Generation of a highly-phase-sensitive polarization-squeezed *N*-photon state by collinear parametric down-conversion and coherent photon subtraction

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It is shown that a highly-phase-sensitive polarization-squeezed $(2n-1)$-photon state can be generated by subtracting a diagonally polarized photon from the $2n$ photon component generated in collinear type II down-conversion. This polarization wedge state has the interesting property that its photon number distribution in the horizontal and vertical polarizations remains sharply defined for phase shifts of up to $1/n$ between the circularly polarized components. Phase shifts at the Heisenberg limit are therefore observed as nearly deterministic transfers of a single photon between the horizontal and vertical polarization components.

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I. INTRODUCTION

One of the most fundamental applications of nonclassical light field states is the improvement of measurement precision beyond the standard quantum limits for classical light sources. Of particular interest is the possible enhancement of phase sensitivity in interferometry [1–7], which could be useful in a wide range of fields, from quantum lithography [8–11] to atomic clocks [12,13]. It is well known that the optimal phase resolution $\Delta\Phi$ that can be achieved using a nonclassical N-photon state is given by the Heisenberg limit of $\Delta\Phi \geqslant 1/N$. Recently, few-photon interferometry at this limit has been accomplished by new methods of generating N-photon path entangled states using parametric down-conversion (PDC) and post-selection [14–18]. Such path entangled states are an equal superposition of the two N-photon states where all photons are located in the same optical mode, $(|N;0\rangle+|0,N\rangle)/\sqrt{2}$. They are therefore ideally suited to obtain N-photon interference fringes with a period of $2\pi/N$ in the optical phase shift between the two paths. In principle, the generation of path entangled states can be extended to higher photon numbers using the methods proposed and realized in Refs. [15–18]. In practice, however, the statistical bottlenecks in the post-selection (or heralding) used to generate the path entangled states rapidly reduce the probabilities of generating an appropriate output as photon number increases. It may therefore be useful to consider alternative few-photon states that can be generated more efficiently from a given number of down-converted photon pairs.

In this paper, it is shown that a highly-phase-sensitive state can be generated by subtracting a single diagonally polarized photon from the $(2n)$-photon state generated in collinear type II down-conversion. Since single photon subtraction can be performed with equal efficiency for any number of input photons, this method could be very helpful in achieving phase resolutions at the Heisenberg limit for higher photon numbers. Moreover, the coherence induced between two adjacent photon number states ensures that the narrowness of the photon number distribution is maintained under phase shifts of up to $1/n$. Phase shifts at the Heisenberg limit can therefore be observed as nearly deterministic transfers of a single photon between the output modes.

II. GENERATION OF THE WEDGE STATE SUPERPOSITION

The proposed experimental setup is shown schematically in Fig. 1. The initial state generated by collinear type II parametric down-conversion is a superposition of photon number states with equal photon number in the horizontal and vertical polarizations,

$$|\mathrm{PDC}\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n|n;n\rangle_{HV}. \tag{1}$$

If it can be assumed that all of the emitted photons will eventually be detected, it is possible to isolate a single $2n$-photon component by post-selecting only outputs where a total of $2n$ photons are detected [19]. Effectively, the input state is then given by $|n;n\rangle_{HV}$. This $2n$-photon input component is reflected at a beam splitter with a reflectivity of R close to one, and one photon is detected in the transmitted light. The components of the $2n$-photon states in the beam splitter output with exactly one transmitted photon are given by

$$\hat{U}_R|n;n\rangle_{HV}\otimes|0;0\rangle_{HV} \approx \sqrt{n(1-R)R^{2n-1}}(|n;n-1\rangle_{HV}\otimes|0;1\rangle_{HV}+|n-1;n\rangle_{HV}\otimes|1;0\rangle_{HV}) + \cdots . \tag{2}$$

The beam splitter thus entangles the polarization of the transmitted one-photon component and the polarization of the reflected $(2n-1)$-photon component. It is now possible to measure the diagonal polarization of the transmitted photon using a $\lambda/2$-plate set at 22.5° and a polarization beam splitter. This measurement projects the state of the transmitted photon onto an equal superposition of horizontal and vertical polarization, resulting in a conditional output state of

$$|\mathrm{wedge}\rangle = \frac{1}{\sqrt{2}}(|n;n-1\rangle_{HV}+|n-1;n\rangle_{HV}) \tag{3}$$

in the reflected light.

