

Quantum Filter for Nonlocal Polarization Properties of Photonic Qubits

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We present an optical filter that transmits photon pairs only if they share the same horizontal or vertical polarization, without decreasing the quantum coherence between these two possibilities. Various applications for entanglement manipulations and multiphoton qubits are discussed.

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The manipulation and control of polarization entanglement between individual photons is an important challenge for optical implementations of quantum information concepts. Recently, significant progress has been achieved by proposals for phase gates using only linear optical elements, single photon sources, and highly efficient detectors [1,2]. These proposals show that strong nonlinear effects can be implemented by exploiting postselection strategies based on recently developed single photon technologies [3–7]. In the following, we apply the same principles to realize a filter sensitive only to the relative polarization of two photons. Since this filter is not sensitive to the individual polarization of the photons, it can preserve and even create entanglement between the photons passed through it. The function of this filter is one of the most elementary operations on the four dimensional Hilbert space of a pair of photonic qubits. It can thus be applied to a wide range of problems, from quantum nondemolition measurements of entanglement to the generation of multiphoton quantum codes.

The basic element of our filter is a simple beam splitter of reflectivity $R = 1/2$. The operation of this beam splitter can be described by a unitary transformation $\hat{U}_{1/2}$ acting on the joint quantum state of the two input modes. We now consider only cases where one of the two modes has a one photon input and a one photon output. This case can be realized by using a single photon source to supply one photon coinciding with the input field state in the other mode, and by postselecting the one photon output events on the same side of the beam splitter with a highly efficient photon detector. The operation of this device on the input state is then described by the operator \hat{S}_{11} obtained from the appropriate matrix elements of the beam splitter operator $\hat{U}_{1/2}$,

$$\begin{aligned} \langle m; 1 | \hat{U}_{1/2} | n; 1 \rangle &= \langle m | \hat{S}_{11} | n \rangle \\ &= \delta_{n,m} \left(\frac{i}{\sqrt{2}} \right)^{n+1} (n-1). \end{aligned} \quad (1)$$

In particular, note that $\hat{S}_{11} | 1 \rangle = 0$, meaning that this operation of the beam splitter eliminates the one photon component of the input state. On the other hand, photon pairs are transmitted since $\hat{S}_{11} | 2 \rangle \neq 0$. By using the postselection condition, we have thus turned the simple beam split-

ter into a nonlinear filter transmitting photons only if they come in pairs.

This filter can now be applied to photon pairs in orthogonal input modes by using the photon bunching properties of the $R = 1/2$ beam splitter, specifically

$$\hat{U}_{1/2} | 1; 1 \rangle = \frac{i}{\sqrt{2}} (| 0; 2 \rangle + | 2; 0 \rangle), \quad (2)$$

$$\hat{U}_{1/2} \frac{i}{\sqrt{2}} (| 0; 2 \rangle + | 2; 0 \rangle) = -| 1; 1 \rangle.$$

These two transformations show that photon pairs entering a Mach-Zehnder interferometer setup from different sides of the input beam splitter will always be found in the same path of the interferometer but will separate again at the output. By using the nonlinear filter on both paths of a Mach-Zehnder setup, it is therefore possible to transmit photons only if one photon enters from each side of the input beam splitter. The operator for this two mode transformation reads

$$\begin{aligned} \hat{U}_{1/2}(1,2) [\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)] \hat{U}_{1/2}(1,2) | 1; 1 \rangle &= -\frac{1}{4} | 1; 1 \rangle, \\ \hat{U}_{1/2}(1,2) [\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)] \hat{U}_{1/2}(1,2) | 0; 1 \rangle &= 0, \\ \hat{U}_{1/2}(1,2) [\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)] \hat{U}_{1/2}(1,2) | 1; 0 \rangle &= 0, \\ \hat{U}_{1/2}(1,2) [\hat{S}_{11}(1) \otimes \hat{S}_{11}(2)] \hat{U}_{1/2}(1,2) | 0; 0 \rangle &= \frac{1}{2} | 0; 0 \rangle. \end{aligned} \quad (3)$$

Note that the two photon component is both phase shifted and attenuated with respect to the vacuum component. The phase shift is easily compensated by a linear element operating on either one of the output fields. The attenuation can be compensated when the filter is applied to the polarization of a two photon input.

