Thick domain walls intersecting a black hole

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We discuss the gravitationally interacting system of a thick domain wall and a black hole. We numerically solve the scalar field equation in the Schwarzschild space-time and show that there exist scalar field configurations representing thick domain walls intersecting the black hole.

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I. INTRODUCTION

Topological defects arise during spontaneous symmetry breaking associated with phase transitions, and cosmological evolution of them is considered to have played an important role in cosmology (see, e.g., [1]). Topological defects produced in the early universe might give us some information on high energy phenomena, which cannot be reached by accelerator experiments, and those produced at the late time phase transitions [2] also have attracted attention as a potential source of the cosmic structures. To study the topological defects is therefore crucially important in cosmology and elementary particle physics.

In general relativity, topological defects have interesting features. Topological defects are extended and relativistic objects due to their large tension. In particular, the gravitational field produced by an infinitely thin domain wall shows a repulsive nature [3,4]. On the other hand, although there are many studies about the properties of thick domain walls in the flat and de Sitter spacetime backgrounds [1], little is known about the existence of thick wall configurations on an inhomogeneous, strongly curved background such as a black hole space-time. Thus it is intriguing to study the gravitational interaction between two extended relativistic objects: a topological defect and a black hole.

In most studies of a defect–black-hole system, topological defects have been approximately treated as infinitely thin and nongravitating objects whose dynamics are governed by Nambu-Goto action. Within this context, the scattering problem of a Nambu-Goto string by a background black hole has been studied in detail [5–9]. In the domain wall case, recently Christensen, Frolov, and Larsen [10,11] considered Nambu-Goto walls embedded in the Schwarzschild black hole space-time and found the static axisymmetric solutions. They showed that there exist a family of infinitely thin walls which intersect the black hole event horizon. However, little attention has been given to the system of a thick defect interacting with a black hole although a defect as a topologically stable configuration of a scalar field has a finite thickness.

Now we shall consider the validity of a thin-wall approximation in the system of a topological defect and a black hole. In such a system, there are two characteristic scales: the thickness w of the defect and the black hole radius R_g. In the case of the system of an astrophysical black hole with the mass $\sim M_\odot$ and a defect formed during a grand unified theory phase transition, the thickness of the defect is much smaller than the black hole radius and therefore thin-wall approximation would be valid. However, it is not so hard to consider the situation where the thickness of defects cannot be negligible as compared with the size of a small black hole. Over the last few decades, many people have studied the formation of small black holes called primordial black holes (PBH's) and their cosmological implications. For example, studying the contribution of PBH's to cosmic rays enables one to place limits on the spectrum of density fluctuations in the early universe (see e.g., [12–14]). On the other hand, the possibility of thick defects and their roles in cosmology, has been discussed e.g., as a source of large-scale structure in the universe [2], or as a candidate for some kind of dark matter [15]. For example, it is thought that the typical mass of PBH's which evaporate at the present epoch is 10^{15} g, so $R_g \sim 10^{-13}$ cm. When one considers the topological defects formed during a phase transition at $\lesssim 100$ MeV, such defects become thicker than the size of PBH's and thin-wall approximation is no longer valid.

In this paper, we investigate the gravitational interaction between a domain wall and a black hole taking the thickness of the wall into account. We deal with scalar fields in the Schwarzschild black hole space-time with ϕ^4 and sine-Gordon potentials, which have a discrete set of degenerate minima. We explicitly show that static axisymmetric thick domain walls intersecting the black hole do exist by numerical investigation. We consider a nongravitating domain wall for simplicity. This test wall assumption might be valid when the symmetry braking scale of the scalar field is much lower than Planck scale as will be shown later by dimensional analysis.

This paper is organized as follows. In Sec. II, we derive the basic equation and discuss the boundary conditions which represent the situation we want to study. In Sec. III, we show the numerical result. We summarize our work in Sec. IV. We also discuss the validity of the assumption that

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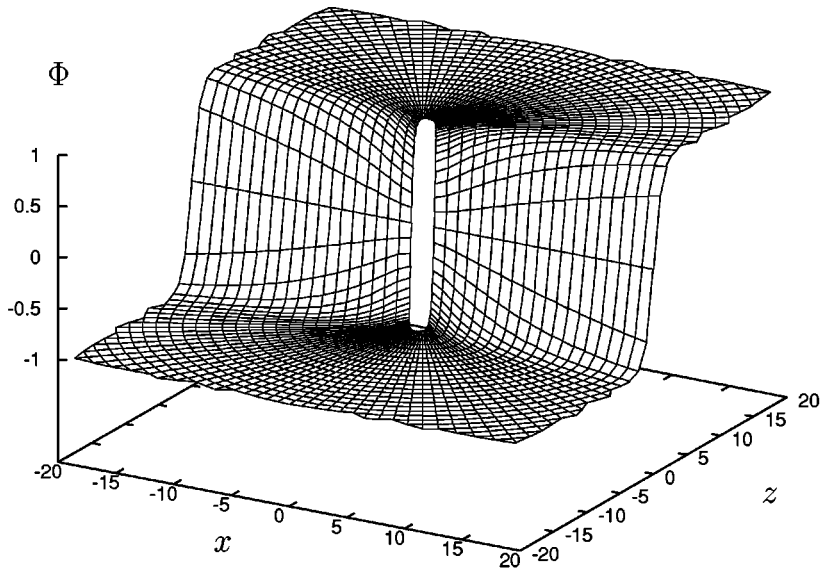