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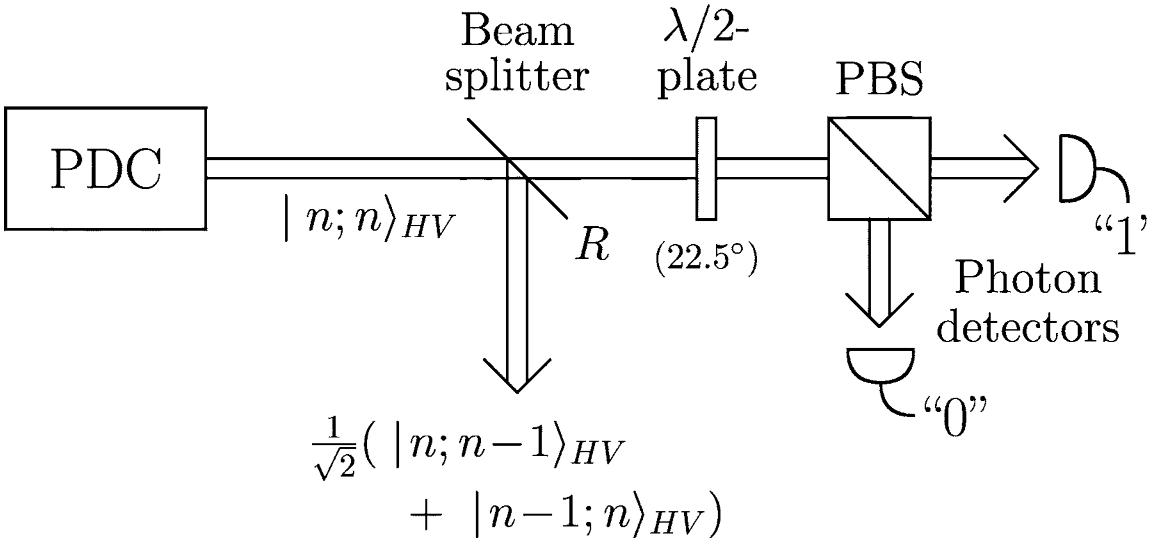

FIG. 1. Sketch of the experimental setup generating the highly-phase-sensitive polarization wedge state. The $(2n-1)$-photon state is generated by parametric down-conversion of n-photon pairs, followed by a reflection of $2n-1$ photons at a beam splitter of reflectivity R and detection of the transmitted photon in a diagonally polarized state using a $\lambda/2$-plate set at 22.5° and a polarization beam splitter (PBS).

Equation (2) shows that the probability of successfully subtracting exactly one photon from the $(2n)$-photon input is given by $n(1-R)R^{2n-1}$. This value can be optimized independently for any desired photon number by varying the reflectivity R. The maximal efficiency of photon subtraction is obtained at $R=1-1/(2n)$. The probability of successful photon subtraction is then equal to $[1-1/(2n)]^{2n-1}/2$. Interestingly, this maximal probability decreases only slightly with photon number, from an initial value of 25% at $n=1$ towards a value of $1/(2e)\approx 18.4\%$ for extremely high photon numbers. By selecting an optimized reflectivity of $R=1-1/(2n)$, it is thus possible to achieve post-selection probabilities greater than 18% for any number of input photons. The efficiency of photon subtraction is therefore almost independent of photon number.

