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COSMOLOGICAL X-RAY FLASHES IN THE OFF-AXIS JET MODEL

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ABSTRACT

The $\langle V/V_{\rm max}\rangle$ of the cosmological X-ray flashes detected by the Wide Field Cameras on *BeppoSAX* is calculated theoretically in a simple jet model. The total emission energy from the jet is assumed to be constant. We find that if the jet opening half-angle is smaller than 0.03 radian, the theoretical $\langle V/V_{\rm max}\rangle$ for fixed opening half-angle is less than $\sim$0.4, which is consistent with the recently reported observational value of 0.27 ± 0.16 at the 1 σ level. This suggests that the off-axis gamma-ray burst jet with the small opening half-angle at the cosmological distance can be identified as the cosmological X-ray flash.

Subject headings: gamma rays: bursts — gamma rays: theory

1. INTRODUCTION

The X-ray flash (XRF) is a class of X-ray transients (Heise et al. 2001; see also Barraud et al. 2003). Some properties of XRFs, such as the observed event rate, the duration, and the isotropic distribution, are similar to those of gamma-ray bursts (GRBs), while the spectral hardness of XRFs characterized by the peak flux ratio, the fluence ratio, and the photon index is softer than that of GRBs. This class represents a large portion of the whole GRB population. Recently, the observational value of $\langle V/V_{\rm max}\rangle$, which is the measure of the homogeneity of spatial distribution (Schmidt, Higdon, & Hueter 1988; see also Chang & Yi 2001; Kim, Chang, & Yi 2001), has been updated from 0.56 ± 0.12 (Heise 2000) to 0.27 ± 0.16 (Heise 2002). The updated value of $\langle V/V_{\rm max}\rangle$ suggests that XRFs take place at a cosmological distance.

Various models accounting for the nature of the XRFs have been proposed (Yamazaki, Ioka, & Nakamura 2002; Heise et al. 2001; Dermer, Chiang, & Böttcher 1999; Huang, Dai, & Lu 2002; Mészáros et al. 2002; Mochkovitch et al. 2003; Daigne & Mochkovitch 2003). Heise et al. (2001) proposed that XRFs could be *GRBs at high redshift*. The redshifts of XRF 011030 and XRF 020427 have an upper limit of $z \lesssim 3.5$ (Bloom et al. 2003; Heise 2002), which suggests that XRFs take place at not so high redshift but the same as that of GRBs. *The photosphere-dominated fireball model* may account the nature of the XRFs with peak energy E_p more than $\sim$20 keV (Mészáros et al. 2002; Ramirez-Ruiz & Lloyd-Ronning 2002). However, further considerations are needed to explain the event with E_p of approximately a few keV, such as XRF 020427 (Amati 2002), XRF 020903, and XRF 010213 (Kawai 2002). The models with small Lorentz factors, such as *the dirty fireball model* (Dermer et al. 1999; Huang et al. 2002) or *the structured-jet model* (Rossi, Lazzati, & Rees 2002; Woosley, Zhang, & Heger 2002; Zhang & Mészáros 2002a), also have possibilities to explain the properties of the XRF, with the implicit assumption that the XRF does not arise from internal shocks (Zhang & Mészáros 2002b). The models for internal shocks with *small contrast of high Lorentz factors* might be the origin of XRFs (Mochkovitch et al. 2003; Daigne & Mochkovitch 2003).

For the other possibility, we have studied *the off-axis jet model* and proposed that if we observe the GRB jet with a large viewing angle, it looks like an XRF (Yamazaki et al. 2002; Yamazaki, Ioka, & Nakamura 2003).

In Yamazaki et al. (2002) the value of the jet opening half-angle was adopted as $\Delta\theta = 0.1$. In this model the distance to the farthest XRF ever detected is about 2 Gpc ($z \sim 0.4$), so the cosmological effect is small and $\langle V/V_{\rm max}\rangle \sim 0.5$. Recent observations suggest that GRBs with relatively small opening angle exist, while the distribution of $\Delta\theta$ is not yet clear (Panaitescu & Kumar 2002). If we assume the total emission energy to be constant, as in the previous paper, the intrinsic luminosity is larger for the smaller opening half-angle. Such GRBs at the cosmological distance observed from off-axis viewing angle may be seen as XRFs and $\langle V/V_{\rm max}\rangle$ is expected to be smaller than 0.5.

