ARTICLES

Measurement of the B hadron energy distribution in Z^0 decays

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We have measured the B hadron energy distribution in Z^0 decays using a sample of semileptonic B decays recorded in the SLD experiment at SLAC. The energy of each tagged B hadron was reconstructed using information from the lepton and a partially reconstructed charm-decay vertex. We compared the scaled energy distribution with several models of heavy quark fragmentation. The average scaled energy of primary B hadrons was found to be $\langle x_{E_B} \rangle = 0.716 \pm 0.011(\mathrm{stat})^{+0.021}_{-0.022}(\mathrm{syst})$. [S0556-2821(97)03221-9]

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I. INTRODUCTION

The production of heavy hadrons (H) in e^+e^- annihilation provides a laboratory for the study of heavy-quark (Q) jet fragmentation. This is commonly characterized in terms of the observable $x_{E_H} \equiv 2E_H/\sqrt{s}$, where E_H is the energy of a B or D hadron containing a b or c quark, respectively, and $\sqrt{s}$ is the c.m. energy. In contrast with light-quark jet fragmentation one expects [1] the distribution of x_{E_H}, $D(x_{E_H})$, to peak at an x_{E_H}value significantly above 0. Since the hadronization process is intrinsically nonperturbative $D(x_{E_H})$ cannot be calculated directly using perturbative quantum chromodynamics (QCD). However, the distribution of the closely related variable $x_{E_Q} \equiv 2E_Q/\sqrt{s}$ can be calculated perturbatively [2–4] and related, via model-dependent assumptions, to the observable quantity $D(x_{E_H})$; a number of such models of heavy quark fragmentation have been proposed [5–7]. Measurements of $D(x_{E_H})$ thus serve to constrain both perturbative QCD and the model predictions. Furthermore, the measurement of $D(x_{E_H})$ at different c.m. energies can be used to test QCD evolution, and comparison of $D(x_{E_B})$ with $D(x_{E_D})$ can be use to test heavy quark symmetry [8]. Finally, the uncertainty on the forms of $D(x_{E_D})$ and $D(x_{E_B})$ must be taken into account in studies of the production and decay of heavy quarks, see, e.g., [9]; more accurate measurements of these forms will allow increased precision in tests of the electroweak heavy-quark sector.

Here we consider measurement of the B hadron scaled energy distribution $D(x_{E_B})$ in Z^0 decays. Earlier studies [10] used the momentum spectrum of the lepton from semileptonic B decays to constrain the mean value $\langle x_{E_B} \rangle$ and found it to be approximately 0.70; this is in agreement with the results of similar studies at $\sqrt{s}=29$ and 35 GeV [11]. In more recent analyses [12,13] the scaled energy distribution $D(x_{E_B})$ has been measured by reconstructing B hadrons via their $B \rightarrow DlX$ decay mode; we have applied a similar technique. We used the precise SLC Large Detector (SLD) tracking system to select jets containing a $B \rightarrow DlX$ decay, where the charmed hadron D was identified semiinclusively from a secondary decay vertex formed from charged tracks. Each hadronic vertex was then associated with a lepton l ($l=e$ or μ) with large momentum transverse to the jet direction. Neutral energy depositions measured in the hermetic calorimeter, as well as the energies of charged tracks, that were not associated with the Dl system were subtracted from the jet energy to yield the reconstructed B hadron energy. This measurement technique may be useful to B-lifetime or B-mixing analyses [14] where the proper time $t=L/\sqrt{\gamma^2-1}$, where $\gamma = E_B/m_B$, m_B is the B hadron mass and L is the decay length, must be known accurately. We then compared the B energy distribution with the perturbative QCD and phenomenological model predictions.

II. APPARATUS AND HADRONIC EVENT SELECTION

The e^+e^- annihilation events produced at the Z^0 resonance by the SLAC Linear Collider (SLC) were recorded using the SLC Large Detector (SLD). A general description of the SLD can be found elsewhere [15]. This analysis used charged tracks measured in the central drift chamber (CDC) [16] and in the vertex detector (VXD) [17], energy clusters measured in the liquid argon calorimeter (LAC) [18], and muons measured in the warm iron calorimeter (WIC) [19]. Electron identification utilizes CDC tracks and LAC clusters [20].

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Momentum measurement is provided by a uniform axial magnetic field of 0.6 T. The CDC and VXD give a momentum resolution of $\sigma_{p_\perp}/p_\perp = 0.01 \oplus 0.0026 p_\perp$, where $p_\perp$ is the track momentum transverse to the beam axis in GeV/c. Including the uncertainty on the primary interaction point (IP), the resolution on the charged-track impact parameter (d) projected in the plane perpendicular to the beamline is $\sigma_d = 11 \oplus 70/(p_\perp \sqrt{\sin\theta})$ μm, where θ is the polar angle with respect to the beamline. This results in a mean resolution on reconstructed two-prong vertices (Sec. III) of $\sigma_{V_{\parallel(\perp)}} = 400(25)$ μm for the projection on an axis along (perpendicular to) the vertex flight direction. The LAC electromagnetic energy scale was calibrated from the measured $\pi^0 \to \gamma\gamma$ signal [21,22]; the electromagnetic energy resolution is $\sigma_E/E \approx 0.15\sqrt{E(\mathrm{GeV})}$.

