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Visco-elastic Analysis of a Dental Wax by 4-elements Models

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ABSTRACT

Visco-elastic behavior of a dental wax was analyzed by beam bending creep test. The results were simulated by 4-elements Voigt model composed of a simple Maxwell model and a simple Voigt model connected in series. Constant values for this model were determined from the results. These constants were E_M=844 MPa, E_V=1050 MPa, $\eta_M=1.44\times10^8$ poise, and $\eta_V=3.58\times10^7$ poise, where E_i is the elastic constant and η_i the coefficient of viscosity.

This model simulated the creep behavior of the wax well. Constants for 4-elements Maxwell model were also determined by comparing the governing equations for 4-elements Maxwell model and 4-elements Voigt model.

INTRODUCTION

Waxes are commonly used in dental field as pattern materials for crowns and dentures. Waxes, however, have low resistance to deformation because of their small elastic moduli, low melting points, and high coefficients of thermal expansion, and visco-elasticity plays an important role in deformation. Thus, it is important to determine visco-elastic properties of wax.

Katakura *et al.* reported numerical method to analyze residual stresses in wax patterns[1]. Simple Maxwell model or Voigt model both composed of a spring and a dashpot element is used for analysis of visco-elastic behavior of materials, though neither of them is precise enough to simulate them. In this study, 4-elements Voigt model composed of a simple Maxwell model and a simple Voigt model connected in series and 4-elements Maxwell model composed of two simple Maxwell models connected in parallel were employed to analyze visco-elastic behavior of wax (Fig. 1). These models were employed to apply the results to visco-elastic analysis by finite element method (FEM) later.

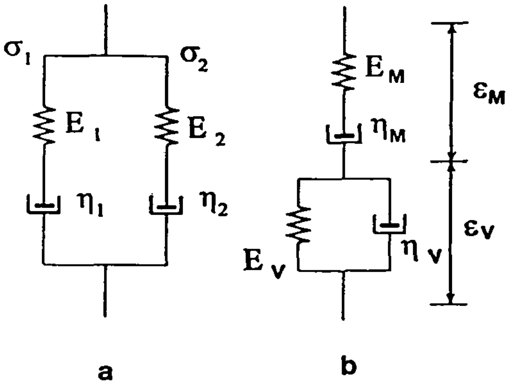

Figure 1 a) 4-elemental Maxwell model
b) 4-elemental Voigt model

MATERIALS AND METHODS

G-C inlay wax was used for measurements. The creep behavior of wax was measured by beam bending method shown in Fig. 2. This method has already been used for measurement of viscosity of dental porcelains by DeHoff and Anusavice[2]. Three wax specimens were prepared as follows. Melted wax was poured into a metal flame, then a glass plate was placed on the surface of the wax when the surface of the wax began to be solid. After specimen became rigid, specimen was removed from the metal flame. Final size of the specimen was 0.45 cm wide, 0.45 cm high, and 7.0 cm long.

Before bending measurement all wax specimens were

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immersed in hot water at 45°C for 30 minutes to release residual stresses. Then, specimen was placed on the supports situating 5 cm apart as shown *in Fig. 2, and* 220 g of load was applied at the middle of the specimen. Deflections were measured at every 0.5 minute for 10 minutes at 25°C.

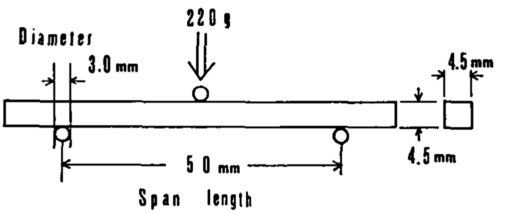

Figure 2 Specimen's size and apparatus for measurements

According to the elementary theory of elasticity, deflection is given by the following equation[3].

$$Y=\frac{PL^3}{48I}\frac{1}{E} \tag{1}$$

where Y is the deflection, L the span length, P the load applied at the middle of wax specimen, E the elastic constant, and I the centroidal moment of inertia.