In this paper, we will show that our off-axis model has a possibility of accounting for the observational value of $\langle V/V_{\rm max}\rangle$ if we change some of the model parameters from the previous paper (Yamazaki et al. 2002). This paper is organized as follows. In § 2, we describe a simple jet model including the effect of cosmological expansion. We assume the uniform jet with sharp edges. Although one may consider the structured jet motivated by the simulation of the collapsar model, we cannot conclude, both observationally and theoretically, which model is preferable. The $\langle V/V_{\rm max}\rangle$ for the XRFs detected by the Wide Field Cameras (WFCs) on *BeppoSAX* is calculated in § 3. Section 4 is devoted to a discussion. Throughout this paper, we adopt the flat universe with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.65$.

2. EMISSION MODEL OF X-RAY FLASHES

We consider a simple jet model of XRFs in the previous papers (Yamazaki et al. 2002; Ioka & Nakamura 2001) taking into account the cosmological effect. A general formula to calculate the observed flux from an optically thin material is derived by Granot, Piran, & Sari (1999) and Woods & Loeb (1999). Here we adopt their formulations and notations. Let us use a spherical coordinate system $\boldsymbol{r} = (r, \theta, \phi)$ in the central engine frame, where the $\theta = 0$ axis points toward the detector and the central engine is located at the origin. Consider a photon emitted at time t and place $\boldsymbol{r}$ in the central engine frame. It will reach the detector at a time T given by

$$T = (1+z)T_z = (1+z)\left(t - \frac{r\mu}{c}\right), \quad (1)$$

where $\mu \equiv \cos\theta$ and z is the cosmological redshift of the source; $T = 0$ was chosen as the time of arrival at the detector of a photon emitted at the origin at $t = 0$. Then the observed flux at the observed time T and observed frequency ν, measured in ergs s^{-1} cm^{-2} Hz^{-1}, is given by

$$F_\nu(T) = \frac{1+z}{d_L^2}\int_0^{2\pi} d\phi \int_{-1}^{1} d\mu \int_0^{\infty} r^2 dr \frac{j'_{\nu'}(\Omega'_d, \boldsymbol{r}, T_z + r\mu/c)}{\gamma^2(1-\beta\mu)^2}, \quad (2)$$

where d_L, Ω'_d, and $j'_{\nu'}$ are the luminosity distance to the source, the direction toward the detector measured in the frame comoving with the jet (comoving frame), and the comoving frame emissivity in units of ergs s^{-1} cm^{-3} Hz^{-1} sr^{-1}. The frequency ν', which is measured in the comoving frame, is given by

$$\nu' = \nu_z\gamma(1-\beta\mu) = (1+z)\nu\gamma(1-\beta\mu). \quad (3)$$

We adopt an instantaneous emission of infinitesimally thin shell at $t = t_0$ and $r = r_0$. If the emission is isotropic in the comoving frame, the emissivity has a functional form of

$$j'_{\nu'}(\Omega'_d, \boldsymbol{r}, t) = A_0 f(\nu')\delta(t - t_0)\delta(r - r_0)H(\Delta\theta - |\theta - \theta_v|) \times H\left[\cos\phi - \left(\frac{\cos\Delta\theta - \cos\theta_v\cos\theta}{\sin\theta_v\sin\theta}\right)\right], \quad (4)$$

where $f(\nu')$ and A_0 represent the spectral shape and the amplitude, respectively; $\Delta\theta$ and θ_v are the jet opening half-angle and the viewing angle that the axis of the emission cone makes with the $\theta = 0$ axis. The delta functions describe an instantaneous emission at $t = t_0$ and $r = r_0$, and $H(x)$ is the Heaviside step function which describes that the emission is inside the cone. Then the observed flux of a single pulse is given by

$$F_\nu(T) = \frac{2(1+z)r_0 c\gamma^2 A_0}{d_L^2} \times \frac{\Delta\phi(T) f[\nu_z\gamma(1-\beta\cos\theta(T))]}{[\gamma^2(1-\beta\cos\theta(T))]^2}, \quad (5)$$