It should be noted that this is quite different from the photon bottleneck used to generate the path entangled state $(|N;0\rangle+|0,N\rangle)/\sqrt{2}$, where the corresponding post-selection probability drops rapidly with increasing photon number as more and more beam splitters become necessary to "bunch up" the photons in the single mode bottleneck. In the basic scheme introduced in Ref. [16], the bottleneck efficiency is $2N!/(2N)^N$ for an N-photon state. At five photons, this is an efficiency of only 0.24%, almost 100 times less than the optimal efficiencies of photon subtraction. It should therefore be much easier to increase the output photon number of wedge states than to achieve the same photon number for path entangled states.

III. STOKES PARAMETER STATISTICS OF (2n−1)-PHOTON WEDGE STATES

The complete polarization statistics of N-photon quantum states can be expressed by the three Stokes parameters describing the photon number differences between horizontal (H) and vertical (V), plus (P) and minus (M) diagonal, and right (R) and left (L) circular polarization,

$$\hat{S}_1=\hat{n}_H-\hat{n}_V=\hat{a}_H^\dagger\hat{a}_H-\hat{a}_V^\dagger\hat{a}_V,$$

$$\hat{S}_2=\hat{n}_P-\hat{n}_M=\hat{a}_H^\dagger\hat{a}_V+\hat{a}_V^\dagger\hat{a}_H,$$

$$\hat{S}_3=\hat{n}_R-\hat{n}_L=-i(\hat{a}_H^\dagger\hat{a}_V-\hat{a}_V^\dagger\hat{a}_H). \quad (4)$$

As can be seen from Eq. (3), the Stokes parameter $\hat{S}_1$ describing the HV-polarization takes on values of +1 or −1, with a 50% probability each. The average of $\hat{S}_1$ is therefore zero, and its uncertainty is $\delta S_1^2=1$.

The low uncertainty in $\hat{S}_1$ is a direct consequence of the quantum correlations in parametric down-conversion. In fact, the original $|n;n\rangle_{HV}$ state is an $\hat{S}_1$ eigenstate with an uncertainty of zero, which already provides phase sensitivities at the Heisenberg limit in the photon statistics [1]. Photon subtraction actually increases the $\hat{S}_1$ uncertainty by one, thus increasing the observed photon number noise in the HV basis. However, the essential effect of the photon subtraction on the polarization statistics of the output state is the generation of coherence between the horizontal and vertical polarization components. This effect can be observed in the statistics of the Stokes parameter $\hat{S}_2$, describing the photon number difference between the diagonal polarizations P and M. Due to the coherence between $|n;n-1\rangle_{HV}$ and $|n-1;n\rangle_{HV}$, the expectation value of this Stokes parameter is

$$\langle\hat{S}_2\rangle=\langle\hat{n}_P-\hat{n}_M\rangle=n. \quad (5)$$

Since the total photon number is $N=2n-1$, this means that on average, more than 3/4 of all output photons are polarized along the same diagonal as the transmitted photon [20]. Since the wedge state polarization of $\langle\hat{S}_2\rangle=n$ originates from complete coherence between two adjacent eigenstates of $\hat{S}_1$, it is the maximal diagonal polarization possible at an uncer-

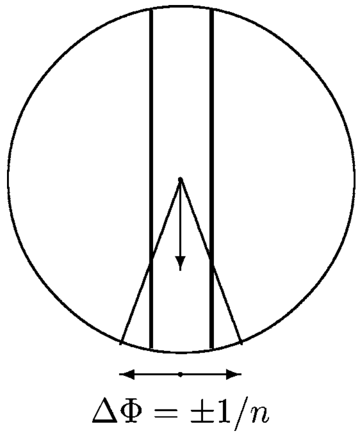

FIG. 2. Schematic illustration of the wedge state statistics in the S_1-S_2 plane of the Poincare sphere. The scale chosen corresponds to five photons ($n=3$). The arrow indicates the average Stokes vector, the thick vertical lines indicate the quantized eigenstates of $\hat{S}_1$. The thin lines at angles of $\pm\Delta\Phi$ illustrate the phase uncertainty of the wedge state. As photon number increases, the eigenstates with $S_1 = \pm 1$ move closer together and the phase distribution of the wedge state becomes narrower.

tainty of only $\delta S_1^2 = 1$ in the difference between horizontally and vertically polarized photons. As shown in Fig. 2, the polarization distribution of the output state thus resembles a quantum mechanically narrow "wedge" inserted between the $\hat{S}_1$ eigenstates from the positive side of the diagonal polarization $\hat{S}_2$—somewhat like the slice of an orange, with a quantum limited thickness of $2\delta S_1 = 2$.

