

TeV Gamma-Rays from Old Supernova Remnants

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ABSTRACT

We study the emission from an old supernova remnant (SNR) with an age of around 10^5 yrs and that from a giant molecular cloud (GMC) encountered by the SNR. When the SNR age is around 10^5 yrs, proton acceleration is efficient enough to emit TeV γ -rays both at the shock of the SNR and that in the GMC. The maximum energy of primarily accelerated electrons is so small that TeV γ -rays and X-rays are dominated by hadronic processes, π^0 -decay and synchrotron radiation from secondary electrons, respectively. However, if the SNR is older than several 10^5 yrs, there are few high-energy particles emitting TeV γ -rays because of the energy loss effect and/or the wave damping effect occurring at low-velocity isothermal shocks. For old SNRs or SNR-GMC interacting systems capable of generating TeV γ -ray emitting particles, we calculated the ratio of TeV γ -ray (1–10 TeV) to X-ray (2–10 keV) energy flux and found that it can be more than $\sim 10^2$. Such a source showing large flux ratio may be a possible origin of recently discovered unidentified TeV sources.

Key words: acceleration of particles — gamma-rays: theory — ISM: clouds — shock waves — supernova remnants — X-rays: ISM

1 INTRODUCTION

The most probable cosmic-ray accelerator in our Galaxy is the young supernova remnant (SNR). The detection of synchrotron X-rays from shells of young SNRs provides us the strong evidence for electron acceleration up to more than ~ 10 TeV (e.g., Koyama et al. 1995, 1997; Bamba et al. 2003a,b, 2005a,b). So far, the evidence for hadron acceleration, however, has not yet been obtained. High energy γ -ray observations may give us important information on the accelerated protons (Naito & Takahara 1994; Drury et al. 1994; Aharonian et al. 1994; Aharonian & Atoyan 1996). For example, TeV γ -rays are detected from the SNRs RX J1713.7–3946 (Enomoto et al. 2002; Aharonian et al. 2004a) and RX J0852.0–4622 (Katagiri et al. 2005; Aharonian et al. 2005c), which can be originated in either the decay of neutral pion, arising from the collision of high energy protons and interstellar matter, or CMB photons up-scattered by accelerated electrons (e.g. Pannuti et al. 2003; Lazendic et al. 2004; Ellison 2001; Bamba et al. 2005b;

Uchiyama et al. 2005). At present, the leptonic process is not yet ruled out.

Recently, a survey of the inner part of our Galaxy has revealed several new TeV γ -ray sources (Aharonian et al. 2002, 2005a,b, 2006). For some of them, no counterpart has been found in any other wave lengths yet. Their spectra are rather hard, and the flux above 1 TeV is around 1×10^{-11} cm⁻²s⁻¹. They are extended with the angular size of around 0.1° . They should be galactic origin because all are located along the galactic plane. In Table 1, we list the properties of the Galactic TeV γ -ray sources that are also observed in the X-ray bands (together with a typical young SNR SN 1006). One of the newly discovered sources, HESS J1303–631 (Aharonian et al. 2005b), was observed by *Chandra*, and no obvious counterpart was revealed (Mukherjee & Halpern 2005). Then, the flux ratio, defined by

$$R_{\text{TeV}/\text{X}} = \frac{F_\gamma(1-10 \text{ TeV})}{F_X(2-10 \text{ keV})}, \quad (1)$$

is more than ~ 2 . On the other hand, young SNRs, Cas A, RX J1713.7–3946 and RX J0852.0–4622, have $R_{\text{TeV}/\text{X}}$ less than ~ 2 . Although at present the lower limit on $R_{\text{TeV}/\text{X}}$

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Table 1. Observed properties of Galactic TeV γ -ray sources and a typical young SNR, SN 1006.

Name	$F_\gamma(1\text{--}10\text{ TeV})^a$	$F_X(2\text{--}10\text{ keV})^b$	$R_{\text{TeV}/X}^c$	Classification ^d	References ^e
Crab	56	2.1×10^4	2.6×10^{-3}	PWN	(1), (2)
RCW 89	16	38	0.41	PWN	(3), (4)
Cas A	1.9	1.3×10^2	0.014	SNR	(5), (6)
SN 1006 (NE)	< 2.1	19	< 0.11	SNR	(7), (8)
RX J1713.7–3946	35	5×10^2	0.07	SNR	(9), (10)
RX J0852.0–4622	69	> 32	< 2.1	SNR	(11), (12)
HESS J1813–178	8.9	7.0	1.3	SNR	(13), (14)
HESS J1640–465	7.1	0.37	19	SNR	(13), (15)
TeV J2032+4130	1.9	< 2	> 1	unID	(16)
HESS J1303–631	10	< 6	> 2	unID	(17), (18)

^a Derived for best fitted values (in units of $10^{-12}\text{erg s}^{-1}\text{cm}^{-2}$).

^b Derived for best fitted values (in units of $10^{-12}\text{erg s}^{-1}\text{cm}^{-2}$).

^c $R_{\text{TeV}/X} = F_\gamma(1\text{--}10\text{ TeV})/F_X(2\text{--}10\text{ keV})$.

^d SNR: Supernova Remnant, PWN: Pulsar Wind Nebula, unID: unidentified TeV γ -ray source.

^e (1) Aharonian et al. (2004b); (2) Willingale et al. (2001); (3) Aharonian et al. (2005e); (4) DeLaney et al. (2006) (5) Aharonian et al. (2001); (6) Allen et al. (1997); (7) Aharonian et al. (2005d); (8) Ozaki & Koyama (1998); (9) Aharonian et al. (2004a); (10) Panuti et al. (2003); (11) Aharonian et al. (2005c); (12) Slane et al. (2001); (13) Aharonian et al. (2006); (14) Brogan et al. (2005); (15) Sugizaki et al. (2001); (16) Aharonian et al. (2002); (17) Aharonian et al. (2005b); (18) Mukherjee & Halpern (2005).

for HESS J1303–631 is comparable to that of young SNRs, it may become much larger with forthcoming deeper X-ray observations. Furthermore, HESS J1640–465 has an X-ray counterpart that is identified as SNR (Aharonian et al. 2006). This object has $R_{\text{TeV}/X} \sim 19$. Unlike young SNRs, these TeV sources with large $R_{\text{TeV}/X}$ may show evidence for hadron acceleration because inverse-Compton scenario requires unusually small magnetic field strength ($\ll 1\ \mu\text{G}$).

A large value of $R_{\text{TeV}/X}$ is expected for old SNRs. As the SNR ages, the shock velocity decreases. In general, primary electron acceleration is limited by synchrotron cooling. Then, the roll-off energy of electron synchrotron radiation is much smaller than that of typical young SNRs, so that small synchrotron X-ray flux is expected (e.g., Sturmer et al. 1997). It is also important to consider the association with a giant molecular cloud (GMC). Because of large volume, old SNRs may encounter the GMC. In this paper, we study how large $R_{\text{TeV}/X}$ can become for single SNRs and the SNR-GMC interacting systems.

