広島大学大学院工学研究科研究報告，53,　1　(2004)
Bulletin of the Graduate School of Engineering, Hiroshima University. Vol. 53, No.1, 2004

Vibration Characteristics and Surface Durability of Non-Involute Helical Gear

Florin G. TUTULAN*, Kazuteru NAGAMURA** and Kiyotaka IKEJO***

Two non-involute helical gears that have an involute-cycloid composite tooth profile and a modified cycloid tooth profile were designed and made to obtain high performance. Using these non-involute helical gears, the circumferential vibration acceleration of the helical gears, the tooth root stress, and the transmission error were measured on running tests. The experimental results showed that the vibration characteristics of the non-involute helical gears do not differ from those of the involute helical gear. Furthermore, we proved from pitting test results that the surface durability of the non-involute helical gears is superior to that of the involute helical gear.

Key words: Gear, Non-Involute Gear, Involute-Cycloid Composite Tooth Profile Gear, Modified Cycloid Tooth Profile Gear, Vibration, Surface Durability, Grinding, Hobbing

1. Introduction

Involute gears have been widely used because of their advantages, such as, ease of manufacture and the fact that the gear speed is unchanged even if the gear center distance changes, etc. However, involute gears have several disadvantages: the sliding on the teeth is greater, the surface durability is lower, and the undercut occurs more frequently in gears having small numbers of teeth compared to other tooth profile gears. In the previous study, we have developed two non- involute spur gears that have an involute-cycloid composite tooth profile and a modified cycloid tooth profile, and proved that these spur gears have greater bending strength and surface durability than the involute spur gear [1), 2)].

In this study, we designed and made an involute-cycloid composite tooth profile helical gear and a modified cycloid tooth profile helical gear to obtain high performance. Using the non-involute helical gears, the circumferential vibration acceleration of the helical gears, the tooth root stress, and the transmission error were measured on running tests, and the vibration characteristics of the non-involute helical gears were discussed. Furthermore, the surface durability of the non-involute helical gears was investigated by carrying out a pitting test.

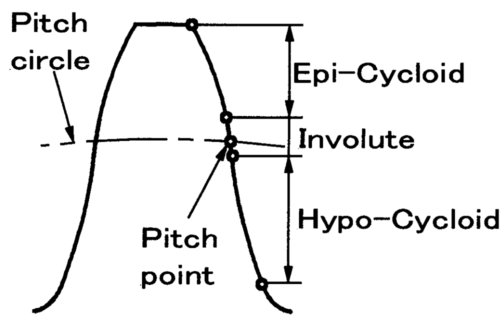
2. Tooth Profile of Non-Involute Helical Gears and Test Gears

It is well known that the involute tooth profile and the cycloid tooth profile are able to transmit a uniform motion. Comparing these tooth profiles, the specific sliding of the involute gear is zero at the pitch point, and has a maximum value at both the tooth tip and the tooth root, while that of the cycloid gear holds a uniform value at the addendum and the dedendum. The cycloid gear has a higher contact strength than the involute gear, as tooth contact holds on concave and convex surfaces. In addition, the cycloid gear is stronger on bending than the involute gear because of the larger tooth thickness at the tooth root critical section. However, the cycloid gear has the serious demerit that the radius of curvature is zero at the pitch point [3)].

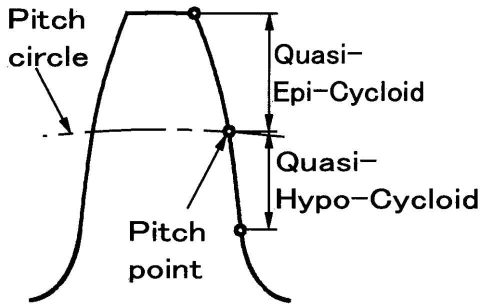
Therefore, we developed two non-involute tooth profiles improving the cycloid tooth profile, as shown in Fig. 1. One is an involute-cycloid composite tooth [Fig. 1(a)], and the other is a modified cycloid tooth [Fig. 1(b)].

The involute-cycloid composite tooth profile consists of an involute tooth profile near the pitch point and a cycloid tooth profile at the addendum and the dedendum. The modified cycloid tooth profile is constructed using the counterpart beside a part near the pitch point on cycloid curves, and conjugating it successively and smoothly.