The trigger and initial selection of hadronic events are described in [23]. A set of cuts was applied to the data to select well-measured tracks and events well contained within the detector acceptance. Charged tracks were required to have a distance of closest approach transverse to the beam axis within 5 cm, and within 10 cm along the axis from the measured interaction point, as well as $|\cos\theta| < 0.80$, and $p_\perp > 0.15$ GeV/c. Events were required to have a minimum of seven such tracks, a thrust axis [24] polar angle θ_T within $|\cos\theta_T| < 0.70$, and a charged visible energy E_{vis} of at least 20 GeV, which was calculated from the selected tracks assigned the charged pion mass. From our 1993–1995 data sample 108 650 events passed these cuts. The efficiency for selecting hadronic events satisfying the $|\cos\theta_T|$ cut was estimated to be above 96%. The background in the selected event sample was estimated to be $0.1 \pm 0.1\%$, dominated by $Z^0 \to \tau^+\tau^-$ events.

Calorimeter clusters used in the subsequent jet-finding analysis (Sec. IV) were required to comprise at least two calorimeter towers, each containing an energy of at least 100 MeV, and to have a total energy greater than 250 MeV. Electromagnetic clusters used in the non-B-associated neutral energy measurement were further required to have less than the smaller of, 25% of their energy and 600 MeV, in the hadronic section of the LAC.

The efficiency for reconstructing B hadrons, the background in the selected sample, and the resolution of the method were evaluated (Secs. III and IV) using a detailed Monte Carlo (MC) simulation. The JETSET 7.4 [25] event generator was used, with parameter values tuned to hadronic e^+e^- annihilation data [26], combined with a simulation of B decays tuned to $\Upsilon(4S)$ data [27] and a simulation of the SLD based on GEANT 3.21 [28]. Inclusive distributions of single particle and event topology observables in hadronic events were found to be well described by the simulation [29]. There is now evidence that roughly 21% of all promptly produced B hadrons in $Z^0 \to b\bar{b}$ events are B^{**} mesons [30]; since JETSET does not produce B^{**} mesons we have corrected the simulation to account for them. Using an event weighting technique we produced a generator-level distribution of B hadron energies in which the energy E_B of 20.7% of all B hadrons was adjusted to be $E_B - E_\pi$, where the pion energy E_π was produced according to an isotropic two-body decay distribution for $B^{**} \to B\pi^\pm$, assuming a B^{**} mass of 5.7 GeV/c^2. Uncertainties in this simulation of B^{**} production were taken into account in the systematic errors (Sec. VII).

III. *B* HADRON SELECTION

Hadronic events were required to contain a lepton candidate within the barrel tracking system with $|\cos\theta| < 0.7$. We then applied the JADE jet-finding algorithm [31] to the LAC clusters in each selected event to define a jet topology. With a jet-resolution criterion of $y_c = 0.07$, 82.9% of the events were classified as two-jet-like and 17.1% as three-jet-like. Kinematic information based on this topological classification was used subsequently (Sec. IV) in the calculation of the B hadron energy. Events in which the lepton had a transverse momentum with respect to its jet axis, p_t, of at least 1 GeV/c were retained for further analysis. In jets containing more than one such lepton only the highest-p_t lepton was labeled for association with a D vertex and any lower-momentum leptons were used in the D-vertex finding.

In each selected jet we then searched for a secondary D vertex among the nonlepton tracks. Tracks were required to comprise at least 40 CDC hits and one VXD hit, to be well contained within the CDC with $|\cos\theta| \leq 0.70$, to have momentum in the range $0.15 < p < 55$ GeV/c, and to have a transverse impact parameter, normalized by its error, of $d/\sigma_d > 1$. Tracks from K_s^0 and Λ^0 decays and γ conversions were suppressed by requiring the distance of closest approach to the IP in the planes both perpendicular to, and containing, the beamline to be less than 1 cm. Two-prong vertices were first formed from all pairs of tracks whose

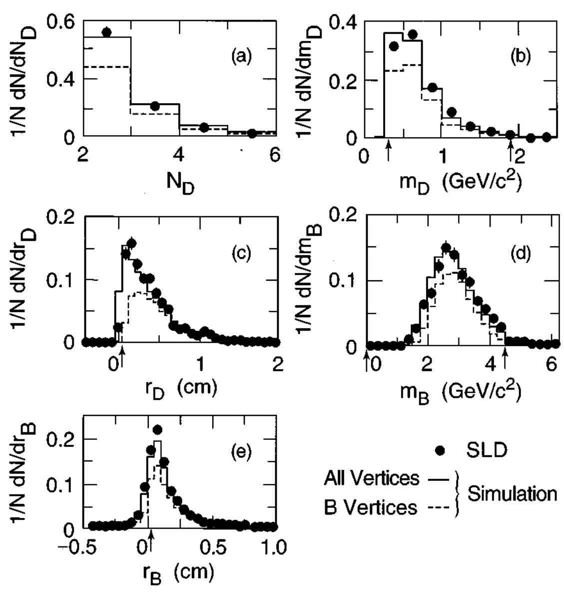

FIG. 1. Candidate D-vertex distributions: (a) number of tracks per vertex; (b) vertex mass; (c) projection of the vertex flight distance from the IP along the jet axis. Candidate B-vertex distributions: (d) vertex mass; (e) projection along the D-vertex momentum vector of the vector between the D vertex and the B vertex. Data (points with error bars) and simulation (solid histogram); the dashed histogram shows the simulated contribution from true $B \to D l X$ decays. In (a) all cuts were applied. In (b)–(e) all cuts were applied except those on the quantity shown, and these latter cut positions (see text) are indicated by arrows.