2 EVOLUTION OF SNR

We consider a simple analytical model of the shock dynamics of SNRs expanding into the uniform ambient medium with the density n_0 . The SNRs evolve through three phases: the free expansion phase, the Sedov-Taylor phase, and the radiative phase. We assume the shock velocity v_s as a function of the SNR age t_{age} as

$$v_s(t_{\text{age}}) = \begin{cases} v_i & (0 < t_{\text{age}} < t_1) \\ v_i(t_{\text{age}}/t_1)^{-3/5} & (t_1 < t_{\text{age}} < t_2) \\ v_i(t_2/t_1)^{-3/5}(t_{\text{age}}/t_2)^{-2/3} & (t_2 < t_{\text{age}}) \end{cases}, \quad (2)$$

where $t_1 = (3E/2\pi m_H n_0 v_i^5)^{1/3} = 2.1 \times 10^2 (E_{51}/n_0)^{1/3} v_{i,9}^{-5/3}$ yrs and $t_2 = 4 \times 10^4 E_{51}^{4/17} n_0^{-9/17}$ yrs (Blondin et al. 1998), and $v_i = v_{i,9} 10^9\text{ cm s}^{-1}$ and

$E = E_{51} 10^{51}$ ergs are the initial velocity, and the initial energy of the ejecta, respectively. Here we adopt the expansion law of the radiative phase, $v_s \propto t_{\text{age}}^{-2/3}$, instead of $v_s \propto t_{\text{age}}^{-5/7}$, because the former gives a better approximation in the epoch around $t_{\text{age}} \sim 10^5$ yrs (Blondin et al. 1998; Bandiera & Petruk 2004). The evolution of the shock radius $R_s = \int v_s dt$ is also calculated. In this paper, we adopt $E_{51} = v_{i,9} = n_0 = 1$ as a fiducial case. Then, we find $R_s = 91\text{ pc}$ and $v_s = 1.1 \times 10^7\text{ cm s}^{-1}$ at $t_{\text{age}} = 3 \times 10^5$ yrs. Note that the effect of the magnetic field on SNR dynamics can be neglected until t_{age} of several 10^5 yrs (e.g., Hanayama & Tomisaka 2006).

3 EMISSION FROM AN SNR

At first, we consider the spectrum of emitting high-energy particles that get their energy via diffusive shock acceleration (Drury 1983; Blandford & Eichler 1987). Generally speaking, in order to obtain the energy spectrum of accelerated particles, time-dependent kinetic equation should be solved as investigated by many authors (e.g., Berezhko et al. 1996), even including the nonlinear effects via accelerated particles (Malkov & Drury 2001). Such calculations well match the observational facts when free parameters such as the injection rate and the magnetic field are chosen appropriately. Instead, we assume, in this paper, that at an arbitrary epoch, the spectral form is a power-law with the index p and the exponential cut-off at E_{max} , i.e., $\propto E^{-p} e^{-E/E_{\text{max}}}$, where E_{max} evolving with time. This approximation may be valid and be useful to extract the basic properties of high-energy emission from a SNR because of the following reasons. High-energy particles are produced by the diffusive shock acceleration and suffer adiabatic expansion after they are transported downstream of the shock and lose their energy. Hence, at the given epoch, the spectrum for high-energy particles is dominated by those which are being

accelerated at that time, in other words, the energy spectrum of particles does not so much depend on the past acceleration history. Especially, we are now interested in the energy region near the upper end of the spectrum because as seen in the following, both X-rays and TeV γ -rays are produced by particles with energy near E_{\max} . In this energy regime, our assumed form of the spectrum may be a good approximation.

Once the SNR dynamics is given, the maximum energy of accelerated particles is calculated. The maximum energy of accelerated protons, $E_{\max,p}$, is determined by

$$t_{\text{acc}} = \min\{t_{\text{age}}, t_{\text{pp}}\} \quad , \quad (3)$$

while that of accelerated electrons, $E_{\max,e}$, is determined by

$$t_{\text{acc}} = \min\{t_{\text{age}}, t_{\text{synch}}\} \quad , \quad (4)$$

where t_{age} , t_{acc} , t_{synch} , and t_{pp} are the age of the SNR, the acceleration time scale, the synchrotron loss time scale, and the pion-production loss time scale, respectively. Since we assume that the energy spectrum does not depend on the past history, it is also a good approximation to determine E_{\max} using Eqs. (3) or (4). Indeed, at least for young SNRs, one can confirm our estimation of E_{\max} coincides with simulation results (Berezhko et al. 2002). For the diffusive shock acceleration, the acceleration time is given as

$$t_{\text{acc}} = \frac{20hcE_{\max}}{eB_d v_s^2} \quad , \quad (5)$$

where

$$h = \frac{0.05r(f + rg)}{r - 1} \quad , \quad (6)$$

and r is the compression ratio, and f and g are functions of the shock angle θ and gyro-factors η_u and η_d that are given as

$$f(\eta_u, \theta) = \eta_u (\cos^2 \theta + r^2 \sin^2 \theta)^{1/2} \left(\cos^2 \theta + \frac{\sin^2 \theta}{1 + \eta_u^2} \right) \quad , \quad (7)$$

$$g(\eta_d, \theta) = \eta_d (\cos^2 \theta + r^2 \sin^2 \theta)^{-1} \left(\cos^2 \theta + \frac{r^2 \sin^2 \theta}{1 + \eta_d^2} \right) \quad , \quad (8)$$

respectively (Yamazaki et al. 2004; Jokipii 1987). The downstream magnetic field is given by $B_d = rB_{\text{ISM}}$, where $B_{\text{ISM}} = 10B_{\text{ISM},-5} \mu\text{G}$ is the ISM magnetic field and we adopt $B_{\text{ISM},-5} = 1$ as a typical value. For high-energy protons, the energy loss time scale through pion-production is given by $t_{\text{pp}} = 5.3 \times 10^7 n^{-1}$ yrs, where n is the number density of the acceleration site. For electrons, the synchrotron loss time scale is given by $t_{\text{synch}} = 1.25 \times 10^4 (E_{\max,e}/10\text{TeV})^{-1} (B_d/10\mu\text{G})^{-2}$ yrs. As long as $t_{\text{age}} < t_{\text{pp}}$, the proton acceleration is age-limited, and we obtain

$$E_{\max,p} = 1.6 \times 10^2 \text{ TeV } h^{-1} v_{s,8}^2 \left(\frac{B_d}{10 \mu\text{G}} \right) \left(\frac{t_{\text{age}}}{10^5 \text{ yrs}} \right) \quad , \quad (9)$$

where $v_{s,8} = v_s/10^8 \text{ cm s}^{-1}$. The electron acceleration is loss-limited when $t_{\text{age}} \gtrsim 10^3$ yrs, and then we derive

$$E_{\max,e} = 14 \text{ TeV } h^{-1/2} v_{s,8} \left(\frac{B_d}{10\mu\text{G}} \right)^{-1/2} \quad . \quad (10)$$

The minimum energy, $E_{\min,e}$ and $E_{\min,p}$ are simply taken as the rest-mass energy of electrons and protons, respectively.