* Doctor student, Mechanical System Engineering, Hiroshima University
** Professor, Mechanical System Engineering, Hiroshima University
*** Research Associate, Mechanical System Engineering, Hiroshima University



(a) Involute-cycloid composite



(b) Modified cycloid

Fig. 1 Tooth profile of non- involute gears

Figure 2(a) shows the basic rack form for the involute-cycloid composite tooth profile gear. The rack for the composite tooth profile gear is of the form PQR' consisting of a straight line PQ and a cycloid curve QR' which is drawn by rolling a circle on the x-axis (the base pitch line). The addendum and the dedendum of the rack are symmetric to each other with respect to the pitch point P because of the interchangeability of gear.

Using the coordinate system shown in Fig. 2, the cycloid curve P'QR' is expressed as follows:

$$\left.\begin{aligned} x &= a(\theta - \sin\theta) + X_0 \\ y &= a(1 - \cos\theta) \end{aligned}\right\} \qquad (1)$$

where a is the radius of the rolling circle; θ is the rotational angle of the rolling circle; and α_0 is the inclination of the straight line PQ to the y-axis, equal to the cutter pressure angle of involute.

In addition, the straight line PR is expressed as follows:

$$y = x\cot\alpha_0 \qquad (2)$$

X_0 shown in Fig. 2(a) and in the equation (1) is the distance between the pitch point P and the point P' where the cycloid curve begins, and Y_0 is the distance between the base pitch line (x-axis) and the point Q connecting the cycloid curve QR' and the straight line PQ. The distances X_0 and Y_0 can be derived from the equations (1) and (2) as follows:

$$\left.\begin{aligned} X_0 &= 2a \,\mathrm{inv}\, \alpha_0 \\ Y_0 &= a(1 - \cos 2\alpha_0) \end{aligned}\right\} \qquad (3)$$

Figure 2(b) shows the basic rack form for the modified cycloid tooth profile gear. The modified cycloid basic rack form consists of an epi-cycloid and a hypo-cycloid curves drawn by rolling a circle on the lines which are shifted above or below the base pitch line (x-axis) by the distance Y_0. The epi-cycloid and the hypo-cycloid curves are smoothly connected so as to have a common tangent line at the pitch point P. The modified cycloid basic rack form is similar to a rack form made by eliminating the part near the pitch point of the pure-cycloid rack. The modified cycloid profile of the basic rack is expressed as follows:

$$\left.\begin{aligned} x &= a(\theta - \sin\theta) - X_0 \\ y &= a(1 - \cos\theta) - Y_0 \end{aligned}\right\} \qquad (4)$$

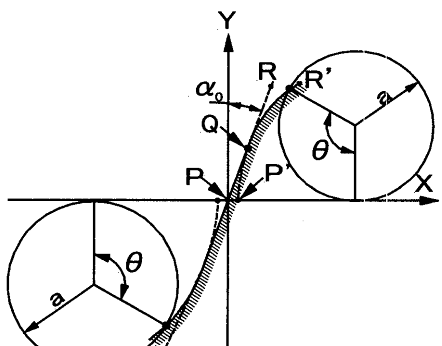
here,

$$\left.\begin{aligned} X_0 &= a(2\alpha_0 - \sin 2\alpha_0) \\ Y_0 &= a(1 - \cos 2\alpha_0) \end{aligned}\right\} \qquad (5)$$

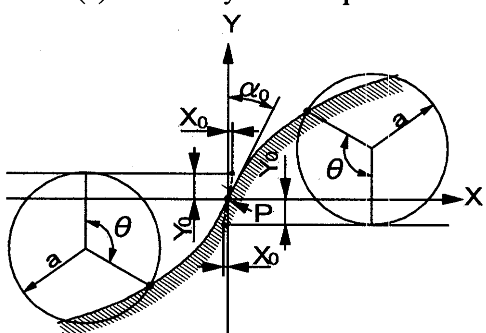
where a is the radius of the rolling circle; θ is the rotational angle of the rolling circle; and α_0 is the pressure angle at the pitch point.

Furthermore, as seen from Fig. 2, the involute basic rack whose form is a straight line can be considered to be the rack obtained when the radius a of the rolling circle for cycloid curve becomes infinite.