where $1 - \beta\cos\theta(T) = (1+z)^{-1}(c\beta/r_0)(T - T_0)$ and $T_0 = (1+z)(t_0 - r_0/c\beta)$. For $\Delta\theta > \theta_v$ and $0 < \theta(T) \leq \Delta\theta - \theta_v$, $\Delta\phi(T) = \pi$, otherwise $\Delta\phi(T) = \cos^{-1}\{[\cos\Delta\theta - \cos\theta(T)\cos\theta_v]/[\sin\theta_v\sin\theta(T)]\}$. For $\theta_v < \Delta\theta$, $\theta(T)$ varies from 0 to $\theta_v + \Delta\theta$, while for $\theta_v > \Delta\theta$, $\theta(T)$ varies from $\theta_v - \Delta\theta$ to $\theta_v + \Delta\theta$. In the latter case, $\Delta\phi(T) = 0$ for $\theta(T) = \theta_v - \Delta\theta$. Pulse starting and ending times are given by

$$T_{\rm start} = T_0 + (1+z)(r_0/c\beta) \times [1 - \beta\cos(\max\{0,\ \theta_v - \Delta\theta\})],$$
$$T_{\rm end} = T_0 + (1+z)(r_0/c\beta) \times [1 - \beta\cos(\theta_v + \Delta\theta)]. \quad (6)$$

The observed spectrum of GRBs is well approximated by the Band spectrum (Band et al. 1993). In order to have a spectral shape similar to the Band spectrum, we adopt the following form of the spectrum in the comoving frame,

$$f(\nu') = \begin{cases} (\nu'/\nu'_0)^{1+\alpha_B}, & \nu' < \nu'_0, \\ (\nu'/\nu'_0)^{1+\beta_B}, & \nu' > \nu'_0, \end{cases} \quad (7)$$

where α_B and β_B are the low- and high- energy power-law indexes, respectively. Equations (5) and (7) are the basic equations to calculate the flux of a single pulse.

In order to study the dependence on the viewing angle θ_v and the jet opening half-angle $\Delta\theta$, we fix the other parameters as $\alpha_B = -1$, $\beta_B = -3$, $\gamma\nu'_0 = 200$ keV, $r_0/c\beta\gamma^2 = 10$ s, and $\gamma = 100$ (Preece et al. 2000). We fix the amplitude A_0 so that the isotropic γ-ray energy $E_{\rm iso} = 4\pi d_L^2(1+z)^{-1}$ (20–2000 keV) satisfies

$$\frac{(\Delta\theta)^2}{2}E_{\rm iso} = 5\times10^{50}\ {\rm ergs}, \quad (8)$$

when $\theta_v = 0$ and $z = 1$ (Frail et al. 2001). Here $S(\nu_1 - \nu_2) = \int_{T_{\rm start}}^{T_{\rm end}} F(T; \nu_1 - \nu_2)dT$ is the fluence in the energy range $\nu_1 - \nu_2$ and $F(T; \nu_1 - \nu_2) = \int_{\nu_1}^{\nu_2} F_\nu(T)d\nu$ is the flux in the same energy range. The values of A_0 for different opening angles are summarized in Table 1. When the jet opening half-angle $\Delta\theta$ becomes smaller, A_0 becomes larger.

3. CALCULATION OF $\langle V/V_{\rm max}\rangle$

The formalism to calculate $\langle V/V_{\rm max}\rangle$ is given by Mao & Paczyński (1992) and Piran (1992). In our case the absolute

TABLE 1

RESULTS OF THE CALCULATION FOR FIXED $\Delta\theta$

$\Delta\theta$	A_0[a]	$\theta_{v,p}$[b]	$z_{\rm max}(\theta_{v,p})$	$z_{\rm min}(\theta_{v,p})$	$\langle V/V_{\rm max}\rangle_{\Delta\theta}$[c]
0.10	0.84	0.103	2.8	1.5	0.46
0.09	1.0	0.095	2.9	1.4	0.45
0.08	1.3	0.086	3.0	1.4	0.44
0.07	1.7	0.077	3.1	1.3	0.44
0.06	2.3	0.068	3.3	1.2	0.44
0.05	3.4	0.060	3.5	1.2	0.44
0.04	5.2	0.052	3.6	1.1	0.43
0.03	9.3	0.045	3.8	0.99	0.40
0.02	22	0.038	4.0	0.89	0.38
0.01	109	0.034	4.1	0.77	0.35

[a] In units of ergs cm^{-2} Hz^{-1}.

[b] The viewing angle at which the weight function $W(\theta_v)$ takes the maximum value.

