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Black holes in three-dimensional Einstein-Born-Infeld-dilaton theory

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The three-dimensional static and circularly symmetric solution of the Einstein-Born-Infeld-dilaton system is derived. The solutions corresponding to low energy string theory are investigated in detail, which include black hole solutions if the cosmological constant is negative and the mass parameter exceeds a certain critical value. Some differences between the Born-Infeld nonlinear electrodynamics and the Maxwell electrodynamics are revealed.

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I. INTRODUCTION

The black hole is one of the fundamental objects in gravity theories. What we shall consider here is a black hole in the Einstein-Born-Infeld-dilaton theory motivated by string theory. We have a clear picture of an astrophysical black hole as a final product of the gravitational collapse owing to the uniqueness theorem of the stationary asymptotically flat black hole solution. On the other hand, there are various kinds of black objects in string theory since there emerge many matter fields as in the present case. It seems hard to reveal the general properties of such stringy black holes. We want to approach this problem.

The Born-Infeld nonlinear electrodynamics has attracted much attention in the context of string theory. It was originally introduced for the purpose of solving various problems of divergence appearing in the Maxwell theory [1]. It has later been shown that the Born-Infeld theory naturally arises in the low energy limit of the open string theory [2,3].

In the open and the closed bosonic string theory, there are four massless states of strings: the dilaton field ϕ, the $U(1)$ gauge field A_i, the gravitational field g_{ij}, and the Kolb-Ramond antisymmetric tensor field B_{ij}. When one considers the Born-Infeld electrodynamics motivated by the string theory, he might include other massless fields together with the electromagnetic field. In this paper, we however neglect B_{ij} for simplicity. Then, the Einstein-Born-Infeld-dilaton system describes interactions among massless fields ϕ, A_i, and g_{ij}.

Recently, numerical studies of the Einstein-Born-Infeld-dilaton system have been done in four-dimensional static and spherically symmetric space-time [4–6]. Here we turn to the three-dimensional case, and show that the analytic solution describing the black hole can be obtained. The three-dimensional gravity is a simple version of general relativity and gives one useful approach to the black hole physics [7–9]. Three-dimensional black holes also arise in some higher dimensional theories [10]. We may anyway get an insight into the property of the Born-Infeld theory by investigating the three-dimensional black hole as the simplest example.

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This paper is organized as follows. In Sec. II, we derive basic equations of the Einstein-Born-Infeld-dilaton system for general dilaton couplings, and give an analytic solution under appropriate assumptions. In Sec. III, we analyze in detail the solution to the low energy string theory. We especially focus on the effects of the nonlinearity of the Born-Infeld field, where some differences between the Maxwell and the Born-Infeld fields are revealed. In Sec. IV, we summarize the results.

II. BASIC EQUATIONS AND SOLUTIONS

We first consider the general form of the Lagrangian which describes nonlinear electrodynamics. We adopt the action which is written in string frame as

$$S=\int \mathrm{d}^3x\sqrt{-g}e^{-2\phi}(R-2\Lambda+4(\partial\phi)^2+\mathcal{L}[F^2]), \tag{1}$$

where $F^2=F_{ij}F^{ij}$ and $\mathcal{L}$ is its functional. The action in Einstein frame is given via the conformal transformation $g_{ij}\mapsto e^{4\phi}g_{ij}$ by

$$S=\int \mathrm{d}^3x\sqrt{-g}(R-2e^{4\phi}\Lambda-4(\partial\phi)^2+e^{4\phi}\mathcal{L}[e^{-8\phi}F^2]). \tag{2}$$

We consider an action with general dilaton coupling constants α, β and $\gamma>0$:

$$S=\int \mathrm{d}^3x\sqrt{-g}(R-2e^{\alpha\phi}\Lambda-\gamma(\partial\phi)^2+e^{\beta\phi}\mathcal{L}[e^{-2\beta\phi}F^2]). \tag{3}$$