The test gears are specified in Table 1. The test gears were designed to have high contact ratios greater than two. The geometry of the involute-cycloid composite gear and the modified cycloid gear were determined according to the non-involute spur gears presented in the previous studies [1), 2)]. They were cut by custom made hobs, and finished by a CNC form grinding machine[4)]. Table 2 shows the surface roughness of the test gears ground and hobbed.



(a) Involute-cycloid composite



(b) Modified cycloid

Fig. 2 Basic rack form of non-involute gears

Table 1 Specification of test gears (in the plane of rotation

Tooth profile	**Involute**	**Involute - cycloid composite**	**Modified cycloid**
Module m [mm]	4	4	4
Number of teeth z_1/z_2	29/29		
Pressure angle α_0 [deg]	22.8	13*	14*
Rolling circle radius a [mm]	-	7	25
Addendum h_a [mm]	0.670m	1.0m	1.0m
Dedendum h_f [mm]	1.264m	1.289m	1.404m
Center distance A [mm]	116.00		
Pitch circle diameter d [mm]	117.20	116.00	116.00
Tip circle diameter d_a [mm]	122.60	124.00	124.00
Modification coefficient x	-0.18	0.00	0.00
Face width b [mm]	20		
Helix angle β [deg]	30		
Total contact ratio ε	2.255	2.129	2.500

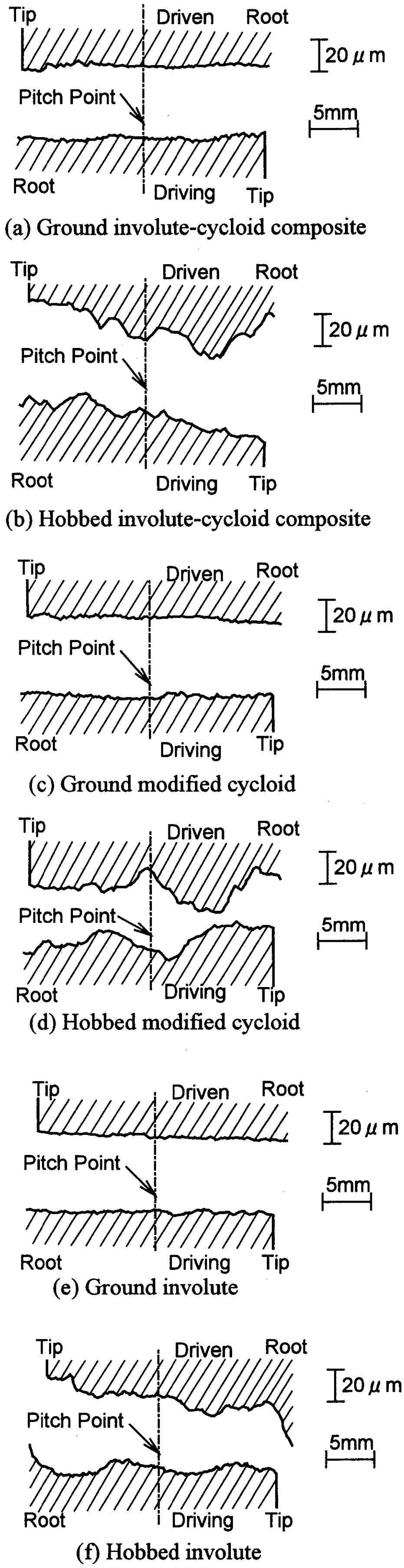
* value at pitch point.

Table 2 Surface roughness R_y of the test gears

Tooth profile	**Involute**	**Involute - cycloid composite**	**Modified cycloid**
SCM415 Carburized Steel & Grinding	7.9μm	7.1μm	7.3μm
S45C Thermal Refining Steel & Hobbing	10.3μm	9.4μm	9.1μm

The tooth profile error of test gears is shown in Fig. 3. The tooth profile errors for the involute–cycloid composite ground and hobbed gears are presented in Figs.3(a) and (b). For the modified cycloid gear and for the involute gear the tooth profile errors are shown in Figs.3(c) and (d), and in Figs.3(e) and (f).