[c] For the XRFs detected by WFCs on *BeppoSAX*.

luminosity and spectrum depend on θ_v and $\Delta\theta$. The observed-integrated number count from the source at redshift z with the jet opening half-angle $\Delta\theta$ and the viewing angle θ_v is given by

$$C(\Delta\theta, \theta_v, z) = \int_{T_{start}}^{T_{end}} dT \int_{\nu_1}^{\nu_2} d\nu \frac{F_\nu(T, \Delta\theta, \theta_v, z)}{h\nu} . \quad (9)$$

Let $z_{\min}$ and $z_{\max}$ be the minimum and the maximum redshift of the XRF for given $\Delta\theta$ and θ_v. In determining $z_{\min}$ and $z_{\max}$, we should note that the operational definition of the XRF detected by *BeppoSAX* is the fast X-ray transient with duration less than $\sim 10^3$ seconds which is detected by WFCs and not detected by the Gamma-Ray Burst Monitor (GRBM). Therefore, if the sources are nearby such that $z < z_{\min}$, they are observed as GRBs because the observed fluence in the γ-ray band (40–700 keV) becomes larger than the limiting sensitivity of GRBM ($\sim 3 \times 10^{-6}$ ergs cm^{-2}). If the sources are too far such that $z > z_{\max}$, they cannot be observed by the WFCs, which have an observation band of 2–28 keV and a limiting sensitivity of about 4×10^{-7} ergs cm^{-2}. Although the detection conditions of the instruments vary with many factors of each event, such as the duration, the spectral index, or the peak photon energy (Band 2002), we adopt very simple criteria here. As shown in Figure 1, both $z_{\max}$ and $z_{\min}$ depend on θ_v and $\Delta\theta$.

For given $z_{\max}$, we can calculate the minimum integrated count $C_{\min} = C(\Delta\theta, \theta_v, z_{\max})$. Then $\langle V/V_{\max}\rangle$ for given $\Delta\theta$ and θ_v is calculated as

$$\left\langle \frac{V}{V_{\max}} \right\rangle_{\Delta\theta,\theta_v} = \frac{\int_{z_{\min}}^{z_{\max}} [C(z)/C_{\min}]^{-3/2} \mathscr{D}(z) dz}{\int_{z_{\min}}^{z_{\max}} \mathscr{D}(z) dz} , \quad (10)$$

where $\mathscr{D}(z)$ is given as

$$\mathscr{D}(z) = \frac{n(z)}{1+z} 4\pi \left(\frac{d_L}{1+z}\right)^2 \frac{d}{dz}\left(\frac{d_L}{1+z}\right) . \quad (11)$$

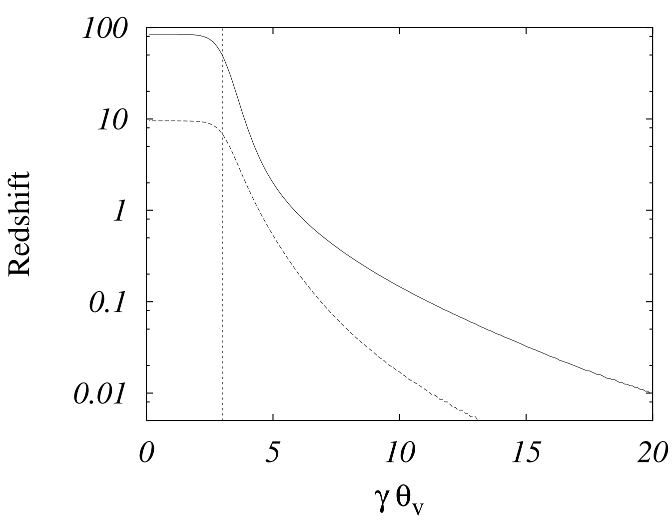

FIG. 1.—Maximum and minimum redshifts of the XRF as a function of the viewing angle $\gamma\theta_v$ are shown in the case of $\Delta\theta = 0.03$. The solid line and dashed line represent the maximum and minimum redshifts, $z_{\max}$ and $z_{\min}$, respectively. The jet emission is observed as the XRF if the source has a redshift z in the range $z_{\min} < z < z_{\max}$ (see text). The vertical dashed line represents $\theta_v = \Delta\theta = 0.03$.

