Non-perturbative renormalization in domain-wall QCD by Schrödinger functional scheme *

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We formulate and numerically test the Schrödinger functional scheme for domain-wall QCD. We then apply it to a non-perturbative calculation of the renormalization factors for vector and axial-vector currents in quenched domain-wall QCD with plaquette and renormalization group improved gauge actions at $a^{-1} \simeq 2$ GeV.

1. Introduction

Recent calculations with domain-wall QCD (DWQCD) have shown that the good chiral property of domain-wall fermions leads to good scaling behavior of physical observables such as quark mass and B_K[1]. Aside from the quenched approximation, the use of perturbative renormalization factors is the largest source of the uncontrolled systematic errors in these calculation. Non-perturbative renormalization is required to reduce the total error to a few percent except for quenching, and we decide to employ the Schrödinger functional(SF) scheme[2], which is a gauge-invariant and finite volume method, toward this goal.

In this report we formulate the SF scheme for DWQCD and calculate the renormalization factors for vector and axial-vector currents as a first step to obtain the renormalization factors for the quark mass and B_K.

2. Schrödinger Functional in DWQCD

We consider DWQCD[3] on an $L^3 \times T \times N_s$ lattice. The SF boundary condition for the domain-wall fermion is given by $\psi(\vec{x}, x_0 = 0, s) = P_+ P_L(s)\rho(\vec{x})$, $\bar{\psi}(\vec{x}, x_0 = 0, s) = \bar{\rho}(\vec{x})P_R(s)P_-$, $\psi(\vec{x}, x_0 = T, s) = P_- P_L(s)\rho'(\vec{x})$ and $\bar{\psi}(\vec{x}, x_0 = T, s) = \bar{\rho}'(\vec{x})P_R(s)P_+$. Here ρ, $\bar{\rho}$ are the boundary quark fields, $P_L(s) = P_L\delta_{s,1} + P_R\delta_{s,N_s}$ and $P_R(s) = P_R\delta_{s,1} + P_L\delta_{s,N_s}$ with $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ and $P_\pm = \frac{1}{2}(1 \pm \gamma_0)$.

We follow the definitions and notations of ref. [4] for the calculation of the renormalization factors.

3. Tests of the formulation

Since the SF boundary condition in time for $\rho = \bar{\rho} = 0$ is identical to the Shamir's domain-wall(Dirichlet) boundary condition in the fifth direction[5], extra zero modes appear near $x_0 = 0$ and T. We check whether these unwanted zero modes induce extra contribution to the low en-
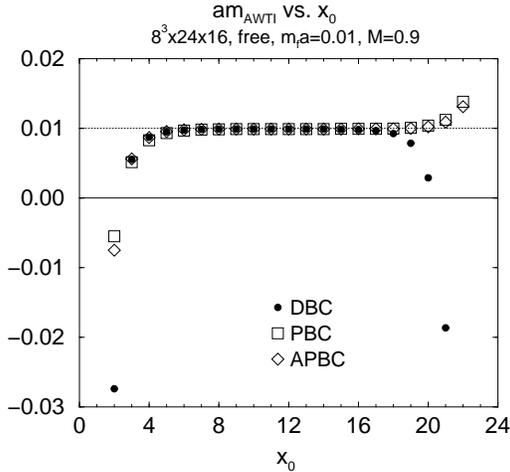
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*Poster presented by S. Aoki

Figure 1. am_{AWTI} as a function of x_0 with Dirichlet(solid circles), periodic(open squares) and antiperiodic(open diamonds) boundary conditions.

Figure 2. am_{AWTI} as a function of x_0 at $\beta = 6.0$ on an $8^3 \times 24 \times N_s$ lattice at $m_f a = 0$(open symbols) and -0.005 (solid circles), together with m_{5q}[7].

ergy observables at $0 \ll x_0 \ll T$. Here we consider the quark mass, am_{AWTI}, defined through the axial Ward-Takahashi identity(AWTI). In Fig. 1, we plot am_{AWTI} for free theory as a function of x_0 with Dirichlet, periodic and antiperiodic boundary conditions at the bare quark mass $m_f a$=0.01, on an $8^3 \times 24 \times 16$ lattice, with the domain-wall height M=0.9. The dependence of am_{AWTI} on the boundary condition, which is visible near the boundaries, disappears away from them. Therefore we conclude that the extra zero modes associated with the Dirichlet boundary condition gives negligible effects to the determination of the renormalization factors evaluated at $x_0 \simeq T/2$. Further analytic investigations on this problem for the free case will be given elsewhere[6].

In Fig. 2, am_{AWTI} for the SF scheme is plotted in quenched DWQCD at $\beta = 6.0$ of the plaquette action. The x_0 dependence is also weak away from the boundaries. Interestingly m_{AWTI} is nonzero even at $m_f a = 0$, and becomes smaller for larger N_s. Moreover the value is consistent with m_{5q}, the explicit breaking of chiral symmetry calculated from the conserved axial vector current of DWQCD[7]. Therefore m_{AWTI} for the SF scheme may be a better alternative as the measure of explicit chiral symmetry breaking in DWQCD, since it can be calculated directly at $m_f a = 0$ with
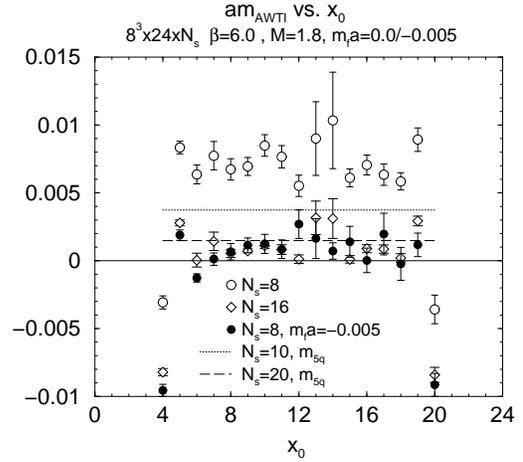
much less computational cost. Note also that the large explicit breaking in m_{AWTI} at $N_s = 8$ (open circles) is well compensated if one takes a negative quark mass of $m_f = -0.005$ (filled circles). Hence the domain-wall fermion at $N_s \neq \infty$ may be considered as a highly improved Wilson fermion[8].