The tooth profile error was measured with an involute tooth profile testing machine. In this case, at the cycloid part of the non- involute gear, the measured value includes the deviation of the cycloid curve from the involute curve in addition to the tooth profile error of the cycloid part. Therefore, the tooth profile error curves shown in Fig.3 were obtained by numerically correcting the deviation between the cycloid curve and the involute curve. For hobbed gears, the tooth profile error is about 18μm and for ground gears it is about 6μm.

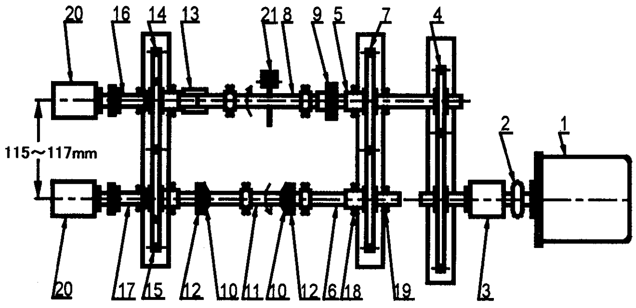


(a) Ground involute-cycloid composite
(b) Hobbed involute-cycloid composite
(c) Ground modified cycloid
(d) Hobbed modified cycloid
(e) Ground involute
(f) Hobbed involute

Fig. 3 Tooth profile error of test gears

3. Vibration Characteristics

The vibration characteristics were investigated with hobbed gears and ground gears using a power circulating-type gear testing machine, as shown in Fig. 4. The main feature of the gear testing machine is to have variable center distance of test gears.



1	Driving motor	8	Torsion bar	15	Test gear
2	Rubber coupling	9	Loading device	16	Gear shaft
3	Speed change gears	10	Barfield type ball joint	17	Gear shaft
4	Speed change gears	11	Torsion bar	18	Ball bearing
5	Gear shaft	12	Coupling	19	Ball bearing
6	Gear shaft	13	Coupling	20	Slip-ring or Optical rotary encoder
7	Slave unit	14	Test gear	21	Index disk & Tachometer

Fig. 4 Power circulating-type gear testing machine

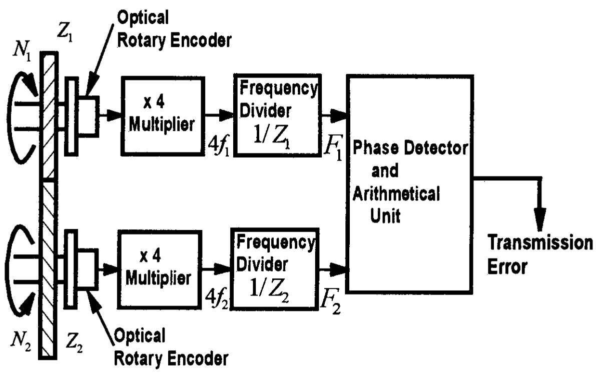


Fig. 5 Measurement of transmission error

Figure 5 shows the measuring method of transmission error of the test gear pair. The pulse signals from two rotary encoders mounted to test gear shafts are coupled as shown in Fig. 5, after correction with the number of teeth and frequency matching. The relative phase differences between F_1 and F_2 are calculated digitally at the arithmetical unit and are the output as the rotational transmission error.

The effect of center distance on the maximum transmission errors of each gear pair is shown in Fig. 6. The maximum transmission errors were measured as peak-to-peak value in a series of data. In Fig. 6, spur gears were examined in a previous study [5]. For involute spur gears and involute helical gears, the maximum transmission error does not vary with the change of the center distance A. On the other hand, for non-involute spur gears the center distance has a significant effect on the maximum transmission error. However, for non-involute helical gears the center distance does not affect significantly the maximum transmission error.

The effects of the center distance on the operational contact ratio are shown in Fig. 7. The operational contact ratio is measured from the stress waveform of the gear tooth. For involute-cycloid composite helical gears, the operational contact ratios decrease rapidly as the center distance increases. Except for involute- cycloid composite helical gears, the contact ratios do not change much with the center distance.

