

Polarized Bhabha Scattering and a Precision Measurement of the Electron Neutral Current Couplings

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Bhabha scattering with polarized electrons at the Z^0 resonance has been measured with the SLD experiment at the SLAC Linear Collider. The first measurement of the left-right asymmetry in Bhabha scattering is presented, yielding the effective weak mixing angle of $\sin^2 \theta_W^{\text{eff}} = 0.2245 \pm 0.0049 \pm 0.0010$. The effective electron couplings to the Z^0 are extracted from a combined analysis of polarized Bhabha scattering and the left-right asymmetry previously published: $v_e = -0.0414 \pm 0.0020$ and $a_e = -0.4977 \pm 0.0045$.

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The standard model of electroweak interactions is a gauge theory based on the $SU(2)_L \times U(1)$ group. The W^+ , W^- , and Z^0 gauge bosons acquire mass, and the neutral gauge fields are mixed through spontaneous symmetry breaking into the physical mass eigenstates γ and Z^0 , described by a single parameter, $\sin^2 \theta_W$. Fermions couple to the Z^0 with both vector and axial vector components, functions of $\sin^2 \theta_W$. Precision measurements of these couplings thereby provide a stringent test of the electroweak theory.

The SLD Collaboration has recently performed the most precise single measurement of the effective electroweak mixing angle, $\sin^2 \theta_W^{\text{eff}}$, by measuring the left-right cross section asymmetry (A_{LR}) in Z boson production at the Z^0 resonance [1]. A_{LR} is a measure of the initial state electron coupling to the Z^0 . For simplicity, the e^+e^- final state (Bhabha scattering) is omitted in the A_{LR} measurement due to the dilution of the asymmetry from the large QED contribution of the t -channel photon exchange. In this Letter, we present two new results: the first measurement of the left-right cross section asymmetry in polarized Bhabha scattering [$A_{LR}^{e^+e^-}(|\cos\theta|)$], and measurements of the effective electron coupling parameters based on a combined analysis of the A_{LR} measurement [1] and the Bhabha cross section and angular distributions. The vector coupling measurement is the most precise yet presented.

In the standard model, measuring the left-right asymmetry yields a value for the quantity A_e , a measure of the degree of parity violation in the neutral current, since

$$A_{LR} = A_e = \frac{2v_e a_e}{v_e^2 + a_e^2} = \frac{2[1 - 4\sin^2 \theta_W^{\text{eff}}]}{1 + [1 - 4\sin^2 \theta_W^{\text{eff}}]^2}, \quad (1)$$

where the effective electroweak mixing parameter is defined as $\sin^2 \theta_W^{\text{eff}} = \frac{1}{4}(1 - v_e/a_e)$, and v_e and a_e are the effective vector and axial vector electroweak coupling parameters of the electron. The partial width for Z^0 decay into e^+e^- is dependent on the coupling parameters

$$\Gamma_{ee} = \frac{G_F M_Z^3}{6\sqrt{2}\pi} (v_e^2 + a_e^2)(1 + \delta_e), \quad (2)$$

where $\delta_e = 3\alpha/4\pi$ is the correction for final state radiation, G_F is the Fermi coupling constant, and M_Z is the Z^0 boson mass [2]. By measuring A_e and Γ_{ee} , the above equations can be used to extract v_e and a_e up to a four-fold ambiguity, which can be resolved from lower energy e^+e^- annihilation data to get $|v_e| < |a_e|$, and $\nu_e e$ scattering data to establish $v_e < 0$ and $a_e < 0$ [3].

The data presented in this Letter were collected during the 1992 and 1993 runs of the SLAC Linear Collider (SLC), which collides unpolarized positrons with longitudinally polarized electrons at a center of mass energy near the Z^0 resonance [4]. The luminosity-weighted center of mass energy was measured to be 91.55 ± 0.02 GeV for the 1992 run and 91.26 ± 0.02 GeV for the 1993 run [1]. The luminosity-weighted electron beam polarization ($\langle P_e \rangle$) was measured to be $(22.4 \pm 0.7)\%$ for the 1992 run and $(63.0 \pm 1.1)\%$ for the 1993 run [1].

The analysis presented here uses the calorimetry systems of the SLD detector. Small angle coverage (28–65 mrad from the beamline) is provided by the finely segmented silicon-diode-tungsten-radiator luminosity calorimeters (LUM) [5]. The LUM measures small angle Bhabha scattering, thereby providing both the absolute luminosity and a check that the left-right luminosity asymmetry is small. Events at larger angles from the beam line (>200 mrad) are measured with the liquid argon calorimeter (LAC) [6]. The LAC covers 98% of the solid angle, and is comprised of a fine sampling electromagnetic section followed by a coarse sampling hadronic section.

