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On the Impact of Characterization of Symmetric Balanced Incomplete Block Designs on Their Complements

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It is trivially known that the complement of a symmetric balanced incomplete block (SBIB) design is again an SBIB design. While charactering all the SBIB designs, Mohan, Kageyama and Nair [1] have obtained a new classification of these SBIB designs. It is found that this classification has a specific impact on the complements of these SBIB designs. This has been explored in this paper in the light of [1]. In addition, at the end, these designs along with their other properties have been tabulated for the ready reference and for practical purposes.

Keywords: Symmetric balanced incomplete block (SBIB) design, Complement of design.

1. Introduction

A balanced incomplete block (BIB) design is an arrangement of v symbols in b sets, each containing $k(< v)$ symbols such that every symbol occurs at most once in a set, every symbol occurs in exactly r sets, and every pair of symbols occurs together in λ sets. The parameters of a BIB design satisfy

$$vr = bk, \quad \lambda(v-1) = r(k-1).$$

A BIB design is said to be symmetric if $v = b$ and consequently $r = k$. It is well known (cf. [2]) that in a symmetric BIB design any two sets have exactly λ symbols in common. These SBIB designs have been studied in a specific outlook by Mohan, Kageyama and Nair [1].

It is clear that the complement of an SBIB design is again an SBIB design. In the light of the theory developed in [1], it is proposed to study the impact of that characterization of SBIB designs on their complements.

In this paper we consider block designs within the scope of parameters $v \leq 111$, $k \leq 55$ and $\lambda \leq 30$, in which there are some SBIB designs. We may take the ranges of parameters to the higher values also if need be. In this attempt the SBIB designs have been classified into three types, where Type I is with $k = n\lambda$ for an integer $n \geq 2$, Type II is with $k = n\lambda+1$ for an integer $n \geq 1$ and Type III is with $k = n\lambda + m$ for an integer $n \geq 1$ and $m \geq 2$, which gave an insight on all SBIB designs in a different way. This type of characterization is entirely distinct and found useful for constructional purposes of designs. Depending on these types when their complements of designs are taken then to find their types and the other properties are found interesting, which have been explored in this paper.

2. Main result

The complements of SBIB designs can be characterized as the following shows.

Theorem 2.1. a) The complement of an SBIB design of Type I is either of Type II or Type

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III. b) The complement of an SBIB design of Type II is either of Type I, Type II or Type III. c) The complement of an SBIB design of Type III is either of Type I, Type II or Type III.

Proof. a) Consider the parameters of an SBIB design of Type I as given in [1] as S_1: $v = b = n^2\lambda - n + 1, r = k = n\lambda, \lambda$ for $n \geq 2$. Its complement S_1^* has parameters $v^* = b^* = n^2\lambda - n + 1$, $r^* = k^* = (n^2\lambda - n + 1) - n\lambda = (n-1)(n\lambda - 1)$, $\lambda^* = n^2\lambda - n + 1 - 2n\lambda + \lambda = (n-1)(n\lambda - 1) - (n-1)\lambda = r^* - (n-1)\lambda$. Hence $r^* = \lambda^* + (n-1)\lambda$, which is of Type III, since $(n-1)\lambda < \lambda^*$, when $n \geq 3$ and $\lambda \geq 1$. In the case when $n = 2$, we have S_1 with $v = b = 4\lambda - 1, r = k = 2\lambda, \lambda$. So S_1^* has $v^* = b^* = 4\lambda - 1, r^* = k^* = 2\lambda - 1, \lambda^* = \lambda - 1$. Therefore $r^* = k^* = 2(\lambda - 1) + 1 = 2\lambda^* + 1$. Hence it is of Type II for any λ.