Deflection due to viscous deformation is expected to be given by the following equation by analogy with the elastic Eq. (1).

$$Y=\frac{PL^3}{48I}\frac{1}{\eta}t \tag{2}$$

where η is the coefficient of viscosity and t is time.

4-elements Voigt model was used to approximate the creep curve. According to the visco-elastic theory creep behavior is given by the following equation for 4-elements Voigt model[4].

$$48I\,Y/PL^3=a_0+a_1t+a_2[1-\exp(-a_3t)] \tag{3}$$

where a_0 is $1/E_M$, a_1 is $1/\eta_M$, a_2 is $1/E_V$, and a_3 is $1/\eta_V$. Constant a_1 governs the later part of the creep curve. When $t\to\infty$, Eq. (3) becomes

$$48I\,Y/PL^3=a_0+a_1t+a_2.$$

Constants a_0+a_2 and a_1 correspond to the crossing with the deflection-axis and slope of the later part of creep curve respectively.

Eq. (3) becomes

$$\frac{48I}{PL^3}\frac{dY}{dt}=a_1+a_2a_3\exp(-a_3t)$$

by differentiating with t. Then

$$\ln\left(\frac{48I}{PL^3}\frac{dY}{dt}-a_1\right)=\ln(a_2a_3)-a_3t \tag{4}$$

By plotting $\ln\left(\frac{48I}{PL^3}\frac{dY}{dt}-a_1\right)$ versus t, constants $\ln(a_2a_3)$ and a_3 were determined by the minimum method of least squares. Then, a_2 and a_3 were determined.

4-elements Maxwell model can be also used to simulate creep behavior. Constants for this model can be determined from the constants for 4-elements Voigt model. Relations of constants for these two different models are obtained by comparing the governing equations for these models shown bellow. The theory of visco-elasticity gives following governing equations (5) and (6).

$$(E_1+E_2)\frac{d^2\varepsilon}{dt^2}+E_1E_2\left(\frac{1}{\eta_1}+\frac{1}{\eta_2}\right)\frac{d\varepsilon}{dt}$$
$$=\left(\frac{E_1}{\eta_1}+\frac{E_2}{\eta_2}\right)\frac{d\sigma}{dt}+\frac{d^2\sigma}{dt^2}+\frac{E_1E_2}{\eta_1\eta_2}\sigma \tag{5}$$

(for 4-elements Maxwell model)

$$E_M\frac{d^2\varepsilon}{dt^2}+\frac{E_ME_V}{\eta_V}\frac{d\varepsilon}{dt}$$
$$=\left(\frac{E_M}{\eta_V}+\frac{E_V}{\eta_V}+\frac{E_M}{\eta_M}\right)\frac{d\sigma}{dt}+\frac{d^2\sigma}{dt^2}+\frac{E_ME_V}{\eta_M\eta_V}\sigma \tag{6}$$

(for 4-elements Voigt model)

Following relations between constants of these models are obtained from Eqs. (5) and (6).

$$(E_1+E_2)=E_M,$$

$$E_1E_2\left(\frac{1}{\eta_1}+\frac{1}{\eta_2}\right)=\frac{E_ME_V}{\eta_V},$$

$$\frac{E_1}{\eta_1}+\frac{E_2}{\eta_2}=\frac{E_M}{\eta_V}+\frac{E_V}{\eta_V}+\frac{E_M}{\eta_M}$$

$$\frac{E_1E_2}{\eta_1\eta_2}=\frac{E_ME_V}{\eta_M\eta_V}$$

RESULTS AND DISCUSSION

Fig. 3 shows a typical time-deflection curve. Constant a_1 in Eq. (3) governs the later part of the creep curve. Results of measurement during 6 and 10 minutes were approximated by a liner line to determine constants a_1 and a_0+a_2 as the slope and the crossing with the deflection-axis of this line respectively. Fig. 4 shows relationship between t (time) and $\ln\left(\frac{48I}{PL^3}\frac{dY}{dt}-a_1\right)$. Values a_3 and $\ln(a_2a_3)$ were obtained as the slope and the crossing with the vertical-axis respectively from Fig. 4. Values a_i and standard deviations are listed in Table 1. Then, con-