The luminosity distance is given by

$$d_L(z) = \frac{c}{H_0}(1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}} . \quad (12)$$

The function $n(z)$ is the comoving GRB rate density. We assume that $n(z)$ is proportional to the comoving rate densities of the star formation $n_{\rm SF}(z)$ in the following form (Madau & Pozzetti 2000; Porciani & Madau 2001):

$$n_{SF}(z) = 0.46h \frac{\exp(3.4z)}{\exp(3.8z) + 45} \times \sqrt{\Omega_M + \Omega_\Lambda(1+z)^{-3}} \; M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3} . \quad (13)$$

When we consider the case of $z_{\min} < z_{\max} \ll 1$, the cosmological effect can be neglected, so that $\mathscr{D} \propto z^2$ and $C/C_{\min} \sim (z/z_{\max})^{-2}$. Then equation (10) becomes

$$\left\langle \frac{V}{V_{\max}} \right\rangle_{\Delta\theta,\theta_v} = 0.5\left[1 + \left(\frac{z_{\min}}{z_{\max}}\right)^3\right] . \quad (14)$$

Therefore, $z_{\min} \ll z_{\max} \ll 1$ implies $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v} \sim 0.5$.

Next, we integrate equation (10) over θ_v as

$$\left\langle \frac{V}{V_{\max}} \right\rangle_{\Delta\theta} = \frac{\int \langle V/V_{\max}\rangle_{\Delta\theta,\theta_v} W(\theta_v) d\theta_v}{\int W(\theta_v) d\theta_v} , \quad (15)$$

where $W(\theta_v)$ is the weight function that is the product of the solid angle factor and the volume factor:

$$W(\theta_v) = 2\pi \sin\theta_v \int_{z_{\min}}^{z_{\max}} \mathscr{D}(z) dz . \quad (16)$$

The results of the numerical integration are summarized in Table 1. For each $\Delta\theta$, $z_{\max}(\theta_{v,p})$ [$z_{\min}(\theta_{v,p})$] means the maximum (minimum) redshift at which $W(\theta_v)$ takes the maximum value. If we take the jet opening half-angle as $\Delta\theta \lesssim 0.03$, $\langle V/V_{\max}\rangle_{\Delta\theta}$ is smaller than ~ 0.4, which is consistent with the observational result at the 1 σ level.

Let us consider the behavior of $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v}$ in the case of the fixed opening half-angle $\Delta\theta = 0.03$. Recent analysis of the GRB afterglows shows that some GRBs have a jet with a opening half-angle of less than 0.05 radian, i.e., the smallest value of $\Delta\theta$ is about 0.03 (Panaitescu & Kumar 2002). Let us decrease the viewing angle θ_v from a sufficiently large value ($\gamma\theta_v \sim 20$). As shown in Figure 1, both $z_{\max}$ and $z_{\min}$ increase monotonically, since the observed flux from the source increases due to the relativistic beaming effect. However, the behavior of $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v}$ is more complicated since the function $\mathscr{D}(z)$ in equation (11) has a maximum value at $z = z_p \sim 1.5$. We plot $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v}$ as a function of $\gamma\theta_v$ in Figure 2. If θ_v is large enough, $z_{\max}$ is smaller than z_p. Then the cosmological effect is small, so that $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v} \sim 0.5$. While if θ_v is small enough, $z_{\min}$ is larger than z_p. Then $\mathscr{D}(z)$ is a decreasing function in the range $z_{\min} < z < z_{\max}$, and the contribution of XRFs at smaller distance to $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v}$ is larger so that the $\langle V/V_{\max}\rangle_{\Delta\theta,\theta_v}$ is small. We note that the above argument does not depend on $\Delta\theta$ so much.

The behavior of the weight function $W(\theta_v)$ is shown in Figure 3. When θ_v is large enough for $z_{\max}$ to be smaller than z_p or when θ_v is small enough, $W(\theta_v)$ is small since $\mathscr{D}(z)$ or the solid angle factor are relatively small in the range $z_{\min} < z < z_{\max}$. One can see that in the case of $\Delta\theta = 0.03$,