The wide-band radiation spectrum is calculated for given $E_{\max,p}$ and $E_{\max,e}$ at a certain t_{age} . We consider radiation processes from primary electrons: the synchrotron, inverse-Compton (IC), and bremsstrahlung emissions, and from primary protons: π^0 decay γ -rays, and synchrotron and bremsstrahlung emissions from secondary electrons arising from the decay of charged pions. We assume the electron-proton ratio, that is the ratio of the electron distribution function to that of protons for a fixed energy in relativistic regimes, of $K_{\text{ep}} = 1 \times 10^{-3}$. For average cosmic-rays in our Galaxy, K_{ep} is about 10^{-2} , which is an order of magnitude larger than our adopted value. However, in general the average value may be different from that at the acceleration site because of the propagation effect. Indeed, as seen in the following, if we adopt $K_{\text{ep}} = 1 \times 10^{-3}$, theoretically calculated flux ratio $R_{\text{TeV}/X}$ for a young SNR is similar to the observed one. For old SNRs, which we are most interested in, the result for $R_{\text{TeV}/X}$ is unchanged even if K_{ep} varies more than one order of magnitude because accelerated electrons do not contribute to TeV and X-ray emission.

Primarily accelerated electrons produce synchrotron emission with the roll-off frequency given by $\nu_{\text{roll}} \sim 1.6 \times 10^{16} (B_d/10\mu\text{G})(E_{\max,e}/10\text{TeV})^2$ Hz (Reynolds & Keohane 1999)¹. Using Eq. (10), ν_{roll} can be rewritten as

$$\nu_{\text{roll}} \sim 3.2 \times 10^{16} h^{-1} v_{s,8}^2 \text{ Hz} \quad , \quad (11)$$

in the loss-limited case (see also Aharonian & Atoyan 1999). Note that ν_{roll} does not depend on B_{ISM} .

3.1 Emission from a young SNR in the Sedov phase

As can be seen from Eqs. (9) and (10), once the SNR dynamics and $B_{\text{ISM}} (= B_d/r)$ are fixed, only necessary are the values of r and h in order to calculate $E_{\max,p}$ and $E_{\max,e}$ for a given time. In the Sedov phase ($t_1 = 2 \times 10^2$ yrs $< t_{\text{age}} < t_2 = 4 \times 10^4$ yrs for $E_{51} = n_0 = v_{i,9} = 1$), the shock of an SNR is strong and adiabatic, so that the density compression ratio, r , is near 4. Then in the Bohm limit case, $\eta_u \sim \eta_d \sim 1$, we find $h \sim 1$ for arbitrary θ . When the nonlinear effects are considered, r becomes as large as ~ 7 (Berezhko et al. 2002), however, in this case, one can find that h weakly depends on r . Hence in this paper, we neglect the nonlinear effects for simplicity. When h is around unity, we can reproduce the observed value of ν_{roll} for young SNRs (e.g., Bamba et al. 2003b, 2005a,b). Hence, for $t_{\text{age}} \lesssim 10^4$ yrs, we adopt $r = 4$ and $h = 1$ as a fiducial value. The power-law index of accelerated particles is fixed as $p = 2.2$, which is typical for young SNRs.

Here we assume that an SNR stores energy, 10^{50} ergs, of high-energy protons. As long as accelerated particles maintains upstream turbulence via streaming instability, the self-confinement of accelerated particles is efficient, so that the escape of them upstream may be neglected. Furthermore, radiative efficiency is also small. As discussed previously, the energy of accelerated particles is reconverted to the expansion energy through the adiabatic expansion, which implies the expansion energy remains almost unchanged. Therefore,

¹ See <http://heasarc.gsfc.nasa.gov/docs/xanadu/xspec> for the erratum of coefficients.

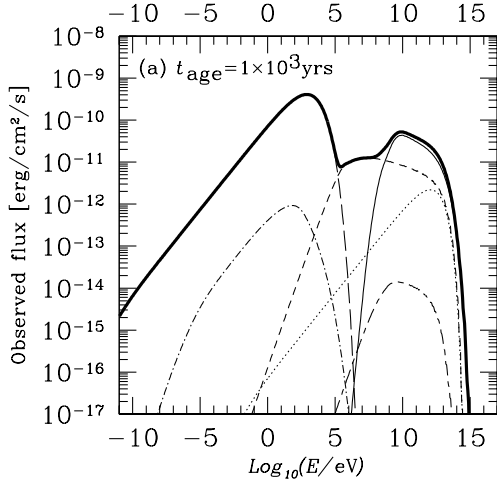


Figure 1. νF_ν spectrum of a single SNR with an age 1×10^3 yrs that stores energy, 10^{50} ergs, of high-energy protons. The thick-solid line shows the total non-thermal flux. Hadronic emissions are π^0 -decay γ -rays (thin-solid), synchrotron (dot-dashed) and bremsstrahlung emission (short-and-long dashed) from secondary electrons produced by charged pion. Leptonic emissions are synchrotron (long-dashed), inverse-Compton (dotted) and bremsstrahlung (short-dashed) emission by primary electrons. The source is located at 1 kpc. We adopt $E_{51} = v_{i,9} = n_0 = h = B_{\text{ISM},-5} = 1$, $p = 2.2$, $r = 4$ and $K_{\text{ep}} = 1 \times 10^{-3}$ (see text for details).

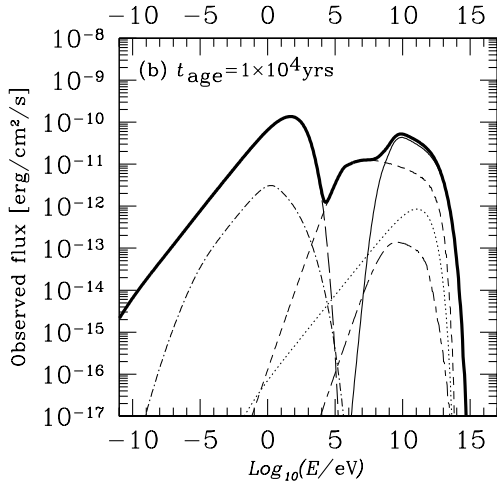


Figure 2. The same as Fig. 1 but for an SNR age of 1×10^4 yrs.

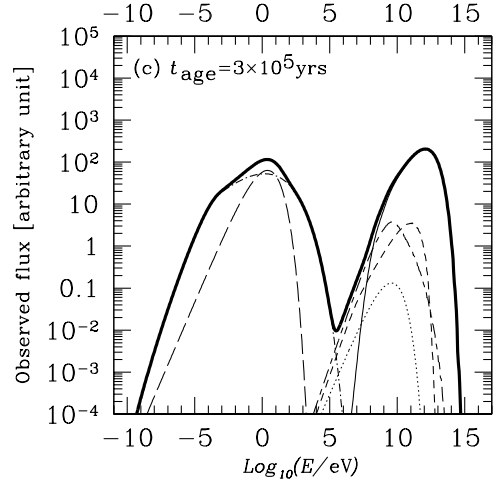


Figure 3. νF_ν spectrum of a single SNR with an age 3×10^5 yrs. Meanings of each lines are the same as those in Fig. 1. The absolute value of the observed flux is uncertain, hence the flux is arbitrary scaled. We adopt $E_{51} = v_{i,9} = n_0 = h = B_{\text{ISM},-5} = 1$, $p = 1.5$, and $K_{\text{ep}} = 1 \times 10^{-3}$ (see text for details).

if the injection rate is time-independent, the energy of high-energy protons is also time-independent.