Figure 1 shows the complete setup for the two photon polarization filter. The filter described by Eq. (3) is applied to the horizontally polarized components of the two input photons. The $| 1; 1 \rangle$ component therefore corresponds to the $| H; H \rangle$ component of the two photon state, and the $| 0; 0 \rangle$ component corresponds to the $| V; V \rangle$ component. The attenuation of the $| V; V \rangle$ component is realized by the beam splitter of reflectivity $R = 3/4$ in one of the vertically polarized paths. A photon detector postselects only polarized events with zero reflected photons, effectively suppressing

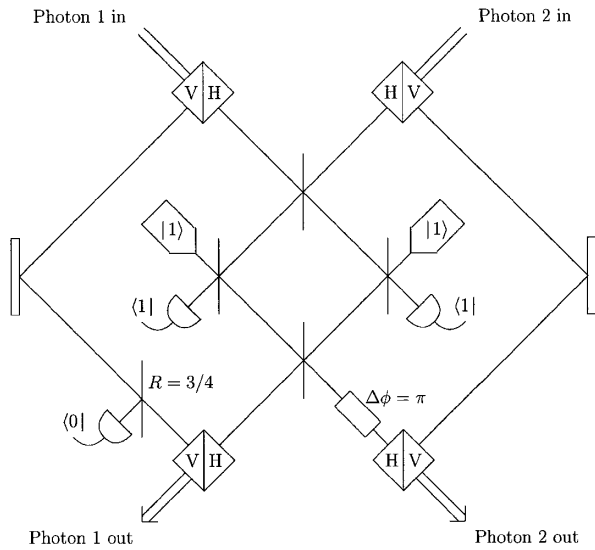


FIG. 1. Schematic setup of the quantum filter for two photon polarization correlations. Vertical lines represent beam splitters with a reflectivity of $R = 1/2$, unless labeled otherwise. The boxes labeled with H and V represent polarization sensitive beam splitters transmitting H polarized photons and reflecting V polarized ones. Quantum states created by single photon sources are given by bra notation, and postselected detector states by ket notation. Note that the $R = 3/4$ beam splitter has a vacuum input not explicitly given in the figure.

the one photon component by an amplitude factor of $1/2$. The components of $|V; H\rangle$ and $|H; V\rangle$ correspond to the $|0; 1\rangle$ and $|1; 0\rangle$ states in Eq. (3) and are therefore completely suppressed by the filter. The operation of this filter can now be summarized in a single operator \hat{S}_{total} describing the effect on an arbitrary two photon input state. It reads

$$\hat{S}_{\text{total}} = \frac{1}{4}(|H; H\rangle\langle H; H| + |V; V\rangle\langle V; V|). \quad (4)$$

This operator projects the input state onto the two dimensional subspace of identical horizontal or vertical polarization, where the factor of $1/4$ is an expression of the postselection efficiency indicating that, even within the transmitted subspace of the parallel polarization components $|H; H\rangle$ and $|V; V\rangle$, only $1/16$ of the input states pass the filter.

The most significant property of this filtering process is that it leaves the entanglement properties of the input intact by preserving the quantum coherence between $|H; H\rangle$

and $|V; V\rangle$. In fact, it can be shown that any maximally entangled photon pair will still be maximally entangled if it passes this filter. In the H, V representation, all maximally entangled states are of the form

$$|\max(c_1, c_2, \phi)\rangle = \frac{1}{\sqrt{2}} [|H\rangle \otimes (c_1|H\rangle + c_2|V\rangle) + e^{-i\phi}|V\rangle \otimes (c_2|H\rangle - c_1|V\rangle)]. \quad (5)$$

The application of the filter to an entangled photon pair of this kind then produces an output of

$$\hat{S}_{\text{total}}|\max(c_1, c_2, \phi)\rangle = \frac{c_1}{4\sqrt{2}} (|H; H\rangle - e^{-i\phi}|V; V\rangle). \quad (6)$$

This property demonstrates that the filter is sensitive only to nonlocal polarization correlations of the two photons. It is therefore a suitable tool for entanglement manipulations.

The clearest physical description of the filter properties is that of a quantum nondemolition measurement for the correlation of the H - V polarization of the photon pair. In this context, it is remarkable that the relative orientations of all other polarizations remain unchanged, even though the local polarizations are necessarily randomized by the measurement backaction. For example, it is possible to put one right and one left circular polarized photon through the filter. The output will still have opposite circular polarization, while all linear polarizations will be parallel,

$$\begin{aligned} \hat{S}_{\text{total}}|R; L\rangle &= \hat{S}_{\text{total}} \frac{1}{\sqrt{2}} (|H\rangle + i|V\rangle) \otimes \frac{1}{\sqrt{2}} (|H\rangle - i|V\rangle) \\ &= \frac{1}{8} (|H; H\rangle + |V; V\rangle) \\ &= \frac{1}{8} (|R; L\rangle + |L; R\rangle). \end{aligned} \quad (7)$$

This filter can thus be used to entangle two previously independent photons. Moreover, the filter can create multiphoton entanglement if it is applied to members of entangled pairs, for example, to photon 2 of an entangled pair (1, 2), and to photon 3 of an entangled pair (3, 4). The output of this filtering process then reads

$$\hat{S}_{\text{total}}(2, 3) \frac{1}{\sqrt{2}} (|H; H\rangle_{1,2} + |V; V\rangle_{1,2}) \otimes \frac{1}{\sqrt{2}} (|H; H\rangle_{3,4} + |V; V\rangle_{3,4}) = \frac{1}{8} (|H; H; H; H\rangle + |V; V; V; V\rangle). \quad (8)$$

While this method of generating multiphoton entanglement may seem a bit more complicated than the simple postselection schemes recently realized to generate three- and four-photon entanglement [8,9], it has the advantage of allowing only single photon states in the output ports. Therefore, it can be used as a reliable source of multiphoton entanglement, even in applications where postselection of one photon events in the output ports is not an option.