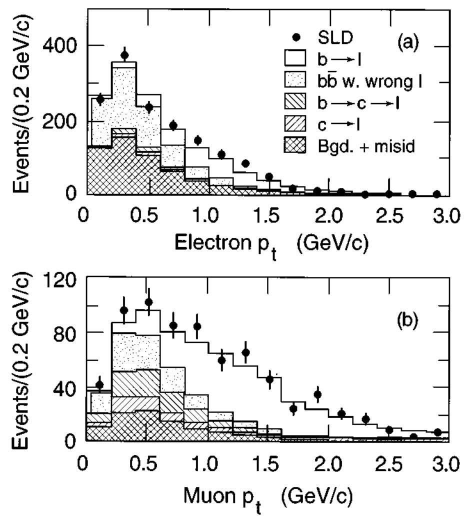

FIG. 2. Distribution of (a) electron and (b) muon transverse momentum with respect to the jet axis in jets containing a selected D vertex and respective lepton. Data (points with error bars) and simulation (histogram). The composition of the simulated distributions in terms of leptons from $B \to l$ decays, cascade $B \to C \to l$ decays, wrongly assigned leptons, promptly produced $C \to l$ decays, and fake leptons is indicated.

distance of closest approach was less than 0.012 cm and whose fit to a vertex satisfied $\chi^2 < 5$. A multiprong D-vertex candidate was then defined to comprise the tracks in all accepted two-prong vertices in the jet, and to be located at the position of the two-prong vertex containing the track with the largest normalized transverse impact parameter d/σ_d.

The tracks in each D vertex were each assigned the charged pion mass and were then combined by adding their four-vectors to obtain the vertex invariant mass m_D and the vertex momentum vector. The vertex flight distance from the IP was projected onto the jet axis to obtain the quantity r_D. Events were retained if at least one jet contained a D vertex with $0.3 < m_D < 1.9$ GeV/c^2, $r_D > 0.05$ cm, r_D normalized by its error larger than unity, and the distance of closest approach between the lepton track and the extrapolated D-vertex momentum vector was less than 0.012 cm. The lepton and D-vertex tracks were then fitted to a common candidate B vertex. The combined D-vertex and lepton invariant mass m_B and the projection of the vector between the B- and D-vertex positions onto the D-vertex momentum vector r_B were calculated. Events were selected in which $m_B < 4.5$ GeV/c^2, $r_B > 0.025$ cm, and r_B normalized by its error was larger than unity.

For the selected events, distributions of the number of tracks per D vertex N_D and of m_D, r_D, m_B, and r_B are shown in Fig. 1. Also shown are the simulated distributions in which the contribution from selected true $B \to DlX$ decays is indicated. In Fig. 2 the distribution of lepton transverse momentum with respect to the jet axis p_t are shown for candidates passing all cuts except the requirement that p_t be above 1 GeV/ c; the simulated distributions are also shown, and the contributions from different processes are indicated. The final sample comprises 597 events, 293 in the muon, and 304 in the electron, channels. Using the simulation we estimate that the purity of this sample, defined to be the fraction of the tagged events whose identified leptons l are from true $B \to DlX$ decays, is 69.2%; a further 18% of the selected events contain B decays with a cascade, punch-through or misidentified lepton, and are still useful. The estimated composition of the $b\bar{b}$ events in terms of the B hadron species is shown in Table I. The remaining 12.8% of the event sample comprises non-$b\bar{b}$ events. The efficiency for selecting B hadron decays in the selected hadronic event sample is shown, as a function of x_{E_B}, in Fig. 3; the overall efficiency is 1.1%.

TABLE I. The composition C of true $B \to DlX$ decays in the final sample; ϵ is the fraction of each species whose D vertices are correctly reconstructed. In all cases the MC statistical errors are less than 2%.

B species	C (%)	ϵ (%)
B_u	43	92
B_d	43	87
B_s	10	89
B baryons	4	87

IV. MEASUREMENT OF THE B ENERGIES

In each selected event we first defined the jet energies by using kinematic information. The two-jet events were divided into two hemispheres by the plane normal to the thrust axis and the jet in each hemisphere was assigned the beam energy. For the three-jet events we corrected the jet energies according to the angles between the jet axes, assuming energy and momentum conversation and massless kinematics. Labeling the jets arbitrarily 1, 2, and 3, and the corresponding interjet angles θ_{23}, θ_{13}, and θ_{12}, the corrected energy of jet 1 is given by

$$E_1 = \sqrt{s}(\sin\theta_{23})/(\sin\theta_{12} + \sin\theta_{23} + \sin\theta_{13}), \tag{1}$$

with corresponding expressions for jets 2 and 3. This procedure results in improved jet energy resolution.

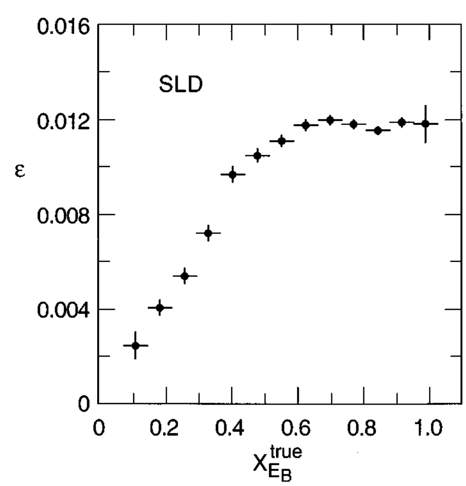

FIG. 3. The efficiency ϵ for selecting B-hadron decays, as a function of scaled energy x_{E_B}. Note that the first bin (no point shown) is beneath the kinematic limit for x_{E_B}.