The equations of motion of the dilaton field ϕ and the electromagnetic field F are

$$2\gamma\phi_{,i}{}^{;i}-2\alpha\Lambda e^{\alpha\phi}+\beta e^{\beta\phi}\mathcal{L}-2\beta e^{-\beta\phi}F^2\dot{\mathcal{L}}=0, \tag{4}$$

$$(e^{-\beta\phi}\dot{\mathcal{L}}F^{ij})_{;j}=0, \tag{5}$$

respectively, where $\dot{\mathcal{L}}[x]=\delta\mathcal{L}/\delta x$. The Einstein equation is

$$R_{ij}=4\phi_{,i}\phi_{,j}+(2e^{\alpha\phi}\Lambda-e^{\beta\phi}\mathcal{L})g_{ij} -2e^{-\beta\phi}\dot{\mathcal{L}}(F_{ik}F_j{}^k-F^2g_{ij}). \tag{6}$$

We consider the Born-Infeld nonlinear electrodynamics, which corresponds to

$$\mathcal{L}[x]=4b^2\left[1-\left(1+\frac{x}{2b^2}\right)^{1/2}\right], \tag{7}$$

where $b>0$ is the Born-Infeld parameter. In the limit of large b, this gives Maxwell Lagrangian, $\mathcal{L}\to -x$.

The three-dimensional static and circularly symmetric space-time can be written in the form

$$g=-f(r)\mathrm{d}t^2+\frac{e^{2\delta(r)}}{f(r)}\mathrm{d}r^2+r^2\mathrm{d}\varphi^2. \tag{8}$$

The electromagnetic field is assumed to have the following pure electrostatic form:

$$F=e^{\beta\phi(r)+\delta(r)}E(r)\mathrm{d}t\wedge\mathrm{d}r. \tag{9}$$

Then, Eq. (5) becomes

$$\left(\frac{rE}{\sqrt{1-E^2/b^2}}\right)'=0, \tag{10}$$

where prime denotes derivative with respect to r. This can be integrated to give

$$E=h^{-1/2}\frac{Q}{r}, \tag{11}$$

where

$$h=1+\frac{Q^2}{b^2r^2}, \tag{12}$$

and Q is an integration constant. The constant Q is the electric charge, namely

$$Q=-\frac{1}{4\pi}\oint_\Gamma e^{-\beta\phi}\dot{\mathcal{L}}{*}F, \tag{13}$$

for any smooth closed spacelike curve Γ enclosing $r=0$. The equation of motion of the dilaton field (4) becomes

$$\frac{\gamma e^{-\delta}}{r}(rfe^{-\delta}\phi')'-\alpha\Lambda e^{\alpha\phi}+2\beta b^2e^{\beta\phi}(1-h^{1/2})=0. \tag{14}$$

The Einstein equations are

$$\left(f''+\frac{f'}{r}-f'\delta'\right)e^{-2\delta}=-4\Lambda e^{\alpha\phi}+8b^2e^{\beta\phi}\times\left(1-h^{1/2}+\frac{Q^2}{2b^2r^2}h^{-1/2}\right), \tag{15}$$

$$\delta'=\gamma r(\phi')^2, \tag{16}$$

$$\frac{f'-f\delta'}{r}e^{-2\delta}=-2\Lambda e^{\alpha\phi}+4b^2e^{\beta\phi}(1-h^{1/2}). \tag{17}$$

Here, we assume the following form of the metric function δ:

$$\delta=n\ln\left(\frac{r}{r_0}\right)\quad\text{for}\ \ r>0, \tag{18}$$

where n and $r_0>0$ are constants. From Eq. (16), it turns out that n must be positive, and then the dilaton field ϕ becomes

$$\phi=\pm\left(\frac{n}{\gamma}\right)^{1/2}\ln\left(\frac{r}{r_1}\right), \tag{19}$$

with a positive constant r_1. Then, Eqs. (14) and (17) become

$$\left(\frac{f}{r^n}\right)'=\pm\frac{r^{n+1}}{(\gamma n)^{1/2}{r_0}^{2n}}\left[\alpha\Lambda\left(\frac{r}{r_1}\right)^{\pm\alpha\sqrt{n/\gamma}}-2\beta b^2\left(\frac{r}{r_1}\right)^{\pm\beta\sqrt{n/\gamma}}(1-h^{1/2})\right] \tag{20}$$