When $t_{\text{age}} = 1 \times 10^3$ yrs (then, $t_1 < t_{\text{age}} < t_2$), we find $E_{\text{max},p} = 96$ TeV, $E_{\text{max},e} = 27$ TeV and $\nu_{\text{roll}} = 4.8 \times 10^{17}$ Hz for the fiducial parameters, and find that nonthermal X-rays are dominated by the synchrotron emission from primary electrons (see Fig. 1). TeV γ -rays are dominated by π^0 -decay process. The flux ratio is $R_{\text{TeV}/X} = 7.6 \times 10^{-2}$, which is consistent with observations for young SNRs.

When $t_{\text{age}} = 1 \times 10^4$ yrs (Fig. 2), an SNR is still in the Sedov phase ($t_1 < t_{\text{age}} < t_2$). For the fiducial parameters, $E_{\text{max},p} = 61$ TeV and $E_{\text{max},e} = 6.9$ TeV are derived. Also in this case, the X-ray band is dominated by the primary synchrotron radiation. As the SNR ages, ν_{roll} becomes small, so that synchrotron radiation flux in the X-ray band becomes small. Hence the flux ratio becomes large, $R_{\text{TeV}/X} = 1.6$.

3.2 Emission from an old SNR in the radiative phase

When an SNR enters into the radiative phase ($t_{\text{age}} > t_2$), the cooling effect becomes important, and the accumulated gas forms a dense, cool shell with the number density $n_{\text{sh}} = r n_0$. Rewriting Eq. (2) as

$$v_s = 2.3 \times 10^7 E_{51}^{11/51} n_0^{-4/17} \left(\frac{t_{\text{age}}}{10^5 \text{ yrs}} \right)^{-2/3} \text{ cm s}^{-1}, \quad (12)$$

we find $v_s \sim 10^7$ cm s $^{-1}$ when $t_{\text{age}} \sim 10^5$ yrs. Note that v_s is independent of v_i .

In the radiative phase, the shock is isothermal. Hence there is a concern that since the downstream temperature is low, the neutral component appears, and Alfvén wave turbulence that scatters nonthermal particles is significantly sup-

pressed via the neutral-ion friction, making the acceleration process inefficient (Drury et al. 1996; Bykov et al. 2000). On the other hand, when the gas remains fully ionized around the shock front, diffusive shock acceleration works well in the radiative phase. Shull & McKee (1979) found that as long as $v_{s,7} \gtrsim 1.1$, where $v_{s,7} = v_s/10^7 \text{ cm s}^{-1}$, UV flux radiated in the downstream region is sufficient enough to keep upstream state fully ionized. For $n_0 = E_{51} = 1$, we find $v_{s,7} = 1.1$ at $t_{\text{age}} = 3 \times 10^5$ yrs. Therefore, when $t_{\text{age}} \lesssim 3 \times 10^5$ yrs, diffusive shock acceleration works well, and we can use Eqs. (9) and (10) in order to estimate the maximum energy of accelerated particles.

The estimation of downstream magnetic field, B_d , which appears in Eqs. (9) and (10), is different from that for the young SNRs. For $n_0 = B_{\text{ISM},-5} = 1$, the magnetic pressure is larger than the gas pressure in the upstream region. Then, solving the shock jump conditions for the quasi-perpendicular ($\theta \sim 90^\circ$) isothermal shock, the compression ratio is given by

$$r \sim \sqrt{2} \frac{v_s}{v_{A1}} = 6.5 B_{\text{ISM},-5}^{-1} n_0^{1/2} v_{s,7} \quad , \quad (13)$$

where $v_{A1} = B_{\text{ISM}}(4\pi m_H n_0)^{-1/2}$ is the upstream Alfvén velocity (Spitzer 1978). Therefore, the downstream magnetic field $B_d = r B_{\text{ISM}} = (8\pi m_H n_0)^{1/2} v_s$ is calculated as

$$B_d = 65 \mu\text{G} n_0^{1/2} v_{s,7} \quad . \quad (14)$$

Recently, using two-dimensional MHD simulation, Hanayama & Tomisaka (2006) have shown that in the case of $n_0 = 0.2$ (or 1) and $E_{51} = B_{\text{ISM}} = 0.5$, the compression ratio at the quasi-perpendicular shock region is about 7 at t_{age} of several 10^5 yrs, which is roughly consistent with our estimation. Using Eqs. (9), (10), (12) and (14), we obtain

$$E_{\text{max},p} = 34 \text{ TeV} h^{-1} E_{51}^{11/34} n_0^{-6/17} v_{s,7}^{3/2} \quad , \quad (15)$$

$$E_{\text{max},e} = 0.55 \text{ TeV} h^{-1/2} n_0^{-1/4} v_{s,7}^{1/2} \quad . \quad (16)$$

Note that Eq. (14) is derived for the limiting case of a small upstream gas pressure compared to the magnetic pressure there. If the explosion occurs where the gas pressure is comparable to or larger than the magnetic pressure, the effect of the gas pressure should be taken into account. Then, the compression ratio becomes large (Spitzer 1978). Furthermore, when the accelerated protons are stored around the shock front, a part of their energy goes into the magnetic field energy, causing the magnetic field amplification (Lucek & Bell 2000). Therefore, Eq. (14) gives, in fact, the lower bound of B_d , so that, Eqs. (15) and (16) give lower and upper bounds of $E_{\text{max},p}$ ($\propto B_d$) and $E_{\text{max},e}$ ($\propto B_d^{-1/2}$), respectively. In the following, however, neglecting such effects for simplicity, we use Eqs. (14), (15) and (16) to determine B_d , $E_{\text{max},p}$ and $E_{\text{max},e}$. One should also remark that Eq. (14) is derived for the quasi-perpendicular shock ($\theta \sim 90^\circ$). Then, one can consider the acceleration at the quasi-perpendicular shock as discussed in Jokipii (1987). However, even if $\theta \sim 0$ upstream, then the accelerated particles penetrate into upstream region and generate waves via streaming instability, resulting in the generation of oblique or quasi-perpendicular magnetic field upstream, because the magnetic field component of generated waves is generally perpendicular to the shock normal and can be as large as the original upstream magnetic field.

When $t_{\text{age}} = 3 \times 10^5$ yrs, we find $r = B_d/B_{\text{ISM}} \sim 7$. Hence, in the case of Bohm limit $\eta_u \sim \eta_d \sim 1$, we find $h \sim 1$ as well as for the young SNRs. Until this epoch, the particle acceleration proceeds, so that the upstream turbulence is maintained and the Bohm limit diffusion can be expected. Therefore, we again take $h = 1$ as a fiducial value. In the test particle approximation, the power-law index of accelerated particles p and the compression ratio r are related as $p = (r+2)/(r-1)$ (Blandford & Eichler 1987). If we take $r \sim 7$ as a typical value, p becomes about 1.5. Hence, we adopt $p = 1.5$ as a fiducial value. However, as can be seen in the following, our conclusion does not depend on p .