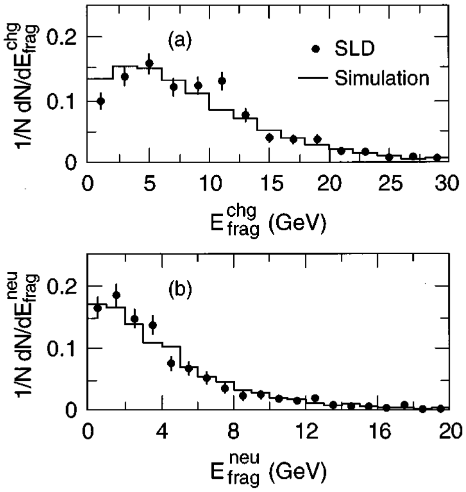

FIG. 4. Distribution of non-B-associated (a) charged and (b) neutral energy in jets containing a candidate $B \rightarrow DlX$ decay. Data (points with error bars) and simulation (histogram).

We then proceeded to reconstruct the B hadron energy E_B^{rec}:

$$E_B^{rec} = E_{jet} - E_{frag}, \tag{2}$$

where E_{jet} is the energy of the jet containing the candidate B vertex and E_{frag} is the energy in the same jet that is not attributed to the B,

$$E_{frag} = f^{chg} E_{frag}^{chg} + f^{neu} E_{frag}^{neu} \tag{3}$$

where E_{frag}^{chg} and E_{frag}^{neu} are the measured charged and neutral energy components respectively, and f^{chg} and f^{neu} are correction factors described below. We define E_{frag}^{chg} to be the sum of the energy, using the momentum and assuming the pion mass, of all the charged tracks in the jet excluding the candidate B-vertex tracks; E_{frag}^{neu} is defined to be the sum of the energy of the electromagnetic calorimeter clusters in the jet that are not associated with charged tracks. A cluster was defined as unassociated if it had no charged track extrapolating to it to within an angle $4\sigma_{cl}$ from its centroid, where $\sigma_{cl} = \sqrt{\sigma_{cl}^{\theta 2} + \sigma_{cl}^{\phi 2}}$ and σ_{cl}^{θ} and σ_{cl}^{ϕ} are the measured cluster widths in polar and azimuthal angle, respectively. The distributions of E_{frag}^{chg} and E_{frag}^{neu} are shown in Fig. 4.

This procedure will *a priori* misassign the energy of any unassociated neutral particle from the D decay to the non-B energy E_{frag}. Similarly, the energy of any charged track from the D decay that is not associated with the reconstructed D vertex will be misassigned to E_{frag}. We have used our MC simulation to study these effects and show in Fig. 5 the correlation between the reconstructed and true values of E_{frag}^{neu} and E_{frag}^{chg}. As expected, both the charged and neutral components are typically slightly overestimated by the reconstruction method. We fitted an *ad hoc* second-order polynomial to each correlation to determine an average energy-dependent correction factor, f^{chg} (f^{neu}) (Eq. 3), which we applied to the non-B charged (neutral) energy component E_{frag}^{chg} (E_{frag}^{neu}) of each tagged jet in the data sample. Uncertainties in these corrections were included in the systematic errors (Sec. VII).

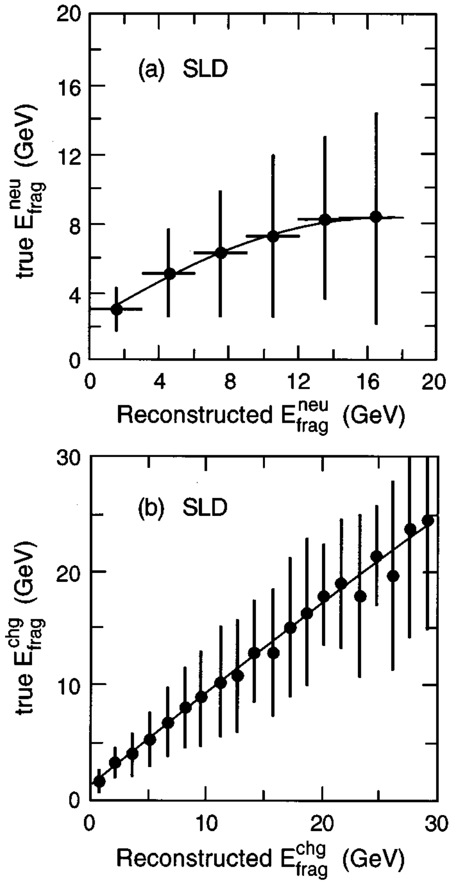

FIG. 5. Simulated correlation between the true and reconstructed values of the non-B-associated (a) neutral and (b) charged energy in jets containing a candidate $B \rightarrow DlX$ decay. In each bin of reconstructed energy the error bar represents the corresponding r.m.s. deviation in the true energy. Each line represents a fit to the correlation (see text).

We have used our simulation to estimate the resolution of the method for reconstructing the B hadron energy. We compared the reconstructed scaled B energy $x_{E_B}^{rec}$ with the input scaled energy $x_{E_B}^{true}$ and show in Fig. 6 the distribution of the quantity $(x_{E_B}^{true} - x_{E_B}^{rec})/x_{E_B}^{true}$. The resolution may be characterized by a parametrization comprising the sum of two Gaussian distributions. The result of such a fit, in which the Gaussian centers, normalizations and widths were allowed to vary, is shown in Fig. 6. The narrower Gaussian of width $\sigma = 0.10$ represents 65% of the fitted area, and the wider Gaussian of width $\sigma = 0.33$ represents the remainder. It can be seen from Fig. 6 that the population corresponding to the "inner core" is somewhat underestimated by this technique since the parametrization does not describe the central bin. We repeated this exercise in subset regions of $x_{E_B}^{true}$ and found the inner core resolution (population) to be 0.27 (84%) for $0.0 < x_{E_B}^{true} < 0.6$, 0.09 (70%) for $0.6 < x_{E_B}^{true} < 0.8$, and 0.06 (79%) for $0.9 < x_{E_B}^{true} < 1.0$; as expected the resolution is better for more energetic B hadrons. Choosing the bin width to be roughly half of our mean resolution we show the measured distribution of $x_{E_B}^{rec}$, $D^{data}(x_{E_B}^{rec})$, in Fig. 7. Also shown in this figure