Due to the narrowness of its $\hat{S}_1$ distribution and due to its comparatively high expectation value $\langle \hat{S}_2 \rangle$, the wedge state is very suitable for measurements of small phase shifts between the right and left circular polarizations, which result in a rotation of the Stokes parameters around the S_3 axis. As indicated in Fig. 2, the phase uncertainty of the polarization wedge can then be estimated by

$$\delta\Phi^2 = \frac{\delta S_1^2}{\langle S_2 \rangle^2} = \frac{1}{n^2}. \tag{6}$$

This phase uncertainty is close to the Heisenberg limit and corresponds to the phase uncertainties of the various phase squeezed states proposed for optimized phase estimation in quantum interferometry [7]. The wedge state is therefore an almost ideal phase squeezed N-photon state.

To clarify the application of this phase sensitivity in interferometry, it may be useful to consider the possibility of converting the polarization modes $\hat{a}_H$ and $\hat{a}_V$ to spatial input modes with equal polarization. As shown in Fig. 3, the two paths inside the interferometer then correspond to the modes $\hat{a}_{R/L} = (\hat{a}_H \pm i\hat{a}_V)/\sqrt{2}$, and the effect of a phase shift Φ between the two paths is to rotate the original HV basis towards the PM basis. In terms of the photon number difference $\hat{S}_1(\text{out})$ observed in the two output ports, this effect can then be expressed as the rotation of the Stokes vector around the S_3 axis mentioned above, with

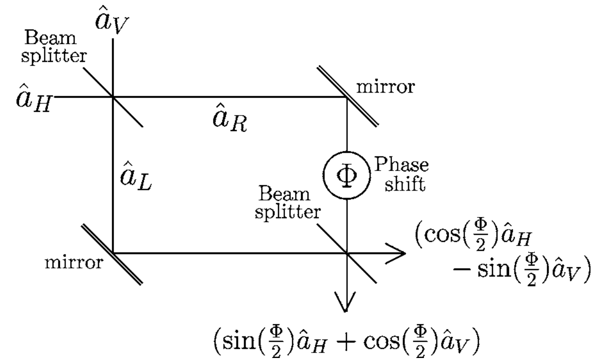

FIG. 3. Illustration of interferometry using the wedge state coherence. By modifying the phases of the RL modes, the original HV input is rotated towards the PM basis. In polarization experiments, the same effect can be achieved by a single half-wave-plate set at $\theta=\phi/4$ followed by a polarization beam splitter.

$$\hat{S}_1(\text{out}) = \cos(\Phi)\hat{S}_1(\text{in}) + \sin(\Phi)\hat{S}_2(\text{in}). \tag{7}$$

The phase shift Φ between the arms of the interferometer thus corresponds directly to the rotation of the linear polarization components obtained, e.g., by a half-wave-plate set at $\theta=\phi/4$. The phase sensitivity of the $(2n-1)$-photon wedge state can then be described by the average $\langle \hat{S}_1(\text{out}) \rangle$ and the variance $\delta S_1^2(\text{out})$ of the measurement result $S_1(\text{out}) = n_H - n_V$,