b) As given in [1], the parameters of an SBIB design of Type II are given by S_2: $v = b = n^2\lambda + n + 1, r = k = n\lambda + 1, \lambda$. In addition, the parameters of its complement S_2^* has parameters $v^* = b^* = n^2\lambda + n + 1$, $r^* = k^* = (n-1)n\lambda + n$, $\lambda^* = r^* - (n-1)\lambda - 1$. Thus $r^* = \lambda^* + (n-1)\lambda + 1$, which is of Type III, considering $(n-1)\lambda + 1$ as some $m > 1$, since $\lambda^* > (n-1)\lambda + 1$ for $n \geq 3$. In particular, when $n = 2$, we have S_2 with $v = b = 4\lambda + 3, r = k = 2\lambda + 1, \lambda$. So S_2^* has $v^* = b^* = 4\lambda + 3, r^* = k^* = 2(\lambda + 1), \lambda^* = \lambda + 1$. Therefore $r^* = 2(\lambda + 1) = 2\lambda^*$, which is of Type I. Hence the complement of an SBIB design of Type II are of Type I when $n = 2$, and of Type III when $n > 2$, as we have $r^* = \lambda^* + (n-1)\lambda + 1$. Consider $(n-1)\lambda + 1$ as some $m > 1$. Here $m >, =, < \lambda^*$ according to the Type of the design. Finally, note the following: In the SBIB designs of Type II, when $n = 1$, the designs are irreducible designs, i.e., $S_2 : v = b = \lambda + 2, r = k = \lambda + 1, \lambda$, and hence its complement S_2^* has $v^* = b^* = \lambda + 2, r^* = k^* = 1, \lambda^* = 0$. Since $r^* = n \cdot 0 + 1$, this is of Type II and all these designs are trivial designs (see Table 4).

c) As given in [1], the parameters of an SBIB design of Type III are given by S_3: $v = b = n^2\lambda + n(2m-1) + 1 + m(m-1)/\lambda, r = k = n\lambda + m, \lambda$. In addition, its complement has the parameters $S_3^* : v^* = b^* = n^2\lambda + n(2m-1) + 1 + m(m-1)/\lambda, r^* = k^* = (n\lambda + m)(n\lambda + m - \lambda - 1)/\lambda + 1 = r(r - \lambda - 1)/\lambda + 1, \lambda^* = b - 2r + \lambda = n^2\lambda + n(2m-1) + 1 + m(m-1)/\lambda - 2(n\lambda + m) + \lambda = (n\lambda + m)[(n-2)\lambda + m - 1]/\lambda + \lambda + 1$. Thus $\lambda^* = r^* - \{(n\lambda + m) - \lambda\} = r^* - \{(n-1)\lambda + m\}$. Hence $r^* = \lambda^* + (n-1)\lambda + m$. When $n = 1$, we have $r^* = \lambda^* + m_1$ (say). Now this m_1 may be as $\lambda^* >, =, < m_1$. If $\lambda^* > m_1$ it is of Type III. If $\lambda^* = m_1$ it is of Type I. If $\lambda^* < m_1$, then this may be of the form $\ell\lambda^*, \ell\lambda^* + 1, \ell\lambda^* + m_2$, where ℓ is an integer > 1, consequently giving out that it will be of Type I, Type II, or Type III, respectively. Similarly, for the m_2 taken above the same procedure may be adopted if needed. When $n \geq 2$, the same treatment as above may be taken and hence we obtain the resulting designs which will be of Type I, Type II or Type III. □

Remark 2.1. In the case of SBIB designs of Type I, when $n = 2$ and $\lambda = 2$, we get an SBIB design with parameters $v = b = 7, r = k = 4, \lambda = 2$, which is of Type I, whose complement has $v^* = b^* = 7, r^* = k^* = 3, \lambda^* = 1$, which can be shown as Type I or Type II. Similarly, in the case of an SBIB design of Type II when $n = 2$ and $\lambda = 1$ we get the parameters $v = b = 7, r = k = 3, \lambda = 1$, which can be taken as Type II, whose complement has $v^* = b^* = 7, r^* = k^* = 4, \lambda^* = 2$, which is of Type I.