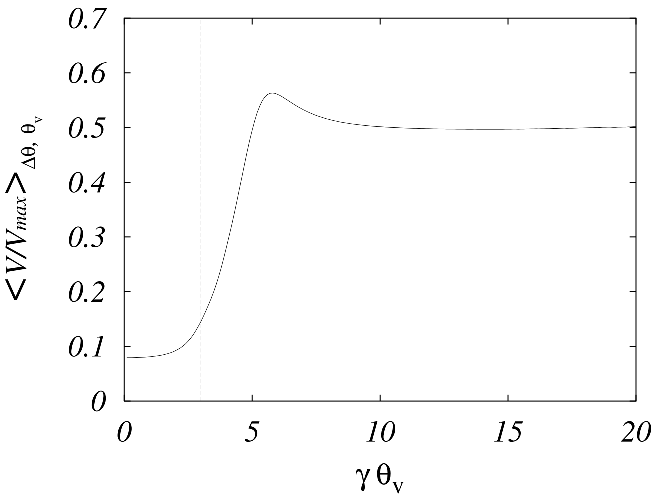

FIG. 2.—$\langle V/V_{\rm max}\rangle_{\Delta\theta,\theta_v}$ for the XRF detected by the WFCs on *BeppoSAX* is shown as a function of the viewing angle $\gamma\theta_v$ in the case of $\Delta\theta = 0.03$.

$W(\theta_v)$ takes the maximum value at $\gamma\theta_{v,p} \sim 4.5$, when $z_{\rm min} \sim z_p$.

The above discussions show that the sources with the viewing angle $\theta_v \sim 0.05$ at $z \sim 1.5$ are the most frequent class of the XRFs in the population for the opening half-angle $\Delta\theta = 0.03$. In other cases in which $\Delta\theta$ is smaller than 0.03, the typical values are $z \sim 1.5$ and $\theta_v \sim \Delta\theta + 0.02$ since $W(\theta_v)$ takes a maximum at $\theta_{v,p} \sim \Delta\theta + 0.02$ (see Table 1).

4. DISCUSSION

We have calculated $\langle V/V_{\rm max}\rangle_{\Delta\theta}$ for the emission from a simple jet model and shown that when the jet opening half-angle $\Delta\theta$ is smaller than about 0.03, $\langle V/V_{\rm max}\rangle_{\Delta\theta}$ for the XRFs detected by WFCs on *BeppoSAX* is smaller than 0.4. The value of $\Delta\theta \sim 0.03$ has been obtained from the fitting of the afterglow light curve (Panaitescu & Kumar 2002; Frail et al. 2001). Such a narrow jet which is inferred by afterglow observations is rare. However, following equation (1) of Frail et al. (2001), the jet break time is given by $t_j \sim 13(\Delta\theta/0.01)^{8/3}$ minutes, and so it requires fast localization to observe the jet break for a narrow jet. Therefore, at present, the small number of GRBs with small $\Delta\theta$ may come from the observational selection effect. In the context of this scenario, we might be able to account for the fact that afterglows of XRFs have been rarely observed since the afterglow at a fixed time gets dimmer for an earlier break time. Furthermore, some "dark GRBs" might be such a small opening angle jet observed with an on-axis viewing angle for the same reason.

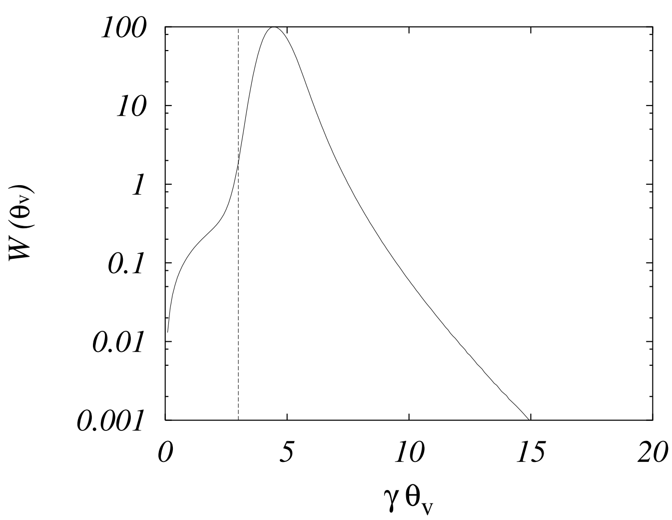

FIG. 3.—Weight function $W(\theta_v)$, which is the relative observed event rate, is shown as a function of the viewing angle $\gamma\theta_v$. Note that the normalization of $W(\theta_v)$ is arbitrary. The vertical dashed line represents $\theta_v = \Delta\theta = 0.03$.

We briefly comment on how the results obtained in this paper will depend on the Lorentz factor of the jet γ. We see that when γ becomes large, $\langle V/V_{\rm max}\rangle_{\Delta\theta}$ becomes small. For example, when we fix $\gamma = 200$, we obtain $\langle V/V_{\rm max}\rangle_{\Delta\theta=0.03} = 0.39$ and $\langle V/V_{\rm max}\rangle_{\Delta\theta=0.1} = 0.43$. This implies that the limitation on $\Delta\theta$ can be relaxed.

