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A Dimensional Analysis of Urban Heat Island Intensity

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REVIEW OF EXISTING RELATIONS

In the surface and planetary boundary layers, dimensional analysis is commonly used to develop universal relations. Similarly, dimensional analysis is one of the most powerful tools in the study of turbulence. In many circumstances, it is able to detect a relation between dependent and independent variables. The outstanding example is the form of the spectrum of turbulent kinetic energy in what is called "inertial subrange". However, it is surprising that dimensional analysis has rarely been used in the study of urban meteorology. In searching the literature, the only available "universal" relation concerning the heat island intensity, which is based on dimensional reasoning, is the proposal by Hanna as following (see Kawamura):

$$\Delta T = [HW\frac{\partial\theta}{\partial Z}/C_p\rho u]^{\frac{1}{2}} \tag{1}$$

where ΔT is the difference between the temperature at the center of the urban area and a representative temperature in the rural areas, H the anthropogenic heat input, W the city width, $\frac{\partial\theta}{\partial Z}$ the rural vertical gradient of potential temperature, Cp specific heat of air at constant pressure, ρ air density, u a representive upwind rural air speed. An obvious defect in the aforementioned relation is that the structural and material complexity of urban area is not considered.

In this work, an attempt is made to establish a new "universal" function for ΔT in light of recent findings and our understanding of urban heat island workings. The rationale behind this work is we believe that under many circumstances the thermal structures of a small city are miniature copies of those for a large city. The intensity of the thermal anomalies may vary only slightly with urban size, even a single building complex will show a different microclimate than an equal piece of land in its natural state. In addition, the artificial heat may not play a dominant role in some cases, particularly during daytime. There are a number of cases where our assertion can be supported. One of them is for Nagoya city. The area of Nagoya city is much smaller than Tokyo metropolitan area. Nevertheless, the heat island intensity in Nagoya, according to a recent report by Mizutori, is about 3^0 C, which is quite the same as observed in Tokyo area. Another good case in point is for Maryland, in Columbia. It is a small town, but with the beginning of a business center and office buildings, the maximum heat island had grown to ΔT = 7°C by 1974. It should be mentioned here that

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in searching the literature, no field measurements could be found which demonstrate a direct link between heat island intensity and city size.

A comparison of temperature rises in a number of United States cities, made by Mitchell, shows that the warming rates during winter time are considerably smaller than those found for summer. This suggests that the major cause for heat island is the change in the radiation balance, and the anthropogenic heat is of second importance. Similar comments are also made by Munn and others. On the other hand, however, a number of studies have provided evidence of considerable contribution of anthropogenic heating to the formation of heat island. In this work, a "unified universal" relation for heat island intensity is proposed, which may conciliate the two opposite views towards the effect of artificial heat.

DIMENSIONAL REASONING

Prior to this dimensional analysis, numerical results obtained from the model described in the preceding section was analyzed to assess the dependence of the heat island intensity on the surface moisture availability. The numerical results indicates that the heat island intensity increases with decreasing the surface relative humidity (RH) as shown in Figure 1. It is seen from the plot 1 that the relation between ΔT and RH could be approximated as linear. If the surface relative humidity (or the surface moisture availability) is interpreted as the fraction of evaporating surfaces, we then arrive at the conclusion that the heat island intensity is linearly proportional to the density of built-up area as schematically shown by the dashed line in Fig.1. This conclusion is in general agreement with the data reported in several studies.

Also found from computation is the ΔT is positively correlated with the surface roughness height, and negatively correlated with the wind speed in an non-linear manner. Besides, the thermal properties of surface materials in urban area make a notable impact on the formation of heat island as discussed by Asaeda and others.

For reasons given above, we can assume:

$$\Delta T=f(P_{\text{built-up}},h,u,C_p,\lambda,\frac{\partial\theta}{\partial Z}) \qquad (2)$$

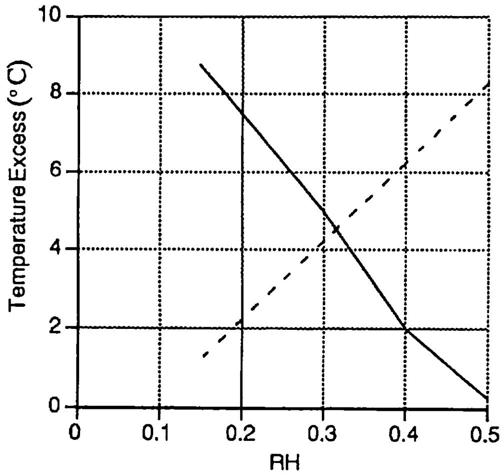

Figure 1: Relation between heat island intensity and surface relative humidity

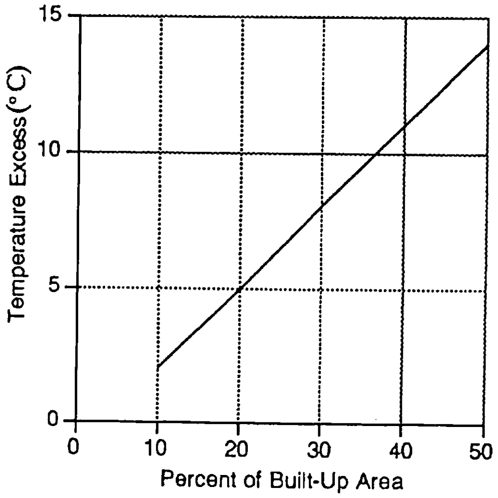

Figure 2: Relation between heat island intensity and percent of built-up area in Maryland