Remark 2.2. (1) In Table 3.1 of [1] there are 178 designs. Out of these designs, there are 58 designs of Type I and among complements of these 58 designs, there are 27 designs of Type II, and 31 designs of Type III. Moreover, out of these 58 complements, 41 designs are in Table 3.1 of [1], and the remaining 17 complements are out of the considered parametric range and hence not in the table mentioned. However, these 17 designs are of Type III only. (2) Again considering the same table, there are 76 designs of Type II and among their 76 complements, 27 designs are of Type I, 29 designs are of Type II, which have not been taken into consideration in the statement of Theorem 2.1. Furthermore, 20 designs are of Type III. In addition, out of these 76 complements, 34 designs are in the said table, and 42 designs are not in the table. Among these 42 designs one design is of Type I, 29 designs are of Type II, which are trivial designs with $\lambda = 0$ and the remaining 12 designs are of Type III. (3) Lastly, considering the same Table 3.1 of [1], out of the 36 complements of SBIB designs of Type III, 14 designs are of Type I, 8 designs are of Type II and 14 designs are of Type III. In addition, out of these 36 complements, 30 designs are in the said table and the remaining 6 designs are not in the table due to the limitations of the parametric range. Moreover, all these 6 designs are of Type III. In Table 3.1 of [1], it seems that a few complements alone are included, but in fact, the above argument addresses this query, clearing that all the complements are included in the table itself. Hence the 178 designs in it include the designs and their complements.

Remark 2.3. In Table 3.1 of [1] we have considered the irreducible designs up to $v = 31$ and they can be further considered to any extent if needed.

Remark 2.4. In the self-complementary designs the complement of a BIB design is again the same design and hence nothing can be said in this regard. The parameters of self-complementary BIB designs are given by $v = 2k, b = 2r, r, k, \lambda$. Furthermore, the complement of an SBIB design is again an SBIB design, but regarding the type of the design to which it belongs is to be determined and there are some exceptional cases also and hence it has been dealt with here. It is also interesting to note that $r^* - \lambda^* = r - \lambda$, where r and λ are the parameters of the original BIB design, while r^* and λ^* are those of its complementary BIB design. Furthermore, the complementary designs are also in the list of the original designs numbering 178 within the chosen parametric ranges but for the trivial designs.

3. Tabulations

For the sake of completeness of the paper we give hereunder a list of these 178 designs along with their complements and their status for ready reference. Trivially, one perceives that the complement of an SBIB design is again an SBIB design. But the following tables depict that complement of which type SBIB design goes to which type SBIB design. In addition, when the complement is taken how the formula A, B, D changes along with the other parameters. The notations that we have used in the tables given are as follows.

1. $t = \{m(m-1) - \lambda(n-1)\}/\lambda$.
2. $t^* = \{m^*(m^*-1) - \lambda^*(n^*-1)\}/\lambda^*$.
3. Different series are given below, which are used in Table 3.1 of [1].

Series	Parameters	Under which type
S_1	$v = b = n^2\lambda - n + 1, r = k = n\lambda, \lambda$	Type I
S_2	$v = b = n^2\lambda + n + 1, r = k = n\lambda + 1, \lambda$	Type II
S_3	$v = b = 4t + 3, r = k = 2t + 1, \lambda = t$	Type II, $n = 2$
S_4	$v = b = s^2 + s + 1, r = k = s + 1, \lambda = 1$	Type I, $n = s + 1, \lambda$ (Also Type II, $n = s, \lambda = 1$)
S_5	$v = b = (s+1)(s^2+1), r = k = s^2 + s + 1,$ $\lambda = s + 1$	Type II, $n = s, \lambda = s + 1$
S_6	$v = b = 4m^2, r = k = 2m^2 + m, \lambda = m^2 + m$	Type III, $n = 1, m = m^2$
S_7	$v = b = 4m^2, r = k = 2m^2 - m, \lambda = m^2 - m$	Type III, $n = 1, m = m^2$
	S_6 and S_7 are complements to each other	