We can estimate the typical observed photon energy as $h\nu_{\rm obs} \sim (1+z)^{-1}\delta\nu'_0$, where $\delta^{-1} = \gamma[1-\beta\cos\tilde{\theta}]$ and $\tilde{\theta} = \max\{0,\ \theta_v - \Delta\theta\}$ (Yamazaki et al. 2002). Since $\gamma \gg 1$ and $\theta_v, \Delta\theta \ll 1$, we obtain

$$h\nu_{\rm obs} \sim \frac{2\gamma\nu'_0}{(1+z)[1+(\gamma\tilde{\theta})^2]} . \quad (17)$$

In § 3, we have shown that for fixed $\Delta\theta \lesssim 0.03$, the typical value of θ_v is $\sim\Delta\theta + 0.02$. Therefore, for the adopted parameters $\gamma\nu'_0 = 200$keV and the typical redshift $z = z_p \sim 1.5$, one can derive $h\nu_{\rm obs} \sim 30$ keV, which is the typical observed peak energy of the XRFs (Kippen et al. 2002). We can propose from our argument that the emissions from the jets with a small opening half-angle such as $\Delta\theta \lesssim 0.03$ are observed as XRFs when they are seen from off-axis viewing angle.

If one can detect the afterglow of the XRF, which has a maximum flux at about several hours after the XRF, the fitting of light curve may give us the key information about the jet opening angle (Granot et al. 2002). Therefore, our theoretical model can be tested by the near-future observations.

We can estimate the observed event rate of the XRF for fixed $\Delta\theta$ as

$$R_{{\rm XRF},\Delta\theta} = \frac{1}{4\pi}\int W(\theta_v)d\theta_v . \quad (18)$$

In order to calculate $R_{{\rm XRF},\Delta\theta}$, we consider the proportionality constant $\mathscr{R} = n(z)/n_{\rm SF}(z)$. One can write it approximately as $\mathscr{R} = r_{>8M_\odot}k$, where $r_{>8M_\odot}$ is the number of stars with masses $M > 8\ M_\odot$ per unit mass. Since we assume all stars with masses $M > 8\ M_\odot$ explode as core-collapse supernovae, k represents the ratio of the number of XRF sources to that of core-collapse supernovae. We adopt the value $k = 1\times 10^{-3}$, which is derived from the result of Porciani & Madau (2001) combined with the effect of the solid angle factor $(\Delta\theta)^2/2$. Using a Salpeter initial mass function $\phi(M)$, we obtain $r_{>8M_\odot} = [\int_{8M_\odot}^{125M_\odot}\phi(M)dM]/[\int_0^{125M_\odot}M\phi(M)dM] = 1.2\times 10^{-2}M_\odot^{-1}$. Then, in the case of $\Delta\theta = 0.03$, we derive $R_{{\rm XRF},\Delta\theta} = 1\times 10^2$ events ${\rm yr}^{-1}(\mathscr{R}/1\times 10^{-5}M_\odot^{-1})$, which is comparable to the observed event rate of the XRF (Heise et al. 2001). Note that the value of $R_{{\rm XRF},\Delta\theta}$ remains unchanged within a factor of 2 when we vary $\Delta\theta$ from 0.01 to 0.07.

When the jet opening half-angle $\Delta\theta$ has a distribution $f_{\Delta\theta}$, we integrate $\langle V/V_{\rm max}\rangle_{\Delta\theta}$ and $R_{{\rm XRF},\Delta\theta}$ over the distribution of $\Delta\theta$ as

$$\langle V/V_{\rm max}\rangle = \frac{\int d(\Delta\theta) f_{\Delta\theta} R_{{\rm XRF},\Delta\theta} \langle V/V_{\rm max}\rangle_{\Delta\theta}}{\int d(\Delta\theta) f_{\Delta\theta} R_{{\rm XRF},\Delta\theta}} , \quad (19)$$

$$R_{\rm XRF} = \frac{\int d(\Delta\theta) f_{\Delta\theta} R_{{\rm XRF},\Delta\theta}}{\int d(\Delta\theta) f_{\Delta\theta}} , \quad (20)$$

