Thermal photon statistics in laser light above threshold

Holger F. Hofmann
Department of Physics, Faculty of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

Ortwin Hess
Institut für Technische Physik, DLR, Pfaffenwaldring 38-40, D-70569 Stuttgart, Germany

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We show that the reduction in photon-number fluctuations at laser threshold often cited as a fundamental laser property does not occur in small semiconductor lasers. The conventional theory of threshold noise is not valid in lasers with a spontaneous emission factor larger than 10^{-8}. If the spontaneous-emission factor is larger than 10^{-4}, the photon-number statistics even remain thermal far above threshold. We therefore conclude that the reduction in photon-number fluctuations is not a fundamental laser property but rather a matter of size and the corresponding relative importance of quantum fluctuations above threshold.

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The matter of photon-number fluctuations in a single-mode laser has been studied more than thirty years ago in a number of pioneering works [1–5]. At that time the question of photon-number fluctuations at the laser threshold was resolved by adiabatically eliminating the excitation dynamics of the gain medium. That procedure allows the formulation of a photon-number rate equation for the single-mode light field that can easily be solved analytically [6]. However, the requirement that at every instant the gain function immediately adjusts to the photon number in the cavity is not necessarily a valid assumption close to the laser threshold. In the following, we show that the validity of the assumption depends on laser size and breaks down as the lasers get smaller. In particular, we derive a general expression for the photon-number fluctuations of both small and large lasers and demonstrate that, indeed, near threshold the photon statistics of small lasers such as typical semiconductor laser diodes are quite different from those of larger lasers.

The dynamics of a single-mode laser can be described by the rate equations

$$\begin{aligned}\frac{d}{dt}N&=j-\frac{1}{\tau_{sp}}N-2\frac{\beta}{\tau_{sp}}(N-N_T)n,\\ \frac{d}{dt}n&=2\frac{\beta}{\tau_{sp}}(N-N_T)n-\frac{1}{\tau_{cav}}n+\frac{\beta}{\tau_{sp}}N.\end{aligned}\tag{1}$$

The dynamical variables are the photon number n in the cavity mode and the excitation number N in the gain medium. The physical properties of the laser device are characterized by four device parameters. These are the spontaneous relaxation rate τ_{sp}^{-1} of the excitations, the spontaneous emission factor β defined as the ratio between the spontaneous emission rate into the laser mode and the total spontaneous relaxation rate of the excitations, the photon lifetime τ_{cav} inside the optical cavity, and the excitation number N_T in the gain medium at transparency. The pump rate is given by j. With respect to electrically pumped semiconductor laser diodes, it will be referred to as the injection current.

The excitation density at transparency N_T/V, the spontaneous emission rate τ_{sp}^{-1} and the dependence of the spontaneous emission factor β on the volume V of the cavity are properties of the gain medium. The order of magnitude of the cavity lifetime τ_{cav} is also defined by the gain medium since the cavity loss rate $(2\tau_{cav})^{-1}$ must be lower than the maximal amplification rate $\beta N_T/\tau_{sp}$ to achieve laser operation. Therefore, the device properties in the rate equations depend mainly on the material properties of the gain medium and on cavity size. For typical semiconductor laser diodes the device parameters are

$$\beta V \approx 10^{-14}\ \mathrm{cm}^3,\tag{2a}$$

$$\frac{N_T}{V}\approx 10^{18}\ \mathrm{cm}^{-3},\tag{2b}$$

$$\tau_{spont}\approx 3\times 10^{-9}\ \mathrm{s},\tag{2c}$$

$$\tau_{cav}>1.5\times 10^{-13}\ \mathrm{s}.\tag{2d}$$

The very fact that semiconductor lasers can be as small as a few μm in size is a direct consequence of the relatively high spontaneous emission rate τ_{sp}^{-1}.

The stable stationary excitation number $\overline{N}$ and the stable stationary photon number $\overline{n}$ may be obtained as a function of injection current j. Transparency is reached at an injection current of $j=N_T/\tau_{sp}$. The photon number at transparency is

$$n_T=\beta N_T\frac{\tau_{cav}}{\tau_{sp}}.\tag{3}$$

This photon number is a measure of the cavity lifetime in units of $\beta N_T/\tau_{sp}=3\times 10^{-13}$ s. Typical values will be between one and two photons corresponding to cavity lifetimes of two to four times the minimum required to achieve laser operation.

