically neutral. However, it can be absorbed by the constant energy shift in the diagonal element of the Hamiltonian without losing the accuracy of the discussion.
[19] N. Gershenfeld and I. Chuang, Science (Washington, DC, U.S.) **275**, 350 (1997).