

A possible observational evidence for θ^{-2} angular distribution of opening half-angle of GRB jets

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ABSTRACT

We propose a method to estimate the pseudo jet opening half-angle of GRBs using the spectral peak energy (E_p)–peak luminosity relation (so called Yonetoku relation) as well as the E_p –collimation-corrected γ -ray energy relation (so called Ghirlanda relation). For bursts with known jet break times and redshifts, we compared the pseudo jet opening half-angle with the standard one and found that the differences are within a factor 2. We apply the method to 689 long GRBs. We found that the distribution function of the pseudo jet opening half-angle obeys $f(\theta_j) \propto \theta_j^{-2.2 \pm 0.2}$ with possible cutoffs for $\theta_j < 0.04$ and $\theta_j > 0.3$ although the log-normal fit is also possible. θ^{-2} distribution is compatible with the structured jet model. From the distribution function we found that the beaming correction for the rate of GRBs is ~ 340 , which means $\sim 10^{-5} \text{ yr}^{-1} \text{ galaxy}^{-1}$ or only one in 10^2 type Ib/c supernovae. We also found the evolution of the distribution function as a function of the redshift.

Key words: gamma rays: bursts — gamma rays: observations — gamma rays: theory.

1 INTRODUCTION

Gamma-ray bursts (GRBs) arise from relativistic jets (e.g., Mészáros 2002; Zhang & Mészáros 2003). Relativistic motion, with Lorentz factor of greater than $\sim 10^2$ is necessary in order to resolve the “Compactness problem”. On the other hand, the evidence of the jet collimation has been derived by the wave-length-independent achromatic break in the observed afterglow light curve. The jet opening half-angle has been actually measured in this context for bursts with successfully measured redshifts and jet break times. However, the small number of samples prevents us from discussing the statistical properties of the opening angle distribution. We have not yet known the maximum and minimum value of the opening angle as well as the slope of the distribution function well (Frail et al. 2001). These quantities are very important to argue the GRB rate, the energetics, the nature of progenitors, and so on.

There is a correlation between the rest-frame spectral peak energy E_p and the isotropic equivalent γ -ray energy E_{iso} of GRBs called the Amati relation (Amati et al. 2002; Atteia 2003; Lamb et al. 2004; Sakamoto et al. 2004). Several authors have argued theoretical interpretations of the relation (Yamazaki et al. 2004; Eichler & Levinson 2004; Lamb et al. 2005). While Nakar & Piran (2004) ar-

gued against the Amati relation because $\sim 40\%$ of 751 BATSE GRBs do not have the solution to the undetermined redshift under the Amati relation and clear outliers such as GRB 980425 and GRB 031203 exist. A similar argument against the Amati relation has also been made by Band & Preece (2005). Ghirlanda et al. (2005a) argued against Nakar & Piran (2004) as well as Band & Preece (2005). They used 442 bright BATSE GRBs with good statistics and pseudo redshift from the lag-luminosity relation (Band, Norris & Bonnel 2004) and obtained the Amati relation with the slightly different power law index and the larger scatter than the original one. They found that the chance probability of the revised Amati relation is 2.1×10^{-65} . Similar analysis can be also seen in Bosnjak et al. (2005).

As for outliers, Yamazaki, Yonetoku & Nakamura (2003) argued that if the intrinsic E_p of GRB 980425 is 2–4 MeV similar to GRB990123 and the viewing angle is $\sim \theta_j + 10\gamma^{-1}$ with θ_j and γ being the jet opening half-angle and the gamma factor of the jet, the observed E_{iso} and E_p of GRB 980425 can be reproduced. If we locate GRB 980425 at the distance larger than 800 Mpc, the event is not observed. Hence, GRB 980425-like events are not included in the bright BATSE GRBs. Similar arguments have been also done for GRB 031203 (Ramirez-Ruiz et al. 2004). Note here that in the classification of *HETE-2*, GRB 980425 and GRB 031203 are X-ray rich GRBs.