The total energy of accelerated protons stored in the SNR is somewhat uncertain. If the confinement of accelerated particles works well until $t_{\text{age}} \sim 10^5$ yrs, the energy of accelerated protons may not be so small compared with the young SNR case. Otherwise, it is much smaller than 10^{50} ergs. In this paper, since we are mainly interested in the value of $R_{\text{TeV}/X}$, the total energy of accelerated particles, which only determines the normalization of the radiated spectra, is not essential for our arguments. Further discussion on determining the amount of accelerated particles will be seen in § 5.

Figure 3 shows the spectrum for $t_{\text{age}} = 3 \times 10^5$ yrs with fiducial parameters (i.e., $B_d = 72 \mu\text{G}$, $E_{\text{max},p} = 42 \text{ TeV}$ and $E_{\text{max},e} = 0.58 \text{ TeV}$). Then the flux ratio is $R_{\text{TeV}/X} = 82$. Since $E_{\text{max},e}$ is small, TeV γ -rays via IC and bremsstrahlung are suppressed, and therefore, the TeV γ -rays come from the π^0 -decay. The roll-off frequency of synchrotron radiation from primary electrons is so small, $\nu_{\text{roll}} \sim 4.0 \times 10^{14} \text{ Hz}$, that the secondary synchrotron radiation dominates the X-ray band. Also in this subsection, we have assumed the electron-to-proton ratio as $K_{\text{ep}} = 1 \times 10^{-3}$. However, even if we adopt much larger K_{ep} , $R_{\text{TeV}/X}$ remains unchanged because as can be seen in Fig. 3, both X-ray and TeV γ -ray bands lie above the cutoff of emission components originating in primary electrons.

We discuss how $R_{\text{TeV}/X}$ varies for different values of parameters. At first, changes of B_{ISM} and v_i do not affect the conclusion because Eqs. (14), (15) and (16) tells us that B_d , $E_{\text{max},p}$ and $E_{\text{max},e}$, which determine the overall shape of the emission spectrum, do not depend on them. As stated above, uncertainty of K_{ep} does not affect the value of $R_{\text{TeV}/X}$. We also find that $R_{\text{TeV}/X}$ only weakly depends on h and E_{51} for reasonable parameter ranges. Hence we discuss the dependence of n_0 and p in the following. Let us first discuss the case of $n_0 = 0.5$ with other parameters unchanged. We consider the epoch in which $v_{s,7} \sim 1.1$ ($t_{\text{age}} \sim 3.8 \times 10^5$ yrs for our parameters) when $R_{\text{TeV}/X}$ becomes maximum since it increases with time. Then, we find $B_d = 51 \mu\text{G}$, $E_{\text{max},p} = 39 \text{ TeV}$ and $E_{\text{max},e} = 0.70 \text{ TeV}$, respectively. Again both the X-rays and TeV γ -rays are dominated by the hadronic emission. Compared with the fiducial case, the secondary synchrotron X-ray emission is dim because $E_{\text{max},p}$ and B_d are small (see the dashed line in Fig. 4), so that the flux ratio becomes large $R_{\text{TeV}/X} = 1.3 \times 10^2$. Next, we consider the case $p = 2$ with other parameters being fiducial (see the dotted line in Fig. 4). TeV γ -rays are produced by protons with energies of $\sim 10 \text{ TeV}$, while X-rays come from secondary electrons with energies of $\sim 30 \text{ TeV}$ originated in $\sim 100 \text{ TeV}$ protons. Then, for a large p , there are less protons with $\sim 100 \text{ TeV}$ and the dimmer

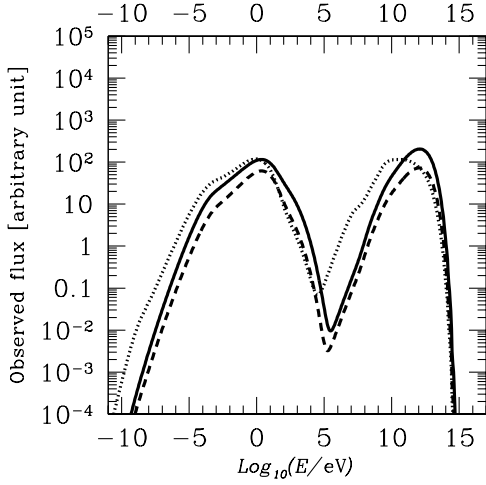


Figure 4. νF_ν spectrum of a single SNR with an age 3×10^5 yrs for various parameter sets. Solid line is the total flux for the fiducial set of parameters ($n_0 = 1$, $p = 1.5$, $B_d = 72 \mu\text{G}$, $E_{\text{max},p} = 42 \text{ TeV}$ and $E_{\text{max},e} = 0.58 \text{ TeV}$), and is the same as the thick solid line in Fig. 3. Dashed and dotted lines are for $n_0 = 0.5$ ($B_d = 51 \mu\text{G}$, $E_{\text{max},p} = 39 \text{ TeV}$ and $E_{\text{max},e} = 0.70 \text{ TeV}$) and $p = 2$ with the other parameters fiducial, respectively.

X-rays. Hence we find the flux ratio becomes as large as $R_{\text{TeV}/X} = 1.6 \times 10^2$. So one can see that when parameters are changed within a reasonable range, the value of $R_{\text{TeV}/X}$ remains unchanged within a factor of two or three.

If $t_{\text{age}} \gtrsim 3 \times 10^5$ yrs, i.e., $v_{s,7} \lesssim 1.1$, then, the ionization around the shock front is incomplete (Shull & McKee 1979) and upstream ion-neutral Alfvén wave damping places significant restriction on shock acceleration (Drury et al. 1996; Bykov et al. 2000). Once the shock acceleration becomes inefficient, there are few high-energy protons emitting TeV γ -rays around the SNR shell, because they escape the SNR shell due to the diffusion; assuming the Bohm diffusion, the escape time for a particle with an energy $E_{\text{cr}} = 10E_{\text{cr},10\text{TeV}} \text{ TeV}$ is estimated as $t_{\text{esc}} \sim 1 \times 10^5 \eta^{-1} (B_d/70\mu\text{G}) E_{\text{cr},10\text{TeV}}^{-1} \Delta_{3\text{pc}}^2$ yrs, where η and $\Delta = 3\Delta_{3\text{pc}}$ pc are the gyro factor and the thickness of the shell, respectively. Therefore, when $t_{\text{age}} \gtrsim 3 \times 10^5$ yrs, TeV γ -rays are significantly suppressed.