Another important feature of the filter is the transfer of quantum coherence from single photon states to multiphoton states. If one of the input photons is polarized along the diagonal between the H and V directions, the filter transfers the quantum coherence of the other input photon to a two photon state,

$$\hat{S}_{\text{total}} \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) \otimes (c_H|H\rangle + c_V|V\rangle) \\ = \frac{1}{4\sqrt{2}} (c_H|H;H\rangle + c_V|V;V\rangle). \quad (9)$$

This process conserves the quantum information carried by the second photon. Effectively, the output represents a very simple form of a two photon quantum code. Such a quantum code could be useful to improve the error resistance of quantum computation and teleportation [1,10]. In particular, this code makes the qubit resistant

$$\hat{S}_{\text{total}}(2,3) \frac{1}{\sqrt{2}} (|H;H\rangle_{1,2} + |V;V\rangle_{1,2}) \otimes (c_H|H\rangle_3 + c_V|V\rangle_3) = \frac{1}{4\sqrt{2}} (c_H|H;H;H\rangle + c_V|V;V;V\rangle). \quad (10)$$

In general, filtering one photon of an $N - 1$ photon entangled state and the original single photon qubit automatically encodes the input qubit into a corresponding N -photon code.

The implementation of quantum codes and the creation of entanglement between previously unentangled inputs are just two examples for a wide range of applications. As can be seen from Eq. (4), the quantum filter described above realizes an elementary form of entanglement manipulation. Moreover, the setup shown in Fig. 1 can be realized using only standard elements of linear optics and photon detectors. However, it should be noted that a high level of precision is required to avoid errors in the filtering process. For instance, it is necessary to avoid dark counts in the detectors. This can be achieved by limiting the time window of the photon detection to the pulse length of the single photon inputs. For example, at a pulse length of 1 ns, the dark count rate of 10 000 counts per second estimated for the detectors presented in [3,4] translates into a mere 10^{-5} counts per pulse. Also, care should be taken to optimize mode matching and pulse timing to achieve maximal photon bunching in the beam splitters. This requirement is typical for experiments using photon entanglement, and previous experimental results suggest that sufficient precision can be achieved using conventional techniques [8,9,11]. A potentially more serious difficulty arises from limited detector efficiencies. The detectors in the H polarized paths need to distinguish between one photon events and two photon events. While the detectors presented in [3,4] should be well suited for this task, the quantum efficiency for this operating regime of the detectors is presently limited to 88%. This means that there is a 19% chance of mistakenly registering a two photon event as a one photon event. Unfortunately, a detailed analysis of this error is beyond the scope of this Letter. However, a good estimate of the error frequency can be obtained by focusing on the errors for the $|H;V\rangle$ input state. For the setup given in Fig. 1, this input state has the highest error frequency, since the $|H;V\rangle$ component is filtered out entirely by a zero or two photon event at the detectors in the H polarized paths. The probability of a two photon event at one of the detectors and a one photon event at the other

against decoherence caused by projective measurements of the circular polarization or of polarization components along the diagonals between the H and V directions. Such errors are typical for the basic implementation of quantum teleportation using polarization entanglement [11]. Therefore, this type of code may be useful for the enhancement of teleportation efficiency. The transfer of quantum information from a single photon qubit to a multiphoton state may be generalized to an arbitrary number of photons by using entangled input photons. For example, quantum coherence is transferred to a three photon state by

is 1/4. Therefore, the total frequency of errors at a quantum efficiency of 88% is given by $19\%/4 \approx 5\%$. While this error rate is still very low, it must be compared to the filter efficiency of $1/16 = 6.25\%$ for $|H;H\rangle$ and $|V;V\rangle$ input states. For example, the operation described in Eq. (7) will produce the correct entangled state with a probability of 3.13%. However, the chance of a mistaken transmission due to the 25% contribution of the $|H;V\rangle$ component in the input is about 1.25%. This mistake would then lead to a mixture of about 70% entangled photon pairs originating from successful filter operations, and about 30% V polarized single photon outputs originating from failure to detect both of two H polarized photons at one of the detectors. This example illustrates the importance of further increasing detector efficiency to distinguish one and two photon events with greater reliability.

In conclusion, a quantum filter for photonic qubit pairs can be realized using only linear optics and single photon counters. Such a filter may be a very useful component for optical quantum information networks. We have presented various applications and discussed the technological requirements for an experimental realization of this scheme. At present, the major technological challenge appears to be the improvement of detector efficiency to distinguish single photon events from two photon events. If this difficulty can be overcome, the quantum filter for non-local polarization properties of photonic qubits may prove to be a very flexible tool for the implementation of photon based quantum information technologies.

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