and

$$\left(\frac{f}{r^n}\right)'=\frac{r^{n+1}}{{r_0}^{2n}}\left[-2\Lambda\left(\frac{r}{r_1}\right)^{\pm\alpha\sqrt{n/\gamma}}+4b^2\left(\frac{r}{r_1}\right)^{\pm\beta\sqrt{n/\gamma}}(1-h^{1/2})\right], \tag{21}$$

respectively. These two equations (20) and (21) are consistent if and only if

$$\alpha=\beta=\mp\sqrt{4n\gamma}. \tag{22}$$

Equation (22) includes the string case ($n=1,\alpha=\beta=\gamma=4$). When the condition (22) is satisfied, Eqs. (20) and (21) become

$$\left(\frac{f}{r^n}\right)'=\left(\frac{r_1}{r_0}\right)^{2n}[(4b^2-2\Lambda)-4b^2h^{1/2}]r^{1-n}. \tag{23}$$

The solutions of Eq. (23) are

$$f=\left(\frac{r_1}{r_0}\right)^{2n}\left[\frac{4b^2-2\Lambda}{2-n}r^2-4b^2r^n\int^r r^{1-n}\left(1+\frac{Q^2}{b^2r^2}\right)^{1/2}\mathrm{d}r\right], \tag{24}$$

for $n\neq 2$, and

$$f=\left(\frac{r_1}{r_0}\right)^4\left[(4b^2-2\Lambda)r^2\ln\left(\frac{r}{r_2}\right)-4b^2r^2\times\int^r r^{-1}\left(1+\frac{Q^2}{b^2r^2}\right)^{1/2}\mathrm{d}r\right], \tag{25}$$

for $n=2$.

TABLE I. Classification of solutions of the three-dimensional Einstein equation. Abbreviations E, BI, M, D and Λ stand for Einstein, Born-Infeld, Maxwell, dilaton and cosmological constant, respectively.

	b	Λ	$\alpha,\ \beta,\ \gamma$	n	M	Q	System
Eqs. (24),(25)			$\alpha=\beta=\mp 2\sqrt{n\gamma}$				EBIDΛ
Eq. (30)			$\alpha=\beta=\gamma=4$	1			EBIDΛ (String)
Cataldo and García (1999)			$\alpha=\beta=0$	0			EBIΛ
Chan and Mann (1994)	$+\infty$		$\alpha=\beta=\mp 2\sqrt{n\gamma}$,	$n\neq 2$			EMDΛ
McGuigan *et al.* (1992)	$+\infty$		$\alpha=\beta=\gamma=4$	1			EMDΛ (String)
Mandal *et al.* (1991)	$+\infty$		$\alpha=\beta=\gamma=4$	1		0	EDΛ (String)
Sa *et al.* (1996)	$+\infty$	<0	$\alpha=\beta=\mathrm{n}\gamma=4$,	$n\neq 1,2$		0	EDΛ
Sa *et al.* (1996)	$+\infty$	<0	$\alpha=\beta=4,\gamma=2$	2	0	0	EDΛ
Barrow *et al.* (1986)	$+\infty$	<0	$\alpha=-\sqrt{2},\gamma=1$	1/2	0	0	EDΛ
Bañados *et al.* (1992)	$+\infty$		$\alpha=\beta=0$	0			EMΛ
Barrow *et al.* (1986)	$+\infty$	0	$\gamma=1$	$\neq 0$		0	ED
Deser and Mazur (1985)	$+\infty$	0	$\alpha=\beta=0$	0			EM
Gott III *et al.* (1986)	$+\infty$	0	$\alpha=\beta=0$	0	0		EM
Bañados *et al.* (1993)	$+\infty$		$\alpha=\beta=0$	0		0	EΛ