$$\langle \hat{S}_1(\text{out}) \rangle = n \sin(\Phi),$$

$$\delta S_1^2(\text{out}) = 1 + (n^2 - 2)\sin^2(\Phi). \tag{8}$$

For phase shifts Φ smaller than $1/n$, $\langle \hat{S}_1 \rangle \approx n\Phi$ and $\delta S_1^2 \approx 1$ corresponds to a phase resolution of $\Delta\Phi = 1/n$, as given by Eq. (6). $(2n-1)$-photon wedge states can thus achieve phase resolutions close to the Heisenberg limit for arbitrarily high photon numbers. Moreover, the variance of the output photon number distribution at phase shifts Φ with $n\sin(\Phi) = \pm 1$ is still smaller than two. Even at phase shifts that change the average output photon number difference by one, the photon number distribution is therefore sharper than the difference of two between two adjacent measurement outcomes of $\hat{S}_1$. This result indicates that phase shifts at the Heisenberg limit are observed as nearly deterministic transfers of a single photon between the horizontal and vertical polarizations, with measurement probabilities greater than 50% of finding a measurement outcome equal to the expectation value of $\langle \hat{S}_1 \rangle = \pm 1$ at $n\sin(\Phi) = \pm 1$.

It is interesting to note that the above argument only relies on averages and variances of the photon number differences $\hat{S}_i$. It is therefore straightforward to estimate the effect of basic photon counting errors. In particular, a photon loss error may occur when the down-conversion actually generates $n+1$ pairs, and two photons are subsequently lost due to limited detector efficiencies. At low detector efficiencies, the probability of such a photon loss error is of the order of $(\tanh r)^2$, corresponding to the ratio of the $n+1$ pair probability to the n pair probability in Eq. (1). Since random (=unpolarized) photon losses do not change the average polarization of the photons, the strong coherence between the H and V polarization given by $\langle \hat{S}_2 \rangle$ is unchanged by this error. However, the polarization fluctuations increase due to the possibility that both of the photons lost had the same polarization. Specifically, the increase in δS_1^2 is equal to 2 times the probability of the photon loss error. For reasonably low error probabilities [e.g., for $(\tanh r)^2 < 0.1$], this additional uncertainty is much smaller than the pure state uncertainty of $\delta S_1^2 = 1$, and the effect on the phase resolution will be negligible. These considerations indicate that the high phase resolution of wedge states is rather robust against photon loss errors. Again, this is an important difference to path entangled states, where the loss of a single photon completely destroys the quantum coherence responsible for the high phase resolution. Due to this robustness against photon loss errors, it may be interesting to investigate wedge state generation at high pump powers even if high detector efficiencies cannot be achieved [21].

IV. PHOTON STATISTICS OF THE FIVE-PHOTON WEDGE STATE

To illustrate the full implications of the phase sensitivity of wedge states at the level of precise photon counting, it may be useful to take a closer look at a specific example. Here, a compromise is necessary between the experimental difficulties and the increasing phase resolution permitted by higher photon numbers. A good choice may be the five-photon wedge state, generated by subtracting one photon from $n=3$ pairs of down-converted photons, since it should be just within reach of present technological possibilities. A phase shift of Φ transforms this state according to

$$\langle 5;0|\hat{U}_\phi|\text{wedge}\rangle = -\frac{\sqrt{10}}{16}\left[\cos\left(\frac{5}{2}\phi - \frac{\pi}{4}\right) + \cos\left(\frac{3}{2}\phi + \frac{\pi}{4}\right) - 2\cos\left(\frac{1}{2}\phi - \frac{\pi}{4}\right)\right],$$

$$\langle 4;1|\hat{U}_\phi|\text{wedge}\rangle = -\frac{\sqrt{2}}{16}\left[5\cos\left(\frac{5}{2}\phi + \frac{\pi}{4}\right) - 3\cos\left(\frac{3}{2}\phi - \frac{\pi}{4}\right) - 2\cos\left(\frac{1}{2}\phi + \frac{\pi}{4}\right)\right],$$

$$\langle 3;2|\hat{U}_\phi|\text{wedge}\rangle = \frac{1}{8}\left[5\cos\left(\frac{5}{2}\phi - \frac{\pi}{4}\right) + \cos\left(\frac{3}{2}\phi + \frac{\pi}{4}\right) + 2\cos\left(\frac{1}{2}\phi - \frac{\pi}{4}\right)\right],$$

$$\langle 2;3|\hat{U}_\phi|\text{wedge}\rangle = \frac{1}{8}\left[5\cos\left(\frac{5}{2}\phi + \frac{\pi}{4}\right) + \cos\left(\frac{3}{2}\phi - \frac{\pi}{4}\right) + 2\cos\left(\frac{1}{2}\phi + \frac{\pi}{4}\right)\right],$$