Figure 8 presents a speed sweep of the dynamic load factor at various center distances. The dynamic load factor is defined as the ratio of the peak dynamic root stress during contact to the peak static root stress. For involute helical gears and modified cycloid helical gears, the dynamic load factor shows little difference, and is not affected by the center distance. For involute-cycloid composite helical gears, the dynamic load factor has a small fluctuation, but its value is under 1.4. For the ground involute-cycloid composite helical gears, the dynamic load factors at the center distance A= 115.9[mm], the shortest one, is larger than at the other center distances. Comparing with the tooth finish methods, the dynamic load factor for hobbed gears is larger than for ground gears in general. Though the dynamic load factor is scattered, as it is less than 1.3, the effect of the center distance on the dynamic load factor is not very significant in the result.

Figure 9 shows the effect of center distance on the circumferential vibration acceleration (root mean square value) of gear as a function of the mesh frequency f_z (gear speed). For the involute-cycloid composite helical gear, the vibration acceleration curves are affected by the center distance, and they are worse at the center distance A= 115.9[mm] than at other center distances. In addition, resonance occurs near f_z=1030[Hz], and the peak of the resonance decreases with widening of the center distance for the hobbed involute-cycloid composite helical gears. For the other helical gears, the center distance has little effect on the vibration acceleration. Also for all the helical gears, grinding decreases the vibration acceleration.

Figure 10 compares the vibration accelerations between helical gears and spur gears at the center distance A =116.0[mm], which is the design value of test gear pairs. For involute gears, the hobbed spur gear has the largest vibration acceleration, and the ground helical gear is best in vibration performance. For the modified cycloid gear on the whole, the vibration acceleration is lower than that of the other tooth profile gears, the ground helical gear has the best vibration performance, and hobbed and ground spur gears have about the same acceleration value as the hobbed helical gear, since these spur gears have a contact ratio over 2. For the involute-cycloid composite helical gear, it is seen that the helical gear has lower acceleration than the spur gear. From the facts described above, we may conclude that the helical gear improves vibration characteristics in the non-involute gears.