4 EMISSION FROM AN OLD SNR INTERACTING WITH A GMC

One may expect that if an old SNR interacts with a GMC, TeV γ -ray flux becomes large because of the large density in the GMC. Here we consider two cases for the emission from old SNR-GMC interacting systems; one comes from a shock running into the GMC, and the other from the GMC illuminated by particles accelerated at the SNR shock. We consider the GMC with the mass $M_c = 10^5 M_5 M_\odot$, and the radius $R_c = 18 \text{ pc } M_5^{1/2}$ (Blitz & Rosolowsky 2004). Assuming the spherical symmetry, the mean number density

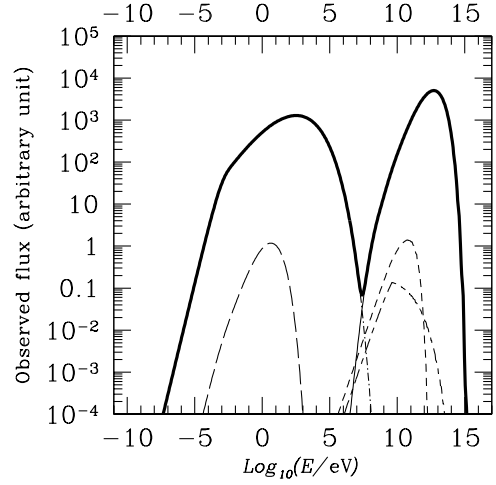


Figure 5. νF_ν spectrum for emission from a shocked GMC that is collided by a SNR with an age of 4.6×10^4 yrs. The GMC has mass of $10^5 M_\odot$, and the meanings of each line are the same as those in Fig. 1. The flux is arbitrarily scaled. We adopt $E_{51} = v_{i,9} = n_0 = h = B_{\text{ISM},-5} = M_5 = 1$, $K_{\text{ep}} = 1 \times 10^{-3}$ and $p = 1$ (see text for details).

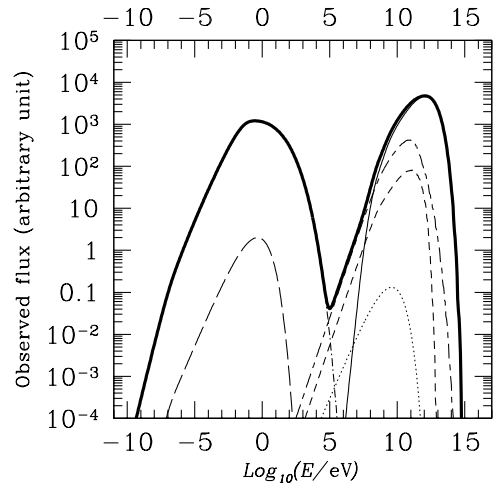


Figure 6. νF_ν emission spectrum from a GMC illuminated by high-energy particles arising from the SNR with an age of 3×10^5 yrs. The GMC has mass of $10^5 M_\odot$, and the meanings of each line are the same as those in Fig. 1. The flux is arbitrarily scaled. We adopt $E_{51} = v_{i,9} = n_0 = h = B_{\text{ISM},-5} = M_5 = 1$, $K_{\text{ep}} = 1 \times 10^{-3}$ and $p = 1.5$ (see text for details).

of the GMC is $n_c = 1.7 \times 10^2 M_5^{-1/2} \text{ cm}^{-3}$. We also assume that the magnetic field of the GMC is $B_c = 1 \mu\text{G} n_c^{1/2}$ (Crutcher 1991). Then we find

$$B_c = 13 \mu\text{G} M_5^{-1/4}, \quad (17)$$

and the Alfvén velocity in the GMC is $v_A = 2.2 \times 10^5 \text{ cm s}^{-1}$. In this section, numerical values are for the fiducial parameter set ($E_{51} = v_{i,9} = n_0 = h = B_{\text{ISM},-5} = M_5 = 1$) unless otherwise stated.

4.1 Emission from a shock running into a GMC

When an SNR shell collides with a GMC with the number density n_c , the shock front is formed in the GMC. In general, since the geometry of SNR shell and the GMC at the collision is unknown, it is hard to precisely determine the dynamics of the shell. Here we perform a very rough calculation. Assuming the momentum conservation, the shell velocity, $v_{s,c}$, in the GMC can be related with the velocity, v_s , of the SNR shell just before the collision as (Chevalier 1999)

$$v_{s,c} \sim \frac{v_s}{1 + (n_c/n_{\text{sh}})^{1/2}}. \quad (18)$$

The radiative cooling is efficient and the shock is isothermal. We find that the velocity of the shock in the GMC is $v_{s,c} \sim 1.1 \times 10^7 \text{ cm s}^{-1}$ when the SNR shell with an age of $4.6 \times 10^4 \text{ yrs}$ ($v_s = 3.9 \times 10^7 \text{ cm s}^{-1}$ and $n_{\text{sh}} = 25 \text{ cm}^{-3}$) collides with the GMC with the mass of $10^5 M_\odot$. Then, the Alfvén Mach number of the shell running in the GMC is ~ 50 , and the compression ratio is estimated as $r \sim \sqrt{2} v_{s,c}/v_A \sim 70$, so that the shock acceleration works well (Bykov et al. 2000). The downstream magnetic field is

$$B_d = (8\pi m_H n_c)^{1/2} v_{s,c} = 9.1 \times 10^2 \mu\text{G}, \quad (19)$$

and we find $E_{\text{max},p} = 79 \text{ TeV}$ and $E_{\text{max},e} = 0.16 \text{ TeV}$. Here we adopt $p = 1$ as a limiting case, because the compression ratio is much larger than unity. The simulated spectrum is shown in Fig. 5. We assume the electron-to-proton ratio $K_{\text{ep}} = 1 \times 10^{-3}$ as in § 3, however, our conclusion does not change if the ratio becomes ten times larger. Since $E_{\text{max},e}$ is small, both X-ray and TeV γ -rays are hadronic origin. Because of the large magnetic field, the secondary synchrotron radiation is strong in the X-ray band, and the flux ratio is not so large, $R_{\text{TeV}/X} = 6.9$.

We calculate $R_{\text{TeV}/X}$ for several cases in which the shocked GMC has $v_{s,c} = 1.1 \times 10^7 \text{ cm s}^{-1}$, and find that the large magnetic field at the emitting region causes bright secondary synchrotron radiation, and that the flux ratio for the emission from shocked GMC does not exceed ~ 20 . We also note that if we adopt $p = 2.0$ instead of $p = 1.0$, with other parameters fiducial, we obtain $R_{\text{TeV}/X} = 15$, which implies the value of p does not affect our conclusion.

If the SNR shell with $t_{\text{age}} \gtrsim 5 \times 10^4 \text{ yrs}$ collides with the GMC with $M_5 = 1$, $v_{s,c}$ is smaller than $1.1 \times 10^7 \text{ cm s}^{-1}$, so that the TeV emission from the shock in the GMC is significantly weak because of the wave damping effect and the energy loss effect as discussed in § 3.2. At this time, the density is so high that high-energy protons lose their energy via pion production. Hence when SNR with $t_{\text{age}} \gtrsim 5 \times 10^4 \text{ yrs}$ collides with GMC, the arising shock in the GMC does not emit TeV γ -rays.