The LUM detectors surround the beam pipe on both sides of the interaction point. Event selection criteria discriminate high energy electromagnetic showers from background. Selected events are narrow and deposit energy throughout the depth of the calorimeter, while the low energy beam backgrounds from the SLC are diffuse.

Electron position is inferred from the energy sharing between adjacent silicon pads.

To minimize systematic uncertainties in the LUM due to detector misalignment and the location of the interaction point, we employ a “gross-precise” method [7] which uses a larger fiducial region on one end of the detector than the other. Bhabha events are counted based on the location of each shower in the respective detector. Events in which both the electron and positron showers are within a tight fiducial region are counted with weight 1, while events in which only one of the two showers is inside the tight fiducial region are given weight 1/2. The resulting misalignment error on the effective number of calculated events is negligible [8]. The effective cross section is calculated by using the Monte Carlo programs BABAMC [9] and BHLUMI [10]. Detector simulation is performed with GEANT [11] and the electromagnetic showers are parametrized using the GFLASH algorithm [12].

The errors on the results of this paper are limited by statistics. For this reason, the systematic error analysis of the luminosity measurement is conservative. A detailed description of the systematic error analysis for the luminosity measurement is given elsewhere [8]. The total systematic uncertainty is 0.93%, which is composed of 0.88% experimental and 0.3% theoretical uncertainty. The integrated luminosity is $\mathcal{L} = 385 \pm 3$ (stat) ± 4 (syst) nb^{-1} for the 1992 polarized SLC run and $\mathcal{L} = 1781 \pm 5$ (stat) ± 17 (syst) nb^{-1} for the 1993 SLC run.

The wide angle Bhabha selection algorithm uses the distinct topology of the e^+e^- final state: two large back-to-back electromagnetic clusters. Two LAC clusters must contain 70% of the center of mass energy with little hadronic energy in either (<3.8 GeV). The normalized energy imbalance (E_{imb}) of events must be less than 0.6 ($E_{\text{imb}} = |\sum \vec{E}_{\text{cluster}}| / |\sum \vec{E}_{\text{cluster}}|$). The number of reconstructed clusters is limited to 9, a rather loose selection permitting multiple clusters from preshowering electrons. Initial state radiation is limited by the energy cuts and a requirement that the absolute value of the rapidity sum of the two main clusters be less than 0.3. The rapidity cut acts as an angle-dependent e^+e^- collinearity cut.

The efficiency and contamination for the wide angle events are calculated from Monte Carlo simulations. Corrections are applied as a function of scattering angle to account for angle-dependent changes in response. The $e^+e^- \rightarrow e^+e^-$ process at large angles is simulated with BHAGEN [13]. The efficiency for accepting wide angle $e^+e^- \rightarrow e^+e^-$ events is found to be 86.7% overall and 93% in the central region of the detector. Two small sources of contamination are $e^+e^- \rightarrow \gamma\gamma$ (1.25%) and $e^+e^- \rightarrow \tau^+\tau^-$ (0.28%), estimated with RADCOR and KORALZ [14,15]. Other sources of contamination such as hadronic Z^0 decays, two-photon processes, cosmic rays, and beam background are all negligible.

This analysis is based on 7926 accepted events in the wide-angle region and 212794 accepted events in the

small-angle region. The asymmetry is defined as

$$A_{LR}^{e^+e^-}(|\cos\theta|) = \frac{1}{\langle \mathcal{P}_e \rangle} (N_L - N_R)/(N_L + N_R),$$

where N_L (N_R) is the number of events tagged with a left- (right-) handed electron beam as a function of the $|\cos\theta|$, where θ is the center of mass scattering angle for the e^+e^- system after initial state radiation. Aside from the charge ambiguity of the calorimeter measurement, θ is derived trivially from the measured electron and positron laboratory scattering angles. The expected asymmetry [$A_{LR}^{e^+e^-}(|\cos\theta|)$] is largest at $\cos\theta = 0$, and may be approximately written as $A_{LR}^{e^+e^-}(|\cos\theta|) = A_e(1 - f_t(|\cos\theta|))$, where $f_t(|\cos\theta|)$ represents the t -channel contribution. For the region $|\cos\theta| < 0.7$, $\langle f_t \rangle \approx 0.12$. The expected asymmetry is very small ($\sim 10^{-4}$) in the small-angle region.