Series	Parameters	Under which type
S_8	$v = b = 4m^2 - 1, r = k = 2m^2 - 1, \lambda = m^2 - 1$	Type II, $n = 2$
S_9	$v = b = 4m^2 - 1, r = k = 2m^2, \lambda = m^2$	Type I, $n = 2$/Type III
	S_8 and S_9 are complements to each other	
S_{10}	$v = b = t^2(t+2), r = k = t(t+1), \lambda = t, t \geq 2$	Type I, $n = t+1$/Type II
S_{11}	$v = b = \{n^2\lambda + n(2m-1) + 1\} + m(m-1)/\lambda$,	
	$r = k = n\lambda + m, \lambda$	Type III
	(The generalized expression of SBIB designs)	

All the series S_3 to S_{10} are especially taken from Raghavarao [2].

4. We have formulated the parameter b in the three types in [1], and we will denote them as follows for the tabulation purpose.

Type I $\quad A = b = r + n(r - \lambda) - (n-1), r = k = n\lambda.$

Type II $\quad B = b = r + n(r - \lambda), r = k = n\lambda + 1.$

Type IIII $\quad D = b = r + n(r - \lambda) + m(n-1) + t, r = k = n\lambda + m,$
where $t = \{m(m-1) - \lambda(n-1)\}/\lambda$ is an integer.

In the following tables, "I" means irreducible, "E" means the design existing, "N" means the design in non-existence, "–" means the design unknown, and F stands for the Formula of blocks as above. Complements of SBIB designs with $v = b \leq 111, r = k \leq 55, \lambda \leq 30$ are given with necessary information as follows.

Table 1: Type I designs whose complements are of Type II

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/E/N	F	Series
1	3	2	1	3	2	0	3	1	0	3	1	0	I		
2	7	4	2	2	0	−1	7	3	1	3	0	−2	E	A	S_1
3	11	6	3	2	0	−1	11	5	2	2	1	−1	E	B	S_1, S_3
4	15	8	4	2	0	−1	15	7	3	2	1	−1	E	B	S_2, S_3, S_5, S_8
5	19	10	5	2	0	−1	19	9	4	2	1	−1	E	B	S_2, S_3
6	23	12	6	2	0	−1	23	11	5	2	1	−1	E	B	S_2, S_3
7	27	14	7	2	0	−1	27	13	6	2	1	−1	E	B	S_2, S_3
8	31	16	8	2	0	−1	31	15	7	2	1	−1	E	B	S_2, S_3
9	35	18	9	2	0	−1	35	17	8	2	1	−1	E	B	S_2, S_3, S_8
10	39	20	10	2	0	−1	39	19	9	2	1	−1	E	B	S_2, S_3
11	43	22	11	2	0	−1	43	21	10	2	1	−1	E	B	S_2, S_3
12	47	24	12	2	0	−1	47	23	11	2	1	−1	E	B	S_2, S_3
13	51	26	13	2	0	−1	51	25	12	2	1	−1	E	B	S_2, S_3
14	55	28	14	2	0	−1	55	27	13	2	1	−1	E	B	S_2, S_3
15	59	30	15	2	0	−1	59	29	14	2	1	−1	E	B	S_2, S_3
16	63	32	16	2	0	−1	63	31	15	2	1	−1	–	B	S_2, S_3, S_8
17	67	34	17	2	0	−1	67	33	16	2	1	−1	–	B	S_2, S_3
18	71	36	18	2	0	−1	71	35	17	2	1	−1	–	B	S_2, S_3
19	75	38	19	2	0	−1	75	37	18	2	1	−1	–	B	S_2, S_3
20	79	40	20	2	0	−1	79	39	19	2	1	−1	–	B	S_2, S_3
21	83	42	21	2	0	−1	83	41	20	2	1	−1	–	B	S_2, S_3
22	87	44	22	2	0	−1	87	43	21	2	1	−1	–	B	S_2, S_3
23	91	46	23	2	0	−1	91	45	22	2	1	−1	–	B	S_2, S_3
24	95	48	24	2	0	−1	95	47	23	2	1	−1	–	B	S_2, S_3
25	99	50	25	2	0	−1	99	49	24	2	1	−1	–	B	S_2, S_3, S_8
26	103	52	26	2	0	−1	103	51	25	2	1	−1	–	B	S_2, S_3
27	107	54	27	2	0	−1	107	53	26	2	1	−1	–	B	S_2, S_3

In the design of No. 1, t is not defined.