The above solutions include known solutions as special cases. When $n=0$, the dilaton field vanishes, and then Eq. (24) becomes

$$f=-M-(\Lambda-2b^2)r^2-2b^2r\sqrt{r^2+Q^2/b^2}-2Q^2\ln(r+\sqrt{r^2+Q^2/b^2}), \tag{26}$$

which has been given by Cataldo and García [11]. In the limit of $b\to\infty$, the Born-Infeld field reduces just to the Maxwell field. Then Eq. (24) gives

$$f=\left(\frac{r_1}{r_0}\right)^{2n}\left[-Ar^n-\frac{2\Lambda}{2-n}r^2+\frac{2Q^2}{n}\right], \tag{27}$$

for $n\neq 0,2$,

$$f=\left(\frac{r_1}{r_0}\right)^{4}\left[-Ar^2-2\Lambda r^2\ln\left(\frac{r}{r_2}\right)+Q^2\right], \tag{28}$$

for $n=2$, and

$$f=-M-\Lambda r^2-2Q^2\ln\left(\frac{r}{r_3}\right), \tag{29}$$

for $n=0$. Equation (27) coincides with the solution obtained by Chan and Mann [12], and Eq. (29) corresponds to the charged Bañados-Teitelboim-Zanelli (BTZ) solution [8]. Other solutions can be obtained by taking special values of parameters in Eqs. (24) or (25). Table I shows the classification of solutions found previously [8,9,11–18].

III. SOLUTIONS TO THE STRING ACTION

The case $n=1$, $\alpha=\beta=\gamma=4$ is particularly important, since the solution with $n=1$ can be converted into the solution for the string frame action via the conformal transformation $g_{ij}\mapsto e^{-4\phi}g_{ij}$. We shall investigate this case in detail. In this case, Eq. (24) is expressed in terms of elementary functions as

$$f(r)=\left(\frac{r}{r_0}\right)\left\{-M-\frac{2\Lambda {r_1}^2}{r_0}r+\frac{4b^2{r_1}^2}{r_0}\left[1-\left(1+\frac{Q^2}{b^2r^2}\right)^{1/2}\right]r+\frac{4b|Q|{r_1}^2}{r_0}\ln\left[\left(1+\frac{Q^2}{b^2r^2}\right)^{1/2}+\frac{|Q|}{br}\right]\right\}. \tag{30}$$

The integration constant M can be regarded as the total mass of the system. In fact, according to Brown and York [19,20]. The (quasi-local) mass function $M(r)$ within the circle of radius r becomes

$$M(r)=2f^{1/2}(f_0^{1/2}-f^{1/2})e^{-\delta}, \tag{31}$$

where $f_0=f_0(r)$ is a function defining the zero of mass corresponding to some background space-time. It will be natural to choose

$$f_0(r)=-\frac{2\Lambda {r_1}^2}{{r_0}^2}r^2, \tag{32}$$

which implies that the Born-Infeld field vanishes in the background space-time. Then, M is the total mass in the sense $M=\lim_{r\to+\infty}M(r)$.

In the following, we analyze the curvature singularity, the causal structure and propagation of light front, where we show some differences between the Born-Infeld and the Maxwell electrodynamics.

A. Curvature singularity

There is a curvature singularity at the center $\{r=0\}$, where the scalar curvature and the energy density diverge. The asymptotic behavior near $\{r=0\}$ of these quantities are shown in Table II. In addition, the cases of the Einstein-Maxwell-dilaton-Λ system [12], the Einstein-Maxwell-Λ

TABLE II. Behavior of the scalar curvature and the energy densities near the central singularity (Einstein frame).

System	R	ρ_{EM}	ρ_D	ρ_Λ
EBID$(Q\neq 0)$	$r^{-3}\ln r$	r^{-3}	$r^{-3}\ln r$	r^{-2}
EMD$(Q\neq 0)$	r^{-4}	r^{-4}	r^{-4}	r^{-2}
EBI$(Q\neq 0)$	r^{-1}	r^{-1}	0	r^0
charged BTZ	r^{-2}	r^{-2}	0	r^0
ED$(Q=0)$	r^{-3}	0	r^{-3}	r^{-2}

system (charged BTZ) [8], the Einstein-Born-Infeld-Λ system [11], and the Einstein-dilaton-Λ system $(Q=0)$ are also shown for comparison. It can be seen that nonlinearity of electrodynamics weakens the divergence of the curvature scalars at least in the case of the dilaton-coupled systems. The curvature scalars also diverge in the string frame. Table III shows the behavior in the string frame.