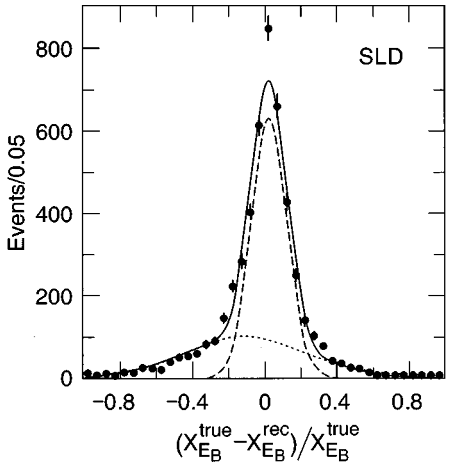

FIG. 6. Distribution of the normalized difference between the true and reconstructed B-hadron energies in simulated events. The solid line is a fit of the sum of two Gaussian distributions (see text). The two component Gaussian distributions are indicated by the dashed lines.

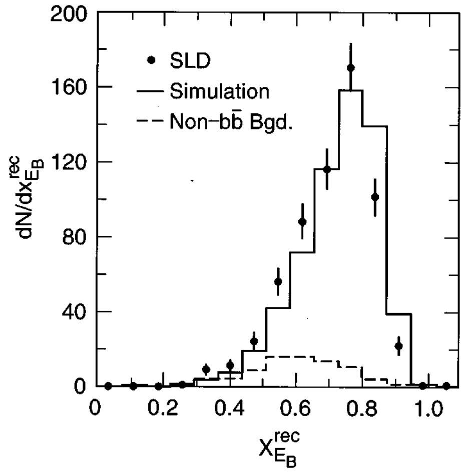

FIG. 7. The distribution of reconstructed scaled energies for B-hadron candidates; data (points with error bars) and simulation (solid histogram). Also shown (dashed histogram) is the simulated contribution from non-$b\bar{b}$ events.

is the simulated distribution in which the background contribution from non-$b\bar{b}$ events is indicated.

V. COMPARISON WITH MODEL PREDICTIONS

It is interesting to compare our measured B hadron energy distribution with the theoretical predictions. The event generator used in our simulation is based on a perturbative QCD ''parton shower'' for production of quarks and gluons, together with the phenomenological Peterson function [6] (Table II) to account for the fragmentation of b and c quarks into B and D hadrons, respectively, within the iterative Lund string hadronization mechanism [25]; this simulation yields a generator-level primary B-hadron energy distribution[1] with $\langle x_{E_B}\rangle = 0.693$. It is apparent (Fig. 7) that this simulation does not reproduce the data well; the χ^2 for the comparison is 36.7 for 15 bins.

We have also considered alternative forms of the fragmentation function based on the phenomenological model of the Lund group [7], the perturbative QCD calculations of Braaten, Cheung, Fleming, and Yuan [4] (BCFY), and of Nason, Colangelo, and Mele [2] (NCM), as well as *ad hoc* parametrizations based on a function used by the ALEPH Collaboration [12] and on a third-order polynomial. These functions are listed in Table II.

In order to make a fair comparison among the models we varied the arbitrary parameter(s) of each function so as to achieve an optimal description of the data; this was done by applying an iterative procedure to our simulated event sample. First, starting values of the parameters were assigned and the corresponding distribution of scaled primary B hadron energies $D^{MC}(x_{E_B}^{true})$ was reproduced in our MC-generated $b\bar{b}$ event sample, *before* simulation of the detector, by weighting events accordingly [32]. Next, the resulting distribution, after simulation of the detector, application of the analysis cuts and background subtraction, of reconstructed B hadron energies D^{MC} $(x_{E_B}^{rec})$ was compared with

TABLE II. Fragmentation functions used in comparison with the data. For the BCFY function $f_1(r) = 3(3-4r)$, $f_2(r) = 12-23r+26r^2$, $f_3(r) = (1-r)(9-11r+12r^2)$, and $f_4(r) = 3(1-r)^2(1-r+r^2)$.

Function name	Functional form $D(x)$	Reference
Peterson	$\frac{1}{x}\left(1-\frac{1}{x}-\frac{\epsilon_b}{1-x}\right)^{-2}$	[6]
Lund	$\frac{1}{x}(1-x)^a \exp(-bm_T^2/x)$	[7]
BCFY	$\frac{x(1-x^2)}{[1-(1-r)x]^6}[3-xf_1(r)+x^2f_2(r)-x^3f_3(r)+x^4f_4(r)]$	[4]
NCM	$\int dy\ g(x,y)y^{\alpha}(1-y)^{\beta}$	[2]
ALEPH	$\frac{1+b(1-x)}{x}\left(1-\frac{c}{x}-\frac{d}{1-x}\right)^{-2}$	[12]
3rd-order polynomial	$1+bx+cx^2+dx^3$	

[1]We used a value of the Peterson function parameter $\epsilon_b = 0.006$ [32].

TABLE III. Results of optimization of fragmentation functions to the reconstructed B hadron energy distribution. For the NCM fit the QCD parameters were fixed at $\Lambda_f = 200$ MeV and $\mu = m_b = 4.5$ GeV.