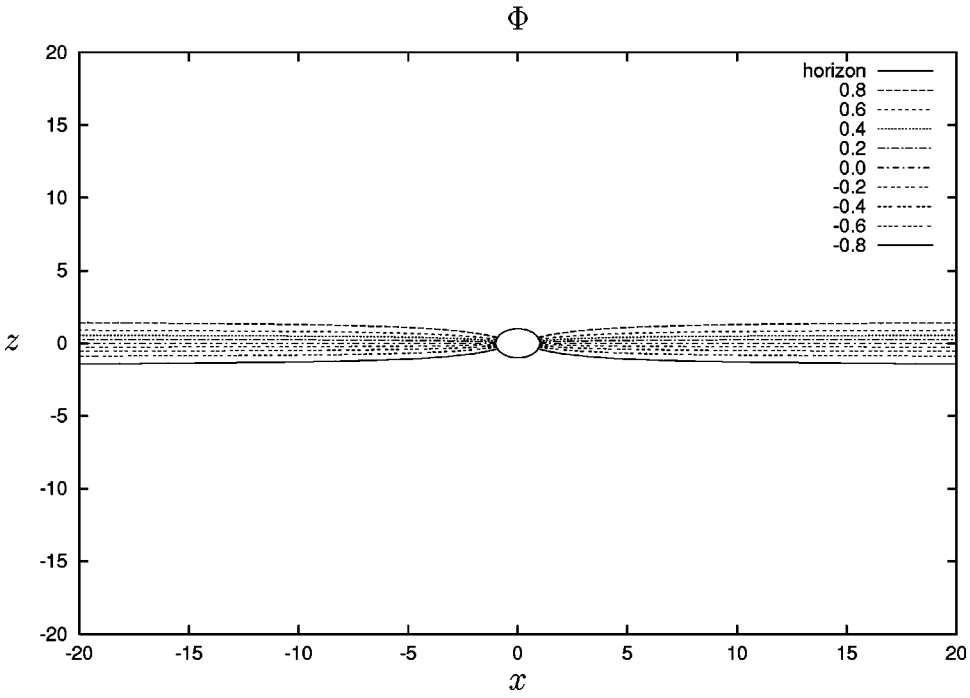

FIG. 1. Numerical solution of Eq. (14) with the ϕ^4 potential for $\epsilon=1$. This solution expresses a domain wall configuration whose thickness is comparable to the Schwarzschild radius.

the effects of gravity of the domain wall can be ignored near the black hole horizon. Throughout this paper, we use units such that $c=\hbar=G=1$ unless otherwise stated.

II. THE BASIC EQUATION AND THE BOUNDARY CONDITIONS

We consider a static thick domain wall in a black hole space-time. The domain wall is constructed by a scalar field with self-interaction in a given curved space-time. In what follows, we neglect the self-gravity of the scalar field, as will be justified later. As a background space-time, we consider the Schwarzschild black hole

$$g=-\left(1-\frac{2M}{R}\right)dt^2+\left(1-\frac{2M}{R}\right)^{-1}dR^2 +R^2(d\vartheta^2+\sin^2\vartheta\, d\varphi^2). \tag{1}$$

For our purpose, we find that it is more convenient to work in the isotropic coordinates $\{t,r,\vartheta,\varphi\}$, where the new radial coordinate r is defined by

$$R=r\left(1+\frac{M}{2r}\right)^2. \tag{2}$$

We mainly consider the region outside the event horizon in

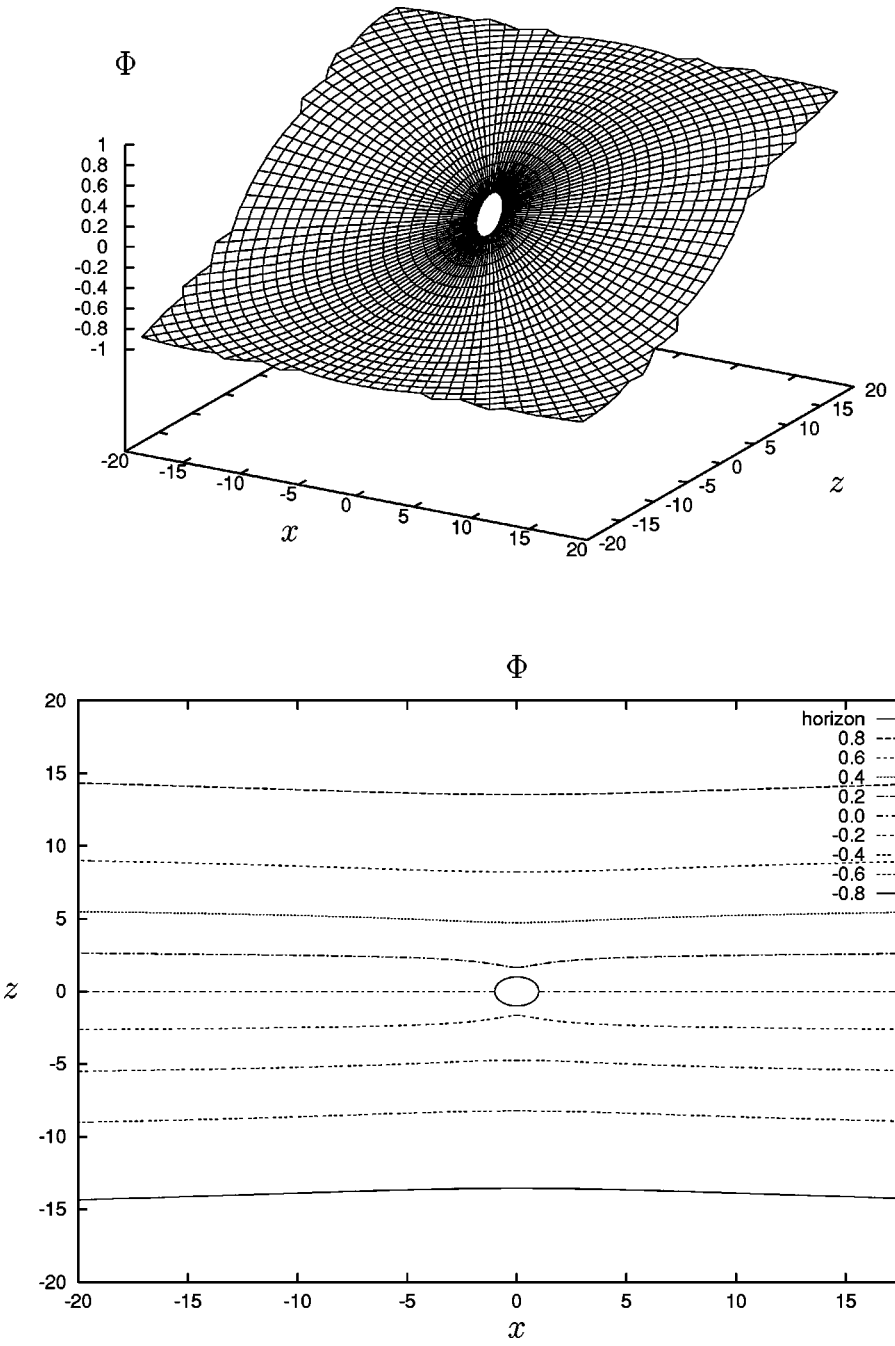

FIG. 2. Numerical solution of Eq. (14) with the ϕ^4 potential for $\epsilon=0.1$. This solution expresses a domain wall configuration whose thickness is one order of magnitude larger than the Schwarzschild radius. The black hole is enveloped in the core region of the wall.

