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# Non-perturbative renormalization for a renormalization group improved gauge action *

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Renormalization constants of vector ($Z_V$) and axial-vector ($Z_A$) currents are determined non-perturbatively in quenched QCD for a renormalization group improved gauge action and a tadpole improved clover quark action using the Schrödinger functional method. Non-perturbative values of $Z_V$ and $Z_A$ turn out to be smaller than the one-loop perturbative values by $O(10\%)$ at $a^{-1} \approx 1$ GeV. A sizable scaling violation of meson decay constants $f_\pi$ and $f_\rho$ observed with the one-loop renormalization factors remains even with non-perturbative renormalization.

## 1. Introduction

Reliable lattice calculations of hadronic matrix elements and quark masses require both high precision numerical simulations and non-perturbative determinations of renormalization constants ($Z$-factors). The CP-PACS collaboration recently carried out a sophisticated spectrum calculation in $N_f = 2$ full QCD [1] using a renormalization group (RG) improved gauge action and a tadpole improved clover quark action. However, non-perturbative $Z$-factors were not available for this action combination. Hence analyses had to rely on one-loop perturbative values.

As a first step toward a systematic study of non-perturbative renormalization for this action, we apply the Schrödinger functional method [2] to calculations of $Z$ factors for vector ($Z_V$) and axial-vector ($Z_A$) currents in quenched QCD with the same improved action. We examine in particular whether a large scaling violation of meson decay constants observed for this action [1] is improved with non-perturbative $Z$-factors. We report preliminary results in these proceedings.

*Talk presented by K. Ide.

## 2. Calculational Method

We follow the method developed by the ALPHA collaboration [3]. Namely, we use a lattice geometry of $L^3 \cdot T$ with $T = 2L$ for $Z_V$ with a vector operator at $t = L$, and $T = 3L$ for $Z_A$ with two axial vector operators at $t = L$ and $t = 2L$, except at $\beta = 2.2$ and 2.4 for $Z_A$ (see sec. 3 for details of this exception). Tree-level values are used for coefficients of boundary counter terms of the action. For improving the axial current, we use the one-loop perturbative value for the coefficient $c_A$.

Values of $Z_V$ and $Z_A$ are determined for $\beta = 2.2 - 8.0$ which almost covers the range of the CP-PACS quenched spectrum calculation [1], $\beta = 2.187 - 2.575$. Physical size is normalized at $\beta = 2.6$ on an $8^3$ lattice. For other $\beta$ values, two lattice sizes are analyzed to match the physical size using the string tension. Our action has $O(a)$ errors since we employ a tadpole improved value of $c_{\rm sw} = (1 - 0.8412/\beta)^{-3/4}$. Therefore we extrapolate/interpolate results linearly in $1/L$. We have analyzed 300–4000 configurations depending on $\beta$ value and lattice size.

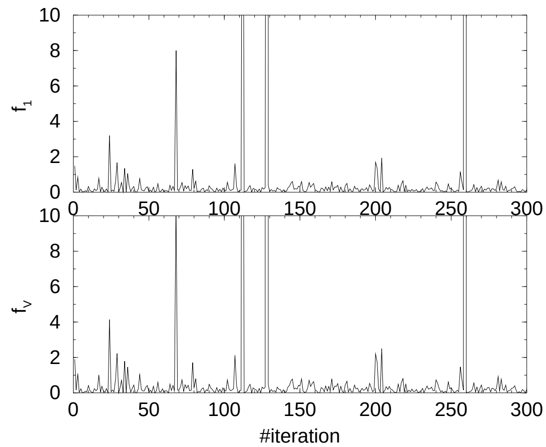


Figure 1. Time history of $f_1$ and $f_V$ at $\beta = 2.4$ on an $8^3 \times 16$ lattice.

## 3. Exceptional Configurations

It is straight-forward to calculate $Z$-factors for $\beta > 2.4$ for $Z_V$ and $\beta > 2.8$ for $Z_A$. We find reasonable plateaux in the ratio of Green functions for the $Z$-factors in spite of the $O(a)$ error of the action, which implies viability of the Schrödinger functional method for our action.

However, for lower $\beta$ values on a large lattice, anomalously large values appear in the ensemble of $f_1$, $f_V$ and $f_{AA}$ where $Z_V = f_1/f_V$ and $Z_A = \sqrt{f_1/f_{AA}}$. This is illustrated with a time history of $f_1$ and $f_V$ at $\beta = 2.4$ in Fig. 1.

In order to estimate $Z$-factors at low $\beta$ values, we have investigated the properties of these "exceptional configurations". We find : i) Large values of $f_1$ and $f_V$ for $Z_V$ and $f_1$ and $f_{AA}$ for $Z_A$ are strongly correlated (see Fig. 1). ii) Histograms of $f$'s have a long tail toward very large values as shown in Fig. 2. We then impose a cutoff in taking the average of the $f$'s, and find that $Z$-factors are stable against change of the cutoff as long as anomalously large values are discarded, as the numerator and denominator for $Z$-factors are correlated and effects mostly cancel out. See Fig. 3.

We then estimate $Z$-factors for low $\beta$ values taking a certain value of the cutoff. For $Z_A$ at $\beta = 2.2$ and 2.4, the lattice geometry is also changed from $T = 3L$ to $T = 2L$ because "exceptional configurations" appear very frequently

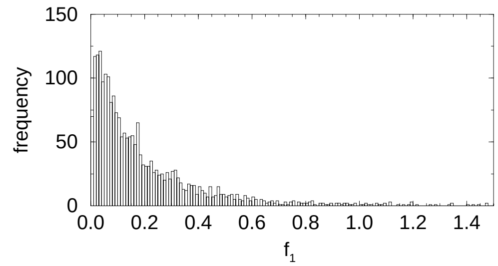


Figure 2. Histogram of $f_1$ at $\beta = 2.4$ on an $8^3 \times 16$ lattice.