respectively. We assume a power-law distribution as $f_{\Delta\theta} \propto (\Delta\theta)^{-q}$. When we adopt $q = 4.54$ (Frail et al. 2001) and integrate over $\Delta\theta$ from 0.01 to 0.2 rad, we find $\langle V/V_{\rm max}\rangle = 0.36$ and $R_{\rm XRF} = 1 \times 10^2$ events yr^{-1}. These values mainly depend on the lower bound of the integration. For example, we obtain $\langle V/V_{\rm max}\rangle = 0.43$ and $R_{\rm XRF} = 3$ events yr^{-1} if the integration is done over $\Delta\theta$ from 0.03 to 0.2 rad. (Note that we may let the value of $R_{\rm XRF}$ be consistent with observed value by adjusting $\mathscr{R}$.) Since the statistics of the observational data will increase in the near future owing to instruments such as *HETE 2* and *Swift*, we will be able to say more than above discussion, including a more accurate functional form of $f_{\Delta\theta}$ than that we have considered above, as well as the relation to the GRB event rate.

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REFERENCES

Amati, L. 2002, talk given in 3rd workshop Gamma-Ray Bursts in the Afterglow Era
Band, D., et al. 1993, ApJ, 413, 281
Band, D. L. 2002, preprint (astro-ph/0212452)
Barraud, C., et al. 2003, A&A, 400, 1021
Bloom, J. S., et al. 2003, preprint (astro-ph/0303514)
Chang, H.-Y., & Yi, I. 2001, ApJ, 554, 12
Daigne, F., & Mochkovitch, R. 2003, MNRAS, submitted (astro-ph/0303287)
Dermer, C. D., Chiang, J., & Böttcher, M. 1999, ApJ, 513, 656
Frail, D., A., et al. 2001, ApJ, 562, L55
Granot, J., Panaitescu, A., Kumar, P., & Woosley, S. E. 2002, ApJ, 570, L61
Granot, J., Piran, T., & Sari, R. 1999, ApJ, 513, 679
Heise, J. 2000, talk given in 2nd workshop Gamma-Ray Bursts in the Afterglow Era
———. 2002, talk given in 3rd workshop Gamma-Ray Bursts in the Afterglow Era
Heise, J., in 't Zand, J., Kippen, R. M., & Woods, P. M. 2001, in Proc. Second Rome Workshop: Gamma-Ray Bursts in the Afterglow Era, ed. E. Costa, F. Frontera, & J. Hjorth (Berlin: Springer), 16
Huang, Y. F., Dai, Z. G., & Lu, T. 2002, MNRAS, 332, 735
Ioka, K., & Nakamura, T. 2001, ApJ, 554, L163
Kawai, N. 2002, talk given in 3rd workshop Gamma-Ray Bursts in the Afterglow Era
Kim, C., Chang, H.-Y., & Yi, I. 2001, ApJ, 548, 532
Kippen, R. M., et al. 2002, in Proc. Woods Hole Gamma-Ray Burst Workshop (astro-ph/0203114)
Madau. P., & Pozzetti, L. 2000, MNRAS, 312, L9
Mao, S., & Paczyński, B. 1992, ApJ, 388, L45
Mészáros, P., Ramirez-Ruiz, E., Rees, M. J., & Zhang, B. 2002, ApJ, 578, 812
Mochkovitch, R., Daigne, F., Barraud, C., & Atteia, J. L. 2003, preprint (astro-ph/0303289)
Panaitescu, A., & Kumar, P. 2002, ApJ, 571, 779
Piran, T. 1992, ApJ, 389, L45
Porciani, C., & Madau, P. 2001, ApJ, 548, 522
Preece, R. D., Briggs, M. S., Mallozzi, R. S., Pendleton, G. N., Paciesas, W. S., & Band, D. L. 2000, ApJS, 126, 19
Ramirez-Ruiz, E., & Lloyd-Ronning, N. M. 2002, NewA, 7, 197
Rossi, E., Lazzati, D., & Rees, M. J. 2002, MNRAS, 332, 945
Schmidt, M., Higdon, J. C., & Hueter, G. 1988, ApJ, 329, L85
Woods, E., & Loeb, A. 1999, ApJ, 523, 187
Woosley, S. E., Zhang, W., & Heger, A. 2002, preprint (astro-ph/0206004)
Yamazaki, R., Ioka, K., & Nakamura, T. 2002, ApJ, 571, L31
———. 2003, ApJ, 591, 283
Zhang, B., & Mészáros, P. 2002a, ApJ, 571, 876
———. 2002b, preprint (astro-ph/0206158)