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Recently, similar relations to the Amati one with the tighter correlation have been found. Yonetoku et al. (2004) have shown that for 16 GRBs with known redshifts detected by BATSE and *BeppoSAX*, there is a tight positive correlation between E_p and the peak luminosity (E_p - L_p relation). The chance probability is extremely low 5.3×10^{-9} . Ghirlanda et al. (2005b) called the relation as the Yonetoku relation and checked the validity of the relation using 442 bright GRBs with the pseudo redshift and confirmed the relation with the same power law index within the error in the original Yonetoku relation. They found that the chance probability of the Yonetoku relation is 1.6×10^{-69} . Yonetoku et al. (2004) used the Yonetoku relation as the redshift indicator for 689 GRB samples without known redshift, and derived the GRB formation rate. On the other hand, for 15 GRBs detected by BATSE, *BeppoSAX* and *HETE-2* with measured redshift and opening half-angle θ_j , Ghirlanda et al. (2004) found that E_p correlates with the collimation-corrected γ -ray energy E_γ . We call the relation as the Ghirlanda relation.

The E_p - L_p diagram of GRBs may correspond to the Hertzsprung-Russell (HR) diagram of stars. Then the Yonetoku as well as the Ghirlanda relations correspond to the main-sequence where stars cluster around a single curve determined by the mass of the star. This suggests the existence of a certain parameter that controls GRBs like the mass of the star in the HR diagram. In the HR diagram the outliers of the main sequence exist such as red giants and white dwarfs. We know the physical reasons for the existence of these outliers so that they do not refute the main sequence relation. In the Yonetoku and the Ghirlanda relations also outliers exist for which we will know the physical reasons in future as suggested in (Yamazaki, Yonetoku & Nakamura 2003; Ramirez-Ruiz et al. 2004).

In this letter, we show that the Yonetoku and the Ghirlanda relations can be used to estimate the opening half-angle of the relativistic jet of GRBs. Main advantage of our jet opening half-angle estimator is that the opening angle can be calculated only from the information of the prompt emission. This letter is organized as follows. In section 2, using the Yonetoku and Ghirlanda relations, we derive an empirical formula to estimate the opening half-angle and discuss its validity. Using the estimator, we derive the distribution of opening half-angle of BATSE-triggered bursts in section 3. Section 4 is devoted to discussions.

2 JET OPENING ANGLE INFERRED FROM THE YONETOKU AND THE GHIRLANDA RELATIONS

Usually the jet opening half-angle is estimated only when both the redshift and the achromatic break time in the afterglow light curve are measured. Under the simple assumption of a uniform ambient matter distribution of number density n_0 , the jet opening half-angle is estimated as

$$\theta_{j, \text{break}} = 0.12 [t_{\text{jet}, d} / (1+z)]^{3/8} (n_0 \eta_\gamma / E_{\text{iso}, 53})^{1/8}, \quad (1)$$

where $E_{\text{iso}, 53} = E_{\text{iso}} / 10^{53}$ erg, and $t_{\text{jet}, d}$ and η_γ are the jet break time in days and the efficiency of the fireball in converting the energy in the ejecta into γ -rays, respectively (Sari et al. 1999). However continuous follow-up observa-

tions are required to measure the achromatic jet-break time, and moreover these kinds of observations are realized only for the bright afterglows so that it is hard to measure the opening half-angle for large amount of GRBs in this method.

We propose here a different method to estimate the opening half-angle using only informations of the prompt gamma-ray emission. Let us assume that the rest-frame spectral peak energy E_p , the peak luminosity L_p , the isotropic equivalent γ -ray energy E_{iso} , the jet opening-half angle θ_j , and the collimation-corrected energy E_γ satisfy the Yonetoku and the Ghirlanda relations as

$$E_p = 2.1 \times 10^2 L_{p, 52}^{0.50} \text{ keV}, \quad (2)$$

$$E_p = 4.8 \times 10^2 E_{\gamma, 51}^{0.71} \text{ keV}, \quad (3)$$

$$E_\gamma = (1 - \cos \theta_j) E_{\text{iso}}, \quad (4)$$

where Q_x denotes $Q/10^x$ in cgs units (Yonetoku et al. 2004; Ghirlanda et al. 2004). From equations (2), (3) and (4), we have,

$$\Omega_j \equiv 1 - \cos \theta_j = 0.30 L_{p, 52}^{0.50/0.71} E_{\text{iso}, 51}^{-1}. \quad (5)$$