4.2 Emission from a GMC illuminated by protons accelerated at an Old SNR shock

If an SNR with $t_{\text{age}} \gtrsim 5 \times 10^4 \text{ yrs}$ interacts with a GMC with $M_5 = 1$, particle acceleration at the GMC shock is inefficient to produce TeV γ -ray emitting particles because $v_{s,c}$ is smaller than $1.1 \times 10^7 \text{ cm s}^{-1}$. Nevertheless, the high density of GMC works as a target of the high-energy protons to produce pions. They are accelerated at the shock front of the SNR and penetrate into the GMC, radiating photons. We have seen in § 3.2 that the SNR shock itself can accelerate TeV- γ -ray-emitting particles until $t_{\text{age}} \sim 3 \times 10^5 \text{ yrs}$. Here we consider the emission from the GMC encountered by the SNR with $t_{\text{age}} = 3 \times 10^5 \text{ yrs}$. The simulated spectrum is shown in Fig. 6. We use $E_{\text{max},p} = 42 \text{ TeV}$ and $E_{\text{max},e} = 0.58 \text{ TeV}$ as seen for the fiducial case in § 3.2, while we adopt the magnetic field of the emission region as $B_c = 13 \mu\text{G}$. Both the X-rays and TeV γ -rays are again hadronic origin. Compared with Fig. 5, the secondary synchrotron radiation is weaker in the X-ray band because of smaller magnetic field. We find $R_{\text{TeV}/X} = 5.9 \times 10^2$. If the GMC is encountered by the SNR with $t_{\text{age}} \gtrsim 3 \times 10^5 \text{ yrs}$, particles emitting TeV γ -rays do not exist around the shock front as seen in § 3.2.

We calculate $R_{\text{TeV}/X}$ for several cases in which parameters are changed within a reasonable range, and find that the value of $R_{\text{TeV}/X}$ remains unchanged within a factor of three. We also note that if we adopt $p = 2.0$ instead of $p = 1.5$, with other parameters fiducial, we obtain $R_{\text{TeV}/X} = 1.4 \times 10^3$, which implies the value of p does not affect our conclusion.

For $B_c = 13 \mu\text{G}$, both TeV γ -rays and X-rays are originated in the accelerated protons with energies of more than 10 TeV. Let $\delta = \delta_{\text{pc}} \text{ pc}$ be the separation between the GMC and the SNR shell producing high-energy particles. The time for a particle with an energy $E_{\text{cr}} = 10 E_{\text{cr},10\text{TeV}} \text{ TeV}$ to reach the GMC is estimated by the diffusion time, $t_{\text{dif}} \sim 2 \times 10^3 \eta^{-1} B_{\text{ISM},-5}^{-1} E_{\text{cr},10\text{TeV}}^{-1} \delta_{\text{pc}}^2 \text{ yrs}$, where η is the gyro factor. If $\delta_{\text{pc}} \sim 1$, we expect t_{dif} is much smaller than the age of the SNR. Hence all particles in the energy regime in which we are interested reach the target GMC almost simultaneously, which implies that the particle spectra at the GMC does not so much depend on the character of propagations from the SNR shell to the GMC. If $\delta_{\text{pc}} \gg 1$, effects of energy-dependent diffusion should be considered. Such a detailed calculation is not considered here.

5 DISCUSSIONS

For an old SNR in the radiative phase, the maximum energy of primary electrons is so small that emissions via leptonic processes are vanishingly small both in the X-ray and TeV γ -ray bands. On the other hand, there still exists accelerated protons with energies of more than $\sim 10 \text{ TeV}$. These particles emit the TeV γ -rays via π^0 -decay and synchrotron X-rays from secondary electrons generated by charged pions. We find that the ratio

$$R_{\text{TeV}/X} = \frac{F_\gamma(1 - 10 \text{ TeV})}{F_X(2 - 10 \text{ keV})}$$

could be more than $\sim 10^2$. Such sources may be an origin of recently discovered unidentified TeV sources, and give us the evidence for hadron acceleration. We might have to consider

Table 2. Calculated flux ratios for various bands.

	$R_{\text{TeV}/X}^a$	$R_{\text{TeV}/\text{GeV}}^b$	$R_{\text{TeV}/\text{radio}}^c$
Fig. 1	7.6×10^{-2}	0.36	9.8
Fig. 2	1.6	0.25	6.6
Fig. 3	82	6.6	47
Fig. 5	6.9	74	9.7×10^3
Fig. 6	5.9×10^2	5.8	3.5×10^2

$$^a R_{\text{TeV}/X} = F_{\gamma}(1-10 \text{ TeV})/F_X(2-10 \text{ keV}).$$

$$^b R_{\text{TeV}/\text{GeV}} = F_{\gamma}(1-10 \text{ TeV})/F_{\text{GeV}}(1-10 \text{ GeV}).$$

$$^c R_{\text{TeV}/\text{radio}} = F_{\gamma}(1-10 \text{ TeV})/F_{\text{radio}}(10^7-10^{11} \text{ Hz}).$$

the interaction between the GMC and the SNR with an age of $\sim 10^5$ yrs. For $t_{\text{age}} \ll 10^5$ yrs, SNR radius is so small that there is only a few SNRs interacting with a GMC. On the other hand, if the SNR with $t_{\text{age}} \gg 10^5$ yrs encounters the GMC, there are few high-energy particles emitting TeV γ -rays around shocks of the SNR and the GMC because of the energy loss effect and/or the wave damping effect occurring at low-velocity isothermal shocks.

Actually detected TeV sources have the energy flux $F_{\gamma}(1-10 \text{ TeV}) \sim 10^{-12}-10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ (Aharonian et al. 2005a, 2006). Hence if $R_{\text{TeV}/X} \lesssim 10^{2-3}$, then $F_X(2-10 \text{ keV}) \gtrsim 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$. Such diffuse, extended source can be detected with current X-ray telescopes (*Suzaku*, *Chandra*, and *XMM-Newton*). Especially, X-ray Imaging Spectrometer (XIS) onboard *Suzaku* is capable of observing dim, diffuse X-ray sources with low background in the hard X-ray band. Less energetic protons emit GeV γ -rays that might have been detected by EGRET or may be detected by GLAST in the future. However, for isothermal shocks, the spectrum of accelerated particles may be hard ($p < 2$). Indeed, some of the newly discovered TeV sources show hard spectrum (Aharonian et al. 2005a, 2006), which may imply $p \lesssim 2$. In such a case, GeV emission becomes dim as shown in Table 2, where we calculate $R_{\text{TeV}/\text{GeV}}$ for the case of Figures 1–3, 5 and 6. Radio emission is also expected. However, as can be seen in Table 2, the sources with large $R_{\text{TeV}/X}$ have large $R_{\text{TeV}/\text{radio}}$, so that the radio emission may be dim. This fact might account for a large scatter of $R_{\text{TeV}/\text{radio}}$ found by Helfand et al. (2005). Furthermore, the source is confined in a galactic plane region and radio emission may be obscured by diffuse components.

In this paper, we consider TeV γ -ray emission from (1) a single old SNR with $t_{\text{age}} \sim 3 \times 10^5$ yrs (§ 3.2), (2) a shocked GMC collided by the old SNR with $t_{\text{age}} \sim 5 \times 10^4$ yrs (§ 4.1), and (3) an unshocked GMC illuminated by high-energy particles arising at the shock of the old SNR with $t_{\text{age}} \sim 3 \times 10^5$ yrs (§ 4.2), respectively. As in the followings, we can roughly estimate, for each case, the expected number of observed TeV sources, N_{TeV} , which have observed flux larger than 3% of the Crab flux ($6 \times 10^{-13} \text{ cm}^{-2} \text{ s}^{-1}$ above 1 TeV) and lie in the inner part of the galactic plane with $-30^\circ < l < 30^\circ$ (cf. Aharonian et al. 2005a, 2006). The number of detected TeV sources gives us important information on the energetics of old SNR emission.