B. Causal structure

A null hypersurface on which the horizon function $f(r)$ vanishes is a Killing horizon. The behavior of the function $f(r)$ determines the causal structure of the space-time. Introduce the following dimensionless quantities $(Q\neq 0)$:

$$x=\frac{b}{|Q|}r,\quad q=\frac{2r_1Q}{r_0},\quad m=\frac{r_0M}{4br_1^2|Q|},\quad \lambda=\frac{\Lambda}{2b^2}. \tag{33}$$

Then, Eq. (30) can be rewritten as

$$F:=f/q^2=x\left\{-m+(1-\lambda)x-(1+x^2)^{1/2}+\ln\left[\frac{1+(1+x^2)^{1/2}}{x}\right]\right\}. \tag{34}$$

It can be seen that the causal structure is determined by essentially two parameters m and λ.

For any m and λ, the behavior of F near $x=0$ is

$$F\sim -x\ln x(>0)(x\to +0). \tag{35}$$

Using Eq. (35), we can see that the conformal radial coordinate $r_*:=\int^r dr\sqrt{-g_{rr}/g_{tt}}$ remains finite as $x\to +0$, which implies the curvature singularity $\{r=0\}$ is timelike. On the other hand, it can be seen that $r_*\to +\infty$ as $x\to +\infty$ for any m and λ. This implies that spatial infinity $\{r=\infty\}$ is a null hypersurface.

TABLE III. Same as in Table II, but in the string frame.

System	R	ρ_{EM}	ρ_D	ρ_Λ
EBID$(Q\neq 0)$	$r^{-1}\ln r$	r^0	$r^{-1}\ln r$	r^0
EMD$(Q\neq 0)$	r^{-2}	r^0	r^{-2}	r^0
ED$(Q=0)$	r^{-1}	0	r^{-1}	r^0

The behavior of F at spatial infinity depends on m and λ. The causal structure is classified into several cases.

(i) $\lambda>0$. In this case, $F\sim -\lambda x^2$ holds asymptotically as $x\to\infty$ for any m. Furthermore, $F=0$ has a simple positive root. This implies that there exists a cosmological horizon.

(ii) $\lambda=0$. The causal structure depends on m. If m is negative, there exists a cosmological horizon, and the causal structure is similar to the $\lambda>0$ case. When m is positive or zero, F is strictly positive for any $x>0$. Therefore, the central singularity is naked.

(iii) $\lambda<0$. The causal structure depends on m. The behavior of F at spatial infinity is same as the $\lambda>0$ case except for the signature. When m satisfies

$$m=m_*(\lambda):=\ln(1-\lambda+\sqrt{\lambda(\lambda-2)}), \tag{36}$$

the equation $F=0$ has a multiple root. Whether the central singularity is covered by the black hole horizon depends on the value of m;

(iii-a) $m<m_*$. In this case, $F>0$ for any $x>0$. This implies that the central singularity is naked.

(iii-b) $m=m_*$. This is the extreme case. The black hole horizon is degenerated, and the causal structure is similar to the four-dimensional extremal Reissner-Nordström space-time. The horizon radius becomes $x=[\lambda(\lambda-2)]^{-1/2}$.

(iii-c) $m>m_*$. $F=0$ has two distinct positive roots. These correspond the inner Cauchy horizon and the black hole horizon, respectively. The causal structure is similar to the four-dimensional nonextremal Reissner-Nordström space-time.