From the observed E_p and L_p , we can determine the redshift under the Yonetoku relation and the given cosmological parameter as in (Yonetoku et al. 2004). Then E_{iso} can be computed using the observed fluence. Therefore, only from the information of the prompt gamma-ray emission, we can estimate the jet opening half-angle for each GRB. This method has a strong advantage compared with the jet-break measurement in afterglows since we can use the large number of BATSE GRBs data.

To show the validity of our method, in figure 1, we compare $\theta_{j, \text{break}}$ estimated by Eq. (1) with θ_j by Eq. (5) for GRBs with measured redshift and jet break time. It is found that there exists a positive correlation with the linear correlation coefficient including weighting factor is 0.760 with 13 degree of freedom (excluding the lower and upper limit samples), which corresponds to the chance probability of 9.63×10^{-4} . Their differences from the linear function (equivalent line) are within a factor of 2. Although we had better use the word ‘‘the pseudo jet opening half-angle’’ for the half-angle from equation (4), we simply use the opening half-angle in this letter for convenience.

3 JET OPENING ANGLE DISTRIBUTION OF BATSE BURSTS

We used the same data set of 689 long GRBs published in (Yonetoku et al. 2004) to measure the distribution function, $f(\theta_j)$. Here θ_j is the jet opening half-angle and $f(\theta_j)d\theta_j$ is the relative number of the jet with the opening half-angle between θ_j and $\theta_j + d\theta_j$. In order to have a better signal-to-noise ratio in our analyses, we selected 605 GRBs with the flux greater than 1×10^{-6} erg cm $^{-2}$ s $^{-1}$. Having obtained the observed peak flux and the spectral indices as well as the E_p , the redshift is estimated using the Yonetoku relation for each event (Yonetoku et al. 2004). Then, we can estimate L_p and E_{iso} . Hence θ_j is also calculated by equation (5). The empirical θ_j estimator shown in equation (5) depends on two parameters; L_p and E_{iso} . This fact means that the two different types of selection effects are mixed. To avoid such doubly truncation effects, we manually set a truncation

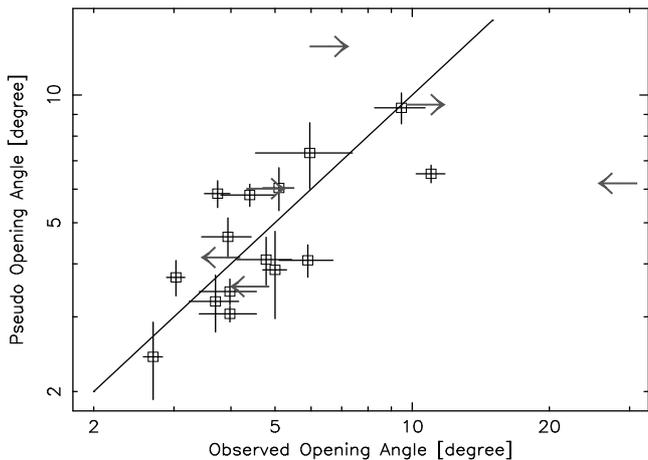


Figure 1. Comparison between jet opening half-angle θ_j estimated by Eq. (5) and $\theta_{j,\text{break}}$ estimated by Eq. (1). The solid line is an equivalent line.