To extract Γ_{ee} and A_e , the data are fit to the e^+e^- cross section using the maximum likelihood method. Two programs are used to calculate the differential e^+e^- cross section: EXPOSTAR [16] and, as a cross check, DMIBA [17]. The EXPOSTAR program calculates the differential cross sections within the framework of the standard model. The DMIBA program calculates the differential e^+e^- cross section in a model independent manner. To extract the maximal amount of information from the differential polarized Bhabha scattering distribution, the fit is performed over the entire angular region accepted by the LAC ($|\cos\theta| < 0.98$). No t -channel subtraction is performed. All 10 lowest order terms in the cross section are included in the fit: the four pure s -channel and t -channel terms for photon and Z^0 exchange, and the six interference terms [18]. The fit also includes initial state radiation. Since the measurement is calorimetric, it is insensitive to final state radiation.

The partial width Γ_{ee} is extracted from the data in two ways: (1) using the full fit to the differential cross section for $|\cos\theta| \leq 0.98$, and (2) measuring the cross section in the central region ($|\cos\theta| < 0.6$) where the systematic errors are smaller, yielding a more precise measurement. Both are normalized by the LUM. For the fits we use $M_Z = 91.187$ GeV/ c^2 and $\Gamma_Z = 2.489$ GeV/ c^2 [19]. Figure 1 shows the fit to the full $e^+e^- \rightarrow e^+e^-$ distribution. This fit has a $\chi^2 = 51.6$ for 39 degrees of freedom, yielding $\Gamma_{ee} = 83.14 \pm 1.03$ (stat) ± 1.95 (syst) MeV. The 2.4% systematic error is dominated (2.1%) by the uncertainty in the efficiency correction factors in the angular region $0.6 < |\cos\theta| < 0.98$, where the LAC response is difficult to model due to materials from interior detector elements [8].

A more precise determination of Γ_{ee} was performed using only the central region of the LAC ($|\cos\theta| < 0.6$) and the small angle region in the LUM [20]. The program MIBA [21] is then used to calculate Γ_{ee} based on the total measured cross section within the defined fiducial region.

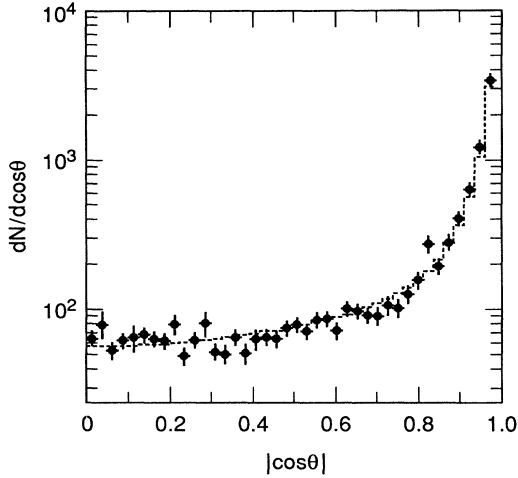


FIG. 1. Differential angular distribution for $e^+e^- \rightarrow e^+e^-$. The points are the corrected data; the dashed line is the fit.

From this method, we find

$$\Gamma_{ee} = 82.89 \pm 1.20 \text{ (stat)} \pm 0.89 \text{ (syst)} \text{ MeV.}$$

The limited fiducial region leads to a much improved systematic error. The sources of the 1.1% systematic error are detector simulation (0.74%), the absolute luminosity (0.52%), contamination (0.3%), M_Z and Γ_Z (0.3%), the cross section calculation (0.3%), and the center of mass energy (0.2%).

To extract A_e from the Bhabha events, the right- and left-handed differential $e^+e^- \rightarrow e^+e^-$ cross sections are fit directly using EXPOSTAR. This yields

$$A_e = 0.202 \pm 0.038 \text{ (stat)} \pm 0.008 \text{ (syst)}.$$

Figure 2 shows the measured left-right cross section asymmetry for $e^+e^- \rightarrow e^+e^-$ [$A_{LR}^{e^+e^-}(|\cos\theta|)$] compared