this paper, which corresponds to $r>M/2$. In this coordinate system, the metric has a spatially conformally flat form

$$g=-\left(\frac{2r-M}{2r+M}\right)^2 dt^2+\left(1+\frac{M}{2r}\right)^4 \times[dr^2+r^2(d\vartheta^2+\sin^2\vartheta\, d\varphi^2)]. \tag{3}$$

Let us consider a real scalar field ϕ with a potential $V[\phi]$, of which Lagrangian is given by

$$\mathcal{L}=-(-\det g)^{1/2}(\tfrac{1}{2}\nabla\phi\cdot\nabla\phi+V[\phi]). \tag{4}$$

The equation of motion for ϕ is

$$\nabla^2\phi-\partial V/\partial\phi=0. \tag{5}$$

In this paper, we consider the following two familiar types of potentials which have a discrete set of degenerate minima: the ϕ^4 potential

$$V_1[\phi]=\frac{\lambda}{4}(\phi^2-\eta^2)^2, \tag{6}$$

and the sine-Gordon potential

$$V_2[\phi]=\lambda\eta^4[1+\cos(\phi/\eta)]. \tag{7}$$

Since the Schwarzschild space-time is asymptotically flat, the asymptotic boundary condition for Eq. (5) would be

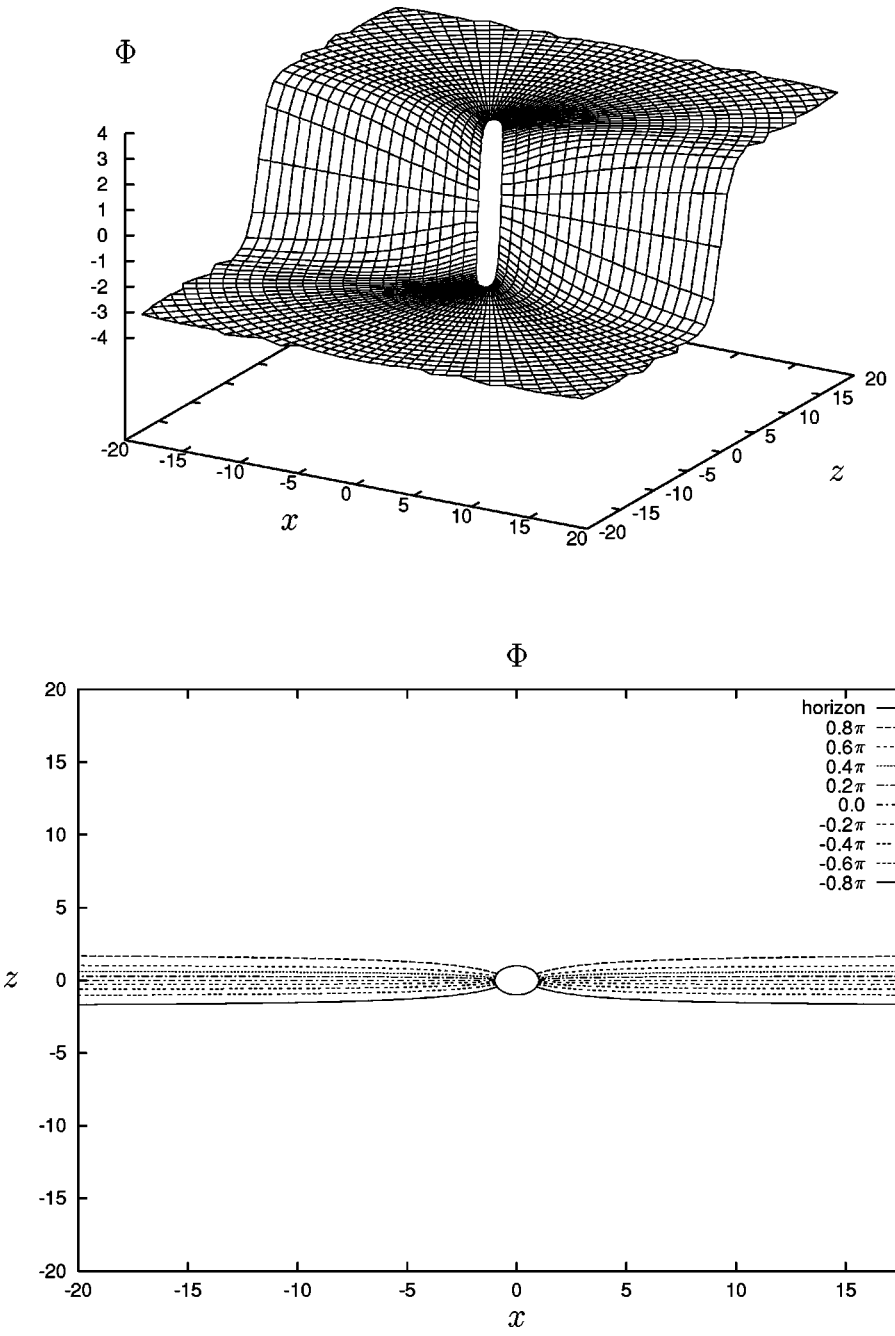

FIG. 3. Numerical solution of Eq. (14) with the sine-Gordon potential for $\epsilon=1$. This solution expresses a domain wall configuration whose thickness is comparable to the Schwarzschild radius.

given by the solution in the flat space-time. The relevant solutions in the flat space-time $g=-dt^2+dx^2+dy^2+dz^2$ are the static and plane-symmetric solutions

$$\phi_1(z)=\eta\tanh\sqrt{\lambda/2}\,\eta z, \tag{8}$$

and

$$\phi_2(z)=\eta[4\arctan\exp(\sqrt{\lambda}\,\eta z)-\pi], \tag{9}$$

for the potentials V_1 and V_2, respectively. These solutions represent domain walls in the flat space-time characterized by the thickness of the wall

$$w=1/\sqrt{\lambda}\,\eta. \tag{10}$$

In the Schwarzschild background, the solution compatible with the above asymptotic boundary condition would have a static and axisymmetric form $\phi=\phi(r,\vartheta)$. Then, the explicit form of the equation of motion (5) becomes

$$\left(\frac{2r}{2r+M}\right)^4\left[\frac{\partial^2}{\partial r^2}+\frac{8r}{(4r^2-M^2)}\frac{\partial}{\partial r}+\frac{1}{r^2}\left(\frac{\partial^2}{\partial\vartheta^2}+\cot\vartheta\frac{\partial}{\partial\vartheta}\right)\right]\phi =\frac{\partial V}{\partial\phi}. \tag{11}$$