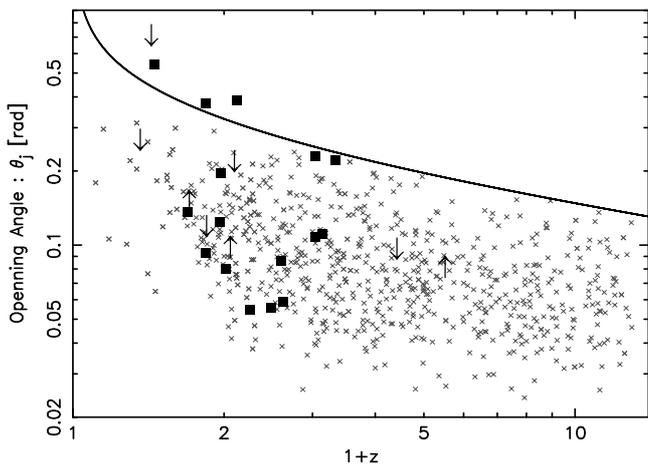


Figure 2. Distribution of the jet opening half-angle θ_j estimated by Eq. (5) vs. redshift derived from the E_p - L_p relation. The solid line shows the truncation of the upper bound of θ_j that is caused by the flux limit of $F_{\text{lim}} = 2 \times 10^{-7} \text{ ergs cm}^{-2} \text{ s}^{-1}$. The cross points are the θ_j measured in the present work. The opening angles measured from the jet-break time in the optical afterglow are also plot on the same figure as squares and arrows.

as the $E_{\text{iso}} > L_p \times 1 \text{ sec}$ at the rest frame of GRBs. In this case, both L_p and E_{iso} are limited only by the peak flux, so we can deal with the data as the simply truncated one.

Figure 2 shows the distribution of the derived jet opening half-angle as a function of the redshift. The θ_j distributes within the range of $0.04 < \theta_j < 0.3$ radian. Using the flux limited samples explained above, we can estimate the opening half-angle evolution and the true distribution of θ_j following the detailed mathematical descriptions (e.g. Lynden-Bell 1971; Petrosian 1993; Maloney & Petrosian 1999; Yonetoku et al. 2004).

First, we estimate the opening half-angle evolution, which is the redshift dependence of $f(\theta_j)$, from the $(1+z, \theta_j)$ distribution shown in figure 2. The data correlation degree in the flux limited samples are calculated by

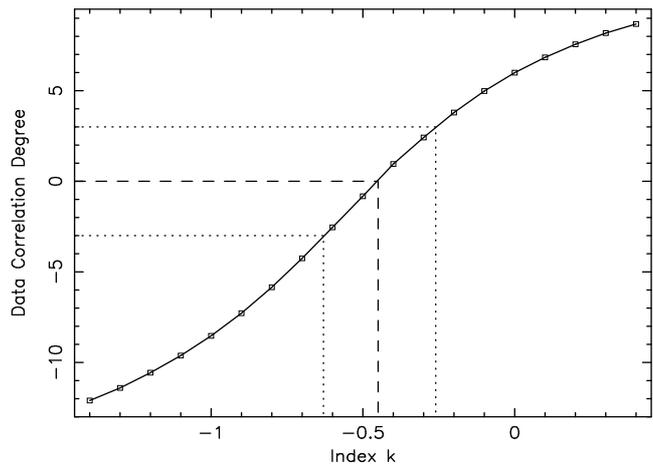


Figure 3. The best index of the jet opening angle evolution measured by the τ -statistical method. When we assume θ_j evolution as the function of $(1+z)^k$, $k = -0.45$ is the best value. We also show 3 sigma upper and lower bound of the index as $k = -0.26$ and $k = -0.63$, respectively. No evolution of θ_j was rejected about 6 σ confidence.

so called τ -statistical method which is very similar to the Kendall's τ statistics. To refer to the previous works (Lloyd-Ronning et al. 2002; Yonetoku et al. 2004), we assume the functional form of $f(\theta_j)$ evolution as $(1+z)^k$, and calculate the data correlation degree for each k value. In figure 3, we show the $(1+z, \theta_j)$ correlation as a function of index k . No θ_j evolution is rejected about 6 sigma confidence level. When we assume the case of $k = -0.45$, θ_j becomes independent of the redshift. In other words, θ_j evolution of $(1+z)^{-0.45}$ is hidden in the $(1+z, \theta_j)$ plane.