where $P_{\text{built-up}}$ is the percentage of built-up area, h the surface roughness height, u the synoptic wind speed, C_p the heat capacity of surface material, λ the thermal conductivity of surface material, $\frac{\partial\theta}{\partial Z}$ the rural vertical gradient of potential temperature.

By dimensional analysis, and considering numerical results, a new relationship for ΔT emerges as:

$$\Delta T = AP_{\text{built-up}}\,\frac{\partial\theta}{\partial Z}\left(\frac{h}{u}\,\frac{\lambda}{C_p}\right)^{\frac{1}{2}} \qquad (3)$$

where A is a coefficient. The value of A may be affected by the topography and location of a city.

In this new relation, the heat island intensity is linearly dependent on the percentage of built-up area. This dependence can be supported by the data reported for Maryland, in Columbia as shown in Figure 2, and additional support comes from the data for Nagoya city reported by Mizutori. It is found that the lowest temperature for typical summer days increases with the percentage of built-up area (Figure 3). The difference in warming rate between Maryland and Nagoya, as can be observed from figures, might be attributed to the location of Nagoya city being closer to sea.

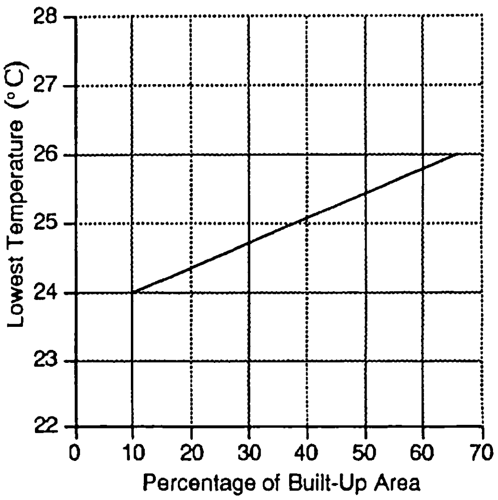

Figure 3: Relation between lowest temperature and percent of built-up area for summer days in Nagoya, adapted from Mizutori.

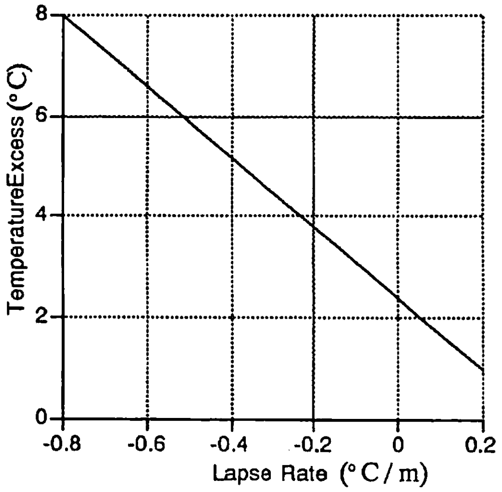

Figure 4: Relation between heat island intensity and vertical temperature gradient, adapted from Ludwig.

The linear dependence of the heat island intensity upon the rural vertical temperature gradient can be verified by field measurements of Ludwig, who has demonstrated a strong relation between the rural lapse rate and the urban heat island intensity as shown in Figure 4. The lapse rate is defined as the minus of the rural vertical temperature gradient. This linear dependence also compares favorably with the work of Leathey, in which he finds from the data for New York that the heat island intensity is proportional to the rural lapse rate times a mixing depth.

Compared to the proposal of Hanna, the thermal properties of surface material in urban area are taken into consideration. Besides, the city width is replaced with the surface roughness height, and the anthropogenic heat is removed in the new relation. However, as mentioned already, a number of studies have shown that the contribution of anthropogenic heating is significant to the formation of urban heat island. For example, the study done by Kimura shows that most of the Tokyo heat island intensity at night is due to anthropogenic heating. Results of field investigations by Leathey in Cincinnati metropolitan area indicate that with a strong inversion over the rural area upwind of the city, the urban boundary layer appears to result primarily from the addition of heat within the urban area rather than from mechanical turbulence generated by the wind passing over the large roughness elements of the city. A major source of the added heat in urban area is anthropogenic heating. Besides, the thermal properties of urban surface would lead to greater daytime heat storage as compared to rural area. The release of this amount of heat into atmosphere might create the so-called "tropical nights" as discussed by Asaeda and others. Therefore, for those cases, we may assume that the heat island intensity is mainly determined by anthropogenic heating, city size, wind speed and surface characteristics in downtown area:

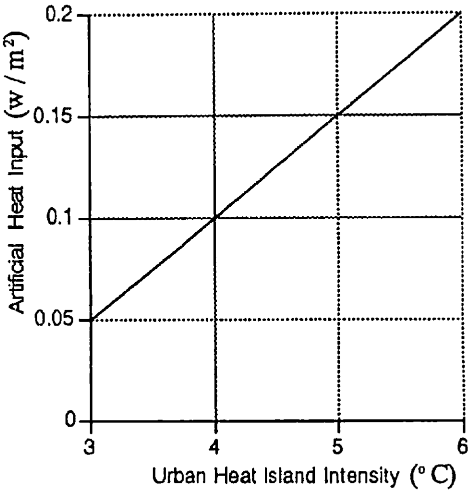

Figure 5: Relation between heat island intensity and artificial heat, adapted from Yu.

$$\Delta T=f(H, W, u, \rho_s, C, C_p, \lambda) \tag{4}$$

where H is the anthropogenic heat input, W the city width, ρ_s, C the density and the specific heat capacity of surface material in downtown area, respectively.
Then, dimensional analysis yields:

$$\Delta T=B\frac{H}{\rho_s C}[\frac{W}{u}\frac{C_p}{\lambda}]^{\frac{1}{2}} \tag{5}$$

A remarkable difference between the relation (3) and (5) is that the heat island intensity decreases with the increase of C_p according to the relation (3), whereas the relation (5) shows that the heat island intensity will be enhanced by larger values of C_p. The reason for this difference can be attributed to the fact that the two relations apply to different regime. Evidence supporting the linear dependence of heat island intensity upon artificial heat can be found in the numerical simulation of Yu as shown in Figure 5.
Based on the above discussions, we propose a "unified universal" relation for the urban heat island intensity ΔT as following:

with a mild inversion over the rural area upwind of a city,

$$\Delta T=AP_{\text{built-up}}\frac{\partial\theta}{\partial Z}(\frac{h}{u}\frac{\lambda}{C_p})^{\frac{1}{2}} \tag{6}$$

with a strong inversion over the rural area upwind of a city,

$$\Delta T = B\frac{H}{\rho_s C}\left[\frac{W}{u}\frac{C_\rho}{\lambda}\right]^{\frac{1}{2}} \tag{7}$$

During night time , a strong inversion usually develops in rural area, so that the relation (7) is applicable to nocturnal heat island. While during daytime a weak inversion is often observed in rural area, so that the relation (6) works better than relation (7) for daytime heat island. It should be stressed howevere, that under the condition of very weak general wind the present analysis is invalid. In the 'ideal' case with calm winds and cloudless skies, Oke found that the heat island intensity is well related to the logarithm of population for many North American and European cities. In view of the fact that the population is a surrogate index of the central building density, one might expect a similar "log-law" between the heat island intensity and the density of buildings under calm condition. The general relation for heat island intensity under calm and near-calm conditions is currently under investigation by means of large-eddy simulation.

From the proposed unified relation, an explanation can be given about the effect of anthropogenic heating. Once a strong inversion develops in rural area, the effect of anthropogenic heating becomes pronounced. The mechanism behind it might be that when a very stable temperature stratification is advected from rural to urban area, it greatly reduces the turbulent mixing, which may result in the accumulation of artificial heat near the urban surface. It is because a strong inversion usually forms in rural area at night that the effect of anthropogenic heating becomes important to nocturnal heat island.

According to relation (6) and (7), the effect of higher C_ρ and lower λ is to reduce the daytime heat island, and meanwhile intensify the night-time heat island, in other words, it reduces the magnitude of diurnal heat wave. This seems justifiable by considering that higher heat capacity and lower heat conductivity will lead to higher heat storage at urban soil sublayer during daytime, and this amount of heat will be released during night. This finding qualitatively agrees with previous studies. It should be pointed out that since the urban surface is made up of a variety of materials due to different land-use classes, the heat capacity C_ρ and the thermal conductivity λ, which appear in the new relation, should be considered as averages of various materials in urban area.

Summary

A dimensional analysis is made to relate the magnitude of urban heat island to synoptic conditions and physical parameters of a city. The proposed relationship is justifiable in view of numerical results and observational data. It shows that the daytime heat island problem could be alleviated by planting more trees and water-fronts, while the nocturnal heat island could be lessened by choosing proper materials for roads and buildings. Although the validity of this proposed relation should be further examined, it is hoped that the refinement of such a "universal" relation can provide a general guideline to urban designers.

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