Function	χ^2/N_{DF}	parameters	$\langle x_{E_B} \rangle$
Peterson	14.0/11	$\epsilon_b = 0.034 \pm 0.006$[a]	0.717
Lund	9.6/10	$a = 1.7 \pm 0.2$	0.743
		$b = 0.19 \pm 0.01$	
BCFY	22.4/11	$r = 0.20 \pm 0.02$	0.705
NCM	15.9/11	$\alpha = 9 \pm 2$	0.687
		$\beta = 44 \pm 8$	
ALEPH	9.7/9	$b = 0.0 \pm 1.0$	0.730
		$c = 0.78 \pm 0.05$	
		$d = 0.042 \pm 0.004$	
3rd-order polynomial	14.9/9	b$= -7.53 \pm 0.04$	
		$c = 16.49 \pm 0.07$	
		$d = -9.98 \pm 0.07$	

[a]This value of ϵ_b refers to the B-hadron energy distribution; it should not be confused with the value of ϵ_b used as input in the JETSET model at the b-quark fragmentation level (Sec. V), which is significantly lower.

the background-subtracted data distribution and the χ^2 value was calculated. The parameter values were then changed, the weighting process repeated in the simulated sample, and the new distribution of reconstructed B hadron energies compared with the data to yield a new χ^2 value. This process was iterated to find the minimum in χ^2, yielding a parameter set that gives an optimal description of the reconstructed data by the input fragmentation function. This procedure was applied for each function listed in Table II. The fitted parameters and minimum χ^2values are listed in Table III, and the corresponding $D^{\rm MC}$ $(x_{E_B}^{\rm rec})$are compared with the data in Fig. 8. Each function reproduces the data. We conclude that, within our resolution and with our current data sample, we are unable to distinguish between these functions. It should be noted, however, that the optimal third-order polynomial function has a small negative minimum point in the region around $x_{E_B}^{\rm true} = 0.2$; since this behavior is unphysical we did not consider this function further in the analysis.

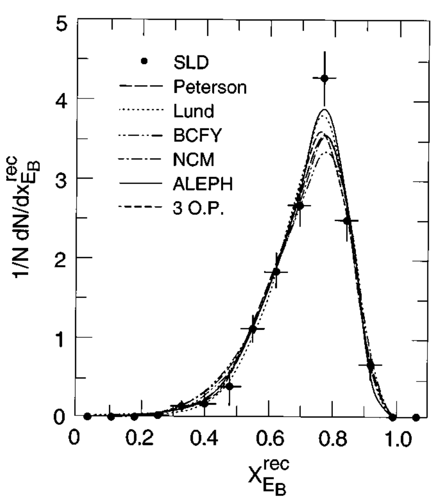

FIG. 8. The background-subtracted distribution of reconstructed scaled B-hadron energy. The data (points with error bars) are compared with simulations based on six different input B-fragmentation functions (see text) represented by lines joining entries at the bin centers.

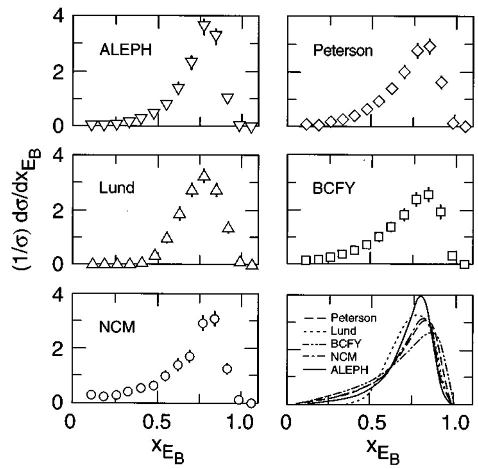

FIG. 9. Data distribution of scaled B-hadron energy corrected using simulations based on different input B-fragmentation functions (see text): (a) ALEPH, (b) Peterson, (c) Lund, (d) BCFY, and (e) NCM functions. Statistical error bars are shown; these are highly correlated between bins and among the five sets of results. (f) The five optimized functional forms used in the correction.

VI. CORRECTION OF THE B ENERGY DISTRIBUTION

In order to compare our results with those from other experiments it is necessary to correct the reconstructed scaled B hadron energy distribution $D^{\rm data}$ $(x_{E_B}^{\rm rec})$ for the effects of non-B backgrounds, detector acceptance, event selection, and analysis bias and initial-state radiation, as well as for bin-to-bin migration effects caused by the finite resolution of the detector and the analysis technique. We also corrected for the effects of B^{**} decays (Sec. II) to derive the primary B hadron energy distribution. We applied a 15×15 matrix unfolding procedure to $D^{\rm data}$ $(x_{E_B}^{\rm rec})$ to obtain an estimate of the true distribution $D^{\rm data}$ $(x_{E_B}^{\rm true})$:

$$D^{\rm data}\ (x_{E_B}^{\rm true}) = \epsilon^{-1}(x_{E_B}^{\rm true}) E(x_{E_B}^{\rm true}, x_{E_B}^{\rm rec})[D^{\rm data}(x_{E_B}^{\rm rec}) - S(x_{E_B}^{\rm rec})], \tag{4}$$

where S is a vector representing the background contribution, E is a matrix to correct for bin-to-bin migrations, and ϵ is a vector representing the efficiency for selecting true B hadron decays for the analysis.

The matrices S, E, and ϵ were calculated from our MC simulation; the elements of ϵ are shown in Fig. 3. The matrix E incorporates a convolution of the input fragmentation function with the resolution of the detector. We used in turn the Peterson, Lund, BCFY, NCM, and ALEPH functions, with the optimized parameters listed in Table III, to produce both a generator-level input primary B hadron energy distribution $D^{\rm MC}$ $(x_{E_B}^{\rm true})$, and a reconstructed distribution $D^{\rm MC}$ $(x_{E_B}^{\rm rec})$, as discussed in the previous section. In each case

TABLE IV. Systematic errors on $\langle x_{E_B} \rangle$.