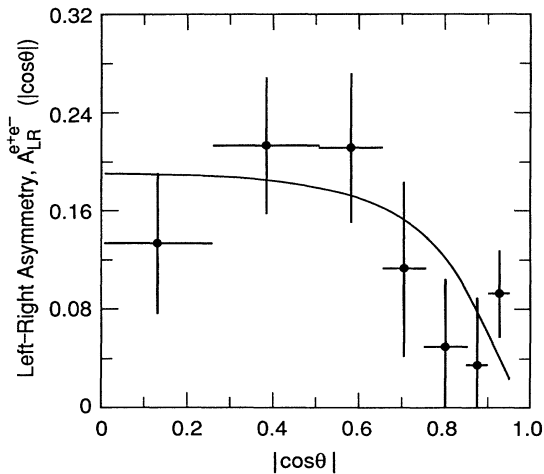


FIG. 2. Left-right asymmetry, $A_{LR}^{e^+e^-}(|\cos\theta|)$, for polarized $e^+e^- \rightarrow e^+e^-$. The points are the corrected data; the solid curve is the fit.

to the fit. The fit shown in Fig. 2 has a χ^2 of 4.36 for 5 degrees of freedom. The measurement of A_e is limited by the statistical uncertainty. The sources of the 3.8% systematic are the angle-dependent response correction factors (3.2%), the polarization (1.7%), SLC asymmetry factors (0.06%) [1,8], the beam energy spread and center of mass energy (0.25%), the EXPOSTAR program (0.7%), and the Z^0 mass and width (0.7%).

The results for Γ_{ee} and A_e from above may now be used in Eqs. (1) and (2) to extract the effective vector and axial vector couplings to the Z^0 : $\nu_e = -0.0507 \pm 0.0096 \text{ (stat)} \pm 0.0020 \text{ (syst)}$, $a_e = -0.4968 \pm 0.0039 \text{ (stat)} \pm 0.0027 \text{ (syst)}$. Figure 3 shows the 1 standard deviation (68%) contour for these electron vector and axial vector coupling measurements. Most of the sensitivity to the electron vector coupling and, hence, $\sin^2\theta_W^{\text{eff}}$ arises from the measurement of A_e , while the sensitivity to the axial vector coupling arises from Γ_{ee} . Also shown are standard model calculations using the program ZFITTER [22].

The effective electroweak mixing angle represented by these vector and axial vector coupling is

$$\sin^2\theta_W^{\text{eff}} = 0.2245 \pm 0.0049 \text{ (stat)} \pm 0.0010 \text{ (syst)}.$$

We reiterate that this measurement derives strictly from the Bhabha events.

The SLD Collaboration has published a more precise measurement of A_e from the left-right cross section asymmetry (A_{LR}) measurement [1]. Combining the Bhabha results with the SLD measurement of A_{LR} gives

$$\nu_e = -0.0414 \pm 0.0020, \quad a_e = -0.4977 \pm 0.0045,$$

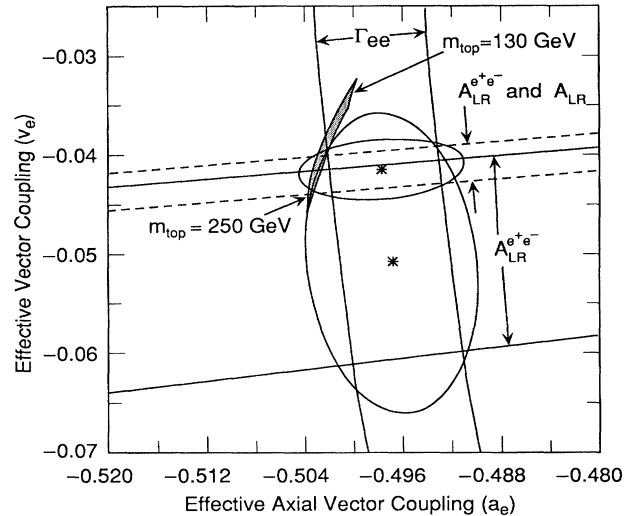


FIG. 3. One-sigma (68%) contour in the a_e, ν_e plane. The large ellipse is for $e^+e^- \rightarrow e^+e^-$, the smaller ellipse includes the measurement of A_{LR} . The shaded region represents the standard model calculation for $130 < m_{\text{top}} < 250 \text{ GeV}$ and $50 < M_{\text{Higgs}} < 1000 \text{ GeV}$.

the most precise measurement of the electron vector coupling to the Z^0 published to date. The v_e, a_e contour including the A_{LR} measurement is also shown in Fig. 3, demonstrating the increased sensitivity in v_e from A_{LR} . The LEP average for the electron coupling parameters to the Z^0 are $v_e = -0.0370 \pm 0.0021$ and $a_e = -0.50093 \pm 0.00064$ [23].

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