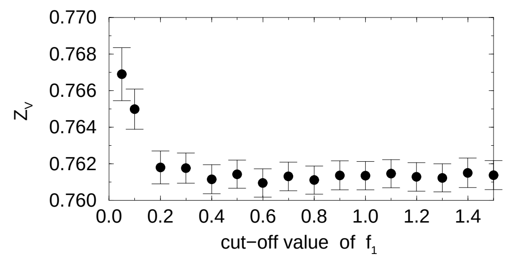


Figure 3. Values of $Z_V$ estimated by discarding configurations with $f_1$ larger than the cutoff value. Data are for an $8^3 \times 16$ lattice at $\beta = 2.4$.

for the original geometry to the extent that the cutoff analysis above does not work. We have checked at $\beta = 2.6$ that the change of geometry does not lead to any significant difference in $Z_A$.

## 4. Results for $Z$-factors

In Fig. 4 we show results of $Z$-factors together with Padé fits (solid curves in the figure) to them. Non-perturbative estimates give values smaller than the one-loop perturbative ones (dashed lines) by about 10 % (6%) for $Z_V$ ($Z_A$) at the largest coupling of the CP-PACS simulation, $\beta = 2.187$.

## 5. Scaling Property of Decay Constants

We compare in Fig. 5 $f_\pi$ and $f_\rho$ determined with non-perturbative (filled circles) and perturbative (open circles) $Z$-factors. Also shown are the results from the standard plaquette and Wilson action (squares)[4] using the perturbative $Z$-factors.

We observe that, even with the non-perturbative $Z$-factors, large scaling violation of meson decay constants remains for the range we have in-

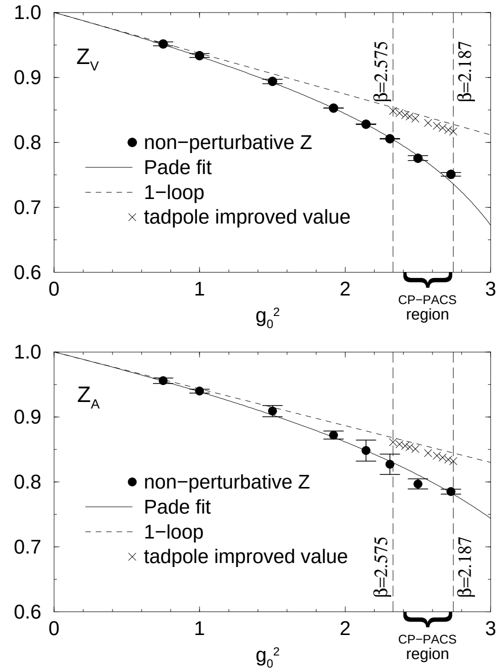


Figure 4. Results for $Z_V$ (top) and $Z_A$ (bottom).

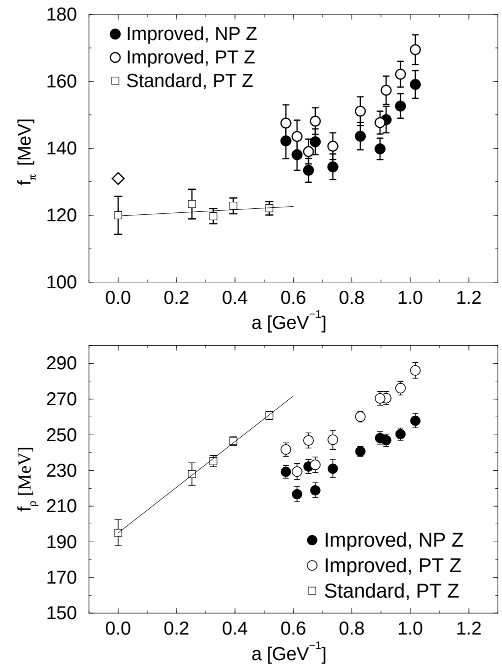


Figure 5. $f_\pi$ (top) and $f_\rho$ (bottom) vs. $a$ for our improved action with non-perturbative (NP) and perturbative (PT) $Z$-factors together with results for the standard action [4].

vestigated. A possible reason is the necessity of non-perturbatively fixing the $O(a)$ and perhaps higher terms in the currents themselves. For the axial vector current, it will be worth investigating if non-perturbative estimates of the $O(a)$ coefficient $c_A$ yield a large value.

## 6. Conclusions

We have successfully applied the Schrödinger functional method to calculations of $Z_V$ and $Z_A$ for the combination of a RG-improved gauge action and a tadpole improved clover quark action down to the lattice spacings $a^{-1} = 1 - 2$ GeV where the quenched CP-PACS data for decay constants were taken.

While $Z$-factors estimated non-perturbatively are smaller by $O(10\%)$ than perturbative ones for this range, there still remain large scaling violations of $O(a)$ and higher in meson decay constants with non-perturbative $Z$-factors. Further work is needed to examine if hadronic matrix elements could be reliably extracted at lattice spacings much coarser than $a^{-1} \approx 2$ GeV with operators improved non-perturbatively at $O(a)$ and beyond.

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