This equation can be parametrized by a single dimensionless parameter

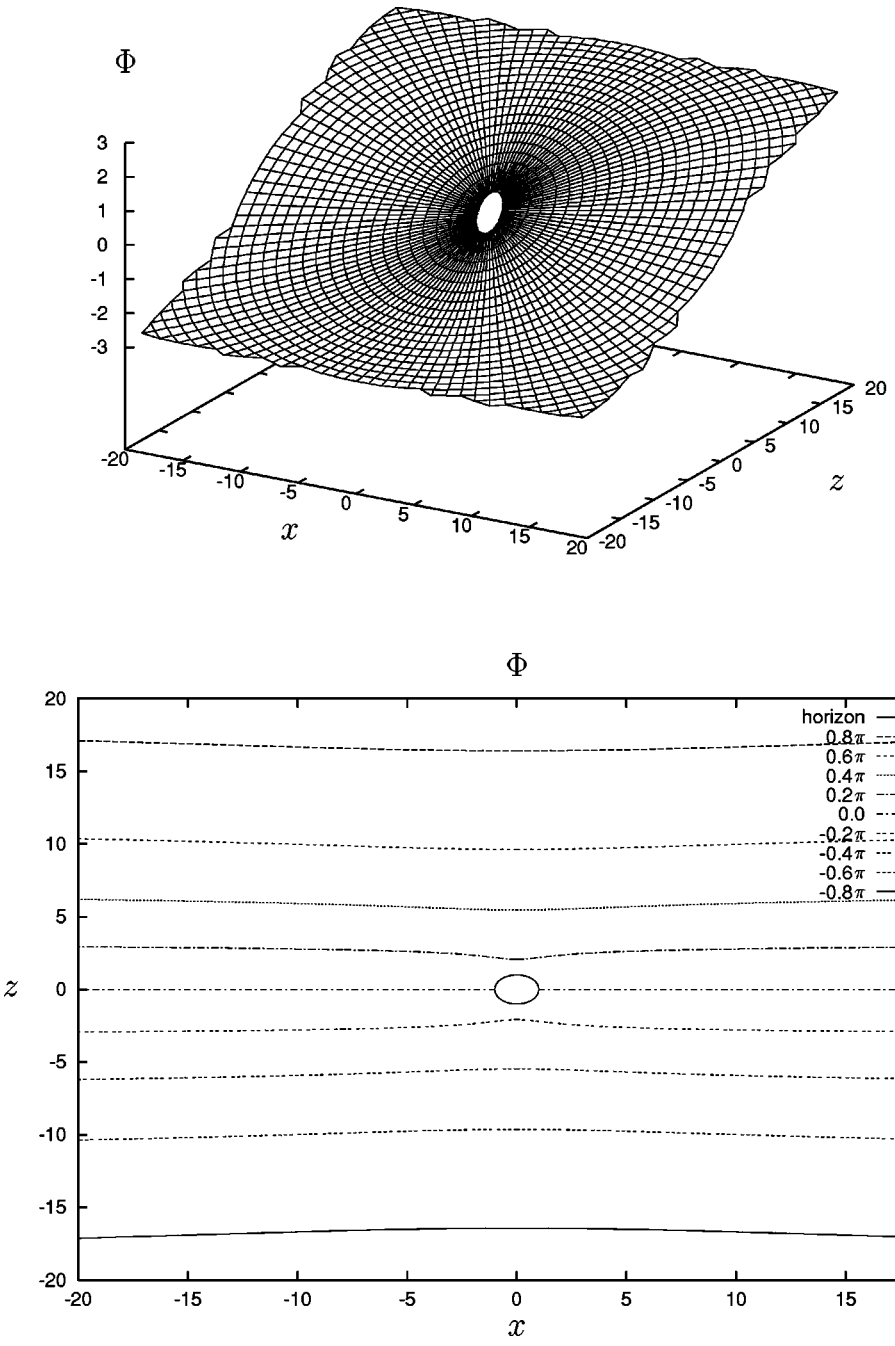

FIG. 4. Numerical solution of Eq. (14) with the sine-Gordon potential for $\epsilon=0.1$. This solution expresses a domain wall configuration whose thickness is one order of magnitude larger than the Schwarzschild radius. The black hole is enveloped in the core region of the wall.

$$\epsilon = M/2w, \tag{12}$$

by introducing dimensionless variables

$$\rho = 2r/M, \quad \Phi(\rho,\vartheta) = \phi(r,\vartheta)/\eta. \tag{13}$$

In terms of these variables, Eq. (11) becomes

$$\left(\frac{\rho}{\rho+1}\right)^4\left[\frac{\partial^2}{\partial\rho^2}+\frac{2\rho}{(\rho^2-1)}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\left(\frac{\partial^2}{\partial\vartheta^2}+\cot\vartheta\frac{\partial}{\partial\vartheta}\right)\right]\Phi = \epsilon^2\frac{\partial U}{\partial\Phi}, \tag{14}$$

where the dimensionless potential $U[\Phi]=V[\phi]/\lambda\eta^4$ is defined. U has minima at $\Phi=\pm 1$ for the ϕ^4 potential and at $\Phi=(2n+1)\pi$ $(n=0,\pm 1,\pm 2,\ldots)$ for the sine-Gordon potential. The parameter ϵ is just a ratio of the horizon radius to the wall thickness measured in the asymptotic region, namely if ϵ is smaller (larger) than unity, then the wall is said to be thick (thin) as compared to the size of the black hole.

We shall confine ourselves to the case where the core of the wall is located at the equatorial plane $\{\vartheta=\pi/2\}$ of the black hole. The solutions without this assumption will be discussed in a separate paper [16]. Accordingly, we impose the Dirichlet boundary condition at the equatorial plane