The non-perturbative renormalization factors for vector and axial-vector currents are defined by $Z_V(1 + b_V m_f a) = f_1/f_V(x_0)$ and $Z_A^2 = 2f_1/f_{AA}(x_0, x_0^+, x_0^-)$, where we fix $x_0^{\pm} = T/2 \pm T/4$ and put $m_f a = 0$ for Z_A. In Fig. 3, Z_V and Z_A are plotted as a function of x_0 at $\beta = 6.0$. Similar to the case of am_{AWTI} a plateau is seen away from the boundaries. The relation $Z_V = Z_A$, exactly valid in perturbation theory, is satisfied within 1–2%. Moreover the magnitude almost agrees with the mean-field(MF) improved one-loop value. We also observe that Z_V is insensitive to boundary parameters such as the 2-loop boundary counter-terms for gauge fields and the parameter θ of the twisted boundary condition for quarks.

4. Results

We have calculated Z_V and Z_A in quenched DWQCD at $a^{-1} \simeq 2$ GeV with the plaquette action ($\beta = 6.0$) and with the renormalization group(RG) improved action ($\beta = 2.6$) on an $8^3 \times 16 \times 16$ lattice with $m_f a = 0$ for

Z_V,A vs. x_0
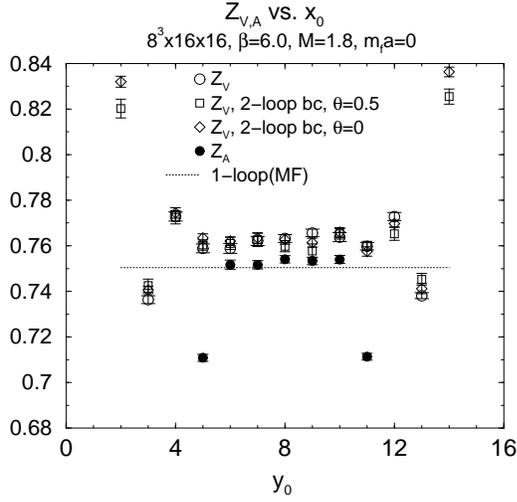
8^3x16x16, β=6.0, M=1.8, $m_f a$=0

Figure 3. $Z_{V,A}$ as a function of x_0 at $\beta = 6$ on an $8^3 \times 16 \times 16$ lattice with $M = 1.8$ and $m_f a = 0$. We compare the results from the boundary counter-terms at tree-level(circles) with those at 2-loop(squares and diamonds) as well as those at $\theta = 0$ with that at $\theta = 0.5$(squares).

$M = 1.0 \sim 2.2$. The results are summarized in Fig. 4, where Z_V and Z_A are plotted as a function of M, together with one-loop perturbative estimates with and without MF improvement. For both gauge actions, $Z_V \simeq Z_A$ holds, and they show a minimum at $M \simeq 1.7$ for the plaquette action or $M \simeq 1.6$ for the RG action. Perturbative estimates without MF improvement fail, particularly for the plaquette action for which the curve can not be placed in the figure. The agreement becomes much better with MF improvement for both actions.

5. Future directions

We are encouraged with the present results to proceed to an extension of the present work to scale-dependent cases such as quark masses and four-quark operators needed for B_K.

8^3x16x16 β=6.0(plaquette), $m_f a$=0.0

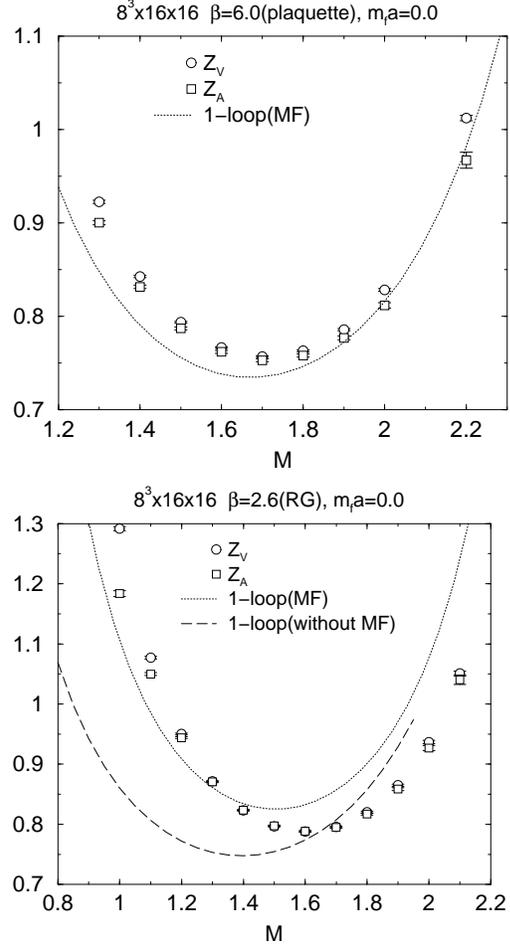**8^3x16x16 β=2.6(RG), $m_f a$=0.0**

Figure 4. Z_V and Z_A vs M for plaquette(upper) and RG(lower) gauge actions.

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