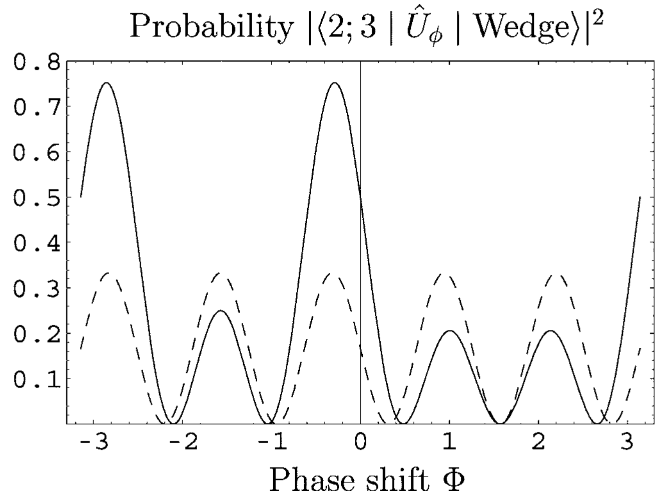

FIG. 4. Probability of finding two photons in the horizontally polarized mode and three photons in the vertically polarized mode as a function of phase shift Φ between the circular polarizations for the five-photon wedge state. The dotted line shows the same probability for a corresponding five-photon path-entangled state.

$$\langle 1;4|\hat{U}_\phi|\text{wedge}\rangle = -\frac{\sqrt{2}}{16}\left[5\cos\left(\frac{5}{2}\phi - \frac{\pi}{4}\right) - 3\cos\left(\frac{3}{2}\phi + \frac{\pi}{4}\right) - 2\cos\left(\frac{1}{2}\phi - \frac{\pi}{4}\right)\right],$$

$$\langle 5;0|\hat{U}_\phi|\text{wedge}\rangle = -\frac{\sqrt{10}}{16}\left[\cos\left(\frac{5}{2}\phi + \frac{\pi}{4}\right) + \cos\left(\frac{3}{2}\phi - \frac{\pi}{4}\right) - 2\cos\left(\frac{1}{2}\phi + \frac{\pi}{4}\right)\right]. \tag{9}$$

Each of these six amplitudes includes a five-photon interference component oscillating at a rate of $5\phi/2$. This five-photon interference effect is particularly strong in the $|3;2\rangle$ and the $|2;3\rangle$ components. Figure 4 shows the interference fringes observed in the $|2;3\rangle$ component. For comparison, the dashed line shows the corresponding fringes of a five-photon path-entangled state. The main difference between the two fringes are the different peak heights of the wedge state indicating the average diagonal polarization of the five-photon state. The wedge state thus combines features of the maximal five-photon interference of path-entangled states with the well-defined polarization direction of a phase squeezed state.

The well-defined polarization direction is particularly visible at phase angles of $\Phi=0.288$ (or 16.5°), where the probability of measuring $|3;2\rangle$ has its maximal value of 75.2%, and at $\Phi=0.623$ (or 35.7°), where the probabilities of measuring $|2;3\rangle$ and $|1;4\rangle$ are both equal to 42.5%. Figure 5 shows these two probability distributions, along with a schematic illustration of the corresponding quantized levels on the Poincare sphere. As indicated by Fig. 5(a), the output photon number distribution can be "switched" between 75.2% $S_1=-1$ ($|2;3\rangle$) at $\Phi=-0.288$ and 75.2% $S_1=+1$ ($|3;2\rangle$) at $\Phi=+0.288$. Likewise, Fig. 5(b) indicates that the photon number distribution at $\Phi=0$ can be shifted by exactly one