$$\Phi\big|_{\vartheta=\pi/2}=0. \tag{15}$$

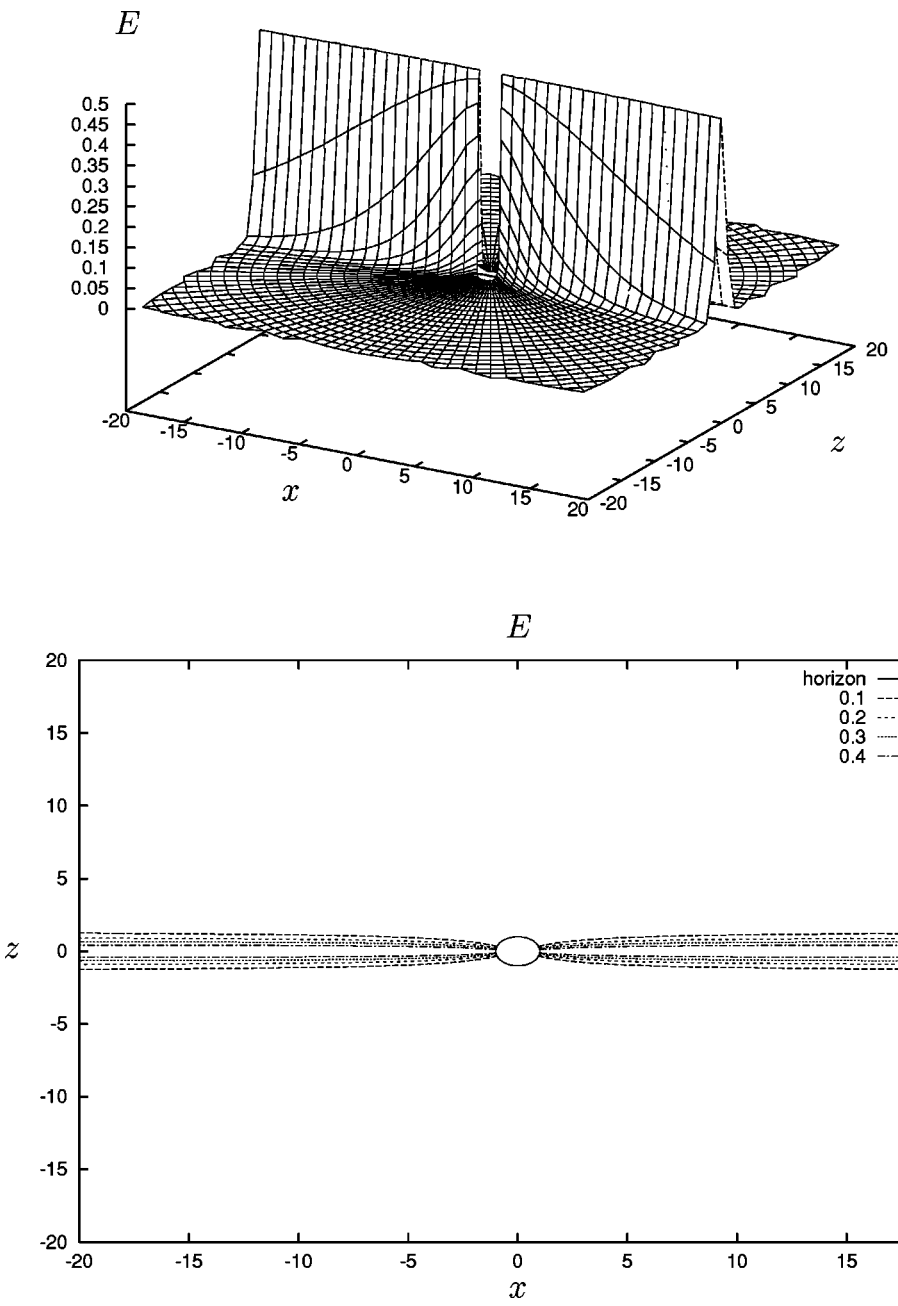

FIG. 5. The energy density $E(x,z)$ of ϕ^4 scalar field for $\epsilon = 1$. This is calculated from the numerical solution shown in Fig. 1.

Now it is sufficient to consider the north region, namely the solution in the south region can be obtained via $\Phi(\rho,\vartheta) = -\Phi(\rho,\pi-\vartheta)$, $\{\pi/2 \leqslant \vartheta \leqslant \pi\}$ of the space-time. The regularity of the scalar field at the symmetric axis is given by the Neumann boundary condition

$$\left.\frac{\partial\Phi}{\partial\vartheta}\right|_{\vartheta=0} = 0. \tag{16}$$

On the other hand, the boundary condition at the event horizon $\{\rho=1\}$ is given by the Neumann boundary condition

$$\left.\frac{\partial\Phi}{\partial\rho}\right|_{\rho=1} = 0. \tag{17}$$

As is shown in the Appendix, the condition Eq. (17) is the consequence of a natural requirement that the energy density observed by a freely falling observer remains finite at the event horizon. In practice, the region of numerical integration should be finite, so that we need an asymptotic boundary condition at $\rho=\rho_{\max}$ for $\rho_{\max} \gg 1$. Taking into account the flat background solutions Eqs. (8) and (9), we impose the Dirichlet boundary condition

$$\Phi_1|_{\rho=\rho_{\max}} = \tanh(2^{-1/2}\epsilon\rho_{\max}\cos\vartheta) \tag{18}$$

and

$$\Phi_2|_{\rho=\rho_{\max}} = 4\arctan\exp(\epsilon\rho_{\max}\cos\vartheta) - \pi, \tag{19}$$

for the ϕ^4 and the sine-Gordon potentials, respectively.