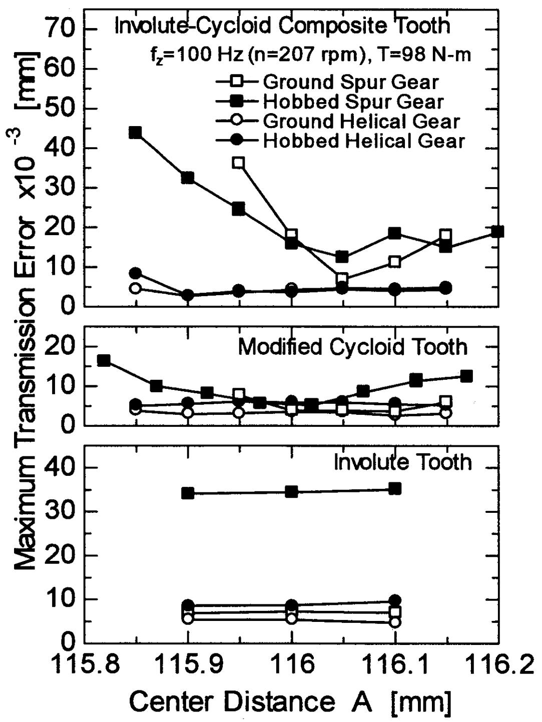


Fig. 6 Maximum transmission error

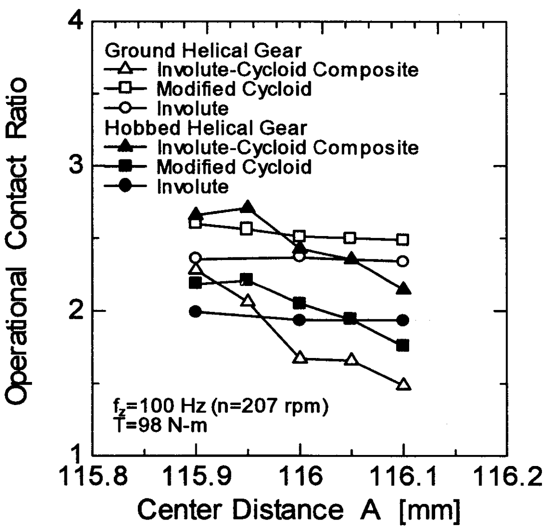


Fig. 7 Operational contact ratio

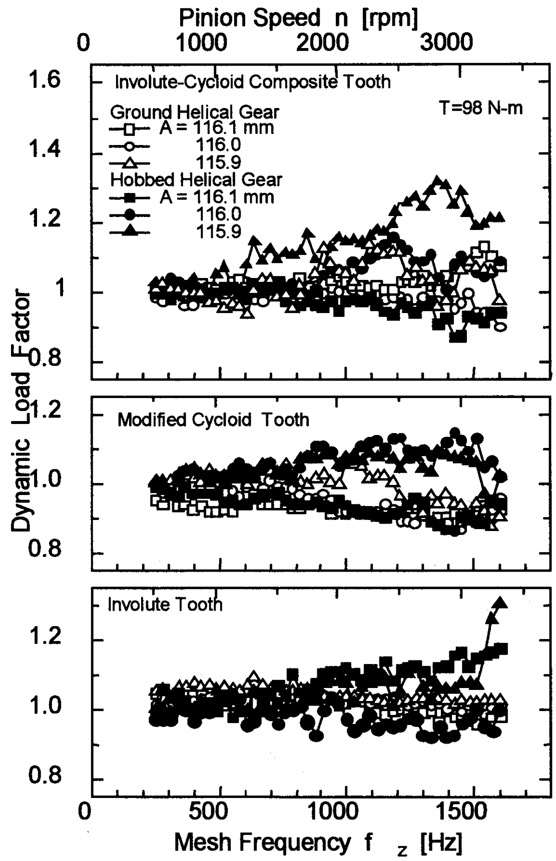


Fig. 8 Dynamic load factor

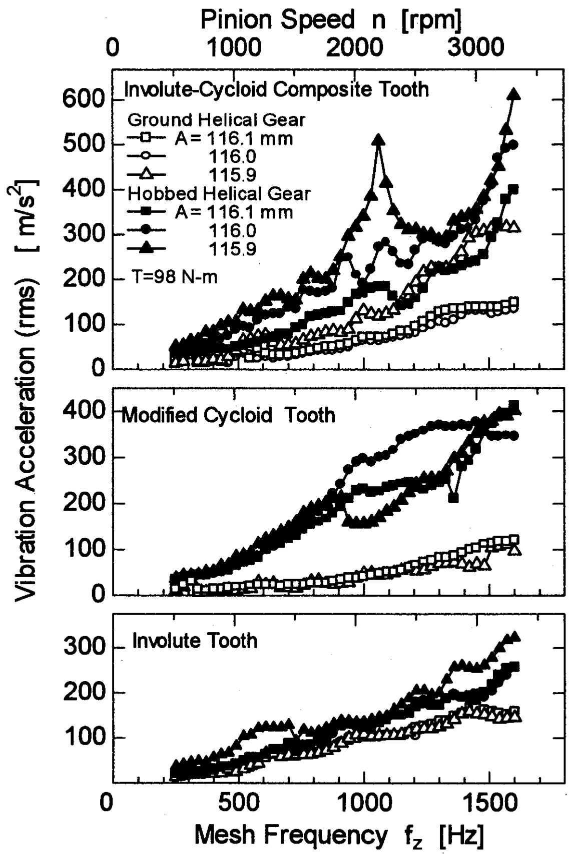


Fig. 9 Vibration acceleration (circumferential direction)