Table 2: Type I designs whose complements are of Type III

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
28	13	4	1	4	0	−3	13	9	6	1	3	1	E	D	S_{11}
29	16	6	2	3	0	−2	16	10	6	1	4	2	E	D	S_{11}
30	21	5	1	5	0	−4	21	16	12	1	4	1	E	D	S_{11}
31	25	9	3	3	0	−2	25	16	10	1	6	3	E	D	S_{11}
32	29	8	2	4	0	−3	29	21	15	1	6	2	N	D	S_{11}
33	31	6	1	6	0	−5	31	25	20	1	5	1	E	D	S_{11}
34	34	12	4	3	0	−2	34	22	14	1	8	4	N	D	S_{11}
35	43	7	1	7	0	−6	43	36	30	1	6	1	N	D	S_{11}
36	43	15	5	3	0	−2	43	28	18	1	10	5	N	D	S_{11}
37	45	12	3	4	0	−3	45	33	24	1	9	3	E	D	S_{11}
38	46	10	2	5	0	−4	46	36	28	1	8	2	N	D	S_{11}
39	52	18	6	3	0	−2	52	34	22	1	12	6	N	D	S_{11}
40	57	8	1	8	0	−7	57	49	42	1	7	1	E	D	S_{11}
41	61	16	4	4	0	−3	61	45	33	1	12	4	E	D	S_{11}
42	61	21	7	3	0	−2	61	40	26	1	14	7	N	D	S_{11}
43	67	12	2	6	0	−5	67	55	45	1	10	2	N	D	S_{11}
44	70	24	8	3	0	−2	70	46	30	1	16	8	–	D	S_{11}
45	71	15	3	5	0	−4	71	56	44	1	12	3	E	D	S_{11}
46	73	9	1	9	0	−8	73	64	56	1	8	1	E	D	S_{11}
47	77	20	5	4	0	−3	77	57	42	1	15	5	N	D	S_{11}
48	79	27	9	3	0	−2	79	52	34	1	18	9	–	D	S_{11}
49	88	30	10	3	0	−2	88	58	38	1	20	10	N	D	S_{11}
50	91	10	1	10	0	−9	91	81	72	1	9	1	E	D	S_{11}
51	92	14	2	7	0	−6	93	78	66	1	12	2	N	D	S_{11}
52	93	24	6	4	0	−3	93	69	51	1	18	6	–	D	S_{11}
53	96	20	4	5	0	−4	96	76	60	1	16	4	E	D	S_{11}
54	97	33	11	3	0	−2	97	64	42	1	22	11	–	D	S_{11}
55	103	18	3	6	0	−5	103	85	70	1	15	3	–	D	S_{11}
56	106	36	12	3	0	−2	106	70	46	1	24	12	–	D	S_{11}
57	109	28	7	4	0	−3	109	81	60	1	21	7	–	D	S_{11}
58	111	11	1	11	0	−10	111	100	90	1	10	1	–	D	S_{11}

Table 3: Type II designs whose complements are of Type I

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
1	7	3	1	2	1	0	7	4	2	2	0	−1	E	A	S_1
2	11	5	2	2	1	−1	11	6	3	2	0	−1	E	A	S_1, S_9
3	15	7	3	2	1	−1	15	8	4	2	0	−1	E	A	S_1
4	19	9	4	2	1	−1	19	10	5	2	0	−1	E	A	S_1
5	23	11	5	2	1	−1	23	12	6	2	0	−1	E	A	S_1
6	27	13	6	2	1	−1	27	14	7	2	0	−1	E	A	S_1
7	31	15	7	2	1	−1	31	16	8	2	0	−1	E	A	S_1
8	35	17	8	2	1	−1	35	18	9	2	0	−1	E	A	S_1, S_9
9	39	19	9	2	1	−1	39	20	10	2	0	−1	E	A	S_1
10	43	21	10	2	1	−1	43	22	11	2	0	−1	E	A	S_1
11	47	23	11	2	1	−1	43	24	12	2	0	−1	E	A	S_1