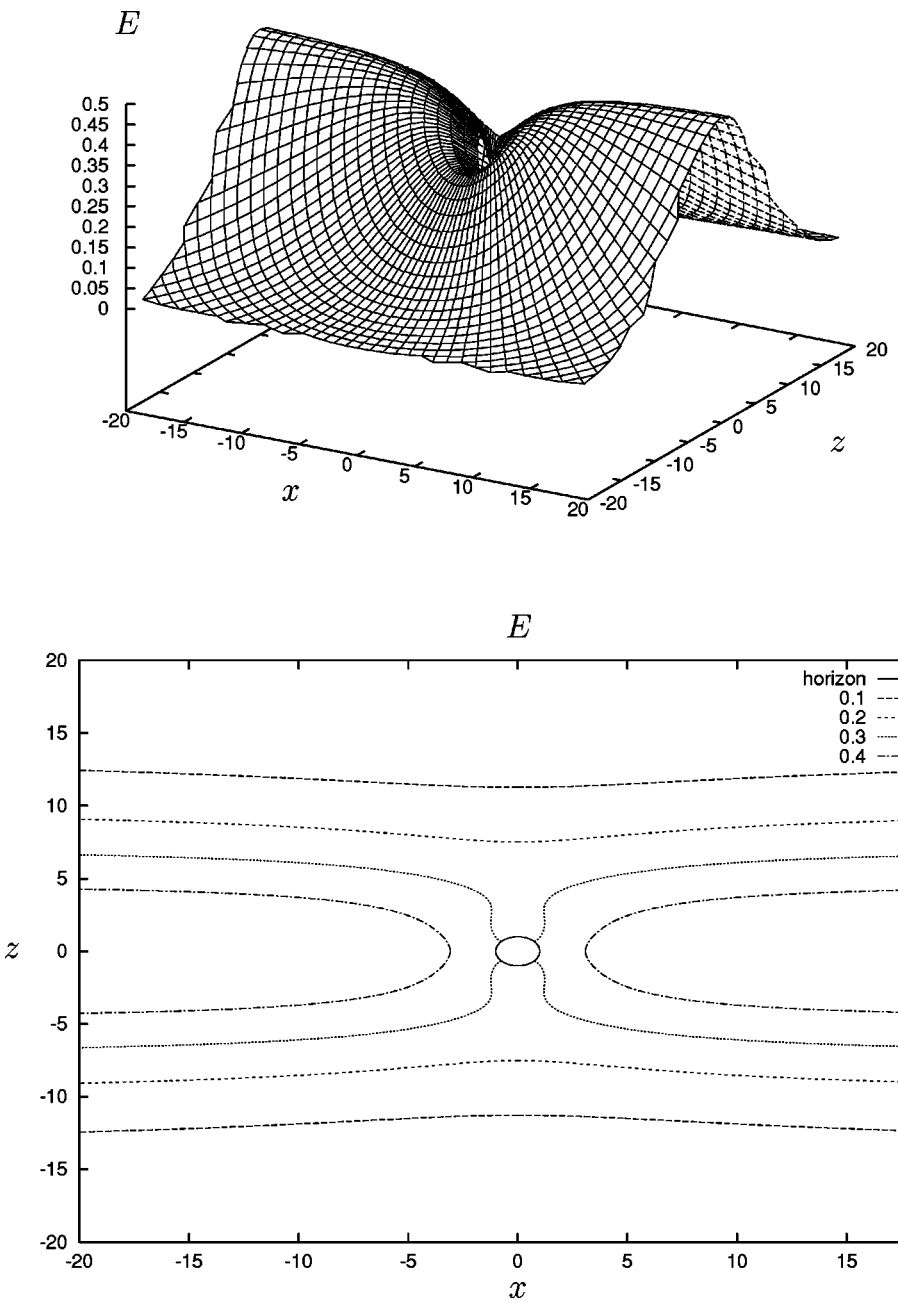

FIG. 6. The energy density $E(x,z)$ of ϕ^4 scalar field for $\epsilon = 0.1$. This is calculated from the numerical solution shown in Fig. 2.

In Sec. III, we numerically integrate the field equation (14) using the relaxation method under these boundary conditions at the equatorial plane Eq. (15), at the symmetric axis (16), at the event horizon Eq. (17), and in the asymptotic region Eqs. (18) and (19) for both the ϕ^4 and the sine-Gordon potentials.

III. NUMERICAL INTEGRATION

The scalar field configurations $\Phi(x,z)$ satisfying Eq. (14) and the boundary conditions are shown in Figs. 1–4, where x and z are the Cartesian coordinates $x=\rho\sin\vartheta$, $z=\rho\cos\vartheta$. Here we show the results typical in two cases; the $\epsilon=1$ case in which the thickness of the kinks Eqs. (18) and (19) at $\rho_{\max}$ is comparable to the Schwarzschild radius (Fig. 1 for ϕ^4 and Fig. 3 for sine-Gordon), and the $\epsilon=0.1$ case in which the thickness of the kinks Eqs. (18) and (19) at $\rho_{\max}$ is one order of magnitude larger than the Schwarzschild radius (Fig. 2 for ϕ^4 and Fig. 4 for sine-Gordon). In both cases, we obtain a domain wall solution as a kink of the scalar field at the equatorial plane $z=0$. Particularly in the case $\epsilon=0.1$, the black hole is enveloped in the core region of the wall.

We also show the energy density E of the scalar field given by

$$E\equiv\frac{T_t{}^t}{\lambda\eta^4}=\frac{1}{2\epsilon^2}\left(\frac{\rho}{\rho+1}\right)^4\left[\left(\frac{\partial\Phi}{\partial\rho}\right)^2+\frac{1}{\rho^2}\left(\frac{\partial\Phi}{\partial\vartheta}\right)^2\right]+U[\Phi] \tag{20}$$

in Figs. 5–8, corresponding to Figs. 1–4, respectively. In all the cases, one can see that the configuration actually has a