Error source	Variation	Error (%)
Detector modeling		
Neutral fragmentation energy:		
cluster energy scale	$\pm 2.2\%$	$^{+0.12}_{-0.27}$
min. clus. energy	100^{-0}_{+100}MeV	$^{+0.00}_{-0.21}$
Tracking efficiency	$2.4 \mp 2.4\%$	$^{+0.2}_{-1.0}$
Lepton mis-ID background	$\pm 25\%$	$^{+0.66}_{-0.65}$
Physics modeling		
B meson/baryon lifetime	$1.55 \pm 0.05/1.10 \pm 0.08$ ps	$^{+0.11}_{-0.12}$
B^{**} production	$20.7 \pm 7\%$	$^{+0.68}_{-0.10}$
B^{**} mass	5.704 ± 0.020 GeV	$^{+0.03}_{-0.00}$
$f^* \equiv \Gamma(B \to D^*)/\Gamma(B \to D)$	$f^{*}{}^{+0}_{-f^*/3}$	$^{+0.32}_{-0.00}$
$f^{**} \equiv \Gamma(B \to D^{**})/\Gamma(B \to D)$	$f^{**} \pm f^{**}/3$	$^{+0.32}_{-0.21}$
$B_u, B_d/B_s/b$-baryon production	$40.1 \pm 20.0\%/11.6 \pm 8.0\%/7.0 \pm 4.0\%$	$^{+0.51}_{-0.48}$
B_u, B_d, B_s, b-baryon decay modes	$\pm 1\sigma$	$^{+0.11}_{-0.12}$
B-decay charged multiplicity	5.3 ± 0.2 tracks	$^{+0.25}_{-0.16}$
c-fragmentation: $\langle x_{E_D} \rangle$	0.484 ± 0.008	± 0.01
$D^0/D^+/D_s/c$-baryon production	$56.0 \pm 5.3\%/23.0 \pm 3.7\%/12.0 \pm 7.0\%/8.9 \pm 0.5\%$	± 0.01
D-decay multiplicity	Ref. [33]	$^{+0.04}_{-0.05}$
$s\bar{s}$ production	$\pm 10\%$	$^{+0.37}_{-0.40}$
R_b	0.2216 ± 0.0010	$^{+0.00}_{-0.01}$
R_c	0.16 ± 0.01	$^{+0.02}_{-0.04}$
$g \to b\bar{b}$ splitting	$\pm 50\%$	$^{+0.23}_{-0.30}$
$g \to c\bar{c}$ splitting	$\pm 50\%$	$^{+0.22}_{-0.25}$
Total		$^{+1.32}_{-1.48}$

E was evaluated by examining the population migrations of true B hadrons between bins of the input scaled B energy, $x_{E_B}^{\mathrm{true}}$ and the reconstructed scaled B energy $x_{E_B}^{\mathrm{rec}}$.

The data were then unfolded according to Eq. (4) to yield $D^{\mathrm{data}}\ (x_{E_B}^{\mathrm{true}})$, which is shown for each input fragmentation function in Fig. 9. It can be seen that the shapes of $D^{\mathrm{data}}\ (x_{E_B}^{\mathrm{true}})$ differ systematically among the assumed input fragmentation functions. These differences were used to assign systematic errors, as discussed in the next section.

VII. SYSTEMATIC ERRORS

We have considered sources of systematic uncertainty that potentially affect our measurement of the B-hadron energy distribution. These may be divided into uncertainties in modeling the detector and uncertainties on experimental measurements serving as input parameters to the underlying physics modeling. For these studies our standard simulation, employing the Peterson fragmentation function, was used.

The uncertainty on the correction of the non-B neutral jet energy component $E_{\mathrm{frag}}^{\mathrm{neu}}$ (Sec. IV) was estimated by changing the LAC cluster-energy selection requirement from 100 to 200 MeV, and by varying the LAC electromagnetic energy scale within our estimated uncertainty of $\pm 2.2\%$ of its nominal value [21]. In each case the difference in results relative to our standard procedure was taken as the systematic uncertainty. A large source of detector modeling uncertainty was found to relate to knowledge of the charged tracking efficiency of the detector, which we varied by our estimated uncertainty of $\pm 2.4\%$. In addition, in each bin of $x_{E_B}^{\mathrm{rec}}$, we varied the estimated contribution from fake leptons in the data sample (Fig. 2) by $\pm 25\%$. These uncertainties were assumed to be uncorrelated and were added in quadrature to obtain the detector modeling uncertainty in each bin of x_{E_B}.

As a cross-check we also varied the event selection requirements. The thrust-axis containment cut was varied in the range $0.65 < |\cos\theta_T| < 0.70$, the minimum number of charged tracks required was increased from 7 to 8, and the total charged-track energy requirement was increased from 20 to 22 GeV. In each case results consistent with the standard selection were obtained. As a further cross-check on jet axis modeling we systematically varied y_c in the range $0.01 \leqslant y_c \leqslant 0.15$ and repeated the analysis; results consistent with the standard analysis were obtained.