Table 4: Type II designs whose complements are of Type II

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
12	4	3	2	1	1	0	4	1	0	1	1		I		
13	5	4	3	1	1	0	5	1	0	1	1		I		
14	6	5	4	1	1	0	6	1	0	1	1		I		
15	7	6	5	1	1	0	7	1	0	1	1		I		
16	8	7	6	1	1	0	8	1	0	1	1		I		
17	9	8	7	1	1	0	9	1	0	1	1		I		
18	10	9	8	1	1	0	10	1	0	1	1		I		
19	11	10	9	1	1	0	11	1	0	1	1		I		
20	12	11	10	1	1	0	12	1	0	1	1		I		
21	13	12	11	1	1	0	13	1	0	1	1		I		
22	14	13	12	1	1	0	14	1	0	1	1		I		
23	15	14	13	1	1	0	15	1	0	1	1		I		
24	16	15	14	1	1	0	16	1	0	1	1		I		
25	17	16	15	1	1	0	17	1	0	1	1		I		
26	18	17	16	1	1	0	18	1	0	1	1		I		
27	19	18	17	1	1	0	19	1	0	1	1		I		

Table 4 (continued)

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
28	20	19	18	1	1	0	20	1	0	1	1		I		
29	21	20	19	1	1	0	21	1	0	1	1		I		
30	22	21	20	1	1	0	22	1	0	1	1		I		
31	23	22	21	1	1	0	23	1	0	1	1		I		
32	24	23	22	1	1	0	24	1	0	1	1		I		
33	25	24	23	1	1	0	25	1	0	1	1		I		
34	26	25	24	1	1	0	26	1	0	1	1		I		
35	27	26	25	1	1	0	27	1	0	1	1		I		
36	28	27	26	1	1	0	28	1	0	1	1		I		
37	29	28	27	1	1	0	29	1	0	1	1		I		
38	30	29	28	1	1	0	30	1	0	1	1		I		
39	31	30	29	1	1	0	31	1	0	1	1		I		
40	32	31	30	1	1	0	32	1	0	1	1		I		

These designs of Nos. 12 to 40 are just trivial designs with $\lambda^* = 0$.

Table 5: Type II designs whose complements are of Type III

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
41	22	7	2	3	1	−2	22	15	10	1	5	2	N	D	S_{11}
42	31	10	3	3	1	−2	31	21	14	1	7	3	E	D	S_{11}
43	37	9	2	4	1	−3	37	28	21	1	7	2	E	D	S_{11}
44	40	13	4	3	1	−2	40	27	18	1	9	4	E	D	S_{11}
45	49	16	5	3	1	−2	49	33	22	1	11	5	E	D	S_{11}
46	53	13	3	4	1	−3	53	40	30	1	10	3	N	D	S_{11}
47	56	11	2	5	1	−4	56	45	36	1	9	2	E	D	S_{11}
48	58	19	6	3	1	−2	58	39	26	1	13	6	N	D	S_{11}
49	67	22	7	3	1	−2	67	45	30	1	15	7	N	D	S_{11}
50	69	17	4	4	1	−3	69	52	39	1	13	4	E	D	S_{11}
51	76	25	8	3	1	−2	76	51	34	1	17	8	N	D	S_{11}
52	79	13	2	6	1	−5	79	66	55	1	11	2	E	D	S_{11}
53	81	16	3	5	1	−4	81	65	52	1	13	3	–	D	S_{11}
54	85	21	5	4	1	−3	85	64	48	1	16	5	E	D	S_{11}
55	85	28	9	3	1	−2	85	57	38	1	19	9	–	D	S_{11}
56	94	31	10	3	1	−2	94	63	42	1	21	10	–	D	S_{11}
57	101	25	6	4	1	−3	101	76	57	1	19	6	–	D	S_{11}
58	103	34	11	3	1	−2	103	69	46	1	23	11	–	D	S_{11}
59	106	15	2	7	1	−6	106	91	78	1	13	2	–	D	S_{11}
60	106	21	4	5	1	−4	106	85	68	1	17	4	–	D	S_{11}