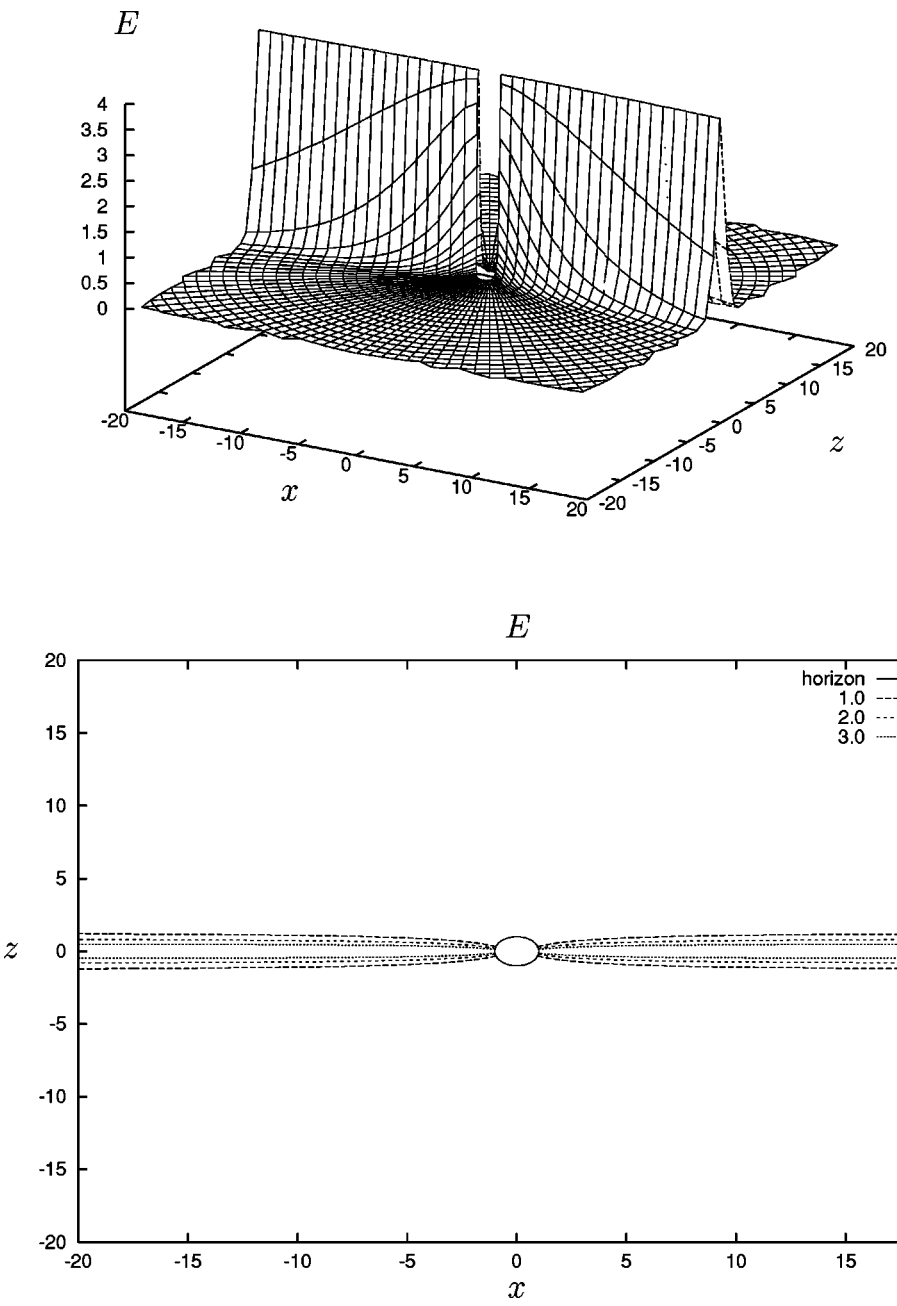

FIG. 7. The energy density $E(x,z)$ of the sine-Gordon scalar field for $\epsilon=1$. This is calculated from the numerical solution shown in Fig. 3.

wall-like structure, namely the energy density is localized around the equatorial plane with a certain thickness corresponding to ϵ.

We shall comment on the computational domain and the grid spacing taken in our calculation. In order that the asymptotic boundary conditions Eqs. (18) and (19) make sense, $\rho_{\max}$ must be large enough. In the above calculation, we take the computational domain which is 50 times as large as the horizon radius (i.e., $\rho_{\max}=51$), and we carry out the integration on a 500×90 grid (the grid spacing in the ρ and ϑ directions is $0.1\times$horizon-radius and $1°$, respectively). Then we clip the region $\{|x|,|z|<20\}$, where the above results are insensitive to the value of $\rho_{\max}$ and the number of grid points. In fact, the results do not change when we extend the computational domain to $\rho_{\max}=101$ when keeping the grid spacing, and the results differ from ones on the finer (1000×180) grid when keeping $\rho_{\max}=51$ at most 1%. We also comment that the reliability of the numerical code is checked in the flat space case and it reproduces the exact solutions Eqs. (8) and (9) with accuracy of 10^{-2}.

IV. SUMMARY AND DISCUSSIONS

In order to answer the question whether or not scalar fields can actually form a topological defect in the vicinity of a black hole, we have numerically solved the equation of motion for real scalar fields with ϕ^4 and sine-Gordon potentials, which have a discrete set of degenerate minima, in the Schwarzschild black hole background. In both ϕ^4 and sine-Gordon potential cases, we showed that there exist the static axisymmetric field configurations which represent thick domain walls intersecting the black hole. In particular, we stud-

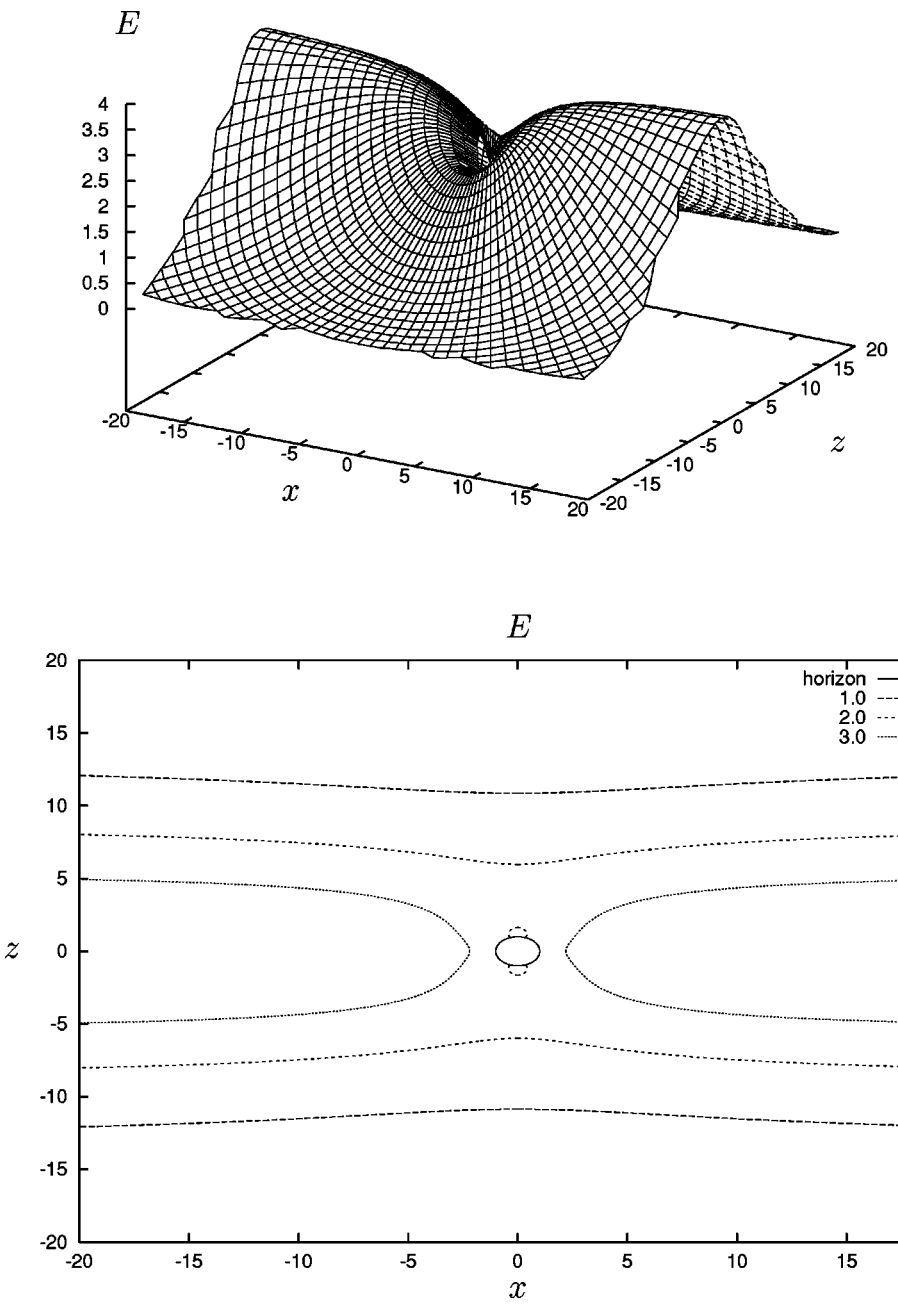

FIG. 8. The energy density $E(x,z)$ of the sine-Gordon scalar field for $\epsilon=0.1$. This is calculated from the numerical solution shown in Fig. 4.