Next, we derive the opening half-angle distribution. One possible method (non-parametric method) developed by Lynden-Bell (1971) is applied in this analysis. We define $\theta'_j \equiv \theta_j / (1+z)^{-0.45}$ which is equivalent to the evolution-removed opening half-angle. Then the observed data randomly distributes in the $(1+z, \theta'_j)$ plane, so we can easily assume the missing data caused by the flux limit. Strictly speaking, for high-redshift GRBs (e.g., $z \gtrsim 5$), the observed fluence may be estimated as the lower value because there may be missing photons behind the background level. On the other hand, the peak flux may be correctly estimated. Hence derived opening half-angles for high- z events may be larger than the actual values. In order to avoid such confusion, we consider 430 events within the estimated redshifts of less than 4.5. Additional merit is that the k-correction factor can be neglected. The differential θ'_j distribution $f(\theta'_j)$ which corresponds to one at $z = 1$, is shown in figure 4. If one would like to obtain the θ_j distribution at each redshift, it is roughly estimated as $f(\theta_j) = f(\theta'_j) \left(\frac{1+z}{2} \right)^{-0.45}$.

The opening half-angle distribution $f(\theta'_j)$ is based on the number of event detected by BATSE. The chance probability to observe narrowly collimated events are much lower than that of the wide opening half-angle events due to the geometrical effect. Therefore, we have to take into account the correction of $f(\theta'_j) / \Omega_j$ when we argue the true probability density function. In figure 5, we show the true opening half-angle distribution. It can be fitted by the power-law

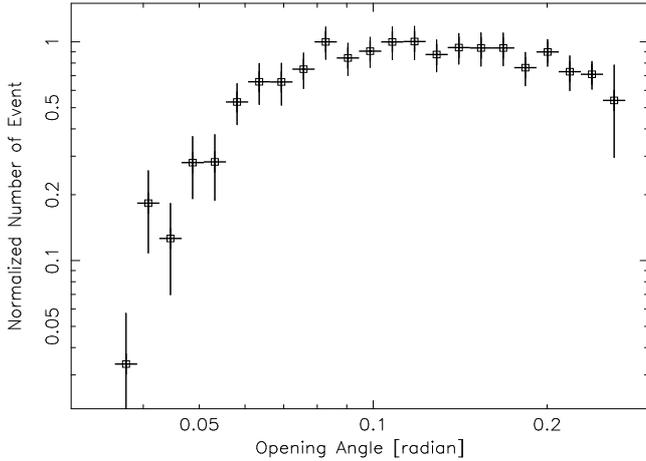


Figure 4. The differential θ'_j distribution at $z = 1$ of the GRBs observed with BATSE detector.

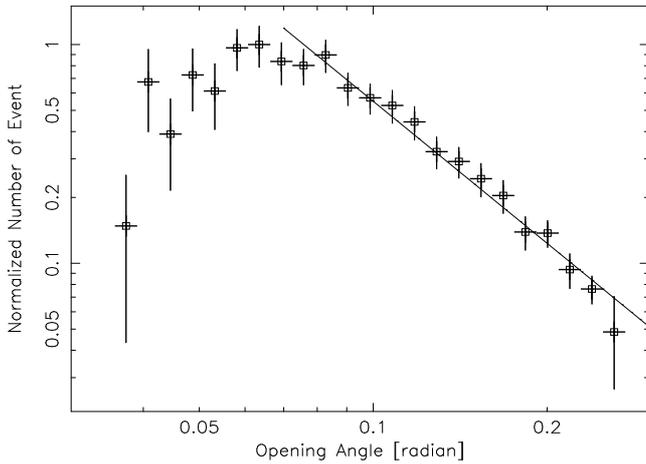


Figure 5. True opening angle distribution at $z = 1$. When we adopt the power-law model for the region $\theta_j > 0.05$ radian, we obtain the best result of $\theta_j'^{-2.2 \pm 0.2}$ with 90 % confidence level.

form in the range $\theta'_j > 0.05$ radian as $\theta_j'^{-2.2 \pm 0.2}$. Clear cut off at ~ 0.04 rad can be seen.