Let us first consider the case (1). Let $E_p = E_{p,50} 10^{50}$ ergs be the energy of accelerated protons stored in the SNR. Then, the observed flux of TeV γ -rays is calcu-

lated as $F(> \text{TeV}) \sim 1 \times 10^{-10} n_{\text{sh},0.86} E_{p,50} d_{\text{kpc}}^{-2} \text{ cm}^{-2} \text{ s}^{-1}$, where $n_{\text{sh}} = 10^{0.86} n_{\text{sh},0.86} \text{ cm}^{-3}$ and $d = d_{\text{kpc}} \text{ kpc}$ are the density of the SNR shell and the distance to the source, respectively. The maximum distance to the source is $d_{\text{max}} \sim 13 n_{\text{sh},0.86}^{1/2} E_{p,50}^{1/2} \text{ kpc}$. Then the volume fraction of the survey region is

$$\beta_1 \sim \frac{(\pi/6)d_{\text{max}}^2}{\pi(10 \text{ kpc})^2} \sim 0.30 n_{\text{sh},0.86} E_{p,50}.$$

Hence, for assumed SN explosion rate of $1 \times 10^{-2} r_{-2} \text{ yr}^{-1}$, we obtain

$$\begin{aligned} N_{\text{TeV}} &\sim 3 \times 10^3 r_{-2} \beta_1 \\ &\sim 9 \times 10^2 r_{-2} n_{\text{sh},0.86} E_{p,50}. \end{aligned}$$

Next, we calculate N_{TeV} for case (3). In order to estimate the number of GMCs in our Galaxy, we adopt the mass function of GMCs, $f(M_c) \propto M_c^{-1.5}$ with the maximum and the minimum mass, $M_{\text{max}} = 2 \times 10^7 M_{\odot}$ and $M_{\text{min}} = 30 M_{\odot}$, respectively (Blitz & Rosolowsky 2004). If the total mass of GMCs in our Galaxy is $2 \times 10^9 M_{\odot}$, then the number of GMCs with mass of around $M_c = 10^5 M_5 M_{\odot}$ is about $7 \times 10^2 M_5^{-1/2}$. The SNR with an age of 3×10^5 yrs has a radius $R_s \sim 90 \text{ pc}$, so that the geometrical factor, representing the fraction of accelerated protons colliding the GMC and emitting γ -rays, is estimated as $\alpha = (R_c/R_s)^2/4 \sim 1 \times 10^{-2}$. The observed flux of TeV γ -rays is calculated as $F(> \text{TeV}) \sim 3 \times 10^{-11} \alpha_{-2} n_{c,2.2} E_{p,50} d_{\text{kpc}}^{-2} \text{ cm}^{-2} \text{ s}^{-1}$, where $\alpha = \alpha_{-2} 10^{-2}$, and $n_c = 10^{2.2} n_{c,2.2} \text{ cm}^{-3}$ is the density of the GMC. The maximum distance to the source is $d_{\text{max}} \sim 6 \alpha_{-2}^{1/2} n_{c,2.2}^{1/2} E_{p,50}^{1/2} \text{ kpc}$. Then the volume fraction of the survey region is

$$\beta_1 \sim \frac{(\pi/6)d_{\text{max}}^2}{\pi(10 \text{ kpc})^2} \sim 7 \times 10^{-2} \alpha_{-2} n_{c,2.2} E_{p,50}.$$

Assuming the volume of the galactic disk of $4 \times 10^{10} \text{ pc}^3$, the mean separation of GMCs is $\ell \sim 4 \times 10^2 M_5^{1/6} \text{ pc}$. Then, the probability that a SNR collides with a GMC is

$$\beta_2 \sim \frac{4\pi}{3} \left(\frac{R_s}{\ell}\right)^3 \sim 5 \times 10^{-2} \ell_{400}^{-3} \left(\frac{R_s}{90 \text{ pc}}\right)^3,$$

where $\ell = 400 \ell_{400} \text{ pc}$. Therefore, we derive

$$\begin{aligned} N_{\text{TeV}} &\sim 3 \times 10^3 r_{-2} \beta_1 \beta_2 \\ &\sim 10 r_{-2} \alpha_{-2} n_{c,2.2} E_{p,50} \ell_{400}^{-3} \left(\frac{R_s}{90 \text{ pc}}\right)^3. \end{aligned}$$

Hence $E_{p,50} \gtrsim 0.1$ is required so that $N_{\text{TeV}} \gtrsim 1$.

Finally, we consider the case (2). We assume that the energy of accelerated protons is $1 \times 10^{48} E'_{p,48}$ ergs, which is much smaller than the thermal energy stored in the shocked GMC. Then, the observed TeV γ -ray flux is calculated as $F(> \text{TeV}) \sim 3 \times 10^{-9} n_{c,d,4.1} E'_{p,48} d_{\text{kpc}}^{-2} \text{ cm}^{-2} \text{ s}^{-1}$, where $n_{c,d} = 10^{4.1} n_{c,d,4.1} \text{ cm}^{-3}$ is the density of shocked GMC. The maximum distance to the source is calculated as $d_{\text{max}} \sim 66 n_{c,d,4.1}^{1/2} E'_{p,48}^{1/2} \text{ kpc}$, which is larger than the size of the Galactic disk. Therefore, the volume fraction is about $\beta_1 \sim 0.3$. Since the SNR with an age of 5×10^4 yrs has a radius $R_s \sim 40 \text{ pc}$, the collision probability is

$$\beta_2 \sim \frac{4\pi}{3} \left(\frac{R_s}{\ell}\right)^3 \sim 4 \times 10^{-3} \ell_{400}^{-3} \left(\frac{R_s}{40 \text{ pc}}\right)^3.$$

Hence we obtain

$$N_{\text{TeV}} \sim 5 \times 10^2 r_{-2} \beta_1 \beta_2$$

$$\sim 0.6 r_{-2} (\beta_1/0.3) \ell_{400}^{-3} \left(\frac{R_s}{40 \text{ pc}} \right)^3 .$$

Taking into account for the uncertainties for R_s and/or ℓ_{400} , we can expect $N_{\text{TeV}} \gtrsim 1$ for reasonable values of parameters.

At present, the number of unidentified TeV sources showing large $R_{\text{TeV}/X}$ is only a few. If the number of actually detected TeV sources does not increase (i.e., the H.E.S.S. galactic plane survey with the flux larger than 0.03 Crab above 1 TeV is nearly complete at present), then $E_{p,50} \lesssim 0.01$ is implied, otherwise N_{TeV} for the case (1) is much larger than a few. This might suggest the escape of high-energy particles from the SNR starts by $t_{\text{age}} \sim 10^5$ yrs, or injection efficiency is low for old SNRs. Then, the number of TeV sources, N_{TeV} , for the case (3) should be small. On the other hand, the case (2) may be still likely because the number of sources for this case can be comparable to that for the case (1). Hence, some TeV sources are associated with the GMC, but others not. In order to clarify whether the TeV source is associated with a GMC, the CO observation may be important.

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