Here we show the effect of the nonlinear electrodynamics. We rewrite Eq. (36) in terms of the original physical quantities. Black hole horizons exist if and only if $\Lambda<0$ and

$$\frac{|Q|}{M}\leqslant\left(\frac{|Q|}{M}\right)_*:=\frac{r_0}{4r_1^2b}\left(\ln\left[1-\frac{\Lambda}{2b^2}+\sqrt{\frac{\Lambda}{2b^2}\left(\frac{\Lambda}{2b^2}-2\right)}\right]\right)^{-1}. \tag{37}$$

To compare with the Einstein-Maxwell-dilaton system [12], we take the Maxwell limit with fixing the other parameters:

$$\left(\frac{|Q|}{M}\right)_*\to\frac{r_0}{4r_1^2}|\Lambda|^{-1/2}(b\to\infty). \tag{38}$$

For finite value of b, $(|Q|/M)_*$ is always larger than this limit value. Therefore, there is a certain set of parameters M and Q for which black hole horizons exist in the Einstein-Born-Infeld-dilaton system but do not exist in the Einstein-Maxwell-dilaton system.

C. Propagation of light front

Finally, we see the propagation of light front Σ, which is a boundary of the region of disturbed electromagnetic field. The motion of this characteristic surface Σ can be investigated by the method similar to [21]. The electromagnetic field is discontinuous in the following sense:

$$[F_{ij}]_\Sigma = 0, \quad [\nabla_k F_{ij}]_\Sigma = f_{ij} k_k, \tag{39}$$

where k_i is a vector normal to Σ. The dilaton field ϕ and its first derivative are continuous at Σ:

$$[\phi]_\Sigma = 0, \quad [\nabla_i \phi]_\Sigma = 0. \tag{40}$$

We apply above conditions to the equation of motion (5). After short calculation, we find that k_i is not tangent to null geodesics of the background space-time metric g^{ij} but of the effective metric

$$g^{ij}_{(eff)} := g^{ij} + 4e^{-2\beta\phi} \frac{\ddot{\mathcal{L}}}{\dot{\mathcal{L}}} F^i{}_k F^{jk}. \tag{41}$$

Using Eqs. (7), (8), (9), and (11), the effective metric can be rewritten as[1]

$$g_{(eff)} = h^{-1}\left(-f dt^2 + \frac{e^{2\delta}}{f} dr^2\right) + r^2 d\varphi^2. \tag{42}$$

A generator of the null geodesic of the effective metric (42) is written as $k^i = \mathrm{d}x^i/\mathrm{d}\tau$. Then, k^i satisfies following equations:

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 + V(r) = 0,$$

$$V(r) := \frac{e^{-2\delta}}{h^2}\left(\frac{hf}{r^2} L^2 - E^2\right), \tag{43}$$

where E and L are conserved quantities defined by

$$E = h^{-1} f \frac{\mathrm{d}t}{\mathrm{d}\tau}, \quad L = r^2 \frac{\mathrm{d}\varphi}{\mathrm{d}\tau}. \tag{44}$$

The light front can exist in the region $V<0$. In the following, we consider the case $n=1$. The behavior of potential V is determined by three parameters m, λ and

$$w := \left(\frac{r_0}{2br_1}\right)^2 \left(\frac{E}{L}\right)^2. \tag{45}$$

Figure 1 shows the region $V<0$. The background parameters m and λ are fixed and satisfy $\lambda<0$ and $m>m_*$, so that the black hole horizon and the inner Cauchy horizon exist. For comparison, we also show a same calculation in the case of the Maxwell limit. When $\mathcal{L}=-x$, the second term on the right hand side of Eq. (41) vanishes. Therefore, k_i is tangent to null geodesics of the background space-time, which is well-known result. Inserting $h=1$ and Eq. (27) with $n=1$ into Eq. (43), we find that the potential function V is determined by the following three parameters:

[1]Covariant components of the effective metric are defined as $g^{(eff)}_{il} g^{lj}_{(eff)} = \delta^j_i$. See Ref. [21].