Ghirlanda et al. (2005a) obtained the opening half-angle distribution using the Ghirlanda relation and the Amati relation. The result looks like a log-normal one. We also tried to fit a log-normal one. We obtained acceptable results with the mean $\log \mu = -1.15$ (0.07 radian) and the standard deviation $\log \sigma = 0.24$ (0.04–0.12 radian). However the data distribution is asymmetric in the logarithmic horizontal scale as shown in Figure 5. To say the power law distribution with cut-offs is better than the log-normal one, we have to confirm that the cut-off at $\theta_j = 0.04$ exists. However we can not conclude the existence because of the large errors in the small θ_j region.

From the distribution function we found the beaming correction for the rate of GRBs ~ 340 at $z = 1$ by computing $\langle \Omega^{-1} \rangle = \int f(\theta'_j) \Omega^{-1} d\theta'_j / \int f(\theta'_j) d\theta'_j$. This rate is about factor of 4 larger than that obtained by Guetta, Piran & Waxman (2004) using a different method, while Frail et al. (2001) obtained this correction ~ 520 us-

ing the same method with smaller number of samples. Since the lower cut-off of the distribution $\theta'_j \sim 0.04$ are similar for both cases, the difference comes from the higher power law index ~ -4.54 of Frail et al. (2001). Since the number of GRBs is much smaller in their analysis, observational biases of the smaller samples of larger opening half-angle causes the steeper power law. Adopting the local GRB rate of $\sim 0.5 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Schmidt 2001), we obtain the true rate of GRB as $\sim 170 \text{ Gpc}^{-3} \text{ yr}^{-1}$ which means $\sim 10^{-5} \text{ yr}^{-1} \text{ galaxy}^{-1}$ with the number density of the galaxy being $\sim 10^{-2} \text{ galaxy Mpc}^{-3}$. Since the rate of type Ib/c supernovae is $\sim 10^{-3} \text{ yr}^{-1} \text{ galaxy}^{-1}$ (Cappellaro et al. 2003), only one in 10^2 type Ib/c supernovae becomes GRB if GRB is the peculiar type Ib/c supernova.

4 DISCUSSIONS

We present an empirical opening half-angle estimator that is inferred from the Yonetoku and the Ghirlanda relations. Our method requires only the data of prompt emission, which is different from the standard method using the break in the afterglow light curve and the redshift. Using the empirical opening angle estimator, we have derived the opening angle distribution of GRBs. The distribution can be fitted by the power-law form in the range $\theta_j > 0.07$ rad as $\theta_j^{-2.2 \pm 0.2}$. The cut off at ~ 0.04 rad can be seen. In the uniform jet model of GRBs, this means that the distribution function happens to be $\sim \theta^{-2}$.

The other possibility is the structured jet model. In the original version of the universal structured jet model (Rossi, Lazzati & Rees 2002; Zhang & Mészáros 2002), the energy per unit solid angle is in proportion to $\sim \theta^{-2}$. The lower and upper angle cutoff exist and the jet structure is essentially the same between the lower and the upper cutoff, which is the origin of the name “structured” jet. The viewing angle corresponds to the jet opening angle in the structured jet model so that we need to argue observationally what will be the opening angle distribution of the structured jet (Perna, Sari & Frail 2003). If all bursts were observable, the distribution would be uniform per unit solid angle and $f(\theta) \propto \theta$. However, E_{iso} for the smaller viewing angle is brighter by a factor of θ^{-2} so that the maximum observable distance is larger by a factor of θ^{-1} which contains a volume larger than θ^{-3} . Then we have $f(\theta) \propto \theta^{-2}$ which is compatible with the result in the present paper. Especially our Fig. 5 looks like Fig. 3 of Perna, Sari & Frail (2003) with appropriate parameters.