The laser threshold is defined by the light-current characteristic

$$\frac{\overline{n}}{\tau_{cav}}=\frac{j-j_{th}-\tau_{cav}^{-1}}{2}+\frac{1}{2}\sqrt{(j-j_{th})^2+\tau_{cav}^{-1}(2j_{th}+\tau_{cav}^{-1})}. \tag{4}$$

via the threshold current j_{th}. The threshold current j_{th} marks the point at which the transition from an almost negligible slope of the light-current characteristic (4) to a slope of one takes place. This clearly corresponds to the intuitive notion of the laser light "turning on" at the laser threshold. In terms of the device parameters (2), the threshold current j_{th} reads

$$j_{th}=\frac{N_T}{\tau_{sp}}\left[\left(1+\frac{1}{2n_T}\right)-\beta\left(1+\frac{1}{n_T}\right)\right], \tag{5}$$

where the cavity lifetime τ_{cav} has been expressed in terms of the photon number at transparency n_T. Note that since lasing requires that $n_T>0.5$, this current is always less than twice the current required to reach transparency. It is thus possible to estimate the spontaneous emission factor directly from the threshold current. In electrically pumped semiconductor laser diodes, the product of the threshold current and the spontaneous emission factor is approximately 0.5 μA, e.g., a typical spontaneous emission factor of 10^{-5} corresponds to a threshold current of 50 mA.

Since for the purpose of photon statistics we will in the following express the point of operation in terms of the average photon number $\overline{n}$ in the cavity, it is the photon number n_{th} at $j=j_{th}$ that defines the laser threshold. Assuming that the spontaneous emission factor β is sufficiently smaller than one, this photon number reads

$$\overline{n}_{th}=\sqrt{\frac{n_T+\frac{1}{2}}{2\beta}}. \tag{6}$$

This photon number is much higher than the photon number at transparency n_T, indicating that even below threshold stimulated processes contribute more to the light-field intensity than spontaneous emissions.

With these definitions, the photon-number fluctuations may now be obtained from the linearized dynamics of the excitation-number fluctuation $\delta N=N-\overline{N}$ and the photon-number fluctuation $\delta n=n-\overline{n}$ that read

$$\frac{d}{dt}\begin{pmatrix}\delta N\\ \delta n\end{pmatrix}=-\begin{pmatrix}\Gamma_N & r\omega_R\\ r^{-1}\omega_R & \gamma_n\end{pmatrix}\begin{pmatrix}\delta N\\ \delta n\end{pmatrix}+\mathbf{q}(t), \tag{7}$$

where γ_n is the relaxation rate of the photon-number fluctuation and Γ_N is the relaxation rate of the excitation-number fluctuation. The coupling rate ω_R describes the rate at which the holeburning effect of a photon-number fluctuation acts back on that fluctuation. The fluctuation ratio r is a measure of the relative importance of photon-number noise with respect to excitation-number fluctuations. In terms of the stationary photon number $\overline{n}$ and the four device parameters N_T, n_T, β, and τ_{sp} the rates and the ratio read

$$\gamma_n=\tau_{sp}^{-1}\frac{\beta N_T}{n_T}\frac{n_T+\frac{1}{2}}{\overline{n}+\frac{1}{2}}, \tag{8a}$$

$$\Gamma_N=\tau_{sp}^{-1}(1+2\beta\overline{n}), \tag{8b}$$

$$\omega_R=\tau_{sp}^{-1}\sqrt{2\beta\frac{\beta N_T}{n_T}(\overline{n}-n_T)}, \tag{8c}$$

$$r=\sqrt{\frac{N_T}{2n_T}\frac{(\overline{n}-n_T)}{\left(\overline{n}+\frac{1}{2}\right)^2}}. \tag{8d}$$

The fluctuation term $\mathbf{q(t)}$ is the shot noise arising from the quantization of excitation energy and light field intensity. Since the excitation number N_T at transparency is usually much larger than the average photon number $\overline{n}$ in the cavity, the ratio r is much larger than one, indicating that the fluctuations in the excitation number are much smaller than the fluctuations in the photon number. It is therefore reasonable to consider only the photon-number contribution. Thus,

$$\mathbf{q}(t)=\begin{pmatrix}0\\ q_n\end{pmatrix},$$

$$\text{with}\quad \langle q_n(t)q_n(t+\Delta t)\rangle=2\overline{n}(\overline{n}+1)\gamma_n\delta(\Delta t). \tag{9}$$

Note that this approximation is not valid for the low-frequency part of the noise spectrum since energy conservation requires that the low-frequency noise is a function of the noise in the injection current at high quantum efficiencies [7].