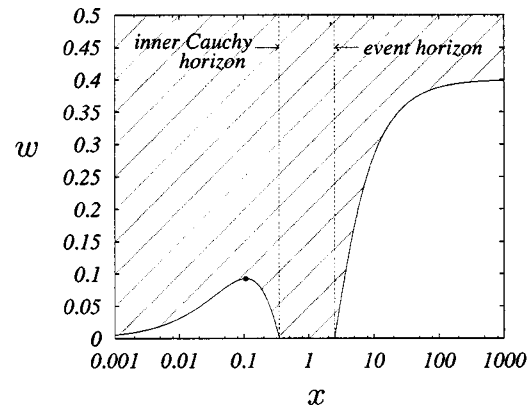

FIG. 1. Allowed region of light fronts (shaded region) is shown in the $x-w$ plane for a fixed background space-time in the case of Einstein-Born-Infeld-dilaton system ($\lambda=-0.4$, $m=1.2$). The filled circle corresponds to the unstable circular orbit.

$$\tilde{m} = \frac{r_0 M}{2r_1 Q^2}, \quad \tilde{\lambda} = \frac{r_1^2}{Q^2}\Lambda, \quad \tilde{w} = \frac{1}{2}\left(\frac{r_0}{Q}\right)^2\left(\frac{E}{L}\right)^2. \tag{46}$$

Figure 2 shows the region $V<0$ with fixed background parameters $\tilde{m}$ and $\tilde{\lambda}$ for which no Killing horizons exist.

We can see some differences between the Born-Infeld and the Maxwell electrodynamics. Let us consider the light fronts coming from infinity to the center $\{r=0\}$. They can arrive at the center in the Born-Infeld case, while they are scattered in the Maxwell case if $L\neq 0$. Furthermore, the unstable circular orbit exist only in the Born-Infeld case. These differences between the Born-Infeld and the Maxwell electrodynamics can also be seen when the naked singularity or the cosmological horizon exist.

IV. CONCLUSION

We have given a seven parameter family of static, circularly symmetric, and pure electrically charged solutions of the three-dimensional Einstein-Born-Infeld-dilaton system. The electric field remains finite near the central singularity, which shows a nonlinear effect of the Born-Infeld field. The

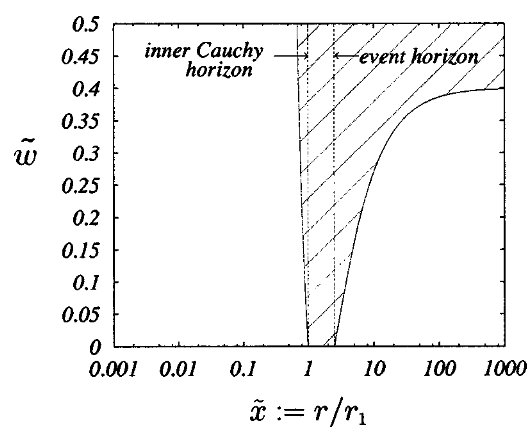

FIG. 2. Same as in Fig. 1, but in the case of Einstein-Maxwell-dilaton system ($\tilde{\lambda}=-0.4$, $\tilde{m}=1.4$).

solution reduces to known black hole solutions of various theories by appropriately taking limits of parameters.

We have studied the low energy string case ($n=1$) in detail. The solution is essentially described by the mass parameter m and the cosmological constant parameter λ. The critical value of mass parameter m_* exists for given λ, such that the black hole horizons exist if and only if $\lambda<0$ and $m\geqslant m_*$ [22]. The causal structure of charged black hole solutions ($\lambda<0$ case) is similar to those of the four-dimensional Reissner-Nordström solution. The spatial infinity is not a timelike but a null hypersurface. This is an effect of the dilaton field [12].

We have revealed some differences between the Born-Infeld nonlinear electrodynamics and the ordinary Maxwell electrodynamics. (i) The curvature scalar and the energy density diverge as $r\to 0$ in both the Einstein frame and the string frame. In the dilaton-coupled system, the divergence of curvature scalars is weaker in the Born-Infeld case than in the Maxwell case. (ii) The lower bound of mass for which black hole horizons exist is lower in the Born-Infeld case, so that it is easier to form a black hole horizon in the Born-Infeld case in the sense that the parameter region corresponding to the black holes is wider. (iii) We have considered the light fronts coming from infinity towards the center with nonzero impact parameter. The light fronts are always scattered in the Maxwell electrodynamics, while they can reach the center in the Born-Infeld electrodynamics.

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