The evolution effect found in the present paper means that the jet opening half-angle becomes narrower for larger redshift. One possible qualitative explanation is that this is due to the metal dependence of the progenitors. Since the metallicity of the star decreases as a function of the redshift, it is expected that the mass loss of the stars decreases as a function of the redshift if the mass loss is derived by the line absorption of photons in the atmosphere of the stars. This suggests that for high redshift progenitors of GRBs, the mass of the envelope is larger so that only the sharper jet can punch a hole in the envelope of the progenitor star.

In this paper, when we calculate the true opening half-angle distribution, we implicitly assumed that the jet emission can be seen only when the jet is seen on-axis. In re-

ality, effects of off-axis viewing GRBs might be important (Yamazaki et al. 2002, 2003, 2004). When the contributions of off-axis emission is considered, true distribution may be modified. However, the beaming correction from off-axis effects is important only for $z \lesssim 1.5$ because of the relativistic beaming effect (Yamazaki et al. 2004).

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REFERENCES

- Amati, L., et al., 2002, *A&A*, 390, 81
 Atteia, J.-L. 2003, *A&A*, 407, L1
 Band, D. L., Norris, J. P., Bonnell, J. T., 2004, *ApJ*, 613, 484
 Band, D. L., Preece, R. D., 2005, *astro-ph/0501559*
 Bloom, J. S., Frail, D. A., Kulkarni, S. R., 2003, *ApJ*, 594, 674
 Bosnjak, Z. et al., 2005, *astro-ph/0502185*
 Cappellaro, E., Barbon, R., Turatto, M., 2003, *astro-ph/0310859*
 Eichler, D. Levinson, A., 2004, *ApJL*, 614, L13
 Frail, D. A. et al., 2001, *ApJ*, 562, L55
 Ghirlanda, G., Ghisellini, G., Firmani, C., 2005a, *astro-ph/0502186*
 Ghirlanda, G. et al., 2005b, *astro-ph/0502488*
 Ghirlanda, G., Ghisellini, G., Lazzati, D., 2004, *ApJ*, 616, 331
 Guetta, D., Piran, T., Waxman, E., 2004, *astro-ph/0311488*
 Ioka, K. Nakamura, T., 2002, *ApJ*, 570, L21
 Lamb, D. Q. et al., 2004, *New Astron.Rev.* 48, 423 (*astro-ph/0309462*)
 Lamb, D. Q., Donaghy, T. Q., Graziani, C., 2005, *ApJ*, 620, 355
 Lloyd-Ronning, N. M., Fryer, C. L. Ramirez-Ruiz, E., 2002, *ApJ*, 574, 554
 Lynden-Bell, D. 1971, *MNRAS*, 155, 95
 Maloney, A., Petrosian, V., 1999, *ApJ*, 518, 32
 Mészáros, P., 2002, *ARA&A*, 40, 137
 Nakar, E. Piran, T. 2004, *astro-ph/0412232*
 Perna, R., Sari, R., Frail, D., 2003, *ApJ*, 594, 379
 Petrosian, V., 1993, *ApJ*, 402, L33
 Ramirez-Ruiz, E. et al., 2004, *astro-ph/0412145*
 Rossi, E., Lazzati, D., Rees, M. J., 2002, *MNRAS*, 332, 945
 Schmidt, M., 2001, *ApJ*, 552, 36
 Sakamoto, T. et al., 2004, *ApJ*, 602, 875
 Sari, R., Piran, T., Halpern, J. P., 1999, *ApJ* 524, L43
 Yamazaki, R., Ioka, K., Nakamura, T., 2002, *ApJ*, 571, L31
 Yamazaki, R., Ioka, K., Nakamura, T., 2003, *ApJ*, 593, 941
 Yamazaki, R., Ioka, K., Nakamura, T., 2004, *ApJ*, 606, L33
 Yamazaki, R., Yonetoku, D., Nakamura, T., 2003, *ApJ*, 594, L79
 Yonetoku, D., et al., 2004, *ApJ*, 609, 935
 Zhang, B., Mészáros, P., 2002, *ApJ*, 571, 876
 Zhang, B., Mészáros, P., 2003, *Int. J. Mod. Phys. A*19, 2385 (*astro-ph/0311321*)