TABLE V. The fully corrected scaled B-hadron energy distribution.

x_{E_B} bin center	$1/\sigma d\sigma/dx_{E_B}$	Stat. error	Syst. error	Unfolding uncertainty
0.037	0.0	0.0	0.0	0.0
0.110	0.104	0.041	0.055	0.041
0.183	0.105	0.050	0.068	0.035
0.256	0.158	0.076	0.095	0.043
0.329	0.248	0.099	0.102	0.064
0.402	0.358	0.115	0.096	0.074
0.475	0.560	0.136	0.095	0.061
0.548	0.951	0.167	0.126	0.033
0.621	1.489	0.204	0.137	0.088
0.694	2.136	0.242	0.164	0.171
0.767	3.011	0.278	0.164	0.191
0.840	2.944	0.285	0.251	0.112
0.913	1.460	0.211	0.319	0.144
0.986	0.164	0.067	0.118	0.041

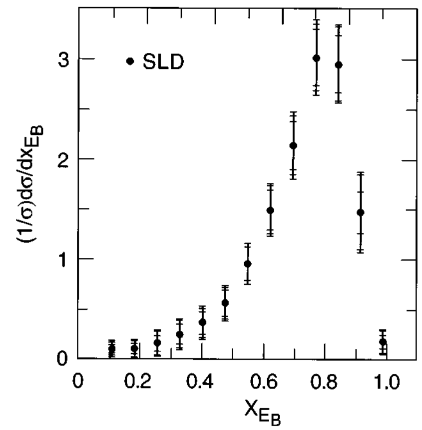

FIG. 10. The final corrected distribution of scaled B-hadron energies. In each bin the statistical error is indicated by the innermost error bar, the quadrature sum of statistical and experimental systematic errors by the middle error bar, and the quadrature sum of statistical, experimental systematic and unfolding errors by the outermost error bar. Note that the first bin (no point shown) is beneath the kinematic limit for x_{E_B}.

A large number of measured quantities relating to the production and decay of charm and bottom hadrons are used as input to our simulation. In $b\bar{b}$ events we have considered the uncertainties on the branching fraction for $Z^0 \to b\bar{b}$; the rates of production of B_u, B_d, and B_s mesons, and B baryons; the rate of production of B^{**} mesons, and the B^{**} mass; the branching ratios for $B \to D^*$ and $B \to D^{**}$; the lifetimes of B mesons and baryons; and the average charged multiplicity of B hadron decays. In $c\bar{c}$ events we have considered the uncertainties on the branching fraction for $Z^0 \to c\bar{c}$; the charmed hadron fragmentation function; the rates of production of D^0, D^+, and D_s mesons, and charmed baryons; and the charged multiplicity of charmed hadron decays. We have also considered the rate of production of $s\bar{s}$ in the jet fragmentation process, and the production of secondary $b\bar{b}$ and $c\bar{c}$ from gluon splitting. The world-average values [9,32] of these quantities used in our simulation, as well as the respective uncertainties, are listed in Table IV.

The variation of each quantity within its uncertainty was produced in turn in our simulated event sample using an event weighting technique [32]. The matrices S and E (Sec. VI) were then reevaluated using the simulated events, and the data were recorrected. In each case the deviation with respect to the standard corrected result was taken as a separate systematic error. These uncertainties were conservatively assumed to be uncorrelated and were added in quadrature to obtain a total physics modeling uncertainty in each bin of x_{E_B}.

The model dependence of the unfolding procedure was estimated by considering the envelope of the unfolded results illustrated in Fig. 9. In each bin of x_{E_B} we calculated the average value of the five unfolded results, as well as the r.m.s. deviation. The average value was taken as our central value in each bin, and the r.m.s. value was assigned as the respective unfolding uncertainty.

VIII. SUMMARY AND CONCLUSIONS

We have used the precise SLD tracking system to reconstruct the energies of B hadrons in $e^+e^- \to Z^0$ events via the $B \to DlX$ decay mode. We estimate our resolution on the B energy to be about 10% for roughly 65% of the reconstructed decays. The distribution of reconstructed scaled B hadron energy $D(x_{E_B}^{\rm rec})$ was compared with perturbative QCD and phenomenological model predictions; the calculations of Braaten, Cheung, Fleming, and Yuan and of Nason, Colangelo, and Mele are consistent with our data, as are the phenomenological models of Peterson *et al.* and of the Lund group. The distribution was then corrected for bin-to-bin migrations caused by the resolution of the method and for selection efficiency, as well as for the effects of B^{**} production, to derive the energy distribution of primary B hadrons produced by Z^0 decays. Systematic uncertainties in the correction were considered. The final corrected x_{E_B} distribution $D(x_{E_B})$ is listed in Table V and shown in Fig. 10; the statistical, experimental systematic, and unfolding uncertainties are indicated separately.

It is conventional to evaluate the mean of this distribution $\langle x_{E_B}\rangle$. For each of the five functions used to correct the data we evaluated $\langle x_{E_B}\rangle$ from the distribution that corresponds to the optimized parameters; these are listed in Table III. We took the average of the five values of $\langle x_{E_B}\rangle$ as our central result, and defined the unfolding uncertainty to be the r.m.s. deviation. We list in Table IV the errors on $\langle x_{E_B}\rangle$ resulting from the study of detector and physics modeling described in Sec. VII. We obtained

$$\langle x_{E_B}\rangle = 0.716 \pm 0.011(\mathrm{stat})^{+0.009}_{-0.011}(\mathrm{exp\ \ syst}) \pm 0.019(\mathrm{unfolding}),$$

where the systematic error is the sum in quadrature of the individual contributions listed in Table IV. It can be seen that $\langle x_{E_B}\rangle$ is relatively insensitive to the variety of allowed forms of the shape of the fragmentation function $D(x_{E_B})$.

Our results are in agreement with a previous measurement of the shape of the primary *B* hadron energy distribution at the Z^0 resonance [12], as well as with measurements of the shape [13] and mean value [10] of the distribution for weakly decaying *B* hadrons, after taking account of our estimate that the latter $\langle x_{E_B} \rangle$ value is about 0.015 lower, since the measured weakly decaying *B* hadron is not always the primary *B* hadron actually produced. Combining all systematic errors in quadrature we obtain $\langle x_{E_B} \rangle = 0.716 \pm 0.011(\mathrm{stat})^{+0.021}_{-0.022}(\mathrm{syst})$.

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