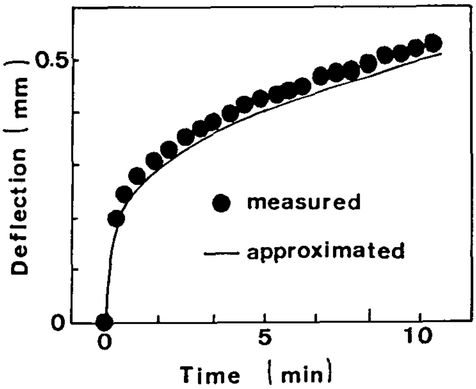

Figure 3 Result of bending creep test and approximation by
$48IY/PL^3 = a_0 + a_1 t + a_2 [1 - \exp(-a_3 t)]$

Table 1 Values of a_i

	a_0	a_1	a_2	a_3
mean	0.177	0.0173	0.142	0.480
S.D.	0.023	0.0015	0.012	0.027

$48IY/PL^3 = a_0 + a_1 t + a_2 [1 - \exp(-a_3 t)]$

Table 2 Visco-elastic properties

4-elements Voigt model

E_M (MPa)	E_V (MPa)	η_M (poise)	η_V (poise)
844	1050	1.44×10^8	3.58×10^7

4-elements Maxwell model

E_1 (MPa)	E_2 (MPa)	η_1 (poise)	η_2 (poise)
420	424	1.33×10^8	7.61×10^6

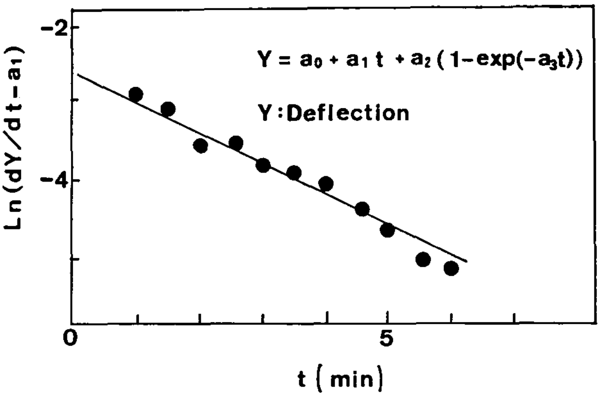

Figure 4 Relationship between $\ln\left(\frac{48I}{PL^3}\frac{dY}{dt} - a_1\right)$ and t

stants for 4-elements Voigt model were $E_M = 844$ MPa, $E_2 = 1050$ MPa, $\eta_M = 1.44 \times 10^8$ poise, and $\eta_V = 3.58 \times 10^7$ poise. Constants for 4-elements Maxwell model were $E_1 = 420$ MPa, $E_2 = 424$ MPa, $\eta_1 = 1.33 \times 10^8$ poise, and $\eta_2 = 7.61 \times 10^6$ poise. These values are listed in Table 2.

Results of calculation by Eq. (3) is also shown in Fig. 3. These curves agreed well each other. By constructing a model with more springs and dashpot elements, we can get a more precise model, however this model would require more complicated process to determine the constants, and physical role of each element becomes vague. Thus, 4-elements Voigt or 4-elements Maxwell models are useful to analyze visco-elastic behavior of dental waxes.

It is well known that wax behavior depends on temperature. To apply results of this study for more practically, it is necessary to determine how temperature affect the visco-elastic behavior. This can be done by measuring bending creep at various temperatures. Those results will determine the temperature dependency of constants of the model in future.

SUMMARY

Bending creep curve was approximated by 4-elements Voigt model and 4-elements Maxwell model. These models simulated the results of creep behavior well. Constant values for these models were $E_M = 844$ MPa, E_V

$=1050$ MPa, $\eta_M=1.44\times10^8$ poise, and $\eta_V=3.58\times10^7$ poise for 4-elements Voigt model and $E_1=420$ MPa, $E_2=424$ MPa, $\eta_1=1.33\times10^8$ poise, and $\eta_2=7.61\times10^6$ poise for 4-elements Maxwell model.

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