ied the specific case where the wall's core is located at the equatorial plane of the Schwarzschild space-time. We introduced the parameter ϵ, which characterizes the domain wall thickness compared to the black hole horizon radius; the smaller than unity ϵ is, the larger the wall width is. We showed the domain wall solutions and their energy densities for $\epsilon=1$ and $\epsilon=0.1$ cases. In summary, we can say that a black hole is not an obstacle for scalar fields to form a domain wall configuration intersecting the black hole.

One might wonder about our present results; one naively expects that the scalar field could have no static distribution around a black hole and inevitably fall into the horizon as usual objects do. However, a domain wall is a relativistic object with a large negative pressure (or large tension) whose magnitude is comparable to that of energy density. Furthermore in our study we examined the domain walls which are extended infinitely in space. Then we can understand that the domain wall is suspended from the asymptotic region and supported against falling into the black hole by its tension, so that the static configuration is realized.

In our analysis, we assumed that the gravitational effect of the domain wall is negligible compared to that of the Schwarzschild black hole. We shall comment on the validity of this assumption. The energy density of a domain wall is given by

$$GT_t^t \sim G\lambda\eta^4 \sim \frac{1}{w^2}\left(\frac{\eta}{m_{\rm Pl}}\right)^2, \tag{21}$$

where $m_{\rm Pl}$ is the Planck mass and G is the Newtonian con-

stant. On the other hand, the curvature strength, or gravitational tidal force, of Schwarzschild space-time is estimated as

$$\frac{GM}{R^3} \sim \epsilon \frac{w}{R^3} \tag{22}$$

at areal radius R. Then a ratio of the gravity, which will be produced by the domain wall to the gravity of the background black hole, is given by

$$\omega \equiv \frac{GT_t^{\ t}}{GM/R^3} \sim \frac{1}{\epsilon}\left(\frac{R}{w}\right)^3 \left(\frac{\eta}{m_{\rm Pl}}\right)^2. \tag{23}$$

When ω becomes much smaller than unity, the gravity of the domain wall is negligible compared to that of the black hole, and our test wall assumption becomes valid. From Eq. (23) we have $\omega \sim \epsilon^2 (\eta/m_{\rm Pl})^2$ near the horizon $(R \sim 2GM)$. Therefore, when we consider domain walls with the symmetry breaking scale being much lower than the Planck scale (i.e., $\eta \ll m_{\rm Pl}$), we have $\omega \ll 1$ and consequently our result gives a good description of shapes of gravitating thick domain walls near the black hole. We can also see from Eq. (23) that, in the thick wall $(w > 2GM)$ case, if $\eta \ll m_{\rm Pl}$, the test wall assumption is still valid even at $R \sim w$.

In the asymptotic region $(R \gg w)$, one may expect that the gravity of the domain wall is no longer negligible and changes the asymptotic geometry drastically. Bonjour, Charmousis, and Gregory have recently investigated the space-time of a thick gravitating domain wall with local planar symmetry and reflection symmetry around the wall's core [17]. They showed that the domain wall space-time becomes spatially compact and has a cosmological horizon as de Sitter space-time does. This suggests that, when a black hole exists and the wall's gravity is taken into account in the region far from the black hole, the whole space-time has a cosmological horizon and an axisymmetric domain wall intersects both the black hole and the cosmological horizons as an equatorial plane in a Schwarzschild–de Sitter space-time. This motivates us to study further the interaction between thick domain walls and black holes in, for example, Schwarzschild–de Sitter background.

The domain wall solutions obtained here are thought to represent a possible final configuration of a gravitational capturing of a domain wall by a black hole. At present, it is far reaching for us to investigate a fully dynamical process such as the scattering and capture of thick domain walls by black holes. However, to get some insight into the problem, it is worth investigating the existence of the static axisymmetric solutions, which represent thick domain walls located away from a black hole.

A cosmic string is also an extended relativistic object with large tension and is thought to play a more important role in cosmology than a domain wall does. Study of a gravitationally interacting system of thick cosmic strings and black holes is an interesting problem as a generalization of the present analysis.

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APPENDIX: THE BOUNDARY CONDITION AT THE EVENT HORIZON

The tangent of a freely falling observer parametrized by its proper time τ is $u^\mu = (dt/d\tau, dr/d\tau, 0, 0)$, and we have

$$-1 = g_{\mu\nu}u^\mu u^\nu = -\left(\frac{2r-M}{2r+M}\right)^2\left(\frac{dt}{d\tau}\right)^2 + \left(1+\frac{M}{2r}\right)^4\left(\frac{dr}{d\tau}\right)^2. \tag{A1}$$

The quantity

$$\alpha \equiv -g_{\mu\nu}\xi^\mu u^\nu = \left(\frac{2r-M}{2r+M}\right)^2\left(\frac{dt}{d\tau}\right) \tag{A2}$$

is a constant of the motion, where $\xi^\mu = (1,0,0,0)$ is the static Killing field. The energy density observed by this observer is

$$T_{\mu\nu}u^\mu u^\nu = -(u\cdot\nabla\phi)^2 - \left(\tfrac{1}{2}\nabla\phi\cdot\nabla\phi + V[\phi]\right). \tag{A3}$$

Since we consider the static configuration and the spatial part of the metric is nonsingular in the isotropic coordinates, the second term of Eq. (A3) is always finite. We have

$$u\cdot\nabla\phi = -\left(1+\frac{M}{2r}\right)^{-2}\left[\alpha^2\left(\frac{2r+M}{2r-M}\right)^2 - 1\right]^{1/2}\frac{\partial\phi}{\partial r} \tag{A4}$$

for the static axisymmetric configuration $\phi(r,\vartheta)$. Thus the requirement that Eq. (A3) is finite at the horizon is reduced to $\partial\phi/\partial r = 0$ at $r = M/2$, or equivalently Eq. (17).

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