Table 6: Type III designs whose complements are of Type I

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
1	13	9	6	1	3	1	13	4	1	4	0	−3	E	A	S_1, S_4
2	16	10	6	1	4	2	16	6	2	3	0	−2	E	A	S_1, S_7, S_{10}
3	21	16	12	1	4	1	21	5	1	5	0	−4	E	A	S_1, S_4
4	25	16	10	1	6	3	25	9	3	3	0	−2	E	A	S_1
5	29	21	15	1	6	2	29	8	2	4	0	−3	N	A	S_1
6	31	25	20	1	5	1	31	6	1	6	0	−5	E	A	S_1
7	34	22	14	1	8	4	34	12	4	3	0	−2	N	A	S_1
8	43	36	30	1	6	1	43	7	1	7	0	−6	N	A	S_1, S_4
9	43	28	18	1	10	5	43	15	5	3	0	−2	–	A	S_1
10	45	33	24	1	9	3	45	12	3	4	0	−3	E	A	S_1, S_{10}
11	46	36	28	1	8	2	46	10	2	5	0	−4	N	A	S_1
12	52	34	22	1	12	6	52	18	6	3	0	−2	N	A	S_1
13	61	40	26	1	14	7	61	21	7	3	0	−2	N	A	S_1
14	70	46	30	1	16	8	70	24	8	3	0	−2	–	A	S_1

Table 7: Type III designs whose complements are of Type II

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
15	22	15	10	1	5	2	22	7	2	3	1	−2	N	B	S_2
16	31	21	14	1	7	3	31	10	3	3	1	−2	E	B	S_2
17	37	28	21	1	7	2	37	9	2	4	1	−3	E	B	S_2
18	40	27	18	1	9	4	40	13	4	3	1	−2	E	B	S_2, S_5
19	49	33	22	1	11	5	49	16	5	3	1	−2	E	B	S_2
20	53	40	30	1	10	3	53	13	3	4	1	−3	N	B	S_2
21	58	39	26	1	13	6	58	19	6	3	1	−2	N	B	S_2
22	67	45	30	1	15	7	67	22	7	3	1	−2	N	B	S_2

Table 8: Type III designs whose complements are of Type III

No.	v	k	λ	n	m	t	v^*	k^*	λ^*	n^*	m^*	t^*	I/N/E	F	Series
23	36	15	6	2	3	0	36	21	12	1	9	6	E	D	S_6, S_{11}
24	41	16	6	2	4	1	41	25	15	1	10	6	E	D	S_{11}
25	61	25	10	2	5	1	61	36	21	1	15	10	E	D	S_{11}
26	64	28	12	2	4	0	64	36	20	1	16	12	E	D	S_6, S_{11}
27	66	26	10	2	6	2	66	40	24	1	16	10	E	D	S_{11}
28	71	21	6	3	3	-1	71	50	35	1	15	6	E	D	S_{11}
29	78	22	6	3	4	0	78	56	40	1	16	6	E	D	S_{11}
30	85	36	15	2	6	1	85	49	28	1	21	15	–	D	S_{11}
31	86	35	14	2	7	2	86	51	30	1	21	14	N	D	S_{11}
32	89	33	12	2	9	5	89	56	35	1	21	12	–	D	S_{11}
33	91	36	14	2	8	3	91	55	33	1	22	14	–	D	S_{11}
34	100	45	20	2	5	0	100	55	30	1	25	20	–	D	S_6, S_{11}
35	105	40	15	2	10	5	105	65	40	1	25	15	–	D	S_{11}
36	111	45	18	2	9	3	111	66	39	1	27	18	–	D	S_{11}

References

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