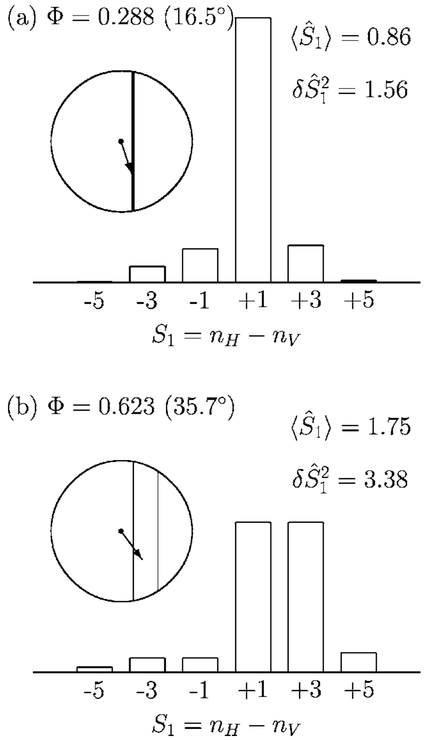

FIG. 5. Measurement probabilities for a five-photon wedge state at phase shifts of (a) $\Phi=0.288$ and (b) $\Phi=0.623$. The corresponding averages and variances of the photon number differences $\hat{S}_1$ are given in the upper right-hand corners. The sketches to the left of the graphs illustrate the quantized levels with the highest measurement probabilities in the S_1-S_2 plane of the Poincare sphere. The arrows represent the average Stokes vector of the rotated state.

photon with only 15% of the outcomes scattered to different photon numbers. Considering the fact that the measurement outcomes are discrete, this shift in the probability distribution is surprisingly smooth. Specifically, the high fidelity of the $|3;2\rangle$ component at $\Phi=+0.288$ shown in Fig. 5(a) suggests a "polarization wedge" that is much sharper than the photon number distribution at $\Phi=0$ [22]. The quantum coherence between the adjacent photon number states induced by the post-selected photon subtraction thus permits a surprisingly high level of control at the single photon level.

Although the detailed calculations presented here only apply to the specific case of five photons, it should be remembered that the basic features of the statistics for a general $(2n-1)$-photon wedge state are defined by the Stokes parameter statistics given in Sec. III. Since the uncertainty of $\hat{S}_1$ is always one, the photon number distributions will be limited to only a few possible measurement outcomes close to $S_1=\pm 1$ for any number of photons. Specifically, the photon number distribution at $\langle \hat{S}_1 \rangle \approx 1$ will always be similar to Fig. 5(a), and the distribution at $\langle \hat{S}_1 \rangle \approx 2$ will be similar to Fig. 5(b). For high n, we can therefore expect a maximal probability of $S_1=+1$ at a phase angle of $\Phi \approx 1/n$, where, according to Eq. (8), $\langle \hat{S}_1 \rangle \approx 1$ and $\delta S_1^2 \approx 2$. The "switch" from $S_1=-1$ to $S_1=+1$ is therefore also observable at higher photon numbers.

V. CONCLUSIONS

In conclusion, it has been shown that it is possible to obtain highly-phase-sensitive $(2n-1)$-photon wedge states by photon subtraction from n-photon pairs generated in collinear type II parametric down-conversion. Since the post-selection condition for the generation of this state can be higher than 18% regardless of photon number, the only limitation in extending this scheme to high photon numbers is the efficiency of the parametric down-conversion. The generation of five-photon wedge states should therefore be well within reach of present technological capabilities. Quantitatively, the phase sensitivity of wedge states is comparable to that of the recently realized path entangled states, with the advantage that a phase shift can be related directly to a shift in the photon number distribution observed in the output. Specifically, phase shifts at the Heisenberg limit appear as nearly deterministic transfers of one photon between the two output ports. $(2n-1)$-photon wedge states should therefore be highly suitable for the determination of phase shifts Φ of about $1/n$. The generation of polarization wedge states by down-conversion and photon subtraction thus provides a simple and effective experimental approach to phase measurements at the Heisenberg limit with nonclassical N-photon inputs.

ACKNOWLEDGMENT

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