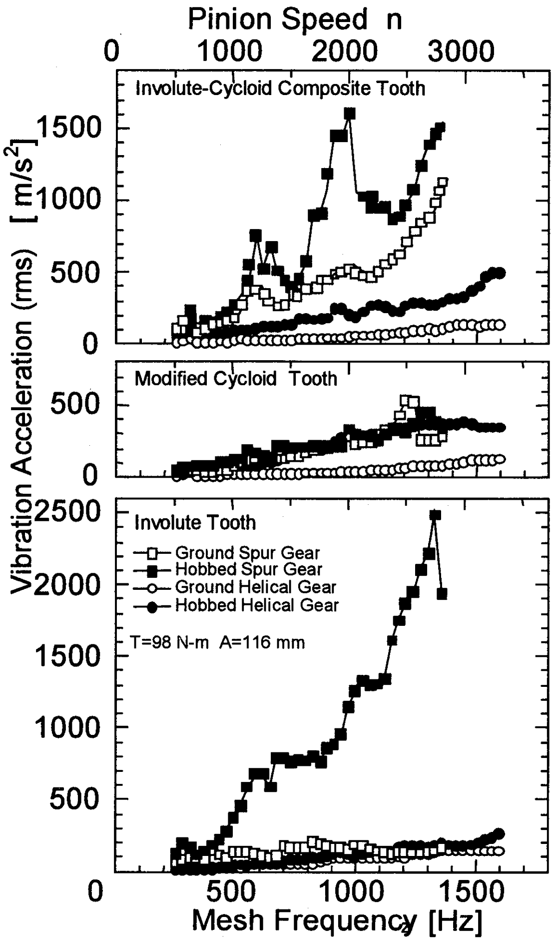


Fig. 10 Comparison of helical gears with spur gears in vibration acceleration (circumferential direction)

4. Surface Durability and Tooth Root Stress

The surface durability tests were carried out using hobbed gears, made of JIS S45C thermal refined steel, which has a hardness of 185HV (HV=Vickers hardness). The surface roughness is shown in Table 2. The ordinary power circulating-type gear testing machine was used, which has a fixed center distance. The gear was lubricated by an oil bath lubrication method, with the turbine oil ISO VG32. The pitting area of each gear tooth was measured in the running test, and the tooth root stress was also measured with a strain gage.

The relationship between applied torque and the maximum tooth root stress is shown in Fig. 11. The maximum tooth root stress of each gear increases in proportion to torque. The tooth root stress for the involute-cycloid composite tooth profile helical gear is the largest, followed by the modified cycloid gear and being the smallest for the involute helical gear at every torque. Therefore, on tooth bending strength of the non-involute gears, it is necessary to review their geometry and specifications.

The pitting area ratio on each helical gear is shown in Fig. 12. The pitting area ratio is defined as the ratio of the whole area of pits to that of tooth surface in the contact. It is evident from this figure that the pitting area ratio of the involute-cycloid composite tooth profile gear is smaller than that of the involute gear at every load cycle N.

Figure 13 shows the pitting endurance limits of the non-involute helical gears and the involute helical gear, which are plotted with the number of cycles N as abscissa and the applied torque T as ordinate, and the generation life of the pitting is defined as 5%. It is found from Fig. 13 that the surface durability of the involute-cycloid composite tooth profile helical gear is larger than that of the involute helical gear by 80% in torque; and that of the modified cycloid helical gear is larger than that of the involute helical gear by 20%. Therefore it is considered that the tooth contact strengths on the involute-cycloid composite tooth profile helical gear and modified cycloid helical gear are superior to that of the involute helical gear.