With these assumptions we obtain the photon-number fluctuations

$$\langle\delta n^2\rangle=\overline{n}(\overline{n}+1)\frac{1}{1+\dfrac{\Gamma_N\omega_R^2}{\gamma_n(\omega_R^2+\Gamma_N\gamma_n+\Gamma_N^2)}}. \tag{10}$$

This function is always lower than the thermal noise limit of $\langle\delta n^2\rangle=\overline{n}(\overline{n}+1)$. If the noise threshold is defined as the point at which the photon-number fluctuation drops below half the thermal noise limit, then this threshold is determined from

$$\Gamma_N\omega_R^2=\gamma_n(\omega_R^2+\Gamma_N\gamma_n+\Gamma_N^2). \tag{11}$$

If $\Gamma_N\gg\gamma_n$ and $\Gamma_N\gg\omega_R$, the excitation dynamics may be adiabatically eliminated. This is the basic assumption made in the conventional derivation of threshold noise [6]. Indeed, one finds that the photon number $\overline{n}_{1/2}$ at which the fluctuations correspond to one-half thermal noise is in that case given by

$$\bar{n}_{1/2}=\sqrt{\frac{n_T+\frac{1}{2}}{2\beta}}=\bar{n}_{th}. \tag{12}$$

The noise threshold is then identical to the laser threshold in agreement with the predictions and observations made in the early days of laser physics [1–5]. However, the requirement that $\Gamma_N \gg \omega_R$ at $\bar{n}=\bar{n}_{th}$ is only valid for

$$\beta \ll \frac{1}{2n_T+1}\left(\frac{n_T}{\beta N_T}\right)^2 \approx 10^{-8}. \tag{13}$$

In electrically pumped semiconductor lasers this would correspond to a threshold current of more than 50 A. Therefore, the assumption that the fluctuations δN in excitation number can be adiabatically eliminated at threshold does not apply to typical semiconductor laser diodes that commonly have threshold currents significantly below 50 A. Thus, for such devices the conventional theory no longer describes the photon-number fluctuations at threshold. Instead, the complete dynamics of the fluctuations both in excitation number and in the number of photons needs to be taken into account.

For typical semiconductor lasers with spontaneous emission factors $\beta \gg 10^{-8}$, the coupling rate ω_R is much larger than the relaxation rate Γ_N at laser threshold. The noise threshold condition (11) then reduces to

$$\Gamma_N=\gamma_n. \tag{14}$$

As long as the total rate of spontaneous emission τ_{sp}^{-1} is still greater than the rate of stimulated emission $2\beta\bar{n}\tau_{sp}^{-1}$, the photon number at the noise threshold is fixed by the properties of the gain medium at

$$\bar{n}_{1/2}=\beta N_T\left(1+\frac{1}{2n_T}\right)\approx 10^4. \tag{15}$$

However, stimulated emission takes over as the major relaxation mechanism in the gain medium at an injection current is twice the threshold current. The noise threshold is located beyond an injection current of two times threshold current in laser devices with

$$\beta > \left[2\beta N_T\left(1+\frac{1}{2n_T}\right)\right]\approx 10^{-4}. \tag{16}$$

This situation should apply in diodes with threshold currents of less than 5 mA. In such devices the noise threshold is given by

$$\bar{n}_{1/2}=\sqrt{\frac{N_T}{2}\left(1+\frac{1}{2n_T}\right)}\approx 10^2\sqrt{\beta}. \tag{17}$$

For semiconductor laser diodes, the dependence of the photon number at the noise threshold $\bar{n}_{1/2}$ on the spontaneous emission factor β may thus be summarized as illustrated in Fig. 1:

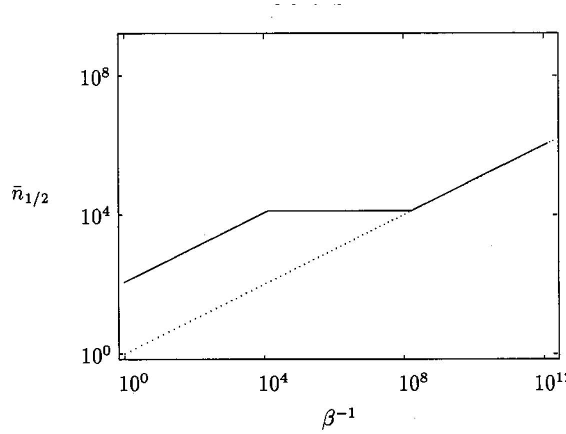

FIG. 1. Noise threshold $\bar{n}_{1/2}$ as a function of the spontaneous emission factor β. The dotted line shows the threshold photon number n_{th}.

$$\bar{n}_{1/2}=\begin{cases}\dfrac{1}{\sqrt{\beta}} & \text{for} \quad \beta<10^{-8},\\ 10^4 & \text{for} \quad 10^{-8}<\beta<10^{-4},\\ \dfrac{10^2}{\sqrt{\beta}} & \text{for} \quad 10^{-4}<\beta.\end{cases} \tag{18}$$

Since the photon number at $j=2j_{th}$ is approximately equal to $(n_T+1/2)/\beta$, the current $j_{1/2}$ at which the photon-number fluctuations drop to one-half their thermal value is approximately given by

$$\frac{j_{1/2}-j_{th}}{j_{th}}=\begin{cases}0 & \text{for} \quad \beta<10^{-8},\\ 10^4\beta & \text{for} \quad 10^{-8}<\beta<10^{-4},\\ 10^2\sqrt{\beta} & \text{for} \quad 10^{-4}<\beta,\end{cases} \tag{19}$$

as illustrated in Fig. 2.

In conclusion, the assumption that above threshold the photon-number fluctuations of laser light are lower than the fluctuations in equally coherent thermal light sources is not valid in typical semiconductor lasers. In particular, laser di-

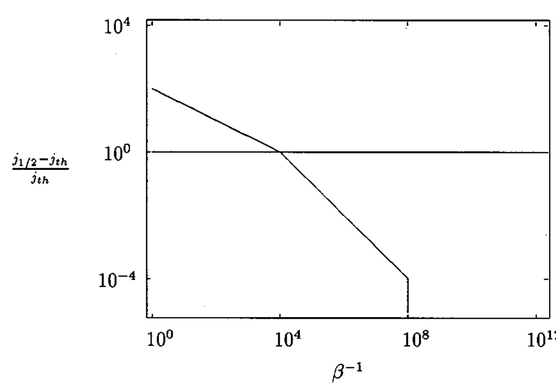

FIG. 2. Normalized difference between the injection current at one-half thermal noise $j_{1/2}$ and the threshold current j_{th} as a function of the spontaneous emission factor β.

odes with a threshold current of less than 5 mA still fluctuate thermally far above threshold. Thus it is not possible to distinguish in principle between lasers and thermal light sources based on the statistical properties of the emitted light field [8]. Therefore ''black box'' laser definitions disregarding the nature of the internal light-matter interaction by which the light field is generated do not apply to typical semiconductor laser diodes. If the definition of laser light is nevertheless based on the photon-number fluctuations as suggested e.g., by Wiseman [9], then the light from most laser diodes could not be considered laser light even though it is definitely generated by laser amplification. Moreover, a laser definition based on the condition that the relaxation rate of the excitations given by Γ_N must be larger than the optical relaxation rate γ_n entirely fails to relate to the original meaning of the acronym laser, i.e., *light amplification by stimulated emission of radiation*.

It therefore seems to be reasonable to distinguish between a thermal laser regime and a saturated laser regime separated by the noise threshold discussed above. In the thermal regime, laser light indeed is indistinguishable from lamp light. In fact, the thermal laser regime naturally connects the saturated laser regime to the black body radiator from which the concepts of spontaneous and stimulated emission originated [10], thus providing a ''missing link'' in the theory of lasers and quantum optics.

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