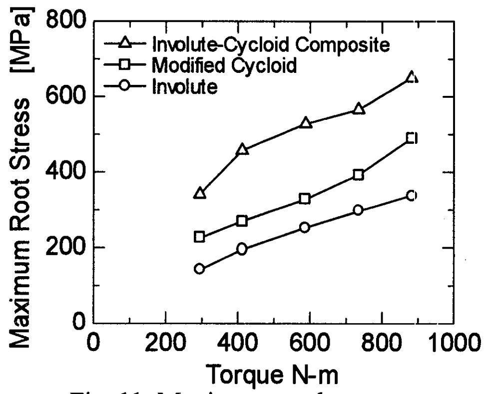


Fig. 11 Maximum tooth root stress

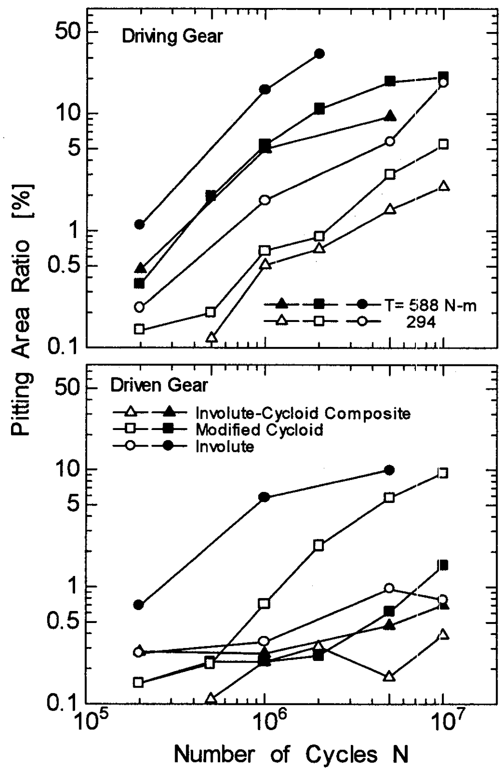


Fig. 12 Pitting area ratio

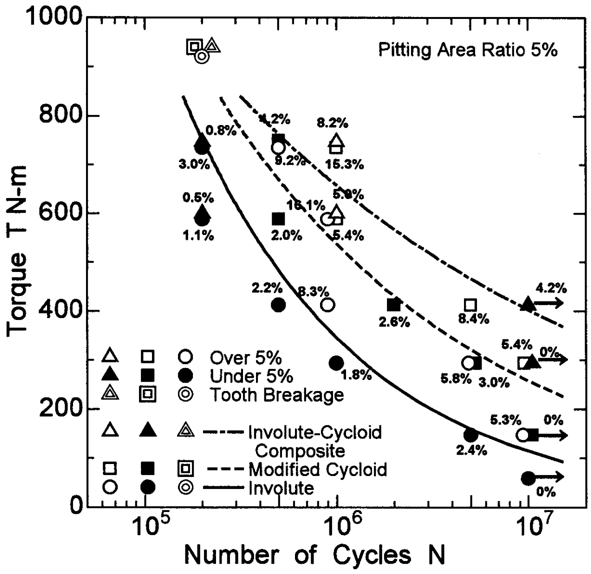


Fig. 13 Endurance limit for pitting

5. Conclusions

The results of this study can be summarized as follows:

(1) The transmission error for the non-involute helical gears in this study is affected significantly by the change of the center distance as well as the involute helical gears. Furthermore, the transmission error of the non-involute helical gears is smaller than that of the non-involute spur gears.

(2) For the involute-cycloid composite tooth profile helical gear, the vibration acceleration is affected by the change of the center distance. On the other hand, for the modified cycloid tooth profile helical gear and the involute helical gear, the center distance does not affect the vibration acceleration.

(3) The non-involute helical gears are superior to the non-involute spur gears in vibration performance, and the ground non-involute helical gears have about the same or a little better vibration performance than the involute helical gear.

(4) It is confirmed experimentally in this study that the tooth contact strength of the non-involute helical gears is superior to the involute helical gear. However, since their tooth bending strength is inferior to the involute helical gear, it is necessary to review their geometry and specifications in future research.

References

1) Terauchi, Y., and Nagamura, K. (1982). On design and Performance of Involute-cycloid Composite Tooth Profile Gear, Bull. JSME, Vol.25, No.199, pp. 118-126.
2) Nagamura, K., Ikejo, K., Matsuyama, K., and Makita, K. (1998). Tooth Surface Durability of Modified Cycloid Gear, Preprint of JSME (in Japanese), No.985-1, pp. 79-80.
3) Litvin, L.F. (1994). Gear Geometry and Applied Theory, Prentice - Hall Inc., pp. 258 - 287.
4) Ikeno, K., Marui, M., and Yoshino, H. (1994). Development of CNC Grinding Machine with Truing Attachment, Jour. of JPSE (in Japanese), Vol.60, No.1, pp.133-137.
5) Nagamura, K., Ikejo, K., Kihara, S., and Sumiyoshi, T. (2001). Vibration Characteristics of Ground Non-Involute Gear, Jour. of JSME, (in Japanese), Vol.36, No.8, pp. 363-370.

Appendix

List of Notations in the Paper

a	rolling circle radius
b	face width
d_a	tip circle diameter
d	pitch circle diameter
h_a	addendum
h_f	dedendum
m	module
n	rotational speed
r	pitch circle radius
x	addendum modification coefficient
y	center distance modification coefficient
z	number of teeth
A	center distance of gear pair
T	driving torque
Y_0	distance from the base pitch line (x-axis)
α_0	cutter pressure angle, or pressure angle at the pitch point
β	helix angle
ε	contact ratio
θ	rotational angle of the rolling